



Montague

## Quiz FM301

# Normal and Oblique Shock Waves

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## PROBLEM DISTRIBUTION

Problems	Subject
1 - 9	Normal shock waves
10 - 20	Oblique shock waves
21 - 22	Prandtl-Meyer expansion waves



Use  $\gamma = 1.4$  in all problems, unless specified otherwise.

## PROBLEMS

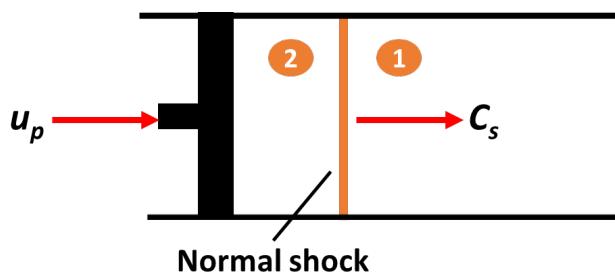
### Problem 1

If the flow Mach number, pressure and temperature ahead of a normal shock are 2.4, 0.6 atm and 300 K, respectively, determine the thermomechanical variables behind the wave. True or false?

- 1.( ) The pressure behind the wave is greater than 3.7 atm.
- 2.( ) The temperature behind the wave is greater than 600 K.
- 3.( ) The Mach number behind the wave is greater than 0.7.
- 4.( ) The flow speed behind the wave is greater than 280 m/s.

### Problem 2

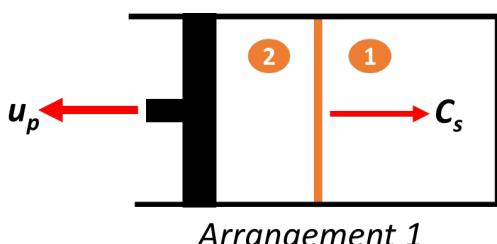
A horizontal tube contains stationary air at 1.2 atm and 310 K. The left end of the tube is closed by a movable piston, which at time  $t = 0$  is moved impulsively at a speed of  $u_p = 130$  m/s to the right. Find the wave speed  $C_s$  and the pressure  $p_2$  on the face of the piston.



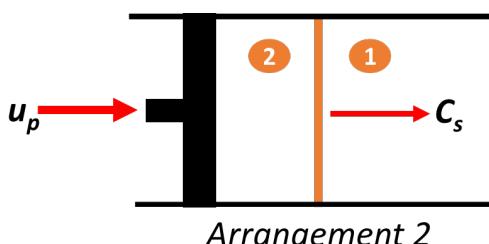
- A)  $C_s = 441$  m/s and  $p_2 = 202$  kPa
- B)  $C_s = 441$  m/s and  $p_2 = 305$  kPa
- C)  $C_s = 508$  m/s and  $p_2 = 202$  kPa
- D)  $C_s = 508$  m/s and  $p_2 = 305$  kPa

### Problem 3

A horizontal tube contains stationary air at 1.25 atm and 340 K. The left end of the tube is closed by a movable piston, which at time  $t = 0$  begins to move impulsively at 150 m/s. In arrangement 1 shown below, the piston motion is to the left, while in arrangement 2 the motion is to the right. Find the pressure on the face of the piston in both cases.



Arrangement 1  
Piston moving to the left



Arrangement 2  
Piston moving to the right

- A)** Pressure on piston in arrangement 1 = 0.390 atm; pressure on piston in arrangement 2 = 2.15 atm
- B)** Pressure on piston in arrangement 1 = 0.390 atm; pressure on piston in arrangement 2 = 3.04 atm
- C)** Pressure on piston in arrangement 1 = 0.691 atm; pressure on piston in arrangement 2 = 2.15 atm
- D)** Pressure on piston in arrangement 1 = 0.691 atm; pressure on piston in arrangement 2 = 3.04 atm

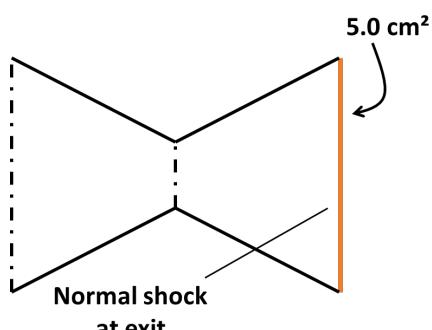
### Problem 4

Consider a pipe in which air at 350 K and  $1.60 \times 10^5 \text{ N/m}^2$  flows uniformly with a speed of 150 m/s. The end of the pipe is suddenly closed by a valve, and a shock wave is propagated back into the pipe. Compute the speed of the wave and the pressure and temperature of the air which has been brought into rest. True or false?

- 1.( )** The speed of the wave is greater than 310 m/s.
- 2.( )** The pressure of the air that has been brought into rest is greater than 250 kPa.
- 3.( )** The temperature of the air that has been brought into rest is greater than 430 K.

### Problem 5.1

A convergent-divergent nozzle of exit area  $5.0 \text{ cm}^2$  is to be designed to generate a Mach 2.5 air stream. If the nozzle is correctly expanded and discharging into the atmosphere, and the stagnation temperature at the entry is 480 K, determine the back pressure required to position a normal shock at the nozzle exit plane.



- A)** Back pressure required to position a shock at nozzle exit = 779 kPa;
- B)** Back pressure required to position a shock at nozzle exit = 804 kPa;
- C)** Back pressure required to position a shock at nozzle exit = 864 kPa;
- D)** Back pressure required to position a shock at nozzle exit = 915 kPa;

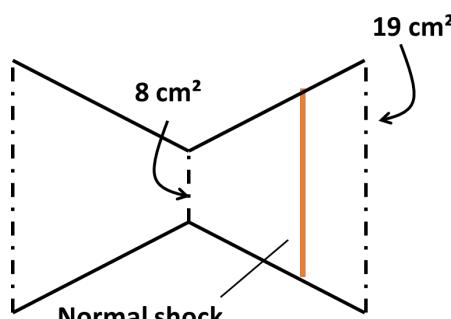
### Problem 5.2

Suppose the backpressure were to be increased for the nozzle in the previous problem until a normal shock wave was formed in the divergent section where  $M = 1.5$ . What back pressure would be necessary to accomplish this, and what would be the resulting velocity and temperature at the nozzle exit? True or false?

- 1.( )** The back pressure required to achieve the conditions mentioned above is greater than 15 atm.
- 2.( )** Under the conditions mentioned in the problem statement, the resulting velocity at the nozzle exit will be greater than 150 m/s.
- 3.( )** Under the conditions mentioned in the problem statement, the resulting temperature at the nozzle exit will be greater than 460 K.

### Problem 6

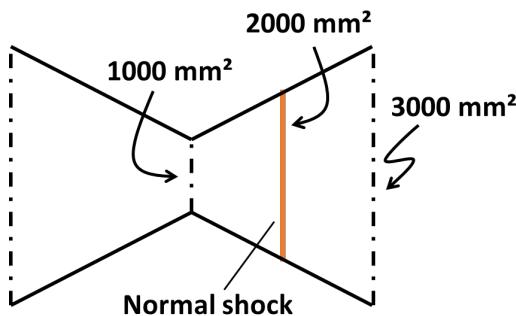
Air from a storage tank at 820 kPa and 525 K is expanded through a frictionless convergent-divergent duct of throat area  $8 \text{ cm}^2$  and exit area  $19 \text{ cm}^2$ . The backpressure is 492 kPa. There is a normal shock in the divergent portion and the Mach number just upstream of the shock is 2.30. Determine the cross-sectional area at the shock location, the exit Mach number, and the back pressure for the flow to be isentropic throughout the duct. True or false?



- 1.( )** The cross-sectional area at the shock location is greater than  $18 \text{ cm}^2$ .
- 2.( )** The exit Mach number is greater than 0.42.
- 3.( )** If the back pressure were 60 kPa, flow would be isentropic throughout the duct.

## Problem 7

A convergent-divergent nozzle with throat area  $A_{th} = 1000 \text{ mm}^2$  and exit area  $A_e = 3000 \text{ mm}^2$  operates at a stagnation condition of 250 kPa and 58°C. If a normal shock is present in the nozzle at a location with area 2000 mm<sup>2</sup>, determine the exit pressure.



- A)  $p_e = 98.8 \text{ kPa}$
- B)  $p_e = 108 \text{ kPa}$
- C)  $p_e = 145 \text{ kPa}$
- D)  $p_e = 179 \text{ kPa}$

## Problem 8

A Pitot tube is placed in an air stream of static pressure 0.95 atm. True or false?

- 1.( ) If the tube records 1.3 atm, the flow Mach number is greater than 0.6.
- 2.( ) If the tube records 2.5 atm, the flow Mach number is greater than 1.5.
- 3.( ) If the tube records 10 atm, the flow Mach number is greater than 3.0.

## Problem 9

A normal shock is positioned inside a convergent-divergent nozzle of throat area 7 cm<sup>2</sup>, run by a settling chamber with air at 6 atm and 345 K. If the pressure loss caused by the shock is 12.4% and the temperature at the nozzle exit is 295 K, determine the Mach number ahead of the shock, the flow speed behind the shock and at the nozzle exit, and the mass flow rate through the nozzle. Use 1000 J/kg as the specific heat at constant pressure of air. True or false?

- 1.( ) The Mach number ahead of the shock is greater than 2.
- 2.( ) The flow speed behind the shock is greater than 220 m/s.
- 3.( ) The flow speed at the nozzle exit is greater than 300 m/s.
- 4.( ) The mass flow rate is greater than 1 kg/s.

## Problem 10

A uniform supersonic air flow at Mach 2.4 passes over a wedge. An oblique shock, making an angle of 42° with the flow direction, is attached to the wedge. If the static pressure and temperature in the freestream are 52 kPa and 2°C, determine the static pressure and temperature behind the wave, the Mach number of the flow passing over the wedge, and the wedge angle. True or false?

- 1.( ) The static pressure behind the wave is greater than 155 kPa.
- 2.( ) The static temperature behind the wave is greater than 375 K.
- 3.( ) The Mach number of the flow passing over the wedge is greater than 1.8.
- 4.( ) The wedge angle is greater than 34°.

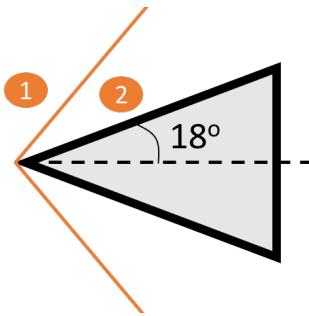
## Problem 11

Air approaches a symmetrical wedge with semi-vertex angle 13° at Mach 2.2. Determine for the strong and weak waves the wave angle with respect to the freestream direction, the pressure ratio across the wave, the density ratio across the wave, and the Mach number downstream of the shock. True or false?

- 1.( ) The pressure ratio for the strong shock solution is greater than 5.
- 2.( ) The density ratio for the strong shock solution is greater than 3.2.
- 3.( ) The temperature ratio for the strong shock solution is greater than 2.
- 4.( ) The Mach number downstream of the shock for the strong shock solution is greater than 0.5.
- 5.( ) The pressure ratio for the weak shock solution is greater than 1.8.
- 6.( ) The density ratio for the weak shock solution is greater than 1.8.
- 7.( ) The temperature ratio for the weak shock solution is greater than 1.1.
- 8.( ) The Mach number downstream of the shock for the weak shock solution is greater than 1.9.

## Problem 12

Air, which is assumed to be a perfect gas, flows in a blow-down wind tunnel with constant stagnation parameters  $T_0 = 320 \text{ K}$  and  $p_0 = 1050 \text{ kPa}$ . A symmetrical wedge of semi-angle  $\theta/2 = 18^\circ$  is placed in the test-section where  $M = 3.2$ . Calculate the static pressure, density, temperature, stagnation pressure, flow Mach number, and flow velocity on the face of the wedge. True or false?



1. ( ) The pressure on the face of the wedge is greater than 68 kPa.
2. ( ) The density of air on the face of the wedge is greater than 2 kg/m<sup>3</sup>.
3. ( ) The temperature on the face of the wedge is greater than 200 K.
4. ( ) The stagnation pressure on the face of the wedge is greater than 925 kPa.
5. ( ) The Mach number on the face of the wedge is greater than 1.8.
6. ( ) The flow velocity on the face of the wedge is greater than 530 m/s.

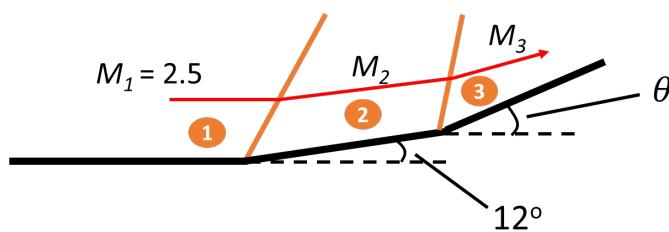
## Problem 13

Consider the following statements.

1. With upstream Mach number  $M_1 = 3.0$ , the maximum deflection angle  $\theta_d$  for which an oblique shock remains attached to a wedge is greater than 30 degrees.
  2. With deflection angle  $\theta_d = 40^\circ$ , the minimum upstream Mach number  $M_1$  for which an oblique shock remains attached to a wedge is greater than 4.8.
- A)** Both statements are true.  
**B)** Statement 1 is true and statement 2 is false.  
**C)** Statement 2 is true and statement 1 is false.  
**D)** Both statements are false.

## Problem 14

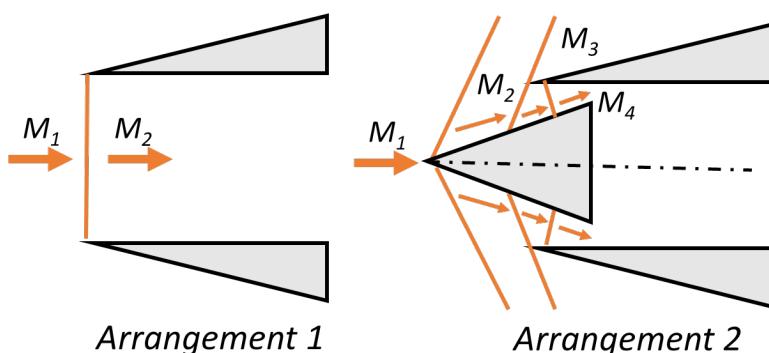
Air at Mach 2.5 passes over two compression corners of angles  $12^\circ$  and  $\theta$ , as shown. Determine the value of  $\theta$  up to which the second shock will remain attached.



- A)**  $\theta_{\max} = 14^\circ$   
**B)**  $\theta_{\max} = 18^\circ$   
**C)**  $\theta_{\max} = 22^\circ$   
**D)**  $\theta_{\max} = 26^\circ$

## Problem 15

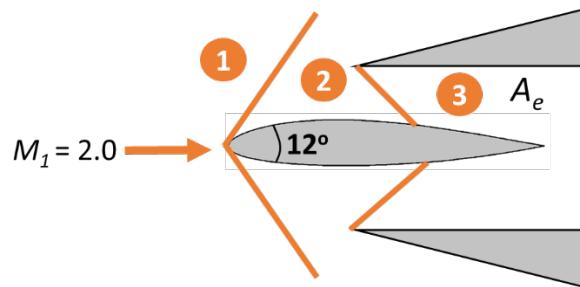
A supersonic inlet is to be designed to operate at Mach 4.0. Two possibilities are considered, as illustrated below. In arrangement 1, compression and deceleration of the flow takes place through a single normal shock. In arrangement 2, a wedge-shaped diffuser is used and the deceleration is through two weak oblique shocks followed by a normal shock wave. The wedge turning angles are  $7^\circ$  each. Find the loss in stagnation pressure for the two cases.



- A)** Pressure loss in arrangement 1 = 86.1%; pressure loss in arrangement 2 = 61.4%;  
**B)** Pressure loss in arrangement 1 = 86.1%; pressure loss in arrangement 2 = 70.8%;  
**C)** Pressure loss in arrangement 1 = 91.1%; pressure loss in arrangement 2 = 61.4%;  
**D)** Pressure loss in arrangement 1 = 91.1%; pressure loss in arrangement 2 = 70.8%;

### Problem 16.1

A supersonic inlet is to be designed to handle air at Mach 2.0 with static pressure and temperature of 72 kPa and 285 K, respectively, as shown below. Determine the diffuser inlet area  $A_i$  if the device is to handle 18 kg/s of air.



- A)**  $A_i = 144 \text{ cm}^2$   
**B)**  $A_i = 257 \text{ cm}^2$   
**C)**  $A_i = 345 \text{ cm}^2$   
**D)**  $A_i = 490 \text{ cm}^2$

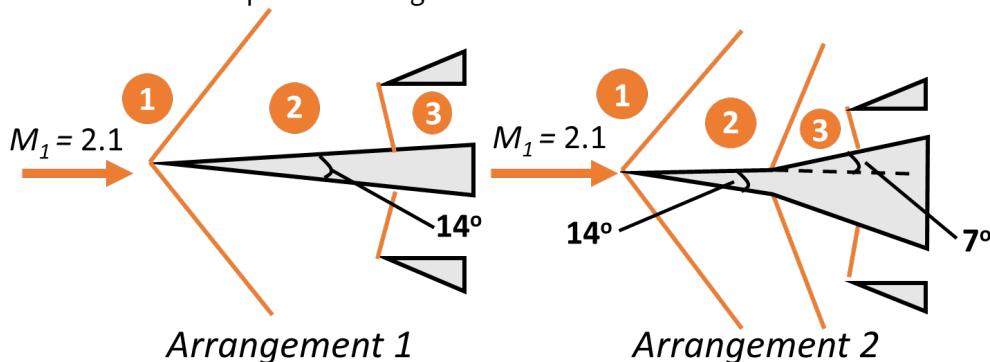
### Problem 16.2

The diffuser has to further decelerate the flow behind the normal shock so that the velocity entering the compressor is not to exceed 35 m/s. Assuming isentropic flow behind the normal shock, determine the area  $A_e$  required and the static pressure  $p_e$  there.

- A)**  $A_e = 0.09 \text{ m}^2$  and  $p_e = 457 \text{ kPa}$   
**B)**  $A_e = 0.09 \text{ m}^2$  and  $p_e = 605 \text{ kPa}$   
**C)**  $A_e = 0.166 \text{ m}^2$  and  $p_e = 457 \text{ kPa}$   
**D)**  $A_e = 0.166 \text{ m}^2$  and  $p_e = 605 \text{ kPa}$

### Problem 17

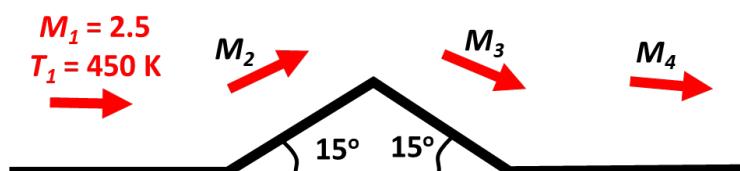
Find the pressure loss experienced by the one-shock spike in arrangement 1 and the two-shock spike in arrangement 2.



- A)** Pressure loss in arrangement 1 = 32.6%; pressure loss in arrangement 2 = 11.8%;  
**B)** Pressure loss in arrangement 1 = 32.6%; pressure loss in arrangement 2 = 23.6%;  
**C)** Pressure loss in arrangement 1 = 49.4%; pressure loss in arrangement 2 = 11.8%;  
**D)** Pressure loss in arrangement 1 = 49.4%; pressure loss in arrangement 2 = 23.6%;

### Problem 18

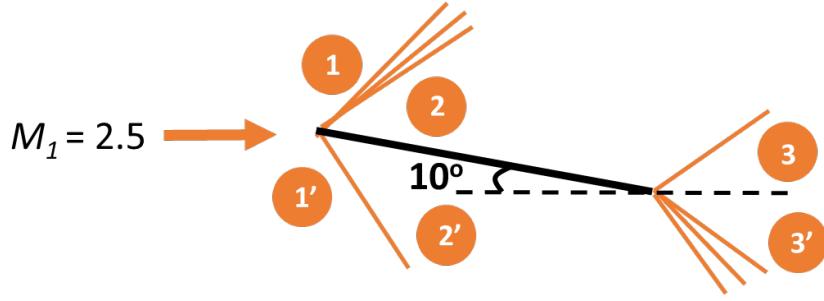
For flow at Mach 2.5 and temperature 450 K over the protrusion shown below, find flow Mach numbers  $M_2$ ,  $M_3$ , and  $M_4$ . Also determine flow temperatures  $T_2$ ,  $T_3$ , and  $T_4$ . True or false?



1. ( ) Mach number  $M_1$  is greater than 2.1.
2. ( ) Temperature  $T_1$  is greater than 620 K.
3. ( ) Mach number  $M_2$  is greater than 3.
4. ( ) Temperature  $T_2$  is greater than 310 K.
5. ( ) Mach number  $M_3$  is greater than 3.4.
6. ( ) Temperature  $T_3$  is greater than 320 K.
7. ( ) Mach number  $M_4$  is greater than 2.2.
8. ( ) Temperature  $T_4$  is greater than 450 K.

### Problem 19

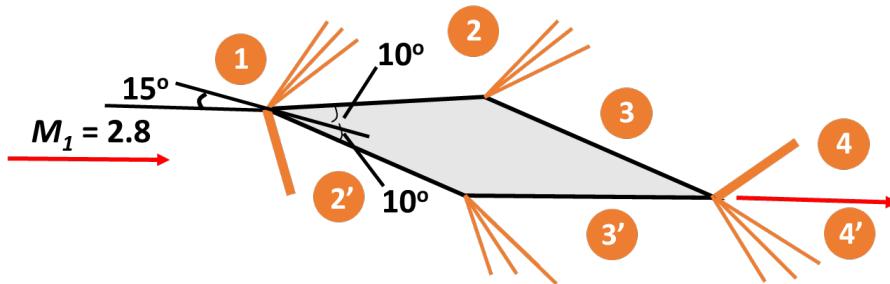
For the flat plate illustrated below, calculate the flow Mach numbers assuming the slipstream deflection to be negligible. True or false?



1. ( ) The flow Mach number at 2 is greater than 3.1.
2. ( ) The flow Mach number at 3 is greater than 2.6.
3. ( ) The flow Mach number at 2' is greater than 2.0.
4. ( ) The flow Mach number at 3' is greater than 2.3.

### Problem 20

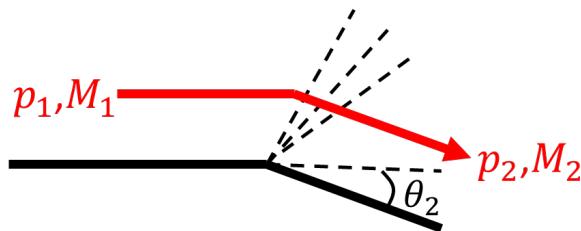
For the double wedge shown below, calculate the flow Mach numbers. True or false?



1. ( ) The flow Mach number at 2 is greater than 3.
2. ( ) The flow Mach number at 3 is greater than 4.8.
3. ( ) The flow Mach number at 4 is greater than 2.7.
4. ( ) The flow Mach number at 2' is greater than 1.5.
5. ( ) The flow Mach number at 3' is greater than 2.6.
6. ( ) The flow Mach number at 4' is greater than 2.3.

### Problem 21

A steady supersonic flow extends from Mach number  $M_1 = 1.8$  and pressure  $p_1$  to pressure  $p_2 = p_1/2$  from a centered rarefaction. Find the Mach number  $M_2$  and flow direction  $\theta_2$ .



- A)  $M_2 = 2.25$  and  $\theta_2 = 12.3^\circ$
- B)  $M_2 = 2.25$  and  $\theta_2 = 17.3^\circ$
- C)  $M_2 = 2.65$  and  $\theta_2 = 12.3^\circ$
- D)  $M_2 = 2.65$  and  $\theta_2 = 17.3^\circ$

### Problem 22

Air at initial pressure  $p_1 = 4 \text{ kPa}$ , temperature  $T_1 = 400 \text{ K}$ , and Mach number  $M_1 = 1.7$  is to be expanded isentropically to  $1 \text{ kPa}$ . Determine the final Mach number, the flow deflection angle, and the temperature of the air after expansion. True or false?

1. ( ) The final Mach number  $M_2$  is greater than 2.8.
2. ( ) The absolute value of the deflection angle is greater than  $20^\circ$ .
3. ( ) The pressure after the shock is greater than 300 K.

## ADDITIONAL INFORMATION

### Equations

→ **Equation 1:** Ratio of pressures for isentropic flow

$$\frac{p}{p_0} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\gamma/(\gamma-1)}$$

→ **Equation 2:** Maximum mass flow rate through a convergent-divergent nozzle for a gas with  $\gamma = 1.4$

$$\dot{m} = \frac{0.6847 p_0 A^*}{\sqrt{RT_0}}$$

where  $A^*$  is the area at the choked location with  $M = 1$ .

→ **Equation 3:** Mach number downstream of a normal shock

$$M_2^2 = \frac{1 + [(\gamma - 1)/2] M_1^2}{\gamma M_1^2 - (\gamma - 1)/2}$$

→ **Equation 4:** Pressure ratio across a normal shock

$$\frac{p_2}{p_1} = 1 + \frac{2\gamma}{\gamma + 1} (M_1^2 - 1)$$

→ **Equation 5:** Density or velocity ratio across a normal shock

$$\frac{\rho_2}{\rho_1} = \frac{V_1}{V_2} = \frac{(\gamma + 1) M_1^2}{(\gamma - 1) M_1^2 + 2}$$

→ **Equation 6:** Temperature ratio across a normal shock

$$\frac{T_2}{T_1} = \left[ 1 + \frac{2\gamma}{\gamma + 1} (M_1^2 - 1) \right] \left[ \frac{2 + (\gamma - 1) M_1^2}{(\gamma + 1) M_1^2} \right]$$

→ **Equation 7:** Rayleigh supersonic Pitot formula

$$\frac{p_1}{p_{02}} = \frac{\left( \frac{2\gamma}{\gamma + 1} M_1^2 - \frac{\gamma - 1}{\gamma + 1} \right)^{1/(\gamma-1)}}{\left( \frac{\gamma + 1}{2} M_1^2 \right)^{\gamma/(\gamma-1)}}$$

→ **Equation 8:**  $\theta - \beta - M$  relation

$$\tan \theta = 2 \cot \beta \left[ \frac{M_1^2 \sin^2 \beta - 1}{M_1^2 (\gamma + \cos 2\beta) + 2} \right]$$

where  $\theta$  is the deflection angle,  $\beta$  is the wave angle, and  $M_1$  is the upstream Mach number.

→ **Equation 9:** Ratio of pressures across an oblique shock

$$\frac{p_2}{p_1} = \frac{2\gamma}{\gamma + 1} M_1^2 \sin^2 \beta - \frac{\gamma - 1}{\gamma + 1}$$

→ **Equation 10:** Ratio of densities across an oblique shock

$$\frac{\rho_2}{\rho_1} = \frac{(\gamma + 1) M_1^2 \sin^2 \beta}{(\gamma - 1) M_1^2 \sin^2 \beta + 2}$$

Alternatively, simply use

$$\frac{\rho_2}{\rho_1} = \frac{\tan \beta}{\tan(\beta - \theta)}$$

→ **Equation 11:** Temperature ratio across an oblique shock

$$\frac{T_2}{T_1} = 1 + \frac{2(\gamma - 1)}{(\gamma + 1)^2} \frac{M_1^2 \sin^2 \beta - 1}{M_1^2 \sin^2 \beta} (\gamma M_1^2 \sin^2 \beta + 1)$$

→ **Equation 12:** Prandtl-Meyer function

$$\nu(M_1) = \sqrt{\frac{\gamma+1}{\gamma-1}} \arctan \sqrt{\frac{\gamma-1}{\gamma+1} (M^2 - 1)} - \arctan \sqrt{M_1^2 - 1}$$

**Tables for isentropic flow, normal shock, and oblique shock**

The following pages are isentropic flow, normal shock, and oblique shock tables drawn from Rathakrishnan, E., 2019, *Applied Gas Dynamics*, 2nd edition. Tables were reproduced with permission from John Wiley and Sons, Inc., 111 River Street, Hoboken, New Jersey, USA.

## Appendix A

**Table A.1** Isentropic flow of perfect gas ( $\gamma = 1.4$ ).

M	$p/p_0$	$T/T_0$	$\rho/\rho_0$	$A/A^*$	$a/a_0$	$M^*$	$\mu$	$\nu$
0.00	1.0000	1.0000	1.0000	$\infty$	1.0000	0.0000		
0.01	0.9999	1.0000	1.0000	57.874	1.0000	0.0110		
0.02	0.9997	0.9999	0.9998	28.942	1.0000	0.0219		
0.03	0.9994	0.9998	0.9996	19.301	0.9999	0.0329		
0.04	0.9989	0.9997	0.9992	14.481	0.9998	0.0438		
0.05	0.9983	0.9995	0.9988	11.591	0.9998	0.0548		
0.06	0.9975	0.9993	0.9982	9.666	0.9996	0.0657		
0.07	0.9966	0.9990	0.9976	8.292	0.9995	0.0766		
0.08	0.9955	0.9987	0.9968	7.262	0.9994	0.0876		
0.09	0.9944	0.9984	0.9960	6.461	0.9992	0.0985		
0.10	0.9930	0.9980	0.9950	5.822	0.9990	0.1094		
0.11	0.9916	0.9976	0.9940	5.299	0.9988	0.1204		
0.12	0.9900	0.9971	0.9928	4.864	0.9986	0.1313		
0.13	0.9883	0.9966	0.9916	4.497	0.9983	0.1422		
0.14	0.9864	0.9961	0.9903	4.182	0.9980	0.1531		
0.15	0.9844	0.9955	0.9888	3.910	0.9978	0.1639		
0.16	0.9823	0.9949	0.9873	3.673	0.9974	0.1748		
0.17	0.9800	0.9943	0.9857	3.464	0.9971	0.1857		
0.18	0.9776	0.9936	0.9840	3.278	0.9968	0.1965		
0.19	0.9751	0.9928	0.9822	3.112	0.9964	0.2074		
0.20	0.9725	0.9921	0.9803	2.964	0.9960	0.2182		
0.21	0.9697	0.9913	0.9783	2.829	0.9956	0.2290		
0.22	0.9668	0.9904	0.9762	2.708	0.9952	0.2398		
0.23	0.9638	0.9895	0.9740	2.597	0.9948	0.2506		
0.24	0.9607	0.9886	0.9718	2.496	0.9943	0.2614		
0.25	0.9575	0.9877	0.9694	2.403	0.9938	0.2722		
0.26	0.9541	0.9867	0.9670	2.317	0.9933	0.2829		
0.27	0.9506	0.9856	0.9645	2.238	0.9928	0.2936		
0.28	0.9470	0.9846	0.9619	2.166	0.9923	0.3043		

(continued)

**Table A.1** (Continued)

$M$	$p/p_0$	$T/T_0$	$\rho/\rho_0$	$A/A^*$	$a/a_0$	$M^*$	$\mu$	$\nu$
0.29	0.9433	0.9835	0.9592	2.098	0.9917	0.3150		
0.30	0.9395	0.9823	0.9564	2.035	0.9911	0.3257		
0.31	0.9355	0.9811	0.9535	1.977	0.9905	0.3364		
0.32	0.9315	0.9799	0.9506	1.922	0.9899	0.3470		
0.33	0.9274	0.9787	0.9476	1.871	0.9893	0.3576		
0.34	0.9231	0.9774	0.9445	1.823	0.9886	0.3682		
0.35	0.9188	0.9761	0.9413	1.778	0.9880	0.3788		
0.36	0.9143	0.9747	0.9380	1.736	0.9873	0.3893		
0.37	0.9098	0.9733	0.9347	1.696	0.9866	0.3999		
0.38	0.9052	0.9719	0.9313	1.659	0.9859	0.4104		
0.39	0.9004	0.9705	0.9278	1.623	0.9851	0.4209		
0.40	0.8956	0.9690	0.9243	1.590	0.9844	0.4313		
0.41	0.8907	0.9675	0.9207	1.559	0.9836	0.4418		
0.42	0.8857	0.9659	0.9170	1.529	0.9828	0.4522		
0.43	0.8807	0.9643	0.9132	1.501	0.9820	0.4626		
0.44	0.8755	0.9627	0.9094	1.474	0.9812	0.4729		
0.45	0.8703	0.9611	0.9055	1.449	0.9803	0.4833		
0.46	0.8650	0.9594	0.9016	1.425	0.9795	0.4936		
0.47	0.8596	0.9577	0.8976	1.402	0.9786	0.5038		
0.48	0.8541	0.9559	0.8935	1.380	0.9777	0.5141		
0.49	0.8486	0.9542	0.8894	1.359	0.9768	0.5243		
0.50	0.8430	0.9524	0.8852	1.340	0.9759	0.5345		
0.51	0.8374	0.9506	0.8809	1.321	0.9750	0.5447		
0.52	0.8317	0.9487	0.8766	1.303	0.9740	0.5548		
0.53	0.8259	0.9468	0.8723	1.286	0.9730	0.5649		
0.54	0.8201	0.9449	0.8679	1.270	0.9721	0.5750		
0.55	0.8142	0.9430	0.8634	1.255	0.9711	0.5851		
0.56	0.8082	0.9410	0.8589	1.240	0.9700	0.5951		
0.57	0.8022	0.9390	0.8544	1.226	0.9690	0.6051		
0.58	0.7962	0.9370	0.8498	1.213	0.9680	0.6150		
0.59	0.7901	0.9349	0.8451	1.200	0.9669	0.6249		
0.60	0.7840	0.9328	0.8405	1.188	0.9658	0.6348		
0.61	0.7778	0.9307	0.8357	1.177	0.9647	0.6447		
0.62	0.7716	0.9286	0.8310	1.166	0.9636	0.6545		
0.63	0.7654	0.9265	0.8262	1.155	0.9625	0.6643		
0.64	0.7591	0.9243	0.8213	1.145	0.9614	0.6740		
0.65	0.7528	0.9221	0.8164	1.136	0.9603	0.6837		
0.66	0.7465	0.9199	0.8115	1.127	0.9591	0.6934		
0.67	0.7401	0.9176	0.8066	1.118	0.9579	0.7031		
0.68	0.7338	0.9153	0.8016	1.110	0.9567	0.7127		

(continued)

**Table A.1** (Continued)

$M$	$p/p_0$	$T/T_0$	$\rho/\rho_0$	$A/A^*$	$a/a_0$	$M^*$	$\mu$	$\nu$
0.69	0.7274	0.9131	0.7966	1.102	0.9555	0.7223		
0.70	0.7209	0.9107	0.7916	1.094	0.9543	0.7318		
0.71	0.7145	0.9084	0.7865	1.087	0.9531	0.7413		
0.72	0.7080	0.9061	0.7814	1.081	0.9519	0.7508		
0.73	0.7016	0.9037	0.7763	1.074	0.9506	0.7602		
0.74	0.6951	0.9013	0.7712	1.068	0.9494	0.7696		
0.75	0.6886	0.8989	0.7660	1.062	0.9481	0.7789		
0.76	0.6821	0.8964	0.7609	1.057	0.9468	0.7883		
0.77	0.6756	0.8940	0.7557	1.052	0.9455	0.7975		
0.78	0.6691	0.8915	0.7505	1.047	0.9442	0.8068		
0.79	0.6625	0.8890	0.7452	1.043	0.9429	0.8160		
0.80	0.6560	0.8865	0.7400	1.038	0.9416	0.8251		
0.81	0.6495	0.8840	0.7347	1.034	0.9402	0.8343		
0.82	0.6430	0.8815	0.7295	1.030	0.9389	0.8433		
0.83	0.6365	0.8789	0.7242	1.027	0.9375	0.8524		
0.84	0.6300	0.8763	0.7189	1.024	0.9361	0.8614		
0.85	0.6235	0.8737	0.7136	1.021	0.9347	0.8704		
0.86	0.6170	0.8711	0.7083	1.018	0.9333	0.8793		
0.87	0.6106	0.8685	0.7030	1.015	0.9319	0.8882		
0.88	0.6041	0.8659	0.6977	1.013	0.9305	0.8970		
0.89	0.5977	0.8632	0.6924	1.011	0.9291	0.9058		
0.90	0.5913	0.8606	0.6870	1.009	0.9277	0.9146		
0.91	0.5849	0.8579	0.6817	1.007	0.9262	0.9233		
0.92	0.5785	0.8552	0.6764	1.006	0.9248	0.9320		
0.93	0.5721	0.8525	0.6711	1.004	0.9233	0.9407		
0.94	0.5658	0.8498	0.6658	1.003	0.9219	0.9493		
0.95	0.5595	0.8471	0.6604	1.002	0.9204	0.9578		
0.96	0.5532	0.8444	0.6551	1.001	0.9189	0.9663		
0.97	0.5469	0.8416	0.6498	1.001	0.9174	0.9748		
0.98	0.5407	0.8389	0.6445	1.000	0.9159	0.9833		
0.99	0.5345	0.8361	0.6392	1.000	0.9144	0.9916		
1.00	0.5283	0.8333	0.6339	1.000	0.9129	1.0000	90.000	0.000
1.01	0.5221	0.8306	0.6287	1.000	0.9113	1.0083	81.931	0.045
1.02	0.5160	0.8278	0.6234	1.000	0.9098	1.0166	78.635	0.126
1.03	0.5099	0.8250	0.6181	1.001	0.9083	1.0248	76.138	0.229
1.04	0.5039	0.8222	0.6129	1.001	0.9067	1.0330	74.058	0.351
1.05	0.4979	0.8193	0.6077	1.002	0.9052	1.0411	72.247	0.487
1.06	0.4919	0.8165	0.6024	1.003	0.9036	1.0492	70.630	0.637
1.07	0.4860	0.8137	0.5972	1.004	0.9020	1.0573	69.160	0.797
1.08	0.4800	0.8108	0.5920	1.005	0.9005	1.0653	67.808	0.968

(continued)

**Table A.1** (Continued)

<i>M</i>	$p/p_0$	$T/T_0$	$\rho/\rho_0$	$A/A^*$	$a/a_0$	$M^*$	$\mu$	$\nu$
1.09	0.4742	0.8080	0.5869	1.006	0.8989	1.0733	66.553	1.148
1.10	0.4684	0.8052	0.5817	1.008	0.8973	1.0812	65.380	1.336
1.11	0.4626	0.8023	0.5766	1.010	0.8957	1.0891	64.277	1.532
1.12	0.4568	0.7994	0.5714	1.011	0.8941	1.0970	63.234	1.735
1.13	0.4511	0.7966	0.5663	1.013	0.8925	1.1048	62.246	1.944
1.14	0.4455	0.7937	0.5612	1.015	0.8909	1.1126	61.306	2.160
1.15	0.4398	0.7908	0.5562	1.017	0.8893	1.1203	60.408	2.381
1.16	0.4343	0.7879	0.5511	1.020	0.8877	1.1280	59.550	2.607
1.17	0.4287	0.7851	0.5461	1.022	0.8860	1.1356	58.727	2.839
1.18	0.4232	0.7822	0.5411	1.025	0.8844	1.1432	57.936	3.074
1.19	0.4178	0.7793	0.5361	1.028	0.8828	1.1508	57.176	3.314
1.20	0.4124	0.7764	0.5311	1.030	0.8811	1.1583	56.443	3.558
1.21	0.4070	0.7735	0.5262	1.033	0.8795	1.1658	55.735	3.806
1.22	0.4017	0.7706	0.5213	1.037	0.8778	1.1732	55.052	4.057
1.23	0.3964	0.7677	0.5164	1.040	0.8762	1.1806	54.391	4.312
1.24	0.3912	0.7648	0.5115	1.043	0.8745	1.1879	53.751	4.569
1.25	0.3861	0.7619	0.5067	1.047	0.8729	1.1952	53.130	4.830
1.26	0.3809	0.7590	0.5019	1.050	0.8712	1.2025	52.528	5.093
1.27	0.3759	0.7561	0.4971	1.054	0.8695	1.2097	51.943	5.359
1.28	0.3708	0.7532	0.4923	1.058	0.8679	1.2169	51.375	5.627
1.29	0.3658	0.7503	0.4876	1.062	0.8662	1.2240	50.823	5.898
1.30	0.3609	0.7474	0.4829	1.066	0.8645	1.2311	50.285	6.170
1.31	0.3560	0.7445	0.4782	1.071	0.8628	1.2382	49.761	6.445
1.32	0.3512	0.7416	0.4736	1.075	0.8611	1.2452	49.251	6.721
1.33	0.3464	0.7387	0.4690	1.080	0.8595	1.2522	48.753	7.000
1.34	0.3417	0.7358	0.4644	1.084	0.8578	1.2591	48.268	7.279
1.35	0.3370	0.7329	0.4598	1.089	0.8561	1.2660	47.795	7.561
1.36	0.3323	0.7300	0.4553	1.094	0.8544	1.2729	47.332	7.844
1.37	0.3277	0.7271	0.4508	1.099	0.8527	1.2797	46.880	8.128
1.38	0.3232	0.7242	0.4463	1.104	0.8510	1.2864	46.439	8.413
1.39	0.3187	0.7213	0.4418	1.109	0.8493	1.2932	46.007	8.699
1.40	0.3142	0.7184	0.4374	1.115	0.8476	1.2999	45.585	8.987
1.41	0.3098	0.7155	0.4330	1.120	0.8459	1.3065	45.171	9.276
1.42	0.3055	0.7126	0.4287	1.126	0.8442	1.3131	44.767	9.565
1.43	0.3012	0.7097	0.4244	1.132	0.8425	1.3197	44.371	9.855
1.44	0.2969	0.7069	0.4201	1.138	0.8407	1.3262	43.983	10.146
1.45	0.2927	0.7040	0.4158	1.144	0.8390	1.3327	43.603	10.438
1.46	0.2886	0.7011	0.4116	1.150	0.8373	1.3392	43.230	10.731
1.47	0.2845	0.6982	0.4074	1.156	0.8356	1.3456	42.865	11.023
1.48	0.2804	0.6954	0.4032	1.163	0.8339	1.3520	42.507	11.317
1.49	0.2764	0.6925	0.3991	1.169	0.8322	1.3583	42.155	11.611

(continued)

**Table A.1** (Continued)

<i>M</i>	$p/p_0$	$T/T_0$	$\rho/\rho_0$	$A/A^*$	$a/a_0$	$M^*$	$\mu$	$\nu$
1.50	0.2724	0.6897	0.3950	1.176	0.8305	1.3646	41.810	11.905
1.51	0.2685	0.6868	0.3909	1.183	0.8287	1.3708	41.472	12.200
1.52	0.2646	0.6840	0.3869	1.190	0.8270	1.3770	41.140	12.495
1.53	0.2608	0.6811	0.3829	1.197	0.8253	1.3832	40.813	12.790
1.54	0.2570	0.6783	0.3789	1.204	0.8236	1.3894	40.493	13.086
1.55	0.2533	0.6754	0.3750	1.212	0.8219	1.3955	40.178	13.381
1.56	0.2496	0.6726	0.3710	1.219	0.8201	1.4015	39.868	13.677
1.57	0.2459	0.6698	0.3672	1.227	0.8184	1.4075	39.564	13.973
1.58	0.2423	0.6670	0.3633	1.234	0.8167	1.4135	39.265	14.269
1.59	0.2388	0.6642	0.3595	1.242	0.8150	1.4195	38.971	14.565
1.60	0.2353	0.6614	0.3557	1.250	0.8133	1.4254	38.682	14.860
1.61	0.2318	0.6586	0.3520	1.258	0.8115	1.4313	38.398	15.156
1.62	0.2284	0.6558	0.3483	1.267	0.8098	1.4371	38.118	15.452
1.63	0.2250	0.6530	0.3446	1.275	0.8081	1.4429	37.843	15.747
1.64	0.2217	0.6502	0.3409	1.284	0.8064	1.4487	37.572	16.043
1.65	0.2184	0.6475	0.3373	1.292	0.8046	1.4544	37.305	16.338
1.66	0.2151	0.6447	0.3337	1.301	0.8029	1.4601	37.043	16.633
1.67	0.2119	0.6419	0.3302	1.310	0.8012	1.4657	36.784	16.928
1.68	0.2088	0.6392	0.3266	1.319	0.7995	1.4713	36.530	17.222
1.69	0.2057	0.6364	0.3232	1.328	0.7978	1.4769	36.279	17.516
1.70	0.2026	0.6337	0.3197	1.338	0.7961	1.4825	36.032	17.810
1.71	0.1996	0.6310	0.3163	1.347	0.7943	1.4880	35.789	18.103
1.72	0.1966	0.6283	0.3129	1.357	0.7926	1.4935	35.549	18.396
1.73	0.1936	0.6256	0.3095	1.367	0.7909	1.4989	35.312	18.689
1.74	0.1907	0.6229	0.3062	1.376	0.7892	1.5043	35.080	18.981
1.75	0.1878	0.6202	0.3029	1.386	0.7875	1.5097	34.850	19.273
1.76	0.1850	0.6175	0.2996	1.397	0.7858	1.5150	34.624	19.565
1.77	0.1822	0.6148	0.2964	1.407	0.7841	1.5203	34.400	19.855
1.78	0.1794	0.6121	0.2931	1.418	0.7824	1.5256	34.180	20.146
1.79	0.1767	0.6095	0.2900	1.428	0.7807	1.5308	33.963	20.436
1.80	0.1740	0.6068	0.2868	1.439	0.7790	1.5360	33.749	20.725
1.81	0.1714	0.6041	0.2837	1.450	0.7773	1.5411	33.538	21.014
1.82	0.1688	0.6015	0.2806	1.461	0.7756	1.5463	33.329	21.302
1.83	0.1662	0.5989	0.2776	1.472	0.7739	1.5514	33.124	21.590
1.84	0.1637	0.5963	0.2745	1.484	0.7722	1.5564	32.921	21.877
1.85	0.1612	0.5936	0.2715	1.495	0.7705	1.5614	32.720	22.163
1.86	0.1587	0.5910	0.2686	1.507	0.7688	1.5664	32.523	22.449
1.87	0.1563	0.5884	0.2656	1.519	0.7671	1.5714	32.328	22.734
1.88	0.1539	0.5859	0.2627	1.531	0.7654	1.5763	32.135	23.019
1.89	0.1516	0.5833	0.2598	1.543	0.7637	1.5812	31.945	23.303
1.90	0.1492	0.5807	0.2570	1.555	0.7620	1.5861	31.757	23.586

(continued)

**Table A.1** (Continued)

$M$	$p/p_0$	$T/T_0$	$\rho/\rho_0$	$A/A^*$	$a/a_0$	$M^*$	$\mu$	$\nu$
1.91	0.1470	0.5782	0.2542	1.568	0.7604	1.5909	31.571	23.869
1.92	0.1447	0.5756	0.2514	1.580	0.7587	1.5957	31.388	24.151
1.93	0.1425	0.5731	0.2486	1.593	0.7570	1.6005	31.207	24.432
1.94	0.1403	0.5705	0.2459	1.606	0.7553	1.6052	31.028	24.712
1.95	0.1381	0.5680	0.2432	1.619	0.7537	1.6099	30.852	24.992
1.96	0.1360	0.5655	0.2405	1.633	0.7520	1.6146	30.677	25.271
1.97	0.1339	0.5630	0.2378	1.646	0.7503	1.6192	30.505	25.549
1.98	0.1318	0.5605	0.2352	1.660	0.7487	1.6239	30.335	25.827
1.99	0.1298	0.5580	0.2326	1.674	0.7470	1.6284	30.166	26.104
2.00	0.1278	0.5556	0.2300	1.688	0.7454	1.6330	30.000	26.380
2.01	0.1258	0.5531	0.2275	1.702	0.7437	1.6375	29.836	26.655
2.02	0.1239	0.5506	0.2250	1.716	0.7420	1.6420	29.673	26.930
2.03	0.1220	0.5482	0.2225	1.730	0.7404	1.6465	29.512	27.203
2.04	0.1201	0.5458	0.2200	1.745	0.7388	1.6509	29.353	27.476
2.05	0.1182	0.5433	0.2176	1.760	0.7371	1.6553	29.196	27.748
2.06	0.1164	0.5409	0.2152	1.775	0.7355	1.6597	29.041	28.020
2.07	0.1146	0.5385	0.2128	1.790	0.7338	1.6640	28.888	28.290
2.08	0.1128	0.5361	0.2104	1.806	0.7322	1.6683	28.736	28.560
2.09	0.1111	0.5337	0.2081	1.821	0.7306	1.6726	28.585	28.829
2.10	0.1094	0.5313	0.2058	1.837	0.7289	1.6769	28.437	29.097
2.11	0.1077	0.5290	0.2035	1.853	0.7273	1.6811	28.290	29.364
2.12	0.1060	0.5266	0.2013	1.869	0.7257	1.6853	28.145	29.630
2.13	0.1043	0.5243	0.1990	1.885	0.7241	1.6895	28.001	29.896
2.14	0.1027	0.5219	0.1968	1.902	0.7225	1.6936	27.859	30.161
2.15	0.1011	0.5196	0.1946	1.919	0.7208	1.6977	27.718	30.425
2.16	0.0996	0.5173	0.1925	1.935	0.7192	1.7018	27.578	30.688
2.17	0.0980	0.5150	0.1903	1.953	0.7176	1.7059	27.441	30.951
2.18	0.0965	0.5127	0.1882	1.970	0.7160	1.7099	27.304	31.212
2.19	0.0950	0.5104	0.1861	1.987	0.7144	1.7139	27.169	31.473
2.20	0.0935	0.5081	0.1841	2.005	0.7128	1.7179	27.036	31.732
2.21	0.0921	0.5059	0.1820	2.023	0.7112	1.7219	26.903	31.991
2.22	0.0906	0.5036	0.1800	2.041	0.7097	1.7258	26.773	32.249
2.23	0.0892	0.5014	0.1780	2.059	0.7081	1.7297	26.643	32.507
2.24	0.0878	0.4991	0.1760	2.078	0.7065	1.7336	26.515	32.763
2.25	0.0865	0.4969	0.1740	2.096	0.7049	1.7374	26.388	33.018
2.26	0.0851	0.4947	0.1721	2.115	0.7033	1.7412	26.262	33.273
2.27	0.0838	0.4925	0.1702	2.134	0.7018	1.7450	26.138	33.527
2.28	0.0825	0.4903	0.1683	2.154	0.7002	1.7488	26.014	33.780
2.29	0.0812	0.4881	0.1664	2.173	0.6986	1.7526	25.892	34.032
2.30	0.0800	0.4859	0.1646	2.193	0.6971	1.7563	25.771	34.283
2.31	0.0787	0.4837	0.1628	2.213	0.6955	1.7600	25.652	34.533

(continued)

**Table A.1** (Continued)

<i>M</i>	$p/p_0$	$T/T_0$	$\rho/\rho_0$	$A/A^*$	$a/a_0$	$M^*$	$\mu$	$\nu$
2.32	0.0775	0.4816	0.1609	2.233	0.6940	1.7637	25.533	34.782
2.33	0.0763	0.4794	0.1592	2.254	0.6924	1.7673	25.416	35.031
2.34	0.0751	0.4773	0.1574	2.274	0.6909	1.7709	25.300	35.279
2.35	0.0740	0.4752	0.1556	2.295	0.6893	1.7745	25.184	35.526
2.36	0.0728	0.4731	0.1539	2.316	0.6878	1.7781	25.070	35.771
2.37	0.0717	0.4709	0.1522	2.338	0.6863	1.7817	24.957	36.017
2.38	0.0706	0.4688	0.1505	2.359	0.6847	1.7852	24.845	36.261
2.39	0.0695	0.4668	0.1488	2.381	0.6832	1.7887	24.734	36.504
2.40	0.0684	0.4647	0.1472	2.403	0.6817	1.7922	24.624	36.747
2.41	0.0673	0.4626	0.1456	2.425	0.6802	1.7956	24.515	36.988
2.42	0.0663	0.4606	0.1439	2.448	0.6786	1.7991	24.407	37.229
2.43	0.0653	0.4585	0.1424	2.471	0.6771	1.8025	24.300	37.469
2.44	0.0643	0.4565	0.1408	2.494	0.6756	1.8059	24.195	37.708
2.45	0.0633	0.4544	0.1392	2.517	0.6741	1.8092	24.090	37.946
2.46	0.0623	0.4524	0.1377	2.540	0.6726	1.8126	23.985	38.183
2.47	0.0613	0.4504	0.1362	2.564	0.6711	1.8159	23.882	38.420
2.48	0.0604	0.4484	0.1346	2.588	0.6696	1.8192	23.780	38.655
2.49	0.0594	0.4464	0.1332	2.612	0.6682	1.8225	23.679	38.890
2.50	0.0585	0.4444	0.1317	2.637	0.6667	1.8257	23.578	39.124
2.51	0.0576	0.4425	0.1302	2.661	0.6652	1.8290	23.479	39.357
2.52	0.0567	0.4405	0.1288	2.686	0.6637	1.8322	23.380	39.589
2.53	0.0559	0.4386	0.1274	2.712	0.6622	1.8354	23.282	39.820
2.54	0.0550	0.4366	0.1260	2.737	0.6608	1.8386	23.185	40.050
2.55	0.0542	0.4347	0.1246	2.763	0.6593	1.8417	23.089	40.280
2.56	0.0533	0.4328	0.1232	2.789	0.6578	1.8448	22.993	40.508
2.57	0.0525	0.4309	0.1218	2.815	0.6564	1.8479	22.899	40.736
2.58	0.0517	0.4289	0.1205	2.842	0.6549	1.8510	22.805	40.963
2.59	0.0509	0.4271	0.1192	2.869	0.6535	1.8541	22.712	41.189
2.60	0.0501	0.4252	0.1179	2.896	0.6521	1.8571	22.620	41.415
2.61	0.0493	0.4233	0.1166	2.923	0.6506	1.8602	22.528	41.639
2.62	0.0486	0.4214	0.1153	2.951	0.6492	1.8632	22.438	41.863
2.63	0.0478	0.4196	0.1140	2.979	0.6477	1.8662	22.348	42.086
2.64	0.0471	0.4177	0.1128	3.007	0.6463	1.8691	22.259	42.307
2.65	0.0464	0.4159	0.1115	3.036	0.6449	1.8721	22.170	42.529
2.66	0.0457	0.4141	0.1103	3.065	0.6435	1.8750	22.082	42.749
2.67	0.0450	0.4122	0.1091	3.094	0.6421	1.8779	21.995	42.968
2.68	0.0443	0.4104	0.1079	3.123	0.6406	1.8808	21.909	43.187
2.69	0.0436	0.4086	0.1067	3.153	0.6392	1.8837	21.823	43.405
2.70	0.0430	0.4068	0.1056	3.183	0.6378	1.8865	21.738	43.621
2.71	0.0423	0.4051	0.1044	3.213	0.6364	1.8894	21.654	43.838
2.72	0.0417	0.4033	0.1033	3.244	0.6350	1.8922	21.571	44.053

(continued)

**Table A.1** (Continued)

$M$	$p/p_0$	$T/T_0$	$\rho/\rho_0$	$A/A^*$	$a/a_0$	$M^*$	$\mu$	$\nu$
2.73	0.0410	0.4015	0.1022	3.275	0.6337	1.8950	21.488	44.267
2.74	0.0404	0.3998	0.1010	3.306	0.6323	1.8978	21.405	44.481
2.75	0.0398	0.3980	0.0999	3.338	0.6309	1.9005	21.324	44.694
2.76	0.0392	0.3963	0.0989	3.370	0.6295	1.9033	21.243	44.906
2.77	0.0386	0.3945	0.0978	3.402	0.6281	1.9060	21.162	45.117
2.78	0.0380	0.3928	0.0967	3.434	0.6268	1.9087	21.083	45.327
2.79	0.0374	0.3911	0.0957	3.467	0.6254	1.9114	21.003	45.537
2.80	0.0368	0.3894	0.0946	3.500	0.6240	1.9140	20.925	45.746
2.81	0.0363	0.3877	0.0936	3.534	0.6227	1.9167	20.847	45.954
2.82	0.0357	0.3860	0.0926	3.567	0.6213	1.9193	20.770	46.161
2.83	0.0352	0.3844	0.0916	3.601	0.6200	1.9219	20.693	46.368
2.84	0.0347	0.3827	0.0906	3.636	0.6186	1.9246	20.617	46.573
2.85	0.0341	0.3810	0.0896	3.671	0.6173	1.9271	20.541	46.778
2.86	0.0336	0.3794	0.0886	3.706	0.6159	1.9297	20.466	46.982
2.87	0.0331	0.3777	0.0877	3.741	0.6146	1.9323	20.391	47.185
2.88	0.0326	0.3761	0.0867	3.777	0.6133	1.9348	20.318	47.388
2.89	0.0321	0.3745	0.0858	3.813	0.6119	1.9373	20.244	47.589
2.90	0.0317	0.3729	0.0849	3.850	0.6106	1.9398	20.171	47.790
2.91	0.0312	0.3712	0.0840	3.887	0.6093	1.9423	20.099	47.990
2.92	0.0307	0.3696	0.0831	3.924	0.6080	1.9448	20.027	48.190
2.93	0.0302	0.3681	0.0822	3.961	0.6067	1.9472	19.956	48.388
2.94	0.0298	0.3665	0.0813	3.999	0.6054	1.9497	19.885	48.586
2.95	0.0293	0.3649	0.0804	4.038	0.6041	1.9521	19.815	48.783
2.96	0.0289	0.3633	0.0796	4.076	0.6028	1.9545	19.745	48.980
2.97	0.0285	0.3618	0.0787	4.115	0.6015	1.9569	19.676	49.175
2.98	0.0281	0.3602	0.0779	4.155	0.6002	1.9593	19.607	49.370
2.99	0.0276	0.3587	0.0770	4.194	0.5989	1.9616	19.539	49.564
3.00	0.0272	0.3571	0.0762	4.235	0.5976	1.9640	19.471	49.757
3.01	0.0268	0.3556	0.0754	4.275	0.5963	1.9663	19.404	49.950
3.02	0.0264	0.3541	0.0746	4.316	0.5951	1.9686	19.337	50.142
3.03	0.0260	0.3526	0.0738	4.357	0.5938	1.9709	19.271	50.333
3.04	0.0256	0.3511	0.0730	4.399	0.5925	1.9732	19.205	50.523
3.05	0.0253	0.3496	0.0723	4.441	0.5913	1.9755	19.139	50.713
3.06	0.0249	0.3481	0.0715	4.483	0.5900	1.9777	19.074	50.902
3.07	0.0245	0.3466	0.0707	4.526	0.5887	1.9800	19.010	51.090
3.08	0.0242	0.3452	0.0700	4.570	0.5875	1.9822	18.946	51.277
3.09	0.0238	0.3437	0.0692	4.613	0.5862	1.9844	18.882	51.464
3.10	0.0234	0.3422	0.0685	4.657	0.5850	1.9866	18.819	51.650
3.11	0.0231	0.3408	0.0678	4.702	0.5838	1.9888	18.756	51.835
3.12	0.0228	0.3393	0.0671	4.747	0.5825	1.9910	18.694	52.020
3.13	0.0224	0.3379	0.0664	4.792	0.5813	1.9931	18.632	52.203

(continued)

**Table A.1** (Continued)

$M$	$p/p_0$	$T/T_0$	$\rho/\rho_0$	$A/A^*$	$a/a_0$	$M^*$	$\mu$	$\nu$
3.14	0.0221	0.3365	0.0657	4.838	0.5801	1.9953	18.571	52.386
3.15	0.0218	0.3351	0.0650	4.884	0.5788	1.9974	18.509	52.569
3.16	0.0215	0.3337	0.0643	4.930	0.5776	1.9995	18.449	52.751
3.17	0.0211	0.3323	0.0636	4.977	0.5764	2.0016	18.388	52.932
3.18	0.0208	0.3309	0.0630	5.025	0.5752	2.0037	18.329	53.112
3.19	0.0205	0.3295	0.0623	5.073	0.5740	2.0058	18.269	53.291
3.20	0.0202	0.3281	0.0617	5.121	0.5728	2.0079	18.210	53.470
3.21	0.0199	0.3267	0.0610	5.170	0.5716	2.0099	18.151	53.649
3.22	0.0196	0.3253	0.0604	5.219	0.5704	2.0119	18.093	53.826
3.23	0.0194	0.3240	0.0597	5.268	0.5692	2.0140	18.035	54.003
3.24	0.0191	0.3226	0.0591	5.319	0.5680	2.0160	17.977	54.179
3.25	0.0188	0.3213	0.0585	5.369	0.5668	2.0180	17.920	54.355
3.26	0.0185	0.3199	0.0579	5.420	0.5656	2.0200	17.863	54.529
3.27	0.0183	0.3186	0.0573	5.472	0.5645	2.0220	17.807	54.704
3.28	0.0180	0.3173	0.0567	5.523	0.5633	2.0239	17.751	54.877
3.29	0.0177	0.3160	0.0561	5.576	0.5621	2.0259	17.695	55.050
3.30	0.0175	0.3147	0.0555	5.629	0.5609	2.0278	17.640	55.222
3.31	0.0172	0.3134	0.0550	5.682	0.5598	2.0297	17.585	55.393
3.32	0.0170	0.3121	0.0544	5.736	0.5586	2.0317	17.530	55.564
3.33	0.0167	0.3108	0.0538	5.790	0.5575	2.0336	17.476	55.734
3.34	0.0165	0.3095	0.0533	5.845	0.5563	2.0355	17.422	55.904
3.35	0.0163	0.3082	0.0527	5.900	0.5552	2.0373	17.368	56.073
3.36	0.0160	0.3069	0.0522	5.956	0.5540	2.0392	17.315	56.241
3.37	0.0158	0.3057	0.0517	6.012	0.5529	2.0411	17.262	56.409
3.38	0.0156	0.3044	0.0511	6.069	0.5517	2.0429	17.209	56.576
3.39	0.0153	0.3032	0.0506	6.126	0.5506	2.0447	17.157	56.742
3.40	0.0151	0.3019	0.0501	6.184	0.5495	2.0466	17.105	56.908
3.41	0.0149	0.3007	0.0496	6.242	0.5484	2.0484	17.053	57.073
3.42	0.0147	0.2995	0.0491	6.301	0.5472	2.0502	17.002	57.237
3.43	0.0145	0.2982	0.0486	6.360	0.5461	2.0520	16.950	57.401
3.44	0.0143	0.2970	0.0481	6.420	0.5450	2.0537	16.900	57.564
3.45	0.0141	0.2958	0.0476	6.480	0.5439	2.0555	16.849	57.726
3.46	0.0139	0.2946	0.0471	6.541	0.5428	2.0573	16.799	57.888
3.47	0.0137	0.2934	0.0466	6.602	0.5417	2.0590	16.749	58.050
3.48	0.0135	0.2922	0.0462	6.664	0.5406	2.0607	16.700	58.210
3.49	0.0133	0.2910	0.0457	6.727	0.5395	2.0625	16.651	58.370
3.50	0.0131	0.2899	0.0452	6.790	0.5384	2.0642	16.602	58.530
3.51	0.0129	0.2887	0.0448	6.853	0.5373	2.0659	16.553	58.689
3.52	0.0127	0.2875	0.0443	6.917	0.5362	2.0676	16.505	58.847
3.53	0.0126	0.2864	0.0439	6.982	0.5351	2.0693	16.456	59.005
3.54	0.0124	0.2852	0.0434	7.047	0.5340	2.0709	16.409	59.162

(continued)

**Table A.1** (Continued)

$M$	$p/p_0$	$T/T_0$	$\rho/\rho_0$	$A/A^*$	$a/a_0$	$M^*$	$\mu$	$\nu$
3.55	0.0122	0.2841	0.0430	7.113	0.5330	2.0726	16.361	59.318
3.56	0.0120	0.2829	0.0426	7.179	0.5319	2.0743	16.314	59.474
3.57	0.0119	0.2818	0.0421	7.246	0.5308	2.0759	16.267	59.629
3.58	0.0117	0.2806	0.0417	7.313	0.5298	2.0775	16.220	59.784
3.59	0.0115	0.2795	0.0413	7.381	0.5287	2.0792	16.174	59.938
3.60	0.0114	0.2784	0.0409	7.450	0.5276	2.0808	16.128	60.091
3.61	0.0112	0.2773	0.0405	7.519	0.5266	2.0824	16.082	60.244
3.62	0.0111	0.2762	0.0401	7.589	0.5255	2.0840	16.036	60.397
3.63	0.0109	0.2751	0.0397	7.659	0.5245	2.0856	15.991	60.549
3.64	0.0108	0.2740	0.0393	7.730	0.5234	2.0871	15.946	60.700
3.65	0.0106	0.2729	0.0389	7.802	0.5224	2.0887	15.901	60.850
3.66	0.0105	0.2718	0.0385	7.874	0.5213	2.0903	15.856	61.001
3.67	0.0103	0.2707	0.0381	7.947	0.5203	2.0918	15.812	61.150
3.68	0.0102	0.2697	0.0378	8.020	0.5193	2.0933	15.768	61.299
3.69	0.0100	0.2686	0.0374	8.094	0.5183	2.0949	15.724	61.447
3.70	0.0099	0.2675	0.0370	8.169	0.5172	2.0964	15.680	61.595
3.71	0.0098	0.2665	0.0367	8.244	0.5162	2.0979	15.637	61.743
3.72	0.0096	0.2654	0.0363	8.320	0.5152	2.0994	15.594	61.889
3.73	0.0095	0.2644	0.0359	8.397	0.5142	2.1009	15.551	62.036
3.74	0.0094	0.2633	0.0356	8.474	0.5132	2.1024	15.508	62.181
3.75	0.0092	0.2623	0.0352	8.552	0.5121	2.1039	15.466	62.326
3.76	0.0091	0.2613	0.0349	8.630	0.5111	2.1053	15.424	62.471
3.77	0.0090	0.2602	0.0345	8.709	0.5101	2.1068	15.382	62.615
3.78	0.0089	0.2592	0.0342	8.789	0.5091	2.1082	15.340	62.758
3.79	0.0087	0.2582	0.0339	8.869	0.5081	2.1097	15.299	62.901
3.80	0.0086	0.2572	0.0335	8.951	0.5072	2.1111	15.258	63.044
3.81	0.0085	0.2562	0.0332	9.032	0.5062	2.1125	15.217	63.186
3.82	0.0084	0.2552	0.0329	9.115	0.5052	2.1140	15.176	63.327
3.83	0.0083	0.2542	0.0326	9.198	0.5042	2.1154	15.135	63.468
3.84	0.0082	0.2532	0.0323	9.282	0.5032	2.1168	15.095	63.608
3.85	0.0081	0.2522	0.0320	9.366	0.5022	2.1182	15.055	63.748
3.86	0.0080	0.2513	0.0316	9.451	0.5013	2.1195	15.015	63.887
3.87	0.0078	0.2503	0.0313	9.537	0.5003	2.1209	14.975	64.026
3.88	0.0077	0.2493	0.0310	9.624	0.4993	2.1223	14.936	64.164
3.89	0.0076	0.2484	0.0307	9.711	0.4984	2.1236	14.896	64.302
3.90	0.0075	0.2474	0.0304	9.799	0.4974	2.1250	14.857	64.440
3.91	0.0074	0.2464	0.0302	9.888	0.4964	2.1263	14.818	64.576
3.92	0.0073	0.2455	0.0299	9.977	0.4955	2.1277	14.780	64.713
3.93	0.0072	0.2446	0.0296	10.067	0.4945	2.1290	14.741	64.848
3.94	0.0071	0.2436	0.0293	10.158	0.4936	2.1303	14.703	64.984
3.95	0.0070	0.2427	0.0290	10.250	0.4926	2.1316	14.665	65.118

(continued)

**Table A.1** (Continued)

$M$	$p/p_0$	$T/T_0$	$\rho/\rho_0$	$A/A^*$	$a/a_0$	$M^*$	$\mu$	$\nu$
3.96	0.0069	0.2418	0.0287	10.342	0.4917	2.1329	14.627	65.253
3.97	0.0069	0.2408	0.0285	10.435	0.4908	2.1342	14.589	65.386
3.98	0.0068	0.2399	0.0282	10.529	0.4898	2.1355	14.552	65.520
3.99	0.0067	0.2390	0.0279	10.623	0.4889	2.1368	14.515	65.652
4.00	0.0066	0.2381	0.0277	10.719	0.4880	2.1381	14.478	65.785
4.01	0.0065	0.2372	0.0274	10.815	0.4870	2.1394	14.441	65.917
4.02	0.0064	0.2363	0.0271	10.912	0.4861	2.1406	14.404	66.048
4.03	0.0063	0.2354	0.0269	11.009	0.4852	2.1419	14.367	66.179
4.04	0.0062	0.2345	0.0266	11.108	0.4843	2.1431	14.331	66.309
4.05	0.0062	0.2336	0.0264	11.207	0.4833	2.1444	14.295	66.439
4.06	0.0061	0.2327	0.0261	11.307	0.4824	2.1456	14.259	66.569
4.07	0.0060	0.2319	0.0259	11.408	0.4815	2.1468	14.223	66.698
4.08	0.0059	0.2310	0.0256	11.509	0.4806	2.1480	14.188	66.826
4.09	0.0058	0.2301	0.0254	11.611	0.4797	2.1493	14.152	66.954
4.10	0.0058	0.2293	0.0252	11.715	0.4788	2.1505	14.117	67.082
4.11	0.0057	0.2284	0.0249	11.819	0.4779	2.1517	14.082	67.209
4.12	0.0056	0.2275	0.0247	11.923	0.4770	2.1529	14.047	67.336
4.13	0.0055	0.2267	0.0245	12.029	0.4761	2.1540	14.012	67.462
4.14	0.0055	0.2258	0.0242	12.135	0.4752	2.1552	13.978	67.588
4.15	0.0054	0.2250	0.0240	12.243	0.4743	2.1564	13.943	67.713
4.16	0.0053	0.2242	0.0238	12.351	0.4735	2.1576	13.909	67.838
4.17	0.0053	0.2233	0.0236	12.460	0.4726	2.1587	13.875	67.963
4.18	0.0052	0.2225	0.0234	12.570	0.4717	2.1599	13.841	68.087
4.19	0.0051	0.2217	0.0231	12.680	0.4708	2.1610	13.808	68.210
4.20	0.0051	0.2208	0.0229	12.792	0.4699	2.1622	13.774	68.333
4.21	0.0050	0.2200	0.0227	12.904	0.4691	2.1633	13.741	68.456
4.22	0.0049	0.2192	0.0225	13.017	0.4682	2.1644	13.708	68.578
4.23	0.0049	0.2184	0.0223	13.131	0.4673	2.1655	13.675	68.700
4.24	0.0048	0.2176	0.0221	13.246	0.4665	2.1667	13.642	68.821
4.25	0.0047	0.2168	0.0219	13.362	0.4656	2.1678	13.609	68.942
4.26	0.0047	0.2160	0.0217	13.479	0.4648	2.1689	13.576	69.063
4.27	0.0046	0.2152	0.0215	13.597	0.4639	2.1700	13.544	69.183
4.28	0.0046	0.2144	0.0213	13.715	0.4631	2.1711	13.512	69.303
4.29	0.0045	0.2136	0.0211	13.835	0.4622	2.1721	13.480	69.422
4.30	0.0044	0.2129	0.0209	13.955	0.4614	2.1732	13.448	69.541
4.31	0.0044	0.2121	0.0207	14.076	0.4605	2.1743	13.416	69.659
4.32	0.0043	0.2113	0.0205	14.198	0.4597	2.1754	13.384	69.777
4.33	0.0043	0.2105	0.0203	14.322	0.4588	2.1764	13.353	69.895
4.34	0.0042	0.2098	0.0202	14.446	0.4580	2.1775	13.321	70.012
4.35	0.0042	0.2090	0.0200	14.571	0.4572	2.1785	13.290	70.129
4.36	0.0041	0.2083	0.0198	14.697	0.4563	2.1796	13.259	70.245

(continued)

**Table A.1** (Continued)

$M$	$p/p_0$	$T/T_0$	$\rho/\rho_0$	$A/A^*$	$a/a_0$	$M^*$	$\mu$	$\nu$
4.37	0.0041	0.2075	0.0196	14.823	0.4555	2.1806	13.228	70.361
4.38	0.0040	0.2067	0.0194	14.951	0.4547	2.1816	13.198	70.476
4.39	0.0040	0.2060	0.0193	15.080	0.4539	2.1827	13.167	70.591
4.40	0.0039	0.2053	0.0191	15.210	0.4531	2.1837	13.137	70.706
4.41	0.0039	0.2045	0.0189	15.341	0.4522	2.1847	13.106	70.820
4.42	0.0038	0.2038	0.0187	15.472	0.4514	2.1857	13.076	70.934
4.43	0.0038	0.2030	0.0186	15.605	0.4506	2.1867	13.046	71.048
4.44	0.0037	0.2023	0.0184	15.739	0.4498	2.1877	13.016	71.161
4.45	0.0037	0.2016	0.0182	15.873	0.4490	2.1887	12.986	71.274
4.46	0.0036	0.2009	0.0181	16.009	0.4482	2.1897	12.957	71.386
4.47	0.0036	0.2002	0.0179	16.146	0.4474	2.1907	12.927	71.498
4.48	0.0035	0.1994	0.0178	16.284	0.4466	2.1917	12.898	71.610
4.49	0.0035	0.1987	0.0176	16.422	0.4458	2.1926	12.869	71.721
4.50	0.0035	0.1980	0.0174	16.562	0.4450	2.1936	12.840	71.832
4.51	0.0034	0.1973	0.0173	16.703	0.4442	2.1946	12.811	71.942
4.52	0.0034	0.1966	0.0171	16.845	0.4434	2.1955	12.782	72.052
4.53	0.0033	0.1959	0.0170	16.988	0.4426	2.1965	12.753	72.162
4.54	0.0033	0.1952	0.0168	17.132	0.4418	2.1974	12.725	72.271
4.55	0.0032	0.1945	0.0167	17.277	0.4411	2.1984	12.696	72.380
4.56	0.0032	0.1938	0.0165	17.423	0.4403	2.1993	12.668	72.489
4.57	0.0032	0.1932	0.0164	17.570	0.4395	2.2002	12.640	72.597
4.58	0.0031	0.1925	0.0163	17.718	0.4387	2.2012	12.612	72.705
4.59	0.0031	0.1918	0.0161	17.867	0.4380	2.2021	12.584	72.812
4.60	0.0031	0.1911	0.0160	18.018	0.4372	2.2030	12.556	72.919
4.61	0.0030	0.1905	0.0158	18.169	0.4364	2.2039	12.528	73.026
4.62	0.0030	0.1898	0.0157	18.322	0.4357	2.2048	12.501	73.132
4.63	0.0029	0.1891	0.0156	18.476	0.4349	2.2057	12.473	73.238
4.64	0.0029	0.1885	0.0154	18.630	0.4341	2.2066	12.446	73.344
4.65	0.0029	0.1878	0.0153	18.786	0.4334	2.2075	12.419	73.449
4.66	0.0028	0.1872	0.0152	18.943	0.4326	2.2084	12.392	73.554
4.67	0.0028	0.1865	0.0150	19.101	0.4319	2.2093	12.365	73.659
4.68	0.0028	0.1859	0.0149	19.261	0.4311	2.2102	12.338	73.763
4.69	0.0027	0.1852	0.0148	19.421	0.4304	2.2110	12.311	73.867
4.70	0.0027	0.1846	0.0146	19.583	0.4296	2.2119	12.284	73.970
4.71	0.0027	0.1839	0.0145	19.746	0.4289	2.2128	12.258	74.073
4.72	0.0026	0.1833	0.0144	19.910	0.4281	2.2136	12.232	74.176
4.73	0.0026	0.1827	0.0143	20.075	0.4274	2.2145	12.205	74.279
4.74	0.0026	0.1820	0.0141	20.241	0.4267	2.2154	12.179	74.381
4.75	0.0025	0.1814	0.0140	20.408	0.4259	2.2162	12.153	74.482
4.76	0.0025	0.1808	0.0139	20.577	0.4252	2.2170	12.127	74.584
4.77	0.0025	0.1802	0.0138	20.747	0.4245	2.2179	12.101	74.685

(continued)

**Table A.1** (Continued)

$M$	$p/p_0$	$T/T_0$	$\rho/\rho_0$	$A/A^*$	$a/a_0$	$M^*$	$\mu$	$\nu$
4.78	0.0025	0.1795	0.0137	20.918	0.4237	2.2187	12.076	74.786
4.79	0.0024	0.1789	0.0135	21.090	0.4230	2.2196	12.050	74.886
4.80	0.0024	0.1783	0.0134	21.264	0.4223	2.2204	12.025	74.986
4.81	0.0024	0.1777	0.0133	21.438	0.4216	2.2212	11.999	75.086
4.82	0.0023	0.1771	0.0132	21.614	0.4208	2.2220	11.974	75.185
4.83	0.0023	0.1765	0.0131	21.792	0.4201	2.2228	11.949	75.285
4.84	0.0023	0.1759	0.0130	21.970	0.4194	2.2236	11.924	75.383
4.85	0.0023	0.1753	0.0129	22.150	0.4187	2.2245	11.899	75.482
4.86	0.0022	0.1747	0.0128	22.331	0.4180	2.2253	11.874	75.580
4.87	0.0022	0.1741	0.0126	22.513	0.4173	2.2261	11.849	75.678
4.88	0.0022	0.1735	0.0125	22.696	0.4166	2.2268	11.825	75.775
4.89	0.0022	0.1729	0.0124	22.881	0.4159	2.2276	11.800	75.872
4.90	0.0021	0.1724	0.0123	23.067	0.4152	2.2284	11.776	75.969
4.91	0.0021	0.1718	0.0122	23.254	0.4145	2.2292	11.751	76.066
4.92	0.0021	0.1712	0.0121	23.443	0.4138	2.2300	11.727	76.162
4.93	0.0021	0.1706	0.0120	23.633	0.4131	2.2308	11.703	76.258
4.94	0.0020	0.1700	0.0119	23.824	0.4124	2.2315	11.679	76.353
4.95	0.0020	0.1695	0.0118	24.017	0.4117	2.2323	11.655	76.449
4.96	0.0020	0.1689	0.0117	24.211	0.4110	2.2331	11.631	76.544
4.97	0.0020	0.1683	0.0116	24.406	0.4103	2.2338	11.608	76.638
4.98	0.0019	0.1678	0.0115	24.603	0.4096	2.2346	11.584	76.732
4.99	0.0019	0.1672	0.0114	24.801	0.4089	2.2353	11.560	76.826
5.00	0.0019	0.1667	0.0113	25.000	0.4082	2.2361	11.537	76.920

Note: In Table A.1  $\mu$  and  $\nu$  values are in degrees.

**Table A.2** Normal shock in perfect gas ( $\gamma = 1.4$ ).

$M_1$	$M_2$	$P_2/P_1$	$\rho_2/\rho_1$	$T_2/T_1$	$a_2/a_1$	$p_{02}/p_{01}$
1.01	0.9901	1.0234	1.0167	1.0066	1.0033	1.0000
1.02	0.9805	1.0471	1.0334	1.0132	1.0066	1.0000
1.03	0.9712	1.0710	1.0502	1.0198	1.0099	1.0000
1.04	0.9620	1.0952	1.0671	1.0263	1.0131	0.9999
1.05	0.9531	1.1196	1.0840	1.0328	1.0163	0.9999
1.06	0.9444	1.1442	1.1009	1.0393	1.0195	0.9998
1.07	0.9360	1.1690	1.1179	1.0458	1.0226	0.9996
1.08	0.9277	1.1941	1.1349	1.0522	1.0258	0.9994
1.09	0.9196	1.2194	1.1520	1.0586	1.0289	0.9992
1.10	0.9118	1.2450	1.1691	1.0649	1.0320	0.9989
1.11	0.9041	1.2708	1.1862	1.0713	1.0350	0.9986

(continued)

**Table A.2** (Continued)

$M_1$	$M_2$	$P_2/P_1$	$\rho_2/\rho_1$	$T_2/T_1$	$a_2/a_1$	$p_{02}/p_{01}$
1.12	0.8966	1.2968	1.2034	1.0776	1.0381	0.9982
1.13	0.8892	1.3230	1.2206	1.0840	1.0411	0.9978
1.14	0.8820	1.3495	1.2378	1.0903	1.0442	0.9973
1.15	0.8750	1.3762	1.2550	1.0966	1.0472	0.9967
1.16	0.8682	1.4032	1.2723	1.1029	1.0502	0.9961
1.17	0.8615	1.4304	1.2896	1.1092	1.0532	0.9953
1.18	0.8549	1.4578	1.3069	1.1154	1.0561	0.9946
1.19	0.8485	1.4854	1.3243	1.1217	1.0591	0.9937
1.20	0.8422	1.5133	1.3416	1.1280	1.0621	0.9928
1.21	0.8360	1.5414	1.3590	1.1343	1.0650	0.9918
1.22	0.8300	1.5698	1.3764	1.1405	1.0680	0.9907
1.23	0.8241	1.5984	1.3938	1.1468	1.0709	0.9896
1.24	0.8183	1.6272	1.4112	1.1531	1.0738	0.9884
1.25	0.8126	1.6562	1.4286	1.1594	1.0767	0.9871
1.26	0.8071	1.6855	1.4460	1.1657	1.0797	0.9857
1.27	0.8016	1.7150	1.4634	1.1720	1.0826	0.9842
1.28	0.7963	1.7448	1.4808	1.1783	1.0855	0.9827
1.29	0.7911	1.7748	1.4983	1.1846	1.0884	0.9811
1.30	0.7860	1.8050	1.5157	1.1909	1.0913	0.9794
1.31	0.7809	1.8354	1.5331	1.1972	1.0942	0.9776
1.32	0.7760	1.8661	1.5505	1.2035	1.0971	0.9758
1.33	0.7712	1.8970	1.5680	1.2099	1.0999	0.9738
1.34	0.7664	1.9282	1.5854	1.2162	1.1028	0.9718
1.35	0.7618	1.9596	1.6028	1.2226	1.1057	0.9697
1.36	0.7572	1.9912	1.6202	1.2290	1.1086	0.9676
1.37	0.7527	2.0230	1.6376	1.2354	1.1115	0.9653
1.38	0.7483	2.0551	1.6549	1.2418	1.1144	0.9630
1.39	0.7440	2.0874	1.6723	1.2482	1.1172	0.9607
1.40	0.7397	2.1200	1.6897	1.2547	1.1201	0.9582
1.41	0.7355	2.1528	1.7070	1.2612	1.1230	0.9557
1.42	0.7314	2.1858	1.7243	1.2676	1.1259	0.9531
1.43	0.7274	2.2190	1.7416	1.2741	1.1288	0.9504
1.44	0.7235	2.2525	1.7589	1.2807	1.1317	0.9476
1.45	0.7196	2.2862	1.7761	1.2872	1.1346	0.9448
1.46	0.7157	2.3202	1.7934	1.2938	1.1374	0.9420
1.47	0.7120	2.3544	1.8106	1.3003	1.1403	0.9390
1.48	0.7083	2.3888	1.8278	1.3069	1.1432	0.9360
1.49	0.7047	2.4234	1.8449	1.3136	1.1461	0.9329
1.50	0.7011	2.4583	1.8621	1.3202	1.1490	0.9298
1.51	0.6976	2.4934	1.8792	1.3269	1.1519	0.9266
1.52	0.6941	2.5288	1.8963	1.3336	1.1548	0.9233

(continued)

**Table A.2** (Continued)

$M_1$	$M_2$	$P_2/P_1$	$\rho_2/\rho_1$	$T_2/T_1$	$a_2/a_1$	$p_{02}/p_{01}$
1.53	0.6907	2.5644	1.9133	1.3403	1.1577	0.9200
1.54	0.6874	2.6002	1.9303	1.3470	1.1606	0.9166
1.55	0.6841	2.6362	1.9473	1.3538	1.1635	0.9132
1.56	0.6809	2.6725	1.9643	1.3606	1.1664	0.9097
1.57	0.6777	2.7090	1.9812	1.3674	1.1694	0.9062
1.58	0.6746	2.7458	1.9981	1.3742	1.1723	0.9026
1.59	0.6715	2.7828	2.0149	1.3811	1.1752	0.8989
1.60	0.6684	2.8200	2.0317	1.3880	1.1781	0.8952
1.61	0.6655	2.8574	2.0485	1.3949	1.1811	0.8915
1.62	0.6625	2.8951	2.0653	1.4018	1.1840	0.8877
1.63	0.6596	2.9330	2.0820	1.4088	1.1869	0.8838
1.64	0.6568	2.9712	2.0986	1.4158	1.1899	0.8799
1.65	0.6540	3.0096	2.1152	1.4228	1.1928	0.8760
1.66	0.6512	3.0482	2.1318	1.4299	1.1958	0.8720
1.67	0.6485	3.0870	2.1484	1.4369	1.1987	0.8680
1.68	0.6458	3.1261	2.1649	1.4440	1.2017	0.8639
1.69	0.6431	3.1654	2.1813	1.4512	1.2046	0.8599
1.70	0.6405	3.2050	2.1977	1.4583	1.2076	0.8557
1.71	0.6380	3.2448	2.2141	1.4655	1.2106	0.8516
1.72	0.6355	3.2848	2.2304	1.4727	1.2136	0.8474
1.73	0.6330	3.3250	2.2467	1.4800	1.2165	0.8431
1.74	0.6305	3.3655	2.2629	1.4873	1.2195	0.8389
1.75	0.6281	3.4062	2.2791	1.4946	1.2225	0.8346
1.76	0.6257	3.4472	2.2952	1.5019	1.2255	0.8302
1.77	0.6234	3.4884	2.3113	1.5093	1.2285	0.8259
1.78	0.6210	3.5298	2.3273	1.5167	1.2315	0.8215
1.79	0.6188	3.5714	2.3433	1.5241	1.2346	0.8171
1.80	0.6165	3.6133	2.3592	1.5316	1.2376	0.8127
1.81	0.6143	3.6554	2.3751	1.5391	1.2406	0.8082
1.82	0.6121	3.6978	2.3909	1.5466	1.2436	0.8038
1.83	0.6099	3.7404	2.4067	1.5541	1.2467	0.7993
1.84	0.6078	3.7832	2.4224	1.5617	1.2497	0.7948
1.85	0.6057	3.8262	2.4381	1.5693	1.2527	0.7902
1.86	0.6036	3.8695	2.4537	1.5770	1.2558	0.7857
1.87	0.6016	3.9130	2.4693	1.5847	1.2588	0.7811
1.88	0.5996	3.9568	2.4848	1.5924	1.2619	0.7765
1.89	0.5976	4.0008	2.5003	1.6001	1.2650	0.7720
1.90	0.5956	4.0450	2.5157	1.6079	1.2680	0.7674
1.91	0.5937	4.0894	2.5310	1.6157	1.2711	0.7627
1.92	0.5918	4.1341	2.5463	1.6236	1.2742	0.7581
1.93	0.5899	4.1791	2.5616	1.6314	1.2773	0.7535

(continued)

**Table A.2** (Continued)

$M_1$	$M_2$	$P_2/P_1$	$\rho_2/\rho_1$	$T_2/T_1$	$a_2/a_1$	$p_{02}/p_{01}$
1.94	0.5880	4.2242	2.5767	1.6394	1.2804	0.7488
1.95	0.5862	4.2696	2.5919	1.6473	1.2835	0.7442
1.96	0.5844	4.3152	2.6069	1.6553	1.2866	0.7395
1.97	0.5826	4.3610	2.6220	1.6633	1.2897	0.7349
1.98	0.5808	4.4071	2.6369	1.6713	1.2928	0.7302
1.99	0.5791	4.4534	2.6518	1.6794	1.2959	0.7255
2.00	0.5774	4.5000	2.6667	1.6875	1.2990	0.7209
2.01	0.5757	4.5468	2.6815	1.6956	1.3022	0.7162
2.02	0.5740	4.5938	2.6962	1.7038	1.3053	0.7115
2.03	0.5723	4.6410	2.7109	1.7120	1.3084	0.7069
2.04	0.5707	4.6885	2.7255	1.7203	1.3116	0.7022
2.05	0.5691	4.7362	2.7400	1.7285	1.3147	0.6975
2.06	0.5675	4.7842	2.7545	1.7369	1.3179	0.6928
2.07	0.5659	4.8324	2.7689	1.7452	1.3211	0.6882
2.08	0.5643	4.8808	2.7833	1.7536	1.3242	0.6835
2.09	0.5628	4.9294	2.7976	1.7620	1.3274	0.6789
2.10	0.5613	4.9783	2.8119	1.7704	1.3306	0.6742
2.11	0.5598	5.0274	2.8261	1.7789	1.3338	0.6696
2.12	0.5583	5.0768	2.8402	1.7875	1.3370	0.6649
2.13	0.5568	5.1264	2.8543	1.7960	1.3402	0.6603
2.14	0.5554	5.1762	2.8683	1.8046	1.3434	0.6557
2.15	0.5540	5.2262	2.8823	1.8132	1.3466	0.6511
2.16	0.5525	5.2765	2.8962	1.8219	1.3498	0.6464
2.17	0.5511	5.3270	2.9101	1.8306	1.3530	0.6419
2.18	0.5498	5.3778	2.9238	1.8393	1.3562	0.6373
2.19	0.5484	5.4288	2.9376	1.8481	1.3594	0.6327
2.20	0.5471	5.4800	2.9512	1.8569	1.3627	0.6281
2.21	0.5457	5.5314	2.9648	1.8657	1.3659	0.6236
2.22	0.5444	5.5831	2.9784	1.8746	1.3691	0.6191
2.23	0.5431	5.6350	2.9918	1.8835	1.3724	0.6145
2.24	0.5418	5.6872	3.0053	1.8924	1.3756	0.6100
2.25	0.5406	5.7396	3.0186	1.9014	1.3789	0.6055
2.26	0.5393	5.7922	3.0319	1.9104	1.3822	0.6011
2.27	0.5381	5.8450	3.0452	1.9194	1.3854	0.5966
2.28	0.5368	5.8981	3.0584	1.9285	1.3887	0.5921
2.29	0.5356	5.9514	3.0715	1.9376	1.3920	0.5877
2.30	0.5344	6.0050	3.0845	1.9468	1.3953	0.5833
2.31	0.5332	6.0588	3.0976	1.9560	1.3986	0.5789
2.32	0.5321	6.1128	3.1105	1.9652	1.4019	0.5745
2.33	0.5309	6.1670	3.1234	1.9745	1.4052	0.5702
2.34	0.5297	6.2215	3.1362	1.9838	1.4085	0.5658

(continued)

**Table A.2** (Continued)

$M_1$	$M_2$	$P_2/P_1$	$\rho_2/\rho_1$	$T_2/T_1$	$a_2/a_1$	$p_{02}/p_{01}$
2.35	0.5286	6.2762	3.1490	1.9931	1.4118	0.5615
2.36	0.5275	6.3312	3.1617	2.0025	1.4151	0.5572
2.37	0.5264	6.3864	3.1743	2.0119	1.4184	0.5529
2.38	0.5253	6.4418	3.1869	2.0213	1.4217	0.5486
2.39	0.5242	6.4974	3.1994	2.0308	1.4251	0.5444
2.40	0.5231	6.5533	3.2119	2.0403	1.4284	0.5401
2.41	0.5221	6.6094	3.2243	2.0499	1.4317	0.5359
2.42	0.5210	6.6658	3.2367	2.0595	1.4351	0.5317
2.43	0.5200	6.7224	3.2489	2.0691	1.4384	0.5276
2.44	0.5189	6.7792	3.2612	2.0788	1.4418	0.5234
2.45	0.5179	6.8362	3.2733	2.0885	1.4452	0.5193
2.46	0.5169	6.8935	3.2855	2.0982	1.4485	0.5152
2.47	0.5159	6.9510	3.2975	2.1080	1.4519	0.5111
2.48	0.5149	7.0088	3.3095	2.1178	1.4553	0.5071
2.49	0.5140	7.0668	3.3215	2.1276	1.4586	0.5030
2.50	0.5130	7.1250	3.3333	2.1375	1.4620	0.4990
2.51	0.5120	7.1834	3.3452	2.1474	1.4654	0.4950
2.52	0.5111	7.2421	3.3569	2.1574	1.4688	0.4911
2.53	0.5102	7.3010	3.3686	2.1674	1.4722	0.4871
2.54	0.5092	7.3602	3.3803	2.1774	1.4756	0.4832
2.55	0.5083	7.4196	3.3919	2.1875	1.4790	0.4793
2.56	0.5074	7.4792	3.4034	2.1976	1.4824	0.4754
2.57	0.5065	7.5390	3.4149	2.2077	1.4858	0.4715
2.58	0.5056	7.5991	3.4263	2.2179	1.4893	0.4677
2.59	0.5047	7.6594	3.4377	2.2281	1.4927	0.4639
2.60	0.5039	7.7200	3.4490	2.2383	1.4961	0.4601
2.61	0.5030	7.7808	3.4602	2.2486	1.4995	0.4564
2.62	0.5022	7.8418	3.4714	2.2590	1.5030	0.4526
2.63	0.5013	7.9030	3.4826	2.2693	1.5064	0.4489
2.64	0.5005	7.9645	3.4937	2.2797	1.5099	0.4452
2.65	0.4996	8.0262	3.5047	2.2902	1.5133	0.4416
2.66	0.4988	8.0882	3.5157	2.3006	1.5168	0.4379
2.67	0.4980	8.1504	3.5266	2.3111	1.5202	0.4343
2.68	0.4972	8.2128	3.5374	2.3217	1.5237	0.4307
2.69	0.4964	8.2754	3.5482	2.3323	1.5272	0.4271
2.70	0.4956	8.3383	3.5590	2.3429	1.5307	0.4236
2.71	0.4949	8.4014	3.5697	2.3536	1.5341	0.4201
2.72	0.4941	8.4648	3.5803	2.3642	1.5376	0.4166
2.73	0.4933	8.5284	3.5909	2.3750	1.5411	0.4131
2.74	0.4926	8.5922	3.6015	2.3858	1.5446	0.4097
2.75	0.4918	8.6562	3.6119	2.3966	1.5481	0.4062

(continued)

**Table A.2** (Continued)

$M_1$	$M_2$	$P_2/P_1$	$\rho_2/\rho_1$	$T_2/T_1$	$a_2/a_1$	$p_{02}/p_{01}$
2.76	0.4911	8.7205	3.6224	2.4074	1.5516	0.4028
2.77	0.4903	8.7850	3.6327	2.4183	1.5551	0.3994
2.78	0.4896	8.8498	3.6431	2.4292	1.5586	0.3961
2.79	0.4889	8.9148	3.6533	2.4402	1.5621	0.3928
2.80	0.4882	8.9800	3.6636	2.4512	1.5656	0.3895
2.81	0.4875	9.0454	3.6737	2.4622	1.5691	0.3862
2.82	0.4868	9.1111	3.6838	2.4733	1.5727	0.3829
2.83	0.4861	9.1770	3.6939	2.4844	1.5762	0.3797
2.84	0.4854	9.2432	3.7039	2.4955	1.5797	0.3765
2.85	0.4847	9.3096	3.7139	2.5067	1.5833	0.3733
2.86	0.4840	9.3762	3.7238	2.5179	1.5868	0.3701
2.87	0.4833	9.4430	3.7336	2.5292	1.5903	0.3670
2.88	0.4827	9.5101	3.7434	2.5405	1.5939	0.3639
2.89	0.4820	9.5774	3.7532	2.5518	1.5974	0.3608
2.90	0.4814	9.6450	3.7629	2.5632	1.6010	0.3577
2.91	0.4807	9.7128	3.7725	2.5746	1.6046	0.3547
2.92	0.4801	9.7808	3.7821	2.5861	1.6081	0.3517
2.93	0.4795	9.8490	3.7917	2.5976	1.6117	0.3487
2.94	0.4788	9.9175	3.8012	2.6091	1.6153	0.3457
2.95	0.4782	9.9862	3.8106	2.6206	1.6188	0.3428
2.96	0.4776	10.0552	3.8200	2.6322	1.6224	0.3398
2.97	0.4770	10.1244	3.8294	2.6439	1.6260	0.3369
2.98	0.4764	10.1938	3.8387	2.6555	1.6296	0.3340
2.99	0.4758	10.2634	3.8479	2.6673	1.6332	0.3312
3.00	0.4752	10.3333	3.8571	2.6790	1.6368	0.3283
3.01	0.4746	10.4034	3.8663	2.6908	1.6404	0.3255
3.02	0.4740	10.4738	3.8754	2.7026	1.6440	0.3227
3.03	0.4734	10.5444	3.8845	2.7145	1.6476	0.3200
3.04	0.4729	10.6152	3.8935	2.7264	1.6512	0.3172
3.05	0.4723	10.6862	3.9025	2.7383	1.6548	0.3145
3.06	0.4717	10.7575	3.9114	2.7503	1.6584	0.3118
3.07	0.4712	10.8290	3.9203	2.7623	1.6620	0.3091
3.08	0.4706	10.9008	3.9291	2.7744	1.6656	0.3065
3.09	0.4701	10.9728	3.9379	2.7865	1.6693	0.3038
3.10	0.4695	11.0450	3.9466	2.7986	1.6729	0.3012
3.11	0.4690	11.1174	3.9553	2.8108	1.6765	0.2986
3.12	0.4685	11.1901	3.9639	2.8230	1.6802	0.2960
3.13	0.4679	11.2630	3.9725	2.8352	1.6838	0.2935
3.14	0.4674	11.3362	3.9811	2.8475	1.6875	0.2910
3.15	0.4669	11.4096	3.9896	2.8598	1.6911	0.2885
3.16	0.4664	11.4832	3.9981	2.8722	1.6948	0.2860

(continued)

**Table A.2** (Continued)

$M_1$	$M_2$	$P_2/P_1$	$\rho_2/\rho_1$	$T_2/T_1$	$a_2/a_1$	$p_{02}/p_{01}$
3.17	0.4659	11.5570	4.0065	2.8846	1.6984	0.2835
3.18	0.4654	11.6311	4.0149	2.8970	1.7021	0.2811
3.19	0.4648	11.7054	4.0232	2.9095	1.7057	0.2786
3.20	0.4643	11.7800	4.0315	2.9220	1.7094	0.2762
3.21	0.4639	11.8548	4.0397	2.9345	1.7131	0.2738
3.22	0.4634	11.9298	4.0479	2.9471	1.7167	0.2715
3.23	0.4629	12.0050	4.0561	2.9598	1.7204	0.2691
3.24	0.4624	12.0805	4.0642	2.9724	1.7241	0.2668
3.25	0.4619	12.1562	4.0723	2.9851	1.7277	0.2645
3.26	0.4614	12.2322	4.0803	2.9979	1.7314	0.2622
3.27	0.4610	12.3084	4.0883	3.0106	1.7351	0.2600
3.28	0.4605	12.3848	4.0963	3.0234	1.7388	0.2577
3.29	0.4600	12.4614	4.1042	3.0363	1.7425	0.2555
3.30	0.4596	12.5383	4.1120	3.0492	1.7462	0.2533
3.31	0.4591	12.6154	4.1198	3.0621	1.7499	0.2511
3.32	0.4587	12.6928	4.1276	3.0751	1.7536	0.2489
3.33	0.4582	12.7704	4.1354	3.0881	1.7573	0.2468
3.34	0.4578	12.8482	4.1431	3.1011	1.7610	0.2446
3.35	0.4573	12.9262	4.1507	3.1142	1.7647	0.2425
3.36	0.4569	13.0045	4.1583	3.1273	1.7684	0.2404
3.37	0.4565	13.0830	4.1659	3.1405	1.7721	0.2383
3.38	0.4560	13.1618	4.1734	3.1537	1.7759	0.2363
3.39	0.4556	13.2408	4.1809	3.1669	1.7796	0.2342
3.40	0.4552	13.3200	4.1884	3.1802	1.7833	0.2322
3.41	0.4548	13.3994	4.1958	3.1935	1.7870	0.2302
3.42	0.4544	13.4791	4.2032	3.2069	1.7908	0.2282
3.43	0.4540	13.5590	4.2105	3.2203	1.7945	0.2263
3.44	0.4535	13.6392	4.2179	3.2337	1.7982	0.2243
3.45	0.4531	13.7196	4.2251	3.2472	1.8020	0.2224
3.46	0.4527	13.8002	4.2323	3.2607	1.8057	0.2205
3.47	0.4523	13.8810	4.2395	3.2742	1.8095	0.2186
3.48	0.4519	13.9621	4.2467	3.2878	1.8132	0.2167
3.49	0.4515	14.0434	4.2538	3.3014	1.8170	0.2148
3.50	0.4512	14.1250	4.2609	3.3150	1.8207	0.2129
3.51	0.4508	14.2068	4.2679	3.3287	1.8245	0.2111
3.52	0.4504	14.2888	4.2749	3.3425	1.8282	0.2093
3.53	0.4500	14.3710	4.2819	3.3562	1.8320	0.2075
3.54	0.4496	14.4535	4.2888	3.3701	1.8358	0.2057
3.55	0.4492	14.5362	4.2957	3.3839	1.8395	0.2039
3.56	0.4489	14.6192	4.3026	3.3978	1.8433	0.2022
3.57	0.4485	14.7024	4.3094	3.4117	1.8471	0.2004

(continued)

**Table A.2** (Continued)

$M_1$	$M_2$	$P_2/P_1$	$\rho_2/\rho_1$	$T_2/T_1$	$a_2/a_1$	$p_{02}/p_{01}$
3.58	0.4481	14.7858	4.3162	3.4257	1.8509	0.1987
3.59	0.4478	14.8694	4.3229	3.4397	1.8546	0.1970
3.60	0.4474	14.9533	4.3296	3.4537	1.8584	0.1953
3.61	0.4471	15.0374	4.3363	3.4678	1.8622	0.1936
3.62	0.4467	15.1218	4.3429	3.4819	1.8660	0.1920
3.63	0.4463	15.2064	4.3496	3.4961	1.8698	0.1903
3.64	0.4460	15.2912	4.3561	3.5103	1.8736	0.1887
3.65	0.4456	15.3762	4.3627	3.5245	1.8774	0.1871
3.66	0.4453	15.4615	4.3692	3.5388	1.8812	0.1855
3.67	0.4450	15.5470	4.3756	3.5531	1.8850	0.1839
3.68	0.4446	15.6328	4.3821	3.5674	1.8888	0.1823
3.69	0.4443	15.7188	4.3885	3.5818	1.8926	0.1807
3.70	0.4439	15.8050	4.3949	3.5962	1.8964	0.1792
3.71	0.4436	15.8914	4.4012	3.6107	1.9002	0.1777
3.72	0.4433	15.9781	4.4075	3.6252	1.9040	0.1761
3.73	0.4430	16.0650	4.4138	3.6397	1.9078	0.1746
3.74	0.4426	16.1522	4.4200	3.6543	1.9116	0.1731
3.75	0.4423	16.2396	4.4262	3.6689	1.9154	0.1717
3.76	0.4420	16.3272	4.4324	3.6836	1.9193	0.1702
3.77	0.4417	16.4150	4.4385	3.6983	1.9231	0.1687
3.78	0.4414	16.5031	4.4447	3.7130	1.9269	0.1673
3.79	0.4410	16.5914	4.4507	3.7278	1.9307	0.1659
3.80	0.4407	16.6800	4.4568	3.7426	1.9346	0.1645
3.81	0.4404	16.7688	4.4628	3.7575	1.9384	0.1631
3.82	0.4401	16.8578	4.4688	3.7723	1.9423	0.1617
3.83	0.4398	16.9470	4.4747	3.7873	1.9461	0.1603
3.84	0.4395	17.0365	4.4807	3.8022	1.9499	0.1589
3.85	0.4392	17.1262	4.4866	3.8172	1.9538	0.1576
3.86	0.4389	17.2162	4.4924	3.8323	1.9576	0.1563
3.87	0.4386	17.3064	4.4983	3.8473	1.9615	0.1549
3.88	0.4383	17.3968	4.5041	3.8625	1.9653	0.1536
3.89	0.4380	17.4874	4.5098	3.8776	1.9692	0.1523
3.90	0.4377	17.5783	4.5156	3.8928	1.9730	0.1510
3.91	0.4375	17.6694	4.5213	3.9080	1.9769	0.1497
3.92	0.4372	17.7608	4.5270	3.9233	1.9807	0.1485
3.93	0.4369	17.8524	4.5326	3.9386	1.9846	0.1472
3.94	0.4366	17.9442	4.5383	3.9540	1.9885	0.1460
3.95	0.4363	18.0362	4.5439	3.9694	1.9923	0.1448
3.96	0.4360	18.1285	4.5494	3.9848	1.9962	0.1435
3.97	0.4358	18.2210	4.5550	4.0003	2.0001	0.1423

(continued)

**Table A.2** (Continued)

$M_1$	$M_2$	$P_2/P_1$	$\rho_2/\rho_1$	$T_2/T_1$	$a_2/a_1$	$p_{02}/p_{01}$
3.98	0.4355	18.3138	4.5605	4.0157	2.0039	0.1411
3.99	0.4352	18.4068	4.5660	4.0313	2.0078	0.1399
4.00	0.4350	18.5000	4.5714	4.0469	2.0117	0.1388
4.01	0.4347	18.5934	4.5769	4.0625	2.0156	0.1376
4.02	0.4344	18.6871	4.5823	4.0781	2.0194	0.1364
4.03	0.4342	18.7810	4.5876	4.0938	2.0233	0.1353
4.04	0.4339	18.8752	4.5930	4.1096	2.0272	0.1342
4.05	0.4336	18.9696	4.5983	4.1253	2.0311	0.1330
4.06	0.4334	19.0642	4.6036	4.1412	2.0350	0.1319
4.07	0.4331	19.1590	4.6089	4.1570	2.0389	0.1308
4.08	0.4329	19.2541	4.6141	4.1729	2.0428	0.1297
4.09	0.4326	19.3494	4.6193	4.1888	2.0467	0.1286
4.10	0.4324	19.4450	4.6245	4.2048	2.0506	0.1276
4.11	0.4321	19.5408	4.6296	4.2208	2.0545	0.1265
4.12	0.4319	19.6368	4.6348	4.2368	2.0584	0.1254
4.13	0.4316	19.7330	4.6399	4.2529	2.0623	0.1244
4.14	0.4314	19.8295	4.6450	4.2690	2.0662	0.1234
4.15	0.4311	19.9262	4.6500	4.2852	2.0701	0.1223
4.16	0.4309	20.0232	4.6550	4.3014	2.0740	0.1213
4.17	0.4306	20.1204	4.6601	4.3176	2.0779	0.1203
4.18	0.4304	20.2178	4.6650	4.3339	2.0818	0.1193
4.19	0.4302	20.3155	4.6700	4.3502	2.0857	0.1183
4.20	0.4299	20.4133	4.6749	4.3666	2.0896	0.1173
4.21	0.4297	20.5115	4.6798	4.3830	2.0936	0.1164
4.22	0.4295	20.6098	4.6847	4.3994	2.0975	0.1154
4.23	0.4292	20.7084	4.6896	4.4159	2.1014	0.1144
4.24	0.4290	20.8072	4.6944	4.4324	2.1053	0.1135
4.25	0.4288	20.9063	4.6992	4.4489	2.1092	0.1126
4.26	0.4286	21.0056	4.7040	4.4655	2.1132	0.1116
4.27	0.4283	21.1051	4.7087	4.4821	2.1171	0.1107
4.28	0.4281	21.2048	4.7135	4.4988	2.1210	0.1098
4.29	0.4279	21.3048	4.7182	4.5155	2.1250	0.1089
4.30	0.4277	21.4050	4.7229	4.5322	2.1289	0.1080
4.31	0.4275	21.5055	4.7275	4.5490	2.1328	0.1071
4.32	0.4272	21.6062	4.7322	4.5658	2.1368	0.1062
4.33	0.4270	21.7071	4.7368	4.5827	2.1407	0.1054
4.34	0.4268	21.8083	4.7414	4.5995	2.1447	0.1045
4.35	0.4266	21.9096	4.7460	4.6165	2.1486	0.1036
4.36	0.4264	22.0113	4.7505	4.6335	2.1525	0.1028
4.37	0.4262	22.1131	4.7550	4.6505	2.1565	0.1020
4.38	0.4260	22.2152	4.7595	4.6675	2.1604	0.1011

(continued)

**Table A.2** (Continued)

$M_1$	$M_2$	$P_2/P_1$	$\rho_2/\rho_1$	$T_2/T_1$	$a_2/a_1$	$p_{02}/p_{01}$
4.39	0.4258	22.3175	4.7640	4.6846	2.1644	0.1003
4.40	0.4255	22.4201	4.7685	4.7017	2.1683	0.0995
4.41	0.4253	22.5229	4.7729	4.7189	2.1723	0.0987
4.42	0.4251	22.6259	4.7773	4.7361	2.1763	0.0979
4.43	0.4249	22.7291	4.7817	4.7533	2.1802	0.0971
4.44	0.4247	22.8326	4.7861	4.7706	2.1842	0.0963
4.45	0.4245	22.9363	4.7904	4.7879	2.1881	0.0955
4.46	0.4243	23.0403	4.7948	4.8053	2.1921	0.0947
4.47	0.4241	23.1445	4.7991	4.8227	2.1961	0.0940
4.48	0.4239	23.2489	4.8034	4.8401	2.2000	0.0932
4.49	0.4237	23.3535	4.8076	4.8576	2.2040	0.0924
4.50	0.4236	23.4584	4.8119	4.8751	2.2080	0.0917
4.51	0.4234	23.5635	4.8161	4.8926	2.2119	0.0910
4.52	0.4232	23.6689	4.8203	4.9102	2.2159	0.0902
4.53	0.4230	23.7745	4.8245	4.9279	2.2199	0.0895
4.54	0.4228	23.8803	4.8287	4.9455	2.2239	0.0888
4.55	0.4226	23.9864	4.8328	4.9632	2.2278	0.0881
4.56	0.4224	24.0926	4.8369	4.9810	2.2318	0.0874
4.57	0.4222	24.1992	4.8410	4.9988	2.2358	0.0867
4.58	0.4220	24.3059	4.8451	5.0166	2.2398	0.0860
4.59	0.4219	24.4129	4.8492	5.0344	2.2438	0.0853
4.60	0.4217	24.5201	4.8532	5.0523	2.2477	0.0846
4.61	0.4215	24.6276	4.8572	5.0703	2.2517	0.0839
4.62	0.4213	24.7353	4.8612	5.0883	2.2557	0.0832
4.63	0.4211	24.8432	4.8652	5.1063	2.2597	0.0826
4.64	0.4210	24.9513	4.8692	5.1243	2.2637	0.0819
4.65	0.4208	25.0597	4.8731	5.1424	2.2677	0.0813
4.66	0.4206	25.1683	4.8771	5.1605	2.2717	0.0806
4.67	0.4204	25.2772	4.8810	5.1787	2.2757	0.0800
4.68	0.4203	25.3863	4.8849	5.1969	2.2797	0.0793
4.69	0.4201	25.4956	4.8887	5.2152	2.2837	0.0787
4.70	0.4199	25.6051	4.8926	5.2335	2.2877	0.0781
4.71	0.4197	25.7149	4.8964	5.2518	2.2917	0.0775
4.72	0.4196	25.8249	4.9002	5.2701	2.2957	0.0769
4.73	0.4194	25.9352	4.9040	5.2885	2.2997	0.0762
4.74	0.4192	26.0457	4.9078	5.3070	2.3037	0.0756
4.75	0.4191	26.1564	4.9116	5.3255	2.3077	0.0750
4.76	0.4189	26.2673	4.9153	5.3440	2.3117	0.0745
4.77	0.4187	26.3785	4.9190	5.3625	2.3157	0.0739
4.78	0.4186	26.4900	4.9227	5.3811	2.3197	0.0733
4.79	0.4184	26.6016	4.9264	5.3998	2.3237	0.0727

(continued)

**Table A.2** (Continued)

$M_1$	$M_2$	$P_2/P_1$	$\rho_2/\rho_1$	$T_2/T_1$	$a_2/a_1$	$p_{02}/p_{01}$
4.80	0.4183	26.7135	4.9301	5.4184	2.3277	0.0721
4.81	0.4181	26.8256	4.9338	5.4372	2.3318	0.0716
4.82	0.4179	26.9380	4.9374	5.4559	2.3358	0.0710
4.83	0.4178	27.0505	4.9410	5.4747	2.3398	0.0705
4.84	0.4176	27.1634	4.9446	5.4935	2.3438	0.0699
4.85	0.4175	27.2764	4.9482	5.5124	2.3478	0.0694
4.86	0.4173	27.3897	4.9518	5.5313	2.3519	0.0688
4.87	0.4172	27.5032	4.9553	5.5502	2.3559	0.0683
4.88	0.4170	27.6170	4.9589	5.5692	2.3599	0.0677
4.89	0.4169	27.7310	4.9624	5.5882	2.3639	0.0672
4.90	0.4167	27.8452	4.9659	5.6073	2.3680	0.0667
4.91	0.4165	27.9596	4.9694	5.6264	2.3720	0.0662
4.92	0.4164	28.0743	4.9728	5.6455	2.3760	0.0657
4.93	0.4162	28.1893	4.9763	5.6647	2.3801	0.0652
4.94	0.4161	28.3044	4.9797	5.6839	2.3841	0.0647
4.95	0.4160	28.4198	4.9831	5.7032	2.3881	0.0642
4.96	0.4158	28.5354	4.9865	5.7225	2.3922	0.0637
4.97	0.4157	28.6513	4.9899	5.7418	2.3962	0.0632
4.98	0.4155	28.7673	4.9933	5.7612	2.4002	0.0627
4.99	0.4154	28.8837	4.9967	5.7806	2.4043	0.0622
5.00	0.4152	29.0002	5.0000	5.8000	2.4083	0.0617

**Table A.3** Oblique shock in perfect gas ( $\gamma = 1.4$ ).

$M_1$	Weak solution			Strong solution			
	$\theta$	$\beta$	$p_2/p_1$	$M_2$	$\beta$	$p_2/p_1$	$M_2$
1.05	0	72.07	0.998	1.052	89.66	1.120	0.953
1.10	0	65.28	0.998	1.101	89.83	1.245	0.912
1.10	1	69.80	1.077	1.039	83.57	1.227	0.925
1.15	0	60.34	0.998	1.151	89.89	1.376	0.875
1.15	1	63.16	1.062	1.102	85.98	1.369	0.880
1.15	2	67.00	1.141	1.043	81.17	1.340	0.901
1.20	0	56.39	0.998	1.201	89.92	1.513	0.842
1.20	1	58.55	1.056	1.158	87.04	1.509	0.845
1.20	2	61.05	1.120	1.111	83.86	1.494	0.855
1.20	3	64.34	1.198	1.056	80.03	1.463	0.876
1.25	0	53.08	0.999	1.251	89.94	1.656	0.813
1.25	1	54.88	1.053	1.211	87.65	1.653	0.815
1.25	2	56.85	1.111	1.170	85.21	1.644	0.821

(continued)

**Table A.3** (Continued)

Weak solution				Strong solution			
$M_1$	$\theta$	$\beta$	$p_2/p_1$	$M_2$	$\beta$	$p_2/p_1$	$M_2$
1.25	3	59.13	1.176	1.124	82.55	1.626	0.832
1.25	4	61.99	1.254	1.072	79.39	1.594	0.852
1.25	5	66.50	1.366	0.999	74.64	1.528	0.895
1.30	0	50.24	0.999	1.301	89.95	1.805	0.786
1.30	1	51.81	1.051	1.263	88.05	1.803	0.787
1.30	2	53.47	1.106	1.224	86.06	1.796	0.792
1.30	3	55.32	1.167	1.184	83.95	1.783	0.800
1.30	4	57.42	1.233	1.140	81.65	1.763	0.812
1.30	5	59.96	1.311	1.090	78.97	1.733	0.831
1.30	6	63.46	1.411	1.027	75.37	1.679	0.864
1.35	0	47.76	0.999	1.351	89.96	1.960	0.762
1.35	1	49.17	1.050	1.314	88.34	1.958	0.763
1.35	2	50.63	1.104	1.277	86.65	1.952	0.766
1.35	3	52.22	1.162	1.239	84.89	1.943	0.772
1.35	4	53.97	1.224	1.199	83.03	1.928	0.781
1.35	5	55.93	1.292	1.157	80.99	1.907	0.793
1.35	6	58.23	1.370	1.109	78.66	1.877	0.811
1.35	7	61.18	1.465	1.052	75.72	1.830	0.839
1.35	8	66.91	1.632	0.954	70.02	1.711	0.909
1.40	0	45.55	0.999	1.401	89.96	2.120	0.740
1.40	1	46.84	1.050	1.365	88.55	2.119	0.741
1.40	2	48.17	1.103	1.329	87.08	2.114	0.743
1.40	3	49.59	1.159	1.293	85.57	2.106	0.748
1.40	4	51.12	1.219	1.255	83.99	2.095	0.754
1.40	5	52.78	1.283	1.216	82.31	2.079	0.764
1.40	6	54.63	1.354	1.174	80.49	2.057	0.776
1.40	7	56.76	1.433	1.128	78.41	2.028	0.793
1.40	8	59.37	1.526	1.074	75.89	1.984	0.818
1.40	9	63.18	1.655	1.003	72.19	1.906	0.863
1.45	0	43.57	0.999	1.451	89.97	2.286	0.720
1.45	1	44.77	1.050	1.416	88.71	2.285	0.720
1.45	2	46.00	1.103	1.381	87.41	2.281	0.722
1.45	3	47.30	1.158	1.345	86.08	2.275	0.726
1.45	4	48.68	1.217	1.309	84.70	2.265	0.732
1.45	5	50.16	1.279	1.272	83.27	2.252	0.739
1.45	6	51.76	1.346	1.232	81.73	2.236	0.749
1.45	7	53.52	1.419	1.191	80.07	2.213	0.761
1.45	8	55.52	1.500	1.146	78.20	2.184	0.778
1.45	9	57.89	1.593	1.095	75.98	2.142	0.801

(continued)

**Table A.3** (Continued)

$M_1$	Weak solution			Strong solution			
	$\theta$	$\beta$	$p_2/p_1$	$M_2$	$\beta$	$p_2/p_1$	$M_2$
1.45	10	61.05	1.711	1.032	72.99	2.076	0.837
1.50	0	41.78	0.999	1.501	89.97	2.458	0.701
1.50	1	42.91	1.050	1.466	88.84	2.457	0.702
1.50	2	44.07	1.103	1.432	87.67	2.454	0.704
1.50	3	45.27	1.158	1.397	86.48	2.448	0.707
1.50	4	46.54	1.216	1.362	85.26	2.440	0.711
1.50	5	47.89	1.278	1.325	83.99	2.430	0.717
1.50	6	49.33	1.343	1.288	82.66	2.415	0.725
1.50	7	50.88	1.413	1.249	81.25	2.398	0.735
1.50	8	52.57	1.489	1.208	79.71	2.375	0.748
1.50	9	54.47	1.572	1.164	78.00	2.345	0.764
1.50	10	56.68	1.666	1.114	76.00	2.305	0.785
1.50	11	59.46	1.781	1.055	73.44	2.245	0.817
1.50	12	64.35	1.967	0.961	68.79	2.115	0.885
1.55	0	40.15	0.999	1.551	89.97	2.636	0.684
1.55	1	41.23	1.051	1.516	88.95	2.635	0.685
1.55	2	42.32	1.104	1.482	87.88	2.632	0.686
1.55	3	43.45	1.159	1.448	86.80	2.628	0.689
1.55	4	44.64	1.217	1.413	85.70	2.620	0.693
1.55	5	45.89	1.278	1.378	84.57	2.611	0.698
1.55	6	47.21	1.343	1.341	83.39	2.599	0.705
1.55	7	48.62	1.411	1.304	82.15	2.584	0.713
1.55	8	50.13	1.484	1.265	80.83	2.565	0.723
1.55	9	51.77	1.563	1.224	79.40	2.541	0.736
1.55	10	53.60	1.649	1.180	77.81	2.511	0.752
1.55	11	55.69	1.746	1.132	75.97	2.471	0.772
1.55	12	58.24	1.860	1.076	73.69	2.415	0.801
1.55	13	61.98	2.018	0.999	70.24	2.316	0.852
1.60	0	38.66	0.999	1.601	89.97	2.820	0.668
1.60	1	39.69	1.051	1.566	89.03	2.819	0.669
1.60	2	40.72	1.105	1.532	88.06	2.817	0.670
1.60	3	41.81	1.160	1.498	87.07	2.812	0.673
1.60	4	42.93	1.219	1.464	86.06	2.806	0.676
1.60	5	44.11	1.280	1.429	85.03	2.798	0.681
1.60	6	45.34	1.345	1.393	83.97	2.787	0.686
1.60	7	46.65	1.412	1.357	82.86	2.774	0.693
1.60	8	48.03	1.484	1.320	81.69	2.758	0.702
1.60	9	49.51	1.561	1.281	80.45	2.738	0.712
1.60	10	51.12	1.643	1.240	79.10	2.713	0.725

(continued)

**Table A.3** (Continued)

Weak solution				Strong solution			
$M_1$	$\theta$	$\beta$	$p_2/p_1$	$M_2$	$\beta$	$p_2/p_1$	$M_2$
1.60	11	52.88	1.732	1.196	77.61	2.682	0.741
1.60	12	54.89	1.832	1.148	75.90	2.643	0.761
1.60	13	57.28	1.947	1.094	73.82	2.588	0.789
1.60	14	60.54	2.097	1.023	70.90	2.500	0.832
1.65	0	37.28	0.999	1.651	89.98	3.010	0.654
1.65	1	38.27	1.052	1.616	89.10	3.009	0.654
1.65	2	39.27	1.106	1.582	88.20	3.006	0.656
1.65	3	40.30	1.162	1.548	87.29	3.002	0.658
1.65	4	41.38	1.221	1.514	86.37	2.997	0.661
1.65	5	42.50	1.283	1.479	85.42	2.989	0.665
1.65	6	43.67	1.347	1.444	84.45	2.980	0.670
1.65	7	44.89	1.415	1.409	83.44	2.968	0.676
1.65	8	46.18	1.487	1.372	82.39	2.954	0.683
1.65	9	47.55	1.562	1.334	81.28	2.937	0.692
1.65	10	49.01	1.643	1.295	80.10	2.916	0.703
1.65	11	50.58	1.729	1.254	78.83	2.890	0.716
1.65	12	52.31	1.822	1.210	77.41	2.859	0.732
1.65	13	54.26	1.926	1.163	75.80	2.818	0.752
1.65	14	56.54	2.044	1.109	73.86	2.764	0.778
1.65	15	59.52	2.192	1.042	71.25	2.681	0.818
1.70	0	36.01	0.999	1.701	89.98	3.205	0.641
1.70	1	36.96	1.052	1.666	89.17	3.204	0.641
1.70	2	37.93	1.107	1.632	88.33	3.202	0.642
1.70	3	38.92	1.164	1.598	87.48	3.198	0.644
1.70	4	39.96	1.224	1.564	86.62	3.193	0.647
1.70	5	41.03	1.286	1.529	85.75	3.186	0.650
1.70	6	42.15	1.351	1.495	84.85	3.178	0.655
1.70	7	43.31	1.420	1.459	83.93	3.167	0.660
1.70	8	44.53	1.491	1.423	82.97	3.154	0.667
1.70	9	45.81	1.567	1.386	81.97	3.139	0.675
1.70	10	47.17	1.647	1.348	80.91	3.121	0.684
1.70	11	48.61	1.731	1.309	79.78	3.099	0.695
1.70	12	50.17	1.822	1.267	78.56	3.072	0.708
1.70	13	51.87	1.919	1.233	77.21	3.040	0.724
1.70	14	53.77	2.027	1.176	75.67	2.998	0.744
1.70	15	55.98	2.150	1.122	73.84	2.944	0.770
1.70	16	58.79	2.300	1.057	71.43	2.863	0.808
1.70	17	64.61	2.585	0.933	65.99	2.647	0.905
1.75	0	34.83	0.999	1.751	89.98	3.406	0.628

(continued)

**Table A.3** (Continued)

Weak solution				Strong solution			
$M_1$	$\theta$	$\beta$	$p_2/p_1$	$M_2$	$\beta$	$p_2/p_1$	$M_2$
1.75	1	35.76	1.053	1.716	89.22	3.406	0.628
1.75	2	36.69	1.109	1.682	88.43	3.404	0.629
1.75	3	37.65	1.167	1.648	87.64	3.400	0.631
1.75	4	38.65	1.227	1.613	86.84	3.395	0.634
1.75	5	39.68	1.290	1.579	86.03	3.389	0.637
1.75	6	40.76	1.356	1.544	85.19	3.381	0.641
1.75	7	41.87	1.425	1.509	84.34	3.361	0.646
1.75	8	43.04	1.497	1.473	83.45	3.360	0.652
1.75	9	44.25	1.573	1.437	82.53	3.349	0.659
1.75	10	45.53	1.653	1.400	81.57	3.329	0.667
1.75	11	46.88	1.737	1.361	80.55	3.310	0.677
1.75	12	48.32	1.826	1.321	79.47	3.287	0.688
1.75	13	49.86	1.922	1.279	78.29	3.259	0.701
1.75	14	51.55	2.024	1.235	76.99	3.225	0.718
1.75	15	53.42	2.137	1.187	75.51	3.183	0.738
1.75	16	55.59	2.265	1.133	73.76	3.127	0.763
1.75	17	58.30	2.419	1.068	71.48	3.046	0.800
1.75	18	62.94	2.667	0.965	67.27	2.873	0.877
1.80	0	33.73	0.998	1.801	89.98	3.613	0.617
1.80	1	34.63	1.054	1.766	89.27	3.613	0.617
1.80	2	35.54	1.110	1.731	88.53	3.611	0.618
1.80	3	36.48	1.169	1.697	87.78	3.608	0.619
1.80	4	37.44	1.231	1.662	87.03	3.603	0.622
1.80	5	38.44	1.295	1.628	86.27	3.597	0.625
1.80	6	39.48	1.361	1.593	85.49	3.590	0.628
1.80	7	40.56	1.431	1.558	84.69	3.581	0.633
1.80	8	41.67	1.504	1.523	83.87	3.570	0.638
1.80	9	42.84	1.581	1.486	83.02	3.557	0.644
1.80	10	44.06	1.661	1.449	82.13	3.542	0.652
1.80	11	45.34	1.745	1.412	81.20	3.525	0.660
1.80	12	46.69	1.834	1.373	80.22	3.504	0.670
1.80	13	48.12	1.929	1.332	79.16	3.480	0.682
1.80	14	49.66	2.029	1.290	78.02	3.450	0.696
1.80	15	51.34	2.138	1.245	76.76	3.415	0.712
1.80	16	53.20	2.257	1.196	75.33	3.371	0.733
1.80	17	55.34	2.391	1.142	73.62	3.313	0.759
1.80	18	57.99	2.551	1.077	71.42	3.230	0.796
1.80	19	62.30	2.797	0.977	67.58	3.063	0.867
1.85	0	32.70	0.998	1.851	89.98	3.826	0.606

(continued)

**Table A.3** (Continued)

Weak solution				Strong solution			
$M_1$	$\theta$	$\beta$	$p_2/p_1$	$M_2$	$\beta$	$p_2/p_1$	$M_2$
1.85	1	33.58	1.055	1.815	89.31	3.826	0.606
1.85	2	34.47	1.112	1.781	88.61	3.824	0.607
1.85	3	35.38	1.172	1.746	87.91	3.821	0.608
1.85	4	36.32	1.234	1.711	87.20	3.817	0.610
1.85	5	37.30	1.299	1.677	86.48	3.811	0.613
1.85	6	38.30	1.367	1.642	85.74	3.704	0.617
1.85	7	39.34	1.438	1.607	84.99	3.796	0.621
1.85	8	40.42	1.512	1.571	84.22	3.786	0.626
1.85	9	41.55	1.590	1.535	83.43	3.774	0.631
1.85	10	42.72	1.671	1.498	82.61	3.760	0.638
1.85	11	43.94	1.756	1.461	81.75	3.744	0.646
1.85	12	45.22	1.845	1.422	80.85	3.725	0.655
1.85	13	46.58	1.940	1.383	79.89	3.703	0.665
1.85	14	48.00	2.039	1.341	78.86	3.677	0.677
1.85	15	49.56	2.146	1.298	77.75	3.646	0.692
1.85	16	51.23	2.261	1.252	76.51	3.609	0.709
1.85	17	53.09	2.386	1.203	75.11	3.562	0.729
1.85	18	55.23	2.527	1.148	73.44	3.502	0.756
1.85	19	67.87	2.696	1.082	71.28	3.415	0.793
1.85	20	62.10	2.952	0.982	67.54	3.244	0.865
1.90	0	31.73	0.998	1.901	89.98	4.045	0.596
1.90	1	32.60	1.056	1.865	89.34	4.044	0.596
1.90	2	33.47	1.114	1.830	88.68	4.043	0.597
1.90	3	34.36	1.175	1.995	88.01	4.040	0.598
1.90	4	35.28	1.238	1.760	87.34	4.036	0.600
1.90	5	36.23	1.304	1.725	86.66	4.031	0.603
1.90	6	37.21	1.373	1.690	85.97	4.024	0.606
1.90	7	38.22	1.446	1.655	85.26	4.016	0.610
1.90	8	39.27	1.521	1.619	84.54	4.007	0.614
1.90	9	30.39	1.600	1.583	83.79	3.996	0.620
1.90	10	41.49	1.682	1.546	83.02	3.983	0.626
1.90	11	42.67	1.768	1.509	82.22	3.968	0.633
1.90	12	43.90	1.858	1.471	81.38	3.950	0.641
1.90	13	45.19	1.953	1.432	80.50	3.930	0.650
1.90	14	46.55	2.053	1.391	79.57	3.907	0.661
1.90	15	48.00	2.159	1.349	78.56	3.879	0.674
1.90	16	49.54	2.272	1.305	77.47	3.847	0.688
1.90	17	51.23	2.393	1.258	76.25	3.807	0.706
1.90	18	53.10	2.526	1.208	74.86	3.758	0.727

(continued)

**Table A.3** (Continued)

Weak solution				Strong solution			
$M_1$	$\theta$	$\beta$	$p_2/p_1$	$M_2$	$\beta$	$p_2/p_1$	$M_2$
1.90	19	55.24	2.676	1.151	73.21	3.693	0.755
1.90	20	57.90	2.856	1.084	71.06	3.601	0.794
1.90	21	62.25	3.132	0.979	67.22	3.414	0.869
1.95	0	30.83	0.998	1.951	89.98	4.270	0.586
1.95	1	31.68	1.057	1.914	89.37	4.269	0.586
1.95	2	32.53	1.116	1.879	88.74	4.267	0.587
1.95	3	33.40	1.178	1.844	88.11	4.265	0.589
1.95	4	34.30	1.242	1.809	87.47	4.261	0.590
1.95	5	35.23	1.310	1.773	86.82	4.256	0.593
1.95	6	36.19	1.380	1.738	86.17	4.250	0.596
1.95	7	37.18	1.454	1.702	85.50	4.242	0.599
1.95	8	38.20	1.530	1.667	84.81	4.233	0.604
1.95	9	39.26	1.610	1.630	84.11	4.223	0.609
1.95	10	40.36	1.694	1.594	83.38	4.211	0.614
1.95	11	41.50	1.781	1.557	82.63	4.197	0.621
1.95	12	42.69	1.873	1.519	81.85	4.180	0.628
1.95	13	43.93	1.968	1.480	81.03	4.162	0.637
1.95	14	45.23	2.069	1.440	80.17	4.140	0.647
1.95	15	46.60	2.175	1.398	79.25	4.115	0.658
1.95	16	48.06	2.288	1.355	78.25	4.086	0.671
1.95	17	49.62	2.408	1.310	77.17	4.051	0.686
1.95	18	51.32	2.537	1.262	75.97	4.009	0.705
1.95	19	53.21	2.678	1.210	74.58	3.956	0.727
1.95	20	55.38	2.838	1.152	72.93	3.887	0.756
1.95	21	58.10	3.030	1.082	70.74	3.787	0.796
1.95	22	62.85	3.346	0.966	66.52	3.565	0.883
2.00	0	29.98	0.998	2.001	89.99	4.500	0.577
2.00	1	30.81	1.058	1.964	89.40	4.499	0.578
2.00	2	31.65	1.118	1.928	88.80	4.498	0.578
2.00	3	32.51	1.181	1.892	88.19	4.495	0.580
2.00	4	33.39	1.247	1.857	87.58	4.492	0.581
2.00	5	34.30	1.315	1.821	86.97	4.487	0.584
2.00	6	35.24	1.387	1.786	86.34	4.481	0.586
2.00	7	36.21	1.462	1.750	85.70	4.474	0.590
2.00	8	37.21	1.540	1.714	85.05	4.465	0.594
2.00	9	38.24	1.621	1.677	84.39	4.455	0.598
2.00	10	39.31	1.707	1.641	83.70	4.444	0.604
2.00	11	40.42	1.795	1.603	82.99	4.431	0.610
2.00	12	41.58	1.888	1.565	82.26	4.415	0.617

(continued)

**Table A.3** (Continued)

Weak solution				Strong solution			
$M_1$	$\theta$	$\beta$	$p_2/p_1$	$M_2$	$\beta$	$p_2/p_1$	$M_2$
2.00	13	42.78	1.986	1.526	81.49	4.398	0.625
2.00	14	44.03	2.087	1.487	80.69	4.378	0.634
2.00	15	45.34	2.195	1.446	79.83	4.355	0.644
2.00	16	46.73	2.307	1.403	78.92	4.328	0.656
2.00	17	48.20	2.427	1.359	77.94	4.296	0.669
2.00	18	49.79	2.554	1.313	76.86	4.259	0.685
2.00	19	51.51	2.692	1.264	75.66	4.214	0.704
2.00	20	53.42	2.843	1.210	74.27	4.157	0.728
2.00	21	55.64	3.014	1.150	72.59	4.082	0.758
2.00	22	58.46	3.223	1.076	70.33	3.971	0.802
2.10	0	28.42	0.998	2.101	89.99	4.978	0.561
2.10	1	29.22	1.060	2.063	89.45	4.978	0.561
2.10	2	30.03	1.122	2.026	88.90	4.976	0.562
2.10	3	30.87	1.187	1.989	88.34	4.974	0.563
2.10	4	31.72	1.256	1.953	87.78	4.971	0.565
2.10	5	32.61	1.327	1.917	87.21	4.966	0.567
2.10	6	33.51	1.402	1.880	86.64	4.961	0.569
2.10	7	34.45	1.480	1.844	86.06	4.954	0.572
2.10	8	35.41	1.561	1.807	85.47	4.946	0.576
2.10	9	36.41	1.646	1.770	84.86	4.937	0.580
2.10	10	37.43	1.734	1.733	84.24	4.926	0.585
2.10	11	38.49	1.827	1.695	83.60	4.914	0.590
2.10	12	39.59	1.923	1.656	82.94	4.901	0.596
2.10	13	40.73	2.024	1.617	82.26	4.885	0.603
2.10	14	41.91	2.129	1.578	81.54	4.867	0.611
2.10	15	43.14	2.239	1.537	80.79	4.847	0.620
2.10	16	44.43	2.355	1.495	80.00	4.823	0.630
2.10	17	45.78	2.476	1.452	79.16	4.796	0.641
2.10	18	47.21	2.604	1.408	78.26	4.765	0.654
2.10	19	48.73	2.740	1.361	77.28	4.729	0.669
2.10	20	50.36	2.885	1.312	76.19	4.685	0.687
2.10	21	52.16	3.042	1.260	74.96	4.632	0.708
2.10	22	54.17	3.215	1.202	73.52	4.564	0.735
2.10	23	56.55	3.415	1.136	71.72	4.472	0.770
2.10	24	59.77	3.674	1.049	69.10	4.324	0.824
2.20	0	27.01	0.998	2.201	89.99	5.480	0.547
2.20	1	27.80	1.062	2.162	89.49	5.480	0.547
2.20	2	28.59	1.127	2.124	88.98	5.478	0.548
2.20	3	29.40	1.194	2.086	88.46	5.476	0.549

(continued)

**Table A.3** (Continued)

Weak solution				Strong solution			
$M_1$	$\theta$	$\beta$	$p_2/p_1$	$M_2$	$\beta$	$p_2/p_1$	$M_2$
2.20	4	30.24	1.265	2.049	87.94	5.473	0.550
2.20	5	31.10	1.340	2.011	87.42	5.468	0.552
2.20	6	31.98	1.417	1.974	86.89	5.463	0.554
2.20	7	32.89	1.498	1.936	86.35	5.457	0.557
2.20	8	33.83	1.583	1.899	85.80	5.450	0.561
2.20	9	34.79	1.672	1.861	85.24	5.441	0.564
2.20	10	35.79	1.764	1.823	84.67	5.431	0.569
2.20	11	36.81	1.860	1.784	84.09	5.420	0.573
2.20	12	37.87	1.961	1.745	83.48	5.407	0.579
2.20	13	38.96	2.066	1.706	82.86	5.393	0.585
2.20	14	40.10	2.176	1.666	82.22	5.376	0.592
2.20	15	41.27	2.290	1.625	81.55	5.358	0.600
2.20	16	42.29	2.409	1.583	80.84	5.337	0.609
2.20	17	43.76	2.535	1.540	80.10	5.313	0.618
2.20	18	45.09	2.666	1.496	79.31	5.286	0.630
2.20	19	46.49	2.804	1.451	78.47	5.254	0.642
2.20	20	47.98	2.949	1.404	77.55	5.217	0.657
2.20	21	49.56	3.104	1.354	76.55	5.174	0.674
2.20	22	51.28	3.270	1.301	75.42	5.122	0.694
2.20	23	53.18	3.451	1.244	74.13	5.057	0.718
2.20	24	55.36	3.655	1.181	72.56	4.973	0.749
2.20	25	58.05	3.899	1.104	70.49	4.850	0.793
2.20	26	62.69	4.291	0.980	66.48	4.581	0.885
2.30	0	25.75	0.998	2.301	89.99	6.005	0.534
2.30	1	26.52	1.064	2.260	89.52	6.005	0.535
2.30	2	27.29	1.131	2.221	89.04	6.003	0.535
2.30	3	28.09	1.201	2.182	88.56	6.001	0.536
2.30	4	28.91	1.275	2.144	88.07	5.998	0.537
2.30	5	29.75	1.353	2.105	87.58	5.994	0.599
2.30	6	30.61	1.434	2.067	87.09	5.989	0.541
2.30	7	31.50	1.518	2.028	86.59	5.983	0.544
2.30	8	32.42	1.607	1.990	86.08	5.976	0.547
2.30	9	33.36	1.699	1.951	85.56	5.968	0.550
2.30	10	34.33	1.796	1.912	85.03	5.959	0.554
2.30	11	35.33	1.897	1.872	84.49	5.948	0.559
2.30	12	36.35	2.002	1.833	83.93	5.936	0.564
2.30	13	37.42	2.112	1.792	83.36	5.922	0.569
2.30	14	38.51	2.226	1.751	82.77	5.907	0.576
2.30	15	39.64	2.345	1.710	82.15	5.890	0.583

(continued)

**Table A.3** (Continued)

Weak solution				Strong solution			
$M_1$	$\theta$	$\beta$	$p_2/p_1$	$M_2$	$\beta$	$p_2/p_1$	$M_2$
2.30	16	40.82	2.470	1.668	81.51	5.870	0.591
2.30	17	42.03	2.600	1.625	80.84	5.849	0.599
2.30	18	43.30	2.736	1.580	80.14	5.824	0.609
2.30	19	44.62	2.878	1.535	79.39	5.796	0.620
2.30	20	46.01	3.028	1.488	78.58	5.763	0.633
2.30	21	47.47	3.185	1.440	77.72	5.726	0.647
2.30	22	49.03	3.351	1.389	76.77	5.682	0.663
2.30	23	50.70	3.529	1.336	75.72	5.629	0.683
2.30	24	52.54	3.721	1.279	74.51	5.565	0.706
2.30	25	54.61	3.934	1.216	73.09	5.482	0.735
2.30	26	57.08	4.182	1.143	71.27	5.368	0.774
2.30	27	60.55	4.513	1.044	68.46	5.173	0.839
2.40	0	24.60	0.998	2.401	89.99	6.553	0.523
2.40	1	25.36	1.066	2.359	89.55	6.553	0.523
2.40	2	26.12	1.136	2.318	89.10	6.552	0.524
2.40	3	26.90	1.209	2.278	88.64	6.550	0.525
2.40	4	27.70	1.286	2.238	88.18	6.547	0.526
2.40	5	28.53	1.366	2.199	87.72	6.543	0.528
2.40	6	29.38	1.450	2.159	87.26	6.538	0.530
2.40	7	30.25	1.539	2.119	86.79	6.532	0.532
2.40	8	31.15	1.631	2.079	86.31	6.525	0.535
2.40	9	32.07	1.728	2.040	85.82	6.518	0.538
2.40	10	33.02	1.829	1.999	85.33	6.509	0.542
2.40	11	34.00	1.935	1.959	84.82	6.499	0.546
2.40	12	35.01	2.045	1.918	84.30	6.487	0.550
2.40	13	36.04	2.160	1.877	83.77	6.474	0.556
2.40	14	37.11	2.280	1.835	83.22	6.460	0.561
2.40	15	38.21	2.405	1.793	82.65	6.443	0.568
2.40	16	39.35	2.535	1.750	82.06	6.425	0.575
2.40	17	40.53	2.671	1.706	81.45	6.405	0.583
2.40	18	41.75	2.813	1.661	80.80	6.382	0.592
2.40	19	43.02	2.961	1.616	80.12	6.356	0.602
2.40	20	44.34	3.115	1.569	79.40	6.326	0.613
2.40	21	45.72	3.277	1.521	78.63	6.292	0.625
2.40	22	47.17	3.448	1.471	77.80	6.253	0.640
2.40	23	48.72	3.628	1.419	76.90	6.208	0.656
2.40	24	50.37	3.819	1.364	75.89	6.154	0.675
2.40	25	52.17	4.026	1.306	74.75	6.088	0.698
2.40	26	54.18	4.252	1.243	73.40	6.005	0.726

(continued)

**Table A.3** (Continued)

Weak solution				Strong solution			
$M_1$	$\theta$	$\beta$	$p_2/p_1$	$M_2$	$\beta$	$p_2/p_1$	$M_2$
2.40	27	56.54	4.511	1.170	71.72	5.892	0.763
2.40	28	59.65	4.838	1.078	69.29	5.713	0.820
2.50	0	23.56	0.998	2.501	89.99	7.125	0.513
2.50	1	24.30	1.068	2.457	89.57	7.125	0.513
2.50	2	25.05	1.140	2.415	89.14	7.123	0.514
2.50	3	25.82	1.216	2.374	88.71	7.121	0.514
2.50	4	26.61	1.296	2.333	88.28	7.118	0.516
2.50	5	27.42	1.380	2.292	87.84	7.115	0.517
2.50	6	28.26	1.468	2.251	87.40	7.110	0.519
2.50	7	29.12	1.560	2.210	86.96	7.104	0.521
2.50	8	30.01	1.657	2.169	86.50	7.098	0.524
2.50	9	30.92	1.758	2.127	86.05	7.090	0.527
2.50	10	31.85	1.864	2.086	85.58	7.082	0.530
2.50	11	32.81	1.974	2.044	85.10	7.072	0.534
2.50	12	33.80	2.090	2.002	84.61	7.061	0.539
2.50	13	34.82	2.211	1.960	84.11	7.048	0.544
2.50	14	35.87	2.336	1.917	83.60	7.034	0.549
2.50	15	36.95	2.467	1.874	83.07	7.019	0.555
2.50	16	38.06	2.604	1.830	82.52	7.001	0.562
2.50	17	39.20	2.746	1.785	81.95	6.982	0.569
2.50	18	40.39	2.895	1.739	81.35	6.960	0.577
2.50	19	41.62	3.049	1.693	80.73	6.936	0.586
2.50	20	42.89	3.211	1.646	80.07	6.908	0.596
2.50	21	44.22	3.379	1.597	79.37	6.877	0.607
2.50	22	45.60	3.556	1.548	78.63	6.841	0.620
2.50	23	47.06	3.741	1.496	77.82	6.800	0.634
2.50	24	48.60	3.936	1.443	76.94	6.753	0.651
2.50	25	50.25	4.143	1.387	75.96	6.696	0.670
2.50	26	52.04	4.365	1.327	74.86	6.627	0.693
2.50	27	54.02	4.609	1.262	73.56	6.541	0.721
2.50	28	56.33	4.884	1.189	71.95	6.425	0.757
2.50	29	59.31	5.225	1.098	69.68	6.246	0.812
2.60	0	22.60	0.998	2.601	89.99	7.720	0.504
2.60	1	23.34	1.071	2.556	89.59	7.720	0.504
2.60	2	24.07	1.145	2.512	89.18	7.718	0.505
2.60	3	24.83	1.224	2.469	88.77	7.716	0.505
2.60	4	25.61	1.307	2.427	88.36	7.714	0.506
2.60	5	26.42	1.394	2.384	87.95	7.710	0.508
2.60	6	27.24	1.486	2.342	87.53	7.705	0.510

(continued)

**Table A.3** (Continued)

$M_1$	Weak solution			Strong solution			
	$\theta$	$\beta$	$p_2/p_1$	$M_2$	$\beta$	$p_2/p_1$	$M_2$
2.60	7	28.09	1.582	2.299	87.10	7.700	0.512
2.60	8	28.97	1.683	2.257	86.67	7.693	0.514
2.60	9	29.87	1.789	2.214	86.24	7.686	0.517
2.60	10	30.79	1.900	2.172	85.79	7.678	0.520
2.60	11	31.74	2.016	2.129	85.34	7.668	0.524
2.60	12	32.71	2.137	2.085	84.88	7.657	0.528
2.60	13	33.72	2.263	2.041	84.41	7.645	0.533
2.60	14	34.75	2.395	1.997	83.92	7.632	0.538
2.60	15	35.81	2.533	1.953	83.42	7.616	0.543
2.60	16	36.90	2.677	1.908	82.91	7.600	0.550
2.60	17	38.03	2.826	1.862	82.37	7.581	0.557
2.60	18	39.19	2.982	1.815	81.82	7.560	0.564
2.60	19	40.38	3.144	1.768	81.24	7.537	0.572
2.60	20	41.62	3.313	1.720	80.63	7.511	0.582
2.60	21	42.91	3.489	1.671	79.98	7.481	0.592
2.60	22	44.24	3.672	1.621	79.30	7.448	0.604
2.60	23	45.64	3.864	1.569	78.57	7.410	0.616
2.60	24	47.10	4.066	1.516	77.78	7.367	0.631
2.60	25	48.65	4.278	1.460	76.92	7.316	0.648
2.60	26	50.31	4.503	1.403	75.96	7.255	0.667
2.60	27	52.10	4.744	1.341	74.87	7.182	0.690
2.60	28	54.09	5.007	1.274	73.59	7.091	0.719
2.60	29	56.39	5.304	1.199	72.01	6.967	0.756
2.60	30	59.35	5.670	1.106	69.78	6.778	0.811
2.70	0	21.72	0.998	2.701	89.99	8.338	0.496
2.70	1	22.44	1.073	2.654	89.61	8.338	0.496
2.70	2	23.17	1.150	2.609	89.22	8.337	0.496
2.70	3	23.92	1.232	2.564	88.83	8.335	0.497
2.70	4	24.70	1.318	2.520	88.43	8.332	0.498
2.70	5	25.49	1.409	2.476	88.03	8.328	0.499
2.70	6	26.31	1.504	2.432	87.63	8.324	0.501
2.70	7	27.15	1.605	2.388	87.23	8.318	0.503
2.70	8	28.02	1.710	2.344	86.82	8.312	0.506
2.70	9	28.91	1.821	2.300	86.40	8.305	0.508
2.70	10	29.82	1.937	2.256	85.98	8.296	0.511
2.70	11	30.76	2.058	2.212	85.55	8.287	0.515
2.70	12	31.73	2.185	2.167	85.11	8.276	0.519
2.70	13	32.72	2.318	2.122	84.66	8.265	0.523
2.70	14	33.74	2.457	2.076	84.20	8.251	0.528

(continued)

**Table A.3** (Continued)

Weak solution				Strong solution			
$M_1$	$\theta$	$\beta$	$p_2/p_1$	$M_2$	$\beta$	$p_2/p_1$	$M_2$
2.70	15	34.79	2.601	2.030	83.73	8.237	0.533
2.70	16	35.86	2.752	1.984	83.24	8.220	0.539
2.70	17	36.97	2.909	1.937	82.74	8.202	0.546
2.70	18	38.11	3.073	1.889	82.21	8.182	0.553
2.70	19	39.28	3.243	1.841	81.67	8.160	0.560
2.70	20	40.50	3.420	1.792	81.10	8.135	0.569
2.70	21	41.75	3.604	1.742	80.50	8.106	0.579
2.70	22	43.05	3.796	1.691	79.86	8.075	0.589
2.70	23	44.40	3.997	1.638	79.19	8.039	0.601
2.70	24	45.81	4.206	1.585	78.47	7.998	0.614
2.70	25	47.29	4.425	1.530	77.69	7.951	0.630
2.70	26	48.85	4.656	1.472	76.83	7.897	0.647
2.70	27	50.52	4.900	1.412	75.88	7.832	0.667
2.70	28	52.33	5.162	1.349	74.79	7.753	0.691
2.70	29	54.35	5.449	1.280	73.51	7.653	0.720
2.70	30	56.69	5.773	1.202	71.92	7.519	0.759
2.70	31	59.72	6.176	1.104	69.63	7.307	0.817
2.80	0	20.91	0.999	2.801	89.99	8.980	0.488
2.80	1	21.62	1.075	2.752	89.63	8.980	0.488
2.80	2	22.34	1.155	2.706	89.25	8.978	0.489
2.80	3	23.09	1.240	2.659	88.87	8.976	0.489
2.80	4	23.85	1.329	2.613	88.49	8.974	0.491
2.80	5	24.64	1.423	2.568	88.11	8.970	0.492
2.80	6	25.46	1.523	2.522	87.73	8.966	0.493
2.80	7	26.29	1.628	2.477	87.34	8.960	0.495
2.80	8	27.15	1.738	2.431	86.95	8.954	0.498
2.80	9	28.03	1.854	2.386	86.55	8.947	0.500
2.80	10	28.94	1.975	2.340	86.14	8.939	0.503
2.80	11	29.87	2.102	2.294	85.73	8.929	0.507
2.80	12	30.83	2.236	2.248	85.31	8.919	0.510
2.80	13	31.81	2.375	2.201	84.88	8.907	0.514
2.80	14	32.82	2.520	2.154	84.44	8.894	0.519
2.80	15	33.86	2.672	2.107	83.99	8.880	0.524
2.80	16	34.92	2.831	2.059	83.53	8.864	0.530
2.80	17	36.02	2.996	2.010	83.05	8.846	0.536
2.80	18	37.14	3.168	1.961	82.55	8.826	0.543
2.80	19	38.30	3.346	1.911	82.04	8.804	0.550
2.80	20	39.49	3.532	1.861	81.50	8.780	0.558
2.80	21	40.72	3.726	1.810	80.93	8.753	0.567

(continued)

**Table A.3** (Continued)

Weak solution				Strong solution			
$M_1$	$\theta$	$\beta$	$p_2/p_1$	$M_2$	$\beta$	$p_2/p_1$	$M_2$
2.80	22	41.99	3.927	1.758	80.34	8.722	0.577
2.80	23	43.31	4.136	1.705	79.71	8.688	0.588
2.80	24	44.68	4.355	1.651	79.04	8.649	0.600
2.80	25	46.10	4.583	1.595	78.33	8.605	0.614
2.80	26	47.60	4.822	1.538	77.55	8.554	0.630
2.80	27	49.19	5.073	1.479	76.69	8.495	0.648
2.80	28	50.89	5.340	1.416	75.73	8.424	0.668
2.80	29	52.73	5.625	1.350	74.63	8.337	0.693
2.80	30	54.79	5.938	1.278	73.33	8.227	0.724
2.80	31	57.20	6.295	1.197	71.68	8.076	0.766
2.80	32	60.43	6.752	1.091	69.21	7.828	0.831
2.90	0	20.15	0.998	2.901	89.99	9.645	0.481
2.90	1	20.86	1.078	2.851	89.64	9.645	0.482
2.90	2	21.58	1.160	2.802	89.28	9.643	0.482
2.90	3	22.32	1.248	2.754	88.91	9.641	0.483
2.90	4	23.08	1.341	2.706	88.55	9.639	0.484
2.90	5	23.86	1.439	2.659	88.18	9.635	0.485
2.90	6	24.67	1.542	2.612	87.81	9.631	0.486
2.90	7	25.50	1.651	2.565	87.44	9.625	0.488
2.90	8	26.35	1.766	2.518	87.06	9.619	0.491
2.90	9	27.23	1.887	2.470	86.67	9.612	0.493
2.90	10	28.13	2.014	2.423	86.28	9.604	0.496
2.90	11	29.06	2.148	2.375	85.89	9.595	0.499
2.90	12	30.01	2.287	2.327	85.49	9.584	0.503
2.90	13	30.98	2.433	2.279	85.07	9.573	0.507
2.90	14	31.99	2.586	2.230	84.65	9.560	0.511
2.90	15	33.01	2.746	2.181	84.22	9.545	0.516
2.90	16	34.07	2.912	2.132	83.78	9.530	0.521
2.90	17	35.15	3.086	2.082	83.32	9.512	0.527
2.90	18	36.26	3.266	2.031	82.85	9.493	0.533
2.90	19	37.41	3.454	1.980	82.36	9.471	0.540
2.90	20	38.58	3.649	1.928	81.85	9.447	0.548
2.90	21	39.80	3.853	1.876	81.31	9.421	0.557
2.90	22	41.04	4.064	1.823	80.70	9.391	0.566
2.90	23	42.34	4.283	1.769	80.16	9.358	0.576
2.90	24	43.67	4.512	1.714	79.54	9.321	0.588
2.90	25	45.06	4.750	1.658	78.87	9.279	0.601
2.90	26	46.51	4.998	1.600	78.14	9.231	0.615
2.90	27	48.04	5.258	1.540	77.36	9.175	0.631

(continued)

**Table A.3** (Continued)

Weak solution				Strong solution			
$M_1$	$\theta$	$\beta$	$p_2/p_1$	$M_2$	$\beta$	$p_2/p_1$	$M_2$
2.90	28	49.65	5.533	1.479	76.49	9.109	0.650
2.90	29	51.39	5.823	1.414	75.52	9.031	0.672
2.90	30	53.27	6.136	1.345	74.39	8.935	0.699
2.90	31	55.40	6.480	1.270	73.04	8.810	0.732
2.90	32	57.93	6.879	1.183	71.29	8.635	0.777
2.90	33	61.57	7.420	1.063	68.44	8.319	0.855
3.00	0	19.45	0.998	3.001	89.99	10.333	0.475
3.00	1	20.16	1.080	2.949	89.65	10.333	0.475
3.00	2	20.87	1.165	2.898	89.30	10.332	0.476
3.00	3	21.60	1.256	2.848	88.95	10.330	0.476
3.00	4	22.35	1.352	2.799	88.60	10.327	0.477
3.00	5	23.13	1.454	2.750	88.24	10.323	0.479
3.00	6	23.94	1.562	2.701	87.88	10.319	0.480
3.00	7	24.76	1.675	2.652	87.52	10.314	0.482
3.00	8	25.61	1.795	2.603	87.16	10.307	0.484
3.00	9	26.49	1.922	2.554	86.79	10.300	0.486
3.00	10	27.38	2.054	2.505	86.41	10.292	0.489
3.00	11	28.30	2.194	2.456	86.03	10.283	0.492
3.00	12	29.25	2.340	2.406	85.64	10.273	0.496
3.00	13	30.22	2.494	2.356	85.24	10.261	0.500
3.00	14	31.22	2.654	2.306	84.84	10.248	0.504
3.00	15	32.24	2.821	2.255	84.42	10.234	0.508
3.00	16	33.29	2.996	2.204	84.00	10.218	0.514
3.00	17	34.36	3.179	2.152	83.56	10.201	0.519
3.00	18	35.47	3.368	2.100	83.11	10.182	0.525
3.00	19	36.60	3.566	2.047	82.64	10.161	0.532
3.00	20	37.76	3.771	1.994	82.15	10.137	0.539
3.00	21	38.96	3.984	1.940	81.64	10.111	0.547
3.00	22	40.19	4.206	1.886	81.11	10.082	0.556
3.00	23	41.46	4.436	1.831	80.55	10.050	0.566
3.00	24	42.78	4.676	1.774	79.96	10.014	0.577
3.00	25	44.14	4.925	1.717	79.33	9.973	0.589
3.00	26	45.55	5.184	1.659	78.65	9.927	0.602
3.00	27	47.03	5.455	1.599	77.92	9.874	0.617
3.00	28	48.59	5.739	1.537	77.13	9.812	0.635
3.00	29	50.24	6.038	1.473	76.24	9.739	0.654
3.00	30	52.01	6.356	1.406	75.24	9.652	0.678
3.00	31	53.96	6.699	1.334	74.07	9.542	0.706
3.00	32	56.18	7.081	1.254	72.64	9.399	0.743

(continued)

**Table A.3** (Continued)

Weak solution				Strong solution			
$M_1$	$\theta$	$\beta$	$p_2/p_1$	$M_2$	$\beta$	$p_2/p_1$	$M_2$
3.00	33	58.91	7.533	1.160	70.71	9.188	0.794
3.00	34	63.67	8.267	1.003	66.75	8.697	0.908
3.10	0	18.80	0.998	3.102	89.99	11.045	0.470
3.10	1	19.50	1.083	3.047	89.66	11.045	0.470
3.10	2	20.20	1.171	2.994	89.32	11.043	0.470
3.10	3	20.93	1.264	2.942	88.98	11.041	0.471
3.10	4	21.68	1.364	2.891	88.64	11.039	0.472
3.10	5	22.46	1.470	2.840	88.30	11.035	0.473
3.10	6	23.26	1.581	2.789	87.95	11.031	0.474
3.10	7	24.08	1.700	2.739	87.60	11.025	0.476
3.10	8	24.93	1.825	2.688	87.24	11.019	0.478
3.10	9	25.80	1.957	2.637	86.89	11.012	0.480
3.10	10	26.69	2.096	2.586	86.52	11.004	0.483
3.10	11	27.61	2.241	2.535	86.15	10.994	0.486
3.10	12	28.55	2.395	2.484	85.78	10.984	0.489
3.10	13	29.52	2.555	2.432	85.39	10.973	0.493
3.10	14	30.51	2.724	2.380	85.00	10.960	0.497
3.10	15	31.53	2.899	2.327	84.60	10.946	0.502
3.10	16	32.57	3.083	2.274	84.19	10.930	0.507
3.10	17	33.64	3.274	2.221	83.77	10.913	0.512
3.10	18	34.74	3.474	2.167	83.33	10.894	0.518
3.10	19	35.86	3.681	2.113	82.88	10.873	0.524
3.10	20	37.02	3.897	2.058	82.42	10.850	0.531
3.10	21	38.20	4.121	2.003	81.93	10.824	0.539
3.10	22	39.42	4.354	1.947	81.42	10.795	0.548
3.10	23	40.67	4.596	1.890	80.89	10.764	0.557
3.10	24	41.97	4.847	1.833	80.33	10.728	0.567
3.10	25	43.31	5.108	1.775	79.73	10.688	0.578
3.10	26	44.69	5.379	1.715	79.09	10.643	0.591
3.10	27	46.14	5.661	1.655	78.41	10.592	0.605
3.10	28	47.65	5.956	1.593	77.67	10.533	0.621
3.10	29	49.24	6.265	1.529	76.85	10.465	0.639
3.10	30	50.93	6.592	1.462	75.94	10.383	0.661
3.10	31	52.77	6.940	1.392	74.90	10.284	0.686
3.10	32	54.80	7.319	1.316	73.66	10.158	0.717
3.10	33	57.15	7.747	1.230	72.11	9.987	0.758
3.10	34	60.20	8.276	1.124	69.87	9.717	0.820
3.20	0	18.19	0.998	3.202	89.99	11.780	0.464
3.20	1	18.89	1.085	3.145	89.67	11.780	0.464

(continued)

**Table A.3** (Continued)

Weak solution				Strong solution			
$M_1$	$\theta$	$\beta$	$p_2/p_1$	$M_2$	$\beta$	$p_2/p_1$	$M_2$
3.20	2	19.59	1.176	3.090	89.34	11.778	0.465
3.20	3	20.31	1.273	3.036	89.01	11.776	0.466
3.20	4	21.06	1.376	2.983	88.68	11.774	0.466
3.20	5	21.83	1.485	2.930	88.34	11.770	0.468
3.20	6	22.63	1.602	2.878	88.01	11.765	0.469
3.20	7	23.45	1.725	2.825	87.67	11.760	0.471
3.20	8	24.29	1.855	2.773	87.32	11.754	0.473
3.20	9	25.16	1.993	2.720	86.97	11.747	0.475
3.20	10	26.05	2.138	2.667	86.62	11.738	0.478
3.20	11	26.97	2.290	2.614	86.26	11.729	0.480
3.20	12	27.91	2.451	2.561	85.90	11.719	0.484
3.20	13	28.87	2.619	2.507	85.53	11.707	0.487
3.20	14	29.86	2.795	2.453	85.15	11.694	0.491
3.20	15	30.88	2.980	2.398	84.76	11.680	0.496
3.20	16	31.92	3.172	2.344	84.36	11.665	0.500
3.20	17	32.98	3.373	2.289	83.96	11.647	0.506
3.20	18	34.07	3.583	2.233	83.54	11.628	0.511
3.20	19	35.19	3.801	2.177	83.10	11.608	0.517
3.20	20	36.34	4.027	2.121	82.65	11.584	0.524
3.20	21	37.51	4.263	2.064	82.18	11.559	0.532
3.20	22	38.72	4.507	2.006	81.70	11.531	0.540
3.20	23	39.96	4.761	1.948	81.19	11.499	0.549
3.20	24	41.24	5.024	1.889	80.65	11.464	0.558
3.20	25	42.56	5.297	1.830	80.08	11.425	0.569
3.20	26	43.92	5.581	1.770	79.48	11.381	0.581
3.20	27	45.34	5.876	1.708	78.83	11.332	0.595
3.20	28	46.81	6.184	1.645	78.13	11.275	0.610
3.20	29	48.36	6.505	1.581	77.37	11.209	0.627
3.20	30	49.99	6.842	1.514	76.53	11.131	0.646
3.20	31	51.74	7.200	1.445	75.58	11.039	0.669
3.20	32	53.65	7.583	1.371	74.48	10.924	0.697
3.20	33	55.79	8.004	1.291	73.15	10.776	0.731
3.20	34	58.35	8.490	1.198	71.41	10.566	0.779
3.20	35	62.06	9.157	1.069	68.52	10.178	0.863
3.30	0	17.62	0.998	3.302	89.99	12.538	0.460
3.30	1	18.31	1.088	3.242	89.68	12.538	0.460
3.30	2	19.01	1.181	3.186	89.36	12.537	0.460
3.30	3	19.73	1.281	3.130	89.04	12.535	0.461
3.30	4	20.48	1.388	3.075	88.71	12.532	0.462

(continued)

**Table A.3** (Continued)

Weak solution				Strong solution			
$M_1$	$\theta$	$\beta$	$p_2/p_1$	$M_2$	$\beta$	$p_2/p_1$	$M_2$
3.30	5	21.25	1.501	3.020	88.39	12.528	0.463
3.30	6	22.04	1.622	2.965	88.06	12.524	0.464
3.30	7	22.86	1.750	2.911	87.73	12.518	0.466
3.30	8	23.70	1.886	2.856	87.39	12.512	0.468
3.30	9	24.57	2.029	2.802	87.05	12.505	0.470
3.30	10	25.46	2.181	2.747	86.71	12.496	0.472
3.30	11	26.37	2.340	2.692	86.36	12.487	0.475
3.30	12	27.31	2.508	2.636	86.01	12.477	0.478
3.30	13	28.27	2.684	2.581	85.65	12.465	0.482
3.30	14	29.26	2.869	2.525	85.28	12.452	0.486
3.30	15	30.27	3.062	2.469	84.90	12.438	0.490
3.30	16	31.31	3.264	2.412	84.52	12.422	0.495
3.30	17	32.37	3.475	2.355	84.12	12.405	0.500
3.30	18	33.46	3.695	2.297	83.72	12.386	0.505
3.30	19	34.57	3.923	2.240	83.30	12.365	0.511
3.30	20	35.71	4.161	2.181	82.86	12.342	0.518
3.30	21	36.88	4.409	2.123	82.41	12.317	0.525
3.30	22	38.08	4.665	2.064	81.94	12.288	0.533
3.30	23	39.31	4.932	2.004	81.45	12.257	0.541
3.30	24	40.57	5.208	1.944	80.93	12.223	0.551
3.30	25	41.88	5.494	1.883	80.39	12.184	0.561
3.30	26	43.22	5.792	1.822	79.81	12.141	0.572
3.30	27	44.61	6.100	1.759	79.20	12.092	0.585
3.30	28	46.06	6.421	1.696	78.54	12.036	0.599
3.30	29	47.57	6.755	1.631	77.82	11.973	0.615
3.30	30	49.16	7.105	1.564	77.03	11.898	0.634
3.30	31	50.85	7.474	1.495	76.15	11.810	0.655
3.30	32	52.67	7.865	1.422	75.15	11.704	0.680
3.30	33	54.67	8.289	1.344	73.97	11.569	0.710
3.30	34	56.96	8.762	1.258	72.50	11.390	0.750
3.30	35	59.85	9.333	1.153	70.45	11.115	0.809
3.40	0	17.09	0.998	3.402	89.99	13.320	0.455
3.40	1	17.77	1.090	3.340	89.69	13.320	0.455
3.40	2	18.47	1.187	3.281	89.37	13.318	0.456
3.40	3	19.19	1.290	3.224	89.06	13.316	0.456
3.40	4	19.93	1.400	3.166	88.74	13.313	0.457
3.40	5	20.70	1.518	3.109	88.43	13.310	0.458
3.40	6	21.49	1.643	3.053	88.10	13.305	0.460
3.40	7	22.31	1.776	2.996	87.78	13.300	0.461

(continued)

**Table A.3** (Continued)

$M_1$	Weak solution			Strong solution			
	$\theta$	$\beta$	$p_2/p_1$	$M_2$	$\beta$	$p_2/p_1$	$M_2$
3.40	8	23.15	1.917	2.940	87.46	13.293	0.463
3.40	9	24.01	2.067	2.883	87.13	13.286	0.465
3.40	10	24.90	2.224	2.826	86.79	13.278	0.468
3.40	11	25.82	2.391	2.769	86.45	13.268	0.471
3.40	12	26.75	2.566	2.712	86.11	13.258	0.474
3.40	13	27.72	2.751	2.654	85.76	13.246	0.477
3.40	14	28.70	2.944	2.596	85.40	13.233	0.481
3.40	15	29.71	3.146	2.537	85.03	13.219	0.485
3.40	16	30.75	3.358	2.479	84.66	13.203	0.489
3.40	17	31.81	3.571	2.420	84.27	13.186	0.494
3.40	18	32.89	3.810	2.360	83.88	13.166	0.500
3.40	19	34.00	4.050	2.301	83.47	13.145	0.506
3.40	20	35.13	4.300	2.241	83.05	13.122	0.512
3.40	21	36.30	4.559	2.180	82.61	13.097	0.519
3.40	22	37.49	4.829	2.120	82.16	13.069	0.526
3.40	23	38.71	5.108	2.058	81.68	13.038	0.535
3.40	24	39.97	5.398	1.997	81.19	13.003	0.544
3.40	25	41.26	5.698	1.934	80.66	12.965	0.554
3.40	26	42.59	6.009	1.872	80.11	12.922	0.565
3.40	27	43.96	6.332	1.808	79.52	12.874	0.577
3.40	28	45.39	6.667	1.744	78.89	12.819	0.590
3.40	29	46.87	7.016	1.678	78.21	12.757	0.605
3.40	30	48.42	7.380	1.611	77.47	12.685	0.623
3.40	31	50.06	7.761	1.541	76.65	12.600	0.642
3.40	32	51.81	8.164	1.469	75.72	12.499	0.665
3.40	33	53.71	8.595	1.393	74.65	12.374	0.693
3.40	34	55.84	9.067	1.310	73.35	12.213	0.728
3.40	35	58.36	9.608	1.215	71.67	11.986	0.775
3.40	36	61.91	10.330	1.088	68.96	11.582	0.856
3.50	0	16.58	0.997	3.502	89.99	14.125	0.451
3.50	1	17.27	1.092	3.438	89.70	14.125	0.451
3.50	2	17.96	1.192	3.377	89.39	14.123	0.452
3.50	3	18.67	1.298	3.317	89.08	14.121	0.452
3.50	4	19.42	1.412	3.257	88.77	14.118	0.453
3.50	5	20.18	1.534	3.198	88.46	14.115	0.454
3.50	6	20.97	1.664	3.140	88.15	14.110	0.456
3.50	7	21.79	1.802	3.081	87.83	14.104	0.457
3.50	8	22.63	1.949	3.022	87.51	14.098	0.459
3.50	9	23.49	2.105	2.963	87.19	14.091	0.461

(continued)

**Table A.3** (Continued)

Weak solution				Strong solution			
$M_1$	$\theta$	$\beta$	$p_2/p_1$	$M_2$	$\beta$	$p_2/p_1$	$M_2$
3.50	10	24.38	2.269	2.904	86.86	14.082	0.463
3.50	11	25.30	2.443	2.845	86.53	14.073	0.466
3.50	12	26.24	2.626	2.786	86.20	14.062	0.469
3.50	13	27.20	2.819	2.726	85.85	14.050	0.473
3.50	14	28.18	3.021	2.666	85.50	14.037	0.476
3.50	15	29.19	3.233	2.605	85.15	14.023	0.480
3.50	16	30.22	3.455	2.545	84.78	14.007	0.485
3.50	17	31.28	3.686	2.484	84.41	13.989	0.489
3.50	18	32.36	3.928	2.422	84.02	13.970	0.495
3.50	19	33.47	4.180	2.361	83.63	13.949	0.500
3.50	20	34.60	4.442	2.299	83.22	13.926	0.506
3.50	21	35.76	4.714	2.236	82.79	13.900	0.513
3.50	22	36.95	4.997	2.174	82.35	13.872	0.521
3.50	23	38.16	5.290	2.111	81.90	13.841	0.529
3.50	24	39.41	5.593	2.048	81.41	13.806	0.537
3.50	25	40.69	5.908	1.984	80.91	13.768	0.547
3.50	26	42.01	6.234	1.920	80.38	13.725	0.557
3.50	27	43.37	6.572	1.855	79.81	13.678	0.569
3.50	28	44.77	6.922	1.789	79.21	13.624	0.582
3.50	29	46.23	7.286	1.723	78.56	13.562	0.596
3.50	30	47.76	7.665	1.655	77.85	13.492	0.613
3.50	31	49.36	8.061	1.585	77.08	13.410	0.631
3.50	32	51.05	8.477	1.513	76.21	13.313	0.653
3.50	33	52.88	8.919	1.438	75.22	13.194	0.678
3.50	34	54.89	9.396	1.357	74.05	13.046	0.710
3.50	35	57.19	9.928	1.268	72.59	12.846	0.750
3.50	36	60.09	10.571	1.159	70.55	12.539	0.810
3.60	0	16.11	0.997	3.602	89.99	14.953	0.447
3.60	1	16.79	1.095	3.536	89.70	14.953	0.448
3.60	2	17.48	1.197	3.472	89.40	14.952	0.448
3.60	3	18.19	1.307	3.410	89.10	14.950	0.448
3.60	4	18.93	1.425	3.348	88.80	14.947	0.449
3.60	5	19.70	1.551	3.287	88.49	14.943	0.450
3.60	6	20.49	1.686	3.226	88.19	14.938	0.452
3.60	7	21.30	1.829	3.165	87.88	14.933	0.453
3.60	8	22.14	1.981	3.104	87.56	14.926	0.455
3.60	9	23.01	2.143	3.043	87.25	14.918	0.457
3.60	10	23.90	2.315	2.982	86.93	14.910	0.460
3.60	11	24.81	2.496	2.921	86.61	14.900	0.462

(continued)

**Table A.3** (Continued)

$M_1$	Weak solution			Strong solution			
	$\theta$	$\beta$	$p_2/p_1$	$M_2$	$\beta$	$p_2/p_1$	$M_2$
3.60	12	25.75	2.687	2.859	86.28	14.889	0.465
3.60	13	26.71	2.888	2.797	85.94	14.878	0.468
3.60	14	27.70	3.100	2.735	85.60	14.864	0.472
3.60	15	28.71	3.322	2.672	85.25	14.850	0.476
3.60	16	29.74	3.554	2.609	84.90	14.834	0.480
3.60	17	30.80	3.796	2.546	84.53	14.816	0.485
3.60	18	31.88	4.050	2.483	84.16	14.796	0.490
3.60	19	32.98	4.314	2.419	83.77	14.775	0.495
3.60	20	34.11	4.588	2.355	83.37	14.752	0.501
3.60	21	35.27	4.873	2.291	82.96	14.726	0.508
3.60	22	36.45	5.170	2.227	82.53	14.698	0.515
3.60	23	37.66	5.477	2.162	82.08	14.666	0.523
3.60	24	38.90	5.795	2.097	81.62	14.632	0.531
3.60	25	40.17	6.125	2.032	81.13	14.594	0.541
3.60	26	41.48	6.466	1.966	80.62	14.551	0.551
3.60	27	42.83	6.820	1.900	80.07	14.504	0.562
3.60	28	44.22	7.186	1.834	79.49	14.450	0.575
3.60	29	45.65	7.566	1.766	78.87	14.389	0.588
3.60	30	47.15	7.961	1.697	78.19	14.320	0.604
3.60	31	48.72	8.372	1.627	77.45	14.240	0.622
3.60	32	50.38	8.803	1.555	76.64	14.145	0.642
3.60	33	52.14	9.259	1.480	75.71	14.032	0.666
3.60	34	54.07	9.746	1.400	74.64	13.892	0.695
3.60	35	56.22	10.279	1.314	73.33	13.709	0.731
3.60	36	58.79	10.894	1.215	71.62	13.450	0.780
3.60	37	62.54	11.738	1.078	68.73	12.963	0.869
3.70	0	15.66	0.997	3.702	89.99	15.805	0.444
3.70	1	16.34	1.098	3.633	89.71	15.805	0.444
3.70	2	17.03	1.203	3.567	89.41	15.803	0.444
3.70	3	17.74	1.316	3.503	89.12	15.801	0.445
3.70	4	18.48	1.438	3.439	88.82	15.798	0.446
3.70	5	19.24	1.568	3.375	88.52	15.794	0.447
3.70	6	20.03	1.707	3.312	88.22	15.790	0.448
3.70	7	20.85	1.856	3.249	87.92	15.784	0.450
3.70	8	21.69	2.014	3.186	87.61	15.777	0.451
3.70	9	22.55	2.183	3.123	87.30	15.770	0.454
3.70	10	23.44	2.361	3.059	86.99	15.761	0.456
3.70	11	24.36	2.550	2.995	86.67	15.751	0.458
3.70	12	25.30	2.750	2.931	86.35	15.740	0.461

(continued)

**Table A.3** (Continued)

Weak solution				Strong solution			
$M_1$	$\theta$	$\beta$	$p_2/p_1$	$M_2$	$\beta$	$p_2/p_1$	$M_2$
3.70	13	26.26	2.960	2.867	86.02	15.728	0.464
3.70	14	27.25	3.181	2.803	85.69	15.715	0.468
3.70	15	28.25	3.412	2.738	85.35	15.700	0.472
3.70	16	29.29	3.655	2.673	85.00	15.684	0.476
3.70	17	30.34	3.909	2.608	84.64	15.666	0.481
3.70	18	31.42	4.174	2.542	84.28	15.646	0.486
3.70	19	32.53	4.451	2.476	83.90	15.624	0.491
3.70	20	33.65	4.738	2.411	83.51	15.601	0.497
3.70	21	34.81	5.037	2.344	83.11	15.575	0.503
3.70	22	35.99	5.347	2.278	82.69	15.546	0.510
3.70	23	37.19	5.669	2.212	82.26	15.515	0.518
3.70	24	38.43	6.002	2.145	81.80	15.480	0.526
3.70	25	39.69	6.348	2.079	81.33	15.442	0.535
3.70	26	40.99	6.705	2.011	80.83	15.399	0.545
3.70	27	42.33	7.075	1.944	80.30	15.352	0.556
3.70	28	43.70	7.458	1.876	79.74	15.298	0.568
3.70	29	45.13	7.854	1.807	79.14	15.238	0.581
3.70	30	46.61	8.266	1.738	78.49	15.169	0.596
3.70	31	48.15	8.694	1.667	77.79	15.090	0.613
3.70	32	49.77	9.142	1.594	77.01	14.998	0.632
3.70	33	51.49	9.612	1.519	76.14	14.888	0.655
3.70	34	53.34	10.112	1.440	75.14	14.754	0.681
3.70	35	55.39	10.653	1.356	73.95	14.584	0.714
3.70	36	57.76	11.259	1.262	72.44	14.352	0.758
3.70	37	60.82	12.007	1.146	70.25	13.982	0.824
3.80	0	15.24	0.997	3.802	89.99	16.680	0.441
3.80	1	15.92	1.100	3.731	89.71	16.680	0.441
3.80	2	16.60	1.208	3.662	89.42	16.678	0.441
3.80	3	17.31	1.325	3.595	89.13	16.676	0.442
3.80	4	18.05	1.450	3.529	88.84	16.673	0.443
3.80	5	18.81	1.585	3.463	88.55	16.669	0.444
3.80	6	19.60	1.729	3.398	88.25	16.664	0.445
3.80	7	20.42	1.883	3.332	87.96	16.658	0.446
3.80	8	21.26	2.048	3.267	87.66	16.652	0.448
3.80	9	22.13	2.223	3.201	87.35	16.644	0.450
3.80	10	23.02	2.409	3.135	87.05	16.635	0.452
3.80	11	23.93	2.605	3.069	86.73	16.625	0.455
3.80	12	24.87	2.813	3.003	86.42	16.614	0.458
3.80	13	25.83	3.032	2.937	86.10	16.602	0.461

(continued)

**Table A.3** (Continued)

$M_1$	Weak solution			Strong solution			
	$\theta$	$\beta$	$p_2/p_1$	$M_2$	$\beta$	$p_2/p_1$	$M_2$
3.80	14	26.82	3.263	2.870	85.77	16.588	0.464
3.80	15	27.83	3.505	2.803	85.44	16.573	0.468
3.80	16	28.86	3.759	2.735	85.09	16.557	0.472
3.80	17	29.92	4.025	2.668	84.74	16.538	0.477
3.80	18	31.00	4.302	2.600	84.39	16.519	0.482
3.80	19	32.10	4.591	2.532	84.02	16.497	0.487
3.80	20	33.23	4.892	2.464	83.64	16.473	0.493
3.80	21	34.38	5.205	2.396	83.24	16.447	0.499
3.80	22	35.56	5.530	2.328	82.84	16.418	0.506
3.80	23	36.76	5.867	2.260	82.41	16.386	0.513
3.80	24	37.99	6.216	2.192	81.97	16.351	0.521
3.80	25	39.25	6.577	2.123	81.51	16.313	0.530
3.80	26	40.54	6.951	2.055	81.02	16.270	0.540
3.80	27	41.87	7.337	1.986	80.51	16.222	0.550
3.80	28	43.23	7.737	1.917	79.97	16.169	0.562
3.80	29	44.64	8.152	1.847	79.39	16.108	0.575
3.80	30	46.10	8.581	1.776	78.76	16.040	0.589
3.80	31	47.63	9.027	1.704	78.09	15.962	0.605
3.80	32	49.22	9.492	1.631	77.34	15.871	0.624
3.80	33	50.90	9.979	1.556	76.52	15.764	0.645
3.80	34	52.70	10.494	1.478	75.57	15.634	0.670
3.80	35	54.67	11.045	1.395	74.47	15.472	0.700
3.80	36	56.89	11.654	1.304	73.12	15.259	0.739
3.80	37	59.60	12.367	1.198	71.28	14.944	0.795
3.80	38	64.18	13.485	1.030	67.57	14.227	0.913
3.90	0	14.84	0.997	3.902	89.99	17.578	0.438
3.90	1	15.51	1.103	3.828	89.72	17.578	0.438
3.90	2	16.20	1.214	3.757	89.43	17.577	0.438
3.90	3	16.91	1.334	3.688	89.15	17.574	0.439
3.90	4	17.64	1.463	3.619	88.86	17.571	0.440
3.90	5	18.41	1.602	3.551	88.57	17.567	0.441
3.90	6	19.20	1.752	3.483	88.28	17.562	0.442
3.90	7	20.01	1.911	3.415	87.99	17.556	0.443
3.90	8	20.85	2.082	3.347	87.70	17.550	0.445
3.90	9	21.72	2.264	3.279	87.40	17.542	0.447
3.90	10	22.61	2.457	3.211	87.10	17.533	0.449
3.90	11	23.53	2.662	3.143	86.79	17.523	0.452
3.90	12	24.47	2.878	3.074	86.48	17.511	0.455
3.90	13	25.44	3.107	3.005	86.16	17.499	0.458

(continued)

**Table A.3** (Continued)

Weak solution				Strong solution			
$M_1$	$\theta$	$\beta$	$p_2/p_1$	$M_2$	$\beta$	$p_2/p_1$	$M_2$
3.90	14	26.42	3.347	2.936	85.84	17.485	0.461
3.90	15	27.43	3.600	2.866	85.51	17.470	0.465
3.90	16	28.47	3.865	2.797	85.18	17.453	0.469
3.90	17	29.53	4.143	2.727	84.84	17.434	0.473
3.90	18	30.61	4.433	2.657	84.49	17.414	0.478
3.90	19	31.71	4.735	2.587	84.12	17.392	0.483
3.90	20	32.83	5.050	2.517	83.75	17.368	0.489
3.90	21	33.98	5.377	2.447	83.37	17.341	0.495
3.90	22	35.16	5.717	2.377	82.97	17.312	0.502
3.90	23	36.36	6.069	2.307	82.55	17.280	0.509
3.90	24	37.58	6.434	2.237	82.12	17.245	0.517
3.90	25	38.84	6.812	2.167	81.67	17.206	0.525
3.90	26	40.13	7.203	2.097	81.20	17.163	0.535
3.90	27	41.45	7.607	2.026	80.70	17.115	0.545
3.90	28	42.80	8.025	1.956	80.17	17.061	0.556
3.90	29	44.20	8.458	1.885	79.61	17.001	0.569
3.90	30	45.65	8.906	1.813	79.01	16.933	0.583
3.90	31	47.15	9.370	1.741	78.36	16.855	0.598
3.90	32	48.72	9.853	1.667	77.64	16.765	0.616
3.90	33	50.37	10.358	1.591	76.85	16.660	0.636
3.90	34	52.13	10.890	1.513	75.96	16.533	0.660
3.90	35	54.03	11.456	1.431	74.92	16.378	0.688
3.90	36	56.15	12.072	1.343	73.68	16.177	0.724
3.90	37	58.64	12.773	1.242	72.06	15.895	0.772
3.90	38	62.08	13.689	1.111	69.50	15.402	0.853
4.00	0	14.46	0.997	4.002	89.99	18.500	0.435
4.00	1	15.13	1.105	3.925	89.72	18.500	0.435
4.00	2	15.81	1.219	3.852	89.44	18.498	0.435
4.00	3	16.52	1.343	3.780	89.16	18.496	0.436
4.00	4	17.26	1.476	3.709	88.88	18.493	0.437
4.00	5	18.02	1.620	3.638	88.60	18.489	0.438
4.00	6	18.81	1.774	3.568	88.31	18.484	0.439
4.00	7	19.63	1.940	3.498	88.02	18.478	0.440
4.00	8	20.47	2.117	3.427	87.73	18.471	0.442
4.00	9	21.34	2.305	3.357	87.44	18.463	0.444
4.00	10	22.23	2.506	3.286	87.14	18.453	0.446
4.00	11	23.15	2.719	3.215	86.84	18.443	0.449
4.00	12	24.10	2.944	3.144	86.54	18.432	0.452
4.00	13	25.06	3.182	3.073	86.23	18.419	0.455

(continued)

**Table A.3** (Continued)

$M_1$	Weak solution			Strong solution			
	$\theta$	$\beta$	$p_2/p_1$	$M_2$	$\beta$	$p_2/p_1$	$M_2$
4.00	14	26.05	3.433	3.001	85.91	18.405	0.458
4.00	15	27.06	3.697	2.929	85.59	18.389	0.462
4.00	16	28.10	3.974	2.857	85.26	18.372	0.466
4.00	17	29.16	4.264	2.785	84.92	18.354	0.470
4.00	18	30.24	4.566	2.713	84.58	18.333	0.475
4.00	19	31.34	4.882	2.641	84.22	18.311	0.480
4.00	20	32.46	5.211	2.569	83.86	18.286	0.485
4.00	21	33.61	5.553	2.497	83.48	18.259	0.491
4.00	22	34.79	5.909	2.425	83.09	18.230	0.498
4.00	23	35.98	6.277	2.353	82.68	18.197	0.505
4.00	24	37.21	6.659	2.281	82.26	18.161	0.513
4.00	25	38.46	7.054	2.209	81.82	18.122	0.521
4.00	26	39.74	7.462	2.137	81.36	18.079	0.530
4.00	27	41.05	7.885	2.066	80.88	18.030	0.540
4.00	28	42.40	8.321	1.994	80.36	17.976	0.551
4.00	29	43.79	8.772	1.921	79.81	17.916	0.563
4.00	30	45.22	9.239	1.849	79.23	17.848	0.577
4.00	31	46.71	9.723	1.775	78.60	17.770	0.592
4.00	32	48.26	10.226	1.701	77.91	17.681	0.609
4.00	33	49.88	10.749	1.625	77.15	17.576	0.628
4.00	34	51.61	11.299	1.546	76.30	17.452	0.651
4.00	35	53.46	11.881	1.465	75.32	17.301	0.678
4.00	36	55.50	12.509	1.378	74.16	17.109	0.711
4.00	37	57.84	13.210	1.281	72.70	16.849	0.754
4.00	38	60.83	14.064	1.164	70.60	16.441	0.820
4.10	0	14.10	0.997	4.102	89.99	19.445	0.432
4.10	1	14.77	1.108	4.023	89.73	19.445	0.432
4.10	2	15.45	1.225	3.947	89.45	19.443	0.433
4.10	3	16.16	1.352	3.872	89.17	19.441	0.433
4.10	4	16.89	1.489	3.798	88.90	19.438	0.434
4.10	5	17.66	1.638	3.725	88.62	19.433	0.435
4.10	6	18.45	1.797	3.652	88.33	19.428	0.436
4.10	7	19.27	1.968	3.580	88.05	19.422	0.438
4.10	8	20.11	2.152	3.507	87.77	19.415	0.439
4.10	9	20.98	2.347	3.434	87.48	19.407	0.441
4.10	10	21.88	2.556	3.360	87.18	19.398	0.444
4.10	11	22.80	2.777	3.287	86.89	19.387	0.446
4.10	12	23.74	3.012	3.213	86.59	19.375	0.449
4.10	13	24.71	3.260	3.139	86.28	19.362	0.452

(continued)

**Table A.3** (Continued)

Weak solution				Strong solution			
$M_1$	$\theta$	$\beta$	$p_2/p_1$	$M_2$	$\beta$	$p_2/p_1$	$M_2$
4.10	14	25.70	3.521	3.065	85.97	19.348	0.455
4.10	15	26.71	3.796	2.991	85.65	19.332	0.459
4.10	16	27.75	4.085	2.916	85.33	19.315	0.462
4.10	17	28.81	4.387	2.842	85.00	19.296	0.467
4.10	18	29.89	4.703	2.768	84.66	19.275	0.471
4.10	19	30.99	5.033	2.693	84.31	19.252	0.476
4.10	20	32.12	5.377	2.619	83.95	19.227	0.482
4.10	21	33.27	5.734	2.545	83.58	19.200	0.488
4.10	22	34.44	6.105	2.471	83.20	19.170	0.494
4.10	23	35.64	6.490	2.397	82.80	19.137	0.501
4.10	24	36.86	6.889	2.323	82.39	19.101	0.509
4.10	25	38.10	7.301	2.250	81.96	19.061	0.517
4.10	26	39.38	7.728	2.177	81.51	19.017	0.526
4.10	27	40.69	8.169	2.103	81.03	18.968	0.536
4.10	28	42.03	8.625	2.030	80.53	18.914	0.547
4.10	29	43.41	9.095	1.956	80.00	18.853	0.558
4.10	30	44.83	9.582	1.883	79.43	18.785	0.572
4.10	31	46.31	10.086	1.808	78.82	18.707	0.586
4.10	32	47.84	10.609	1.733	78.15	18.618	0.603
4.10	33	49.44	11.152	1.656	77.42	18.514	0.621
4.10	34	51.13	11.722	1.578	76.60	18.392	0.643
4.10	35	52.94	12.322	1.496	75.67	18.244	0.669
4.10	36	54.91	12.965	1.410	74.58	18.059	0.700
4.10	37	57.15	13.672	1.316	73.24	17.814	0.739
4.10	38	59.86	14.501	1.207	71.42	17.452	0.796
4.10	39	64.50	15.811	1.031	67.66	16.611	0.919
4.20	0	13.76	0.997	4.202	89.99	20.413	0.430
4.20	1	14.42	1.110	4.120	89.73	20.413	0.430
4.20	2	15.10	1.231	4.041	89.46	20.411	0.430
4.20	3	15.81	1.361	3.964	89.18	20.409	0.431
4.20	4	16.55	1.503	3.888	88.91	20.406	0.432
4.20	5	17.31	1.655	3.812	88.63	20.402	0.433
4.20	6	18.10	1.820	3.736	88.36	20.396	0.434
4.20	7	18.92	1.997	3.661	88.08	20.390	0.435
4.20	8	19.77	2.187	3.586	87.80	20.383	0.437
4.20	9	20.64	2.390	3.510	87.51	20.374	0.439
4.20	10	21.54	2.607	3.434	87.22	20.365	0.441
4.20	11	22.46	2.837	3.358	86.93	20.354	0.443
4.20	12	23.41	3.081	3.282	86.64	20.342	0.446

(continued)

**Table A.3** (Continued)

$M_1$	Weak solution			Strong solution			
	$\theta$	$\beta$	$p_2/p_1$	$M_2$	$\beta$	$p_2/p_1$	$M_2$
4.20	13	24.38	3.339	3.205	86.33	20.329	0.449
4.20	14	25.37	3.611	3.128	86.03	20.314	0.452
4.20	15	26.38	3.897	3.052	85.72	20.298	0.456
4.20	16	27.42	4.198	2.975	85.40	20.281	0.460
4.20	17	28.48	4.513	2.898	85.07	20.261	0.464
4.20	18	29.56	4.843	2.821	84.74	20.240	0.468
4.20	19	30.67	5.187	2.745	84.40	20.217	0.473
4.20	20	31.79	5.546	2.668	84.04	20.191	0.479
4.20	21	32.94	5.919	2.592	83.68	20.164	0.485
4.20	22	34.11	6.306	2.516	83.30	20.133	0.491
4.20	23	35.31	6.708	2.440	82.91	20.100	0.498
4.20	24	36.53	7.124	2.365	82.51	20.063	0.505
4.20	25	37.77	7.555	2.290	82.09	20.023	0.513
4.20	26	39.05	8.000	2.215	81.64	19.978	0.522
4.20	27	40.35	8.461	2.140	81.18	19.929	0.532
4.20	28	41.69	8.936	2.065	80.69	19.874	0.542
4.20	29	43.06	9.427	1.990	80.17	19.813	0.554
4.20	30	44.47	9.934	1.915	79.61	19.744	0.567
4.20	31	45.93	10.459	1.840	79.02	19.666	0.581
4.20	32	47.45	11.002	1.764	78.37	19.577	0.597
4.20	33	49.03	11.567	1.686	77.66	19.474	0.615
4.20	34	50.70	12.157	1.608	76.88	19.352	0.636
4.20	35	52.47	12.777	1.526	75.99	19.207	0.660
4.20	36	54.39	13.437	1.441	74.96	19.026	0.690
4.20	37	56.54	14.156	1.349	73.70	18.793	0.726
4.20	38	59.07	14.977	1.244	72.07	18.461	0.777
4.20	39	62.66	16.072	1.104	69.37	17.859	0.864
4.30	0	13.43	0.997	4.302	89.99	21.405	0.428
4.30	1	14.10	1.113	4.217	89.73	21.405	0.428
4.30	2	14.77	1.236	4.136	89.46	21.403	0.428
4.30	3	15.48	1.370	4.056	89.20	21.401	0.429
4.30	4	16.22	1.516	3.977	88.92	21.397	0.429
4.30	5	16.98	1.674	3.898	88.65	21.393	0.430
4.30	6	17.78	1.844	3.820	88.38	21.388	0.432
4.30	7	18.60	2.027	3.742	88.10	21.381	0.433
4.30	8	19.44	2.223	3.664	87.82	21.374	0.435
4.30	9	20.32	2.434	3.586	87.54	21.365	0.436
4.30	10	21.22	2.658	3.507	87.26	21.356	0.439
4.30	11	22.14	2.897	3.428	86.97	21.345	0.441

(continued)

**Table A.3** (Continued)

Weak solution				Strong solution			
$M_1$	$\theta$	$\beta$	$p_2/p_1$	$M_2$	$\beta$	$p_2/p_1$	$M_2$
4.30	12	23.09	3.151	3.349	86.68	21.333	0.444
4.30	13	24.06	3.419	3.270	86.38	21.319	0.447
4.30	14	25.06	3.702	3.191	86.08	21.304	0.450
4.30	15	26.07	4.000	3.111	85.77	21.288	0.453
4.30	16	27.11	4.314	3.032	85.46	21.270	0.457
4.30	17	28.18	4.642	2.953	85.14	21.250	0.461
4.30	18	29.26	4.986	2.874	84.81	21.228	0.466
4.30	19	30.36	5.345	2.795	84.47	21.205	0.471
4.30	20	31.49	5.719	2.716	84.12	21.179	0.476
4.30	21	32.64	6.108	2.638	83.77	21.150	0.482
4.30	22	33.81	6.512	2.560	83.40	21.120	0.488
4.30	23	35.00	6.931	2.482	83.01	21.086	0.495
4.30	24	36.22	7.366	2.405	82.62	21.048	0.502
4.30	25	37.47	7.815	2.328	82.20	21.007	0.510
4.30	26	38.74	8.280	2.251	81.77	20.962	0.518
4.30	27	40.04	8.759	2.175	81.31	20.912	0.528
4.30	28	41.37	9.255	2.099	80.83	20.857	0.538
4.30	29	42.73	9.766	2.023	80.32	20.795	0.550
4.30	30	44.14	10.295	1.947	79.78	20.726	0.562
4.30	31	45.59	10.841	1.870	79.20	20.647	0.576
4.30	32	47.09	11.406	1.793	78.57	20.558	0.592
4.30	33	48.66	11.993	1.715	77.89	20.455	0.609
4.30	34	50.30	12.604	1.636	77.13	20.334	0.629
4.30	35	52.05	13.245	1.554	76.27	20.190	0.653
4.30	36	53.92	13.924	1.469	75.29	20.013	0.681
4.30	37	56.00	14.658	1.378	74.11	19.787	0.715
4.30	38	58.40	15.481	1.277	72.61	19.477	0.761
4.30	39	61.54	16.506	1.151	70.37	18.969	0.833
4.40	0	13.12	0.997	4.402	90.00	22.420	0.426
4.40	1	13.78	1.116	4.314	89.74	22.419	0.426
4.40	2	14.46	1.242	4.230	89.47	22.418	0.426
4.40	3	15.17	1.380	4.147	89.20	22.416	0.426
4.40	4	15.90	1.529	4.065	88.94	22.412	0.427
4.40	5	16.67	1.692	3.984	88.67	22.408	0.428
4.40	6	17.46	1.867	3.903	88.40	22.402	0.429
4.40	7	18.29	2.057	3.823	88.13	22.396	0.431
4.40	8	19.13	2.260	3.742	87.85	22.388	0.432
4.40	9	20.01	2.478	3.661	87.57	22.379	0.434
4.40	10	20.91	2.711	3.579	87.29	22.370	0.436

(continued)

**Table A.3** (Continued)

$M_1$	Weak solution			Strong solution			
	$\theta$	$\beta$	$p_2/p_1$	$M_2$	$\beta$	$p_2/p_1$	$M_2$
4.40	11	21.84	2.959	3.498	87.01	22.358	0.439
4.40	12	22.79	3.222	3.416	86.72	22.346	0.441
4.40	13	23.76	3.501	3.334	86.43	22.332	0.444
4.40	14	24.76	3.795	3.252	86.13	22.317	0.447
4.40	15	25.78	4.106	3.170	85.83	22.300	0.451
4.40	16	26.82	4.432	3.088	85.52	22.282	0.455
4.40	17	27.89	4.774	3.007	85.20	22.262	0.459
4.40	18	28.97	5.132	2.925	84.88	22.240	0.463
4.40	19	30.08	5.506	2.844	84.54	22.216	0.468
4.40	20	31.20	5.896	2.763	84.20	22.189	0.473
4.40	21	32.35	6.301	2.683	83.85	22.160	0.479
4.40	22	33.53	6.723	2.602	83.48	22.129	0.485
4.40	23	34.72	7.160	2.523	83.11	22.094	0.492
4.40	24	35.94	7.612	2.444	82.72	22.057	0.499
4.40	25	37.18	8.081	2.365	82.31	22.015	0.507
4.40	26	38.45	8.565	2.287	81.88	21.969	0.515
4.40	27	39.74	9.065	2.209	81.43	21.919	0.524
4.40	28	41.07	9.581	2.132	80.96	21.863	0.535
4.40	29	42.43	10.114	2.054	80.47	21.800	0.546
4.40	30	43.83	10.664	1.977	79.94	21.730	0.558
4.40	31	45.27	11.233	1.899	79.37	21.651	0.572
4.40	32	46.76	11.820	1.821	78.76	21.561	0.587
4.40	33	48.31	12.429	1.743	78.09	21.457	0.604
4.40	34	49.94	13.063	1.663	77.35	21.337	0.623
4.40	35	51.66	13.726	1.581	76.53	21.194	0.646
4.40	36	53.50	14.426	1.496	75.58	21.019	0.673
4.40	37	55.51	15.178	1.406	74.47	20.800	0.705
4.40	38	57.81	16.010	1.308	73.07	20.504	0.748
4.40	39	60.68	17.004	1.190	71.11	20.051	0.811
4.50	0	12.82	0.997	4.503	90.00	23.458	0.424
4.50	1	13.49	1.118	4.411	89.74	23.458	0.424
4.50	2	14.16	1.248	4.324	89.48	23.456	0.424
4.50	3	14.87	1.389	4.238	89.21	23.454	0.424
4.50	4	15.61	1.543	4.154	88.95	23.450	0.425
4.50	5	16.37	1.710	4.070	88.68	23.446	0.426
4.50	6	17.17	1.891	3.986	88.42	23.440	0.427
4.50	7	17.99	2.087	3.903	88.15	23.434	0.429
4.50	8	18.84	2.297	3.819	87.88	23.426	0.430
4.50	9	19.72	2.523	3.735	87.60	23.417	0.432

(continued)

**Table A.3** (Continued)

Weak solution				Strong solution			
$M_1$	$\theta$	$\beta$	$p_2/p_1$	$M_2$	$\beta$	$p_2/p_1$	$M_2$
4.50	10	20.62	2.764	3.651	87.32	23.407	0.434
4.50	11	21.55	3.021	3.567	87.04	23.395	0.437
4.50	12	22.50	3.294	3.482	86.76	23.383	0.439
4.50	13	23.48	3.584	3.397	86.47	23.369	0.442
4.50	14	24.48	3.890	3.313	86.18	23.353	0.445
4.50	15	25.50	4.213	3.228	85.88	23.336	0.449
4.50	16	26.55	4.552	3.144	85.57	23.317	0.452
4.50	17	27.61	4.908	3.059	85.26	23.297	0.456
4.50	18	28.70	5.281	2.975	84.94	23.274	0.461
4.50	19	29.81	5.671	2.892	84.61	23.250	0.465
4.50	20	30.94	6.076	2.809	84.27	23.223	0.471
4.50	21	32.09	6.499	2.726	83.92	23.193	0.476
4.50	22	33.26	6.938	2.644	83.57	23.161	0.482
4.50	23	34.45	7.393	2.563	83.19	23.126	0.489
4.50	24	35.67	7.865	2.482	82.81	23.088	0.496
4.50	25	36.91	8.353	2.401	82.41	23.046	0.504
4.50	26	38.17	8.857	2.321	81.99	22.999	0.512
4.50	27	39.47	9.378	2.242	81.55	22.948	0.521
4.50	28	40.79	9.916	2.163	81.09	22.891	0.531
4.50	29	42.15	10.470	2.084	80.60	22.827	0.542
4.50	30	43.54	11.043	2.006	80.08	22.757	0.554
4.50	31	44.97	11.634	1.927	79.52	22.677	0.567
4.50	32	46.45	12.244	1.848	78.92	22.586	0.582
4.50	33	47.99	12.877	1.769	78.27	22.482	0.599
4.50	34	49.60	13.534	1.689	77.56	22.361	0.618
4.50	35	51.30	14.220	1.606	76.76	22.219	0.640
4.50	36	53.10	14.943	1.521	75.85	22.046	0.665
4.50	37	55.07	15.714	1.432	74.79	21.831	0.697
4.50	38	57.29	16.559	1.335	73.47	21.546	0.736
4.50	39	59.98	17.543	1.223	71.70	21.129	0.793
4.50	40	64.33	19.026	1.052	68.25	20.214	0.909
4.60	0	12.54	0.997	4.603	90.00	24.520	0.422
4.60	1	13.20	1.121	4.508	89.74	24.519	0.422
4.60	2	13.88	1.253	4.418	89.48	24.518	0.422
4.60	3	14.58	1.398	4.329	89.22	24.515	0.423
4.60	4	15.32	1.557	4.242	88.96	24.512	0.423
4.60	5	16.09	1.729	4.155	88.70	24.507	0.424
4.60	6	16.88	1.916	4.069	88.43	24.501	0.425
4.60	7	17.71	2.118	3.982	88.17	24.495	0.427

(continued)

**Table A.3** (Continued)

$M_1$	Weak solution			Strong solution			
	$\theta$	$\beta$	$p_2/p_1$	$M_2$	$\beta$	$p_2/p_1$	$M_2$
4.60	8	18.56	2.335	3.896	87.90	24.487	0.428
4.60	9	19.44	2.568	3.809	87.63	24.478	0.430
4.60	10	20.35	2.818	3.722	87.35	24.467	0.432
4.60	11	21.28	3.085	3.635	87.08	24.456	0.435
4.60	12	22.24	3.368	3.547	86.79	24.443	0.437
4.60	13	23.21	3.669	3.460	86.51	24.428	0.440
4.60	14	24.22	3.987	3.372	86.22	24.412	0.443
4.60	15	25.24	4.322	3.285	85.92	24.395	0.446
4.60	16	26.29	4.675	3.198	85.62	24.376	0.450
4.60	17	27.36	5.045	3.111	85.31	24.355	0.454
4.60	18	28.44	5.433	3.025	84.99	24.332	0.458
4.60	19	29.55	5.838	2.939	84.67	24.307	0.463
4.60	20	30.68	6.261	2.853	84.34	24.279	0.468
4.60	21	31.83	6.701	2.769	83.99	24.250	0.474
4.60	22	33.00	7.158	2.685	83.64	24.217	0.480
4.60	23	34.20	7.632	2.601	83.27	24.181	0.486
4.60	24	35.41	8.123	2.518	82.89	24.142	0.493
4.60	25	36.65	8.631	2.436	82.50	24.099	0.501
4.60	26	37.92	9.156	2.355	82.09	24.052	0.509
4.60	27	39.21	9.698	2.274	81.65	23.999	0.518
4.60	28	40.53	10.258	2.193	81.20	23.942	0.528
4.60	29	41.88	10.835	2.114	80.72	23.877	0.539
4.60	30	43.27	11.430	2.034	80.21	23.806	0.550
4.60	31	44.69	12.044	1.954	79.66	23.725	0.563
4.60	32	46.17	12.679	1.874	79.08	23.633	0.578
4.60	33	47.69	13.335	1.794	78.44	23.529	0.594
4.60	34	49.29	14.017	1.713	77.75	23.408	0.613
4.60	35	50.96	14.727	1.630	76.97	23.265	0.634
4.60	36	52.74	15.473	1.545	76.09	23.094	0.659
4.60	37	54.67	16.265	1.457	75.07	22.881	0.689
4.60	38	56.83	17.128	1.361	73.83	22.605	0.726
4.60	39	59.37	18.111	1.253	72.20	22.212	0.778
4.60	40	62.99	19.429	1.107	69.49	21.489	0.868
4.70	0	12.27	0.997	4.703	90.00	25.605	0.420
4.70	1	12.93	1.123	4.605	89.75	25.604	0.420
4.70	2	13.60	1.259	4.512	89.49	25.603	0.420
4.70	3	14.31	1.408	4.420	89.23	25.600	0.421
4.70	4	15.05	1.571	4.330	88.97	25.597	0.422
4.70	5	15.82	1.748	4.240	88.71	25.592	0.422

(continued)

**Table A.3** (Continued)

Weak solution				Strong solution			
$M_1$	$\theta$	$\beta$	$p_2/p_1$	$M_2$	$\beta$	$p_2/p_1$	$M_2$
4.70	6	16.61	1.940	4.151	88.45	25.586	0.424
4.70	7	17.44	2.149	4.061	88.19	25.579	0.425
4.70	8	18.30	2.373	3.972	87.92	25.571	0.427
4.70	9	19.18	2.615	3.882	87.65	25.562	0.428
4.70	10	20.09	2.873	3.792	87.38	25.551	0.430
4.70	11	21.02	3.149	3.702	87.11	25.539	0.433
4.70	12	21.98	3.443	3.612	86.83	25.526	0.435
4.70	13	22.96	3.755	3.522	86.54	25.511	0.438
4.70	14	23.97	4.085	3.431	86.26	25.495	0.441
4.70	15	24.99	4.434	3.341	85.96	25.477	0.444
4.70	16	26.04	4.800	3.251	85.67	25.458	0.448
4.70	17	27.11	5.185	3.162	85.36	25.436	0.452
4.70	18	28.20	5.588	3.073	85.05	25.413	0.456
4.70	19	29.31	6.010	2.985	84.73	25.387	0.461
4.70	20	30.44	6.449	2.897	84.40	25.359	0.466
4.70	21	31.59	6.907	2.810	84.06	25.329	0.472
4.70	22	32.77	7.382	2.724	83.71	25.295	0.477
4.70	23	33.96	7.875	2.639	83.35	25.259	0.484
4.70	24	35.18	8.386	2.554	82.97	25.219	0.491
4.70	25	36.42	8.915	2.470	82.58	25.175	0.498
4.70	26	37.68	9.461	2.387	82.18	25.127	0.506
4.70	27	38.97	10.025	2.305	81.75	25.074	0.515
4.70	28	40.28	10.607	2.223	81.30	25.015	0.525
4.70	29	41.63	11.207	2.142	80.83	24.950	0.535
4.70	30	43.01	11.826	2.061	80.33	24.878	0.547
4.70	31	44.43	12.464	1.980	79.80	24.796	0.560
4.70	32	45.90	13.123	1.899	79.22	24.703	0.574
4.70	33	47.42	13.804	1.818	78.60	24.598	0.590
4.70	34	49.00	14.511	1.737	77.92	24.476	0.608
4.70	35	50.66	15.246	1.654	77.17	24.333	0.629
4.70	36	52.41	16.016	1.568	76.31	24.162	0.653
4.70	37	54.31	16.832	1.480	75.33	23.952	0.682
4.70	38	56.40	17.714	1.385	74.15	23.682	0.717
4.70	39	58.84	18.705	1.280	72.63	23.307	0.765
4.70	40	62.09	19.956	1.146	70.30	22.676	0.842
4.80	0	12.01	0.997	4.803	90.00	26.713	0.418
4.80	1	12.67	1.126	4.701	89.75	26.713	0.418
4.80	2	13.34	1.265	4.605	89.49	26.711	0.419
4.80	3	14.05	1.417	4.511	89.24	26.709	0.419

(continued)

**Table A.3** (Continued)

$M_1$	Weak solution			Strong solution			
	$\theta$	$\beta$	$p_2/p_1$	$M_2$	$\beta$	$p_2/p_1$	$M_2$
4.80	4	14.79	1.585	4.418	88.98	26.705	0.420
4.80	5	15.56	1.767	4.325	88.72	26.700	0.421
4.80	6	16.36	1.965	4.232	88.46	26.694	0.422
4.80	7	17.19	2.180	4.140	88.20	26.687	0.423
4.80	8	18.04	2.412	4.047	87.94	26.679	0.425
4.80	9	18.93	2.662	3.955	87.68	26.669	0.427
4.80	10	19.84	2.929	3.862	87.41	26.658	0.429
4.80	11	20.78	3.215	3.769	87.13	26.646	0.431
4.80	12	21.74	3.520	3.676	86.86	26.632	0.433
4.80	13	22.72	3.843	3.582	86.58	26.617	0.436
4.80	14	23.73	4.186	3.489	86.29	26.601	0.439
4.80	15	24.76	4.547	3.396	86.00	26.583	0.443
4.80	16	25.81	4.928	3.304	85.71	26.563	0.446
4.80	17	26.88	5.328	3.212	85.41	26.541	0.450
4.80	18	27.97	5.747	3.121	85.10	26.517	0.454
4.80	19	29.08	6.185	3.030	84.78	26.491	0.459
4.80	20	30.22	6.641	2.940	84.46	26.462	0.464
4.80	21	31.37	7.117	2.851	84.12	26.431	0.469
4.80	22	32.54	7.611	2.762	83.78	26.397	0.475
4.80	23	33.74	8.124	2.675	83.42	26.360	0.482
4.80	24	34.95	8.655	2.589	83.05	26.319	0.488
4.80	25	36.19	9.205	2.503	82.66	26.275	0.496
4.80	26	37.45	9.773	2.418	82.26	26.226	0.504
4.80	27	38.74	10.359	2.334	81.84	26.172	0.513
4.80	28	40.05	10.964	2.251	81.40	26.112	0.522
4.80	29	41.40	11.588	2.169	80.94	26.046	0.533
4.80	30	42.78	12.231	2.087	80.44	25.972	0.544
4.80	31	44.19	12.894	2.005	79.92	25.889	0.557
4.80	32	45.65	13.578	1.923	79.35	25.796	0.571
4.80	33	47.16	14.284	1.841	78.75	25.689	0.586
4.80	34	48.73	15.016	1.759	78.08	25.566	0.604
4.80	35	50.37	15.777	1.675	77.34	25.422	0.624
4.80	36	52.11	16.573	1.590	76.52	25.252	0.647
4.80	37	53.97	17.413	1.501	75.57	25.043	0.675
4.80	38	56.02	18.317	1.408	74.43	24.777	0.709
4.80	39	58.37	19.321	1.304	73.00	24.416	0.754
4.80	40	61.37	20.542	1.179	70.93	23.842	0.823
4.90	0	11.76	0.997	4.903	90.00	27.845	0.417
4.90	1	12.42	1.129	4.798	89.75	27.844	0.417

(continued)

**Table A.3** (Continued)

Weak solution				Strong solution			
$M_1$	$\theta$	$\beta$	$p_2/p_1$	$M_2$	$\beta$	$p_2/p_1$	$M_2$
4.90	2	13.09	1.271	4.699	89.50	27.843	0.417
4.90	3	13.80	1.427	4.601	89.24	27.840	0.418
4.90	4	14.54	1.599	4.505	88.99	27.836	0.418
4.90	5	15.31	1.786	4.409	88.74	27.831	0.419
4.90	6	16.11	1.990	4.314	88.48	27.825	0.420
4.90	7	16.94	2.212	4.218	88.22	27.818	0.422
4.90	8	17.80	2.451	4.123	87.96	27.809	0.423
4.90	9	18.69	2.709	4.027	87.70	27.800	0.425
4.90	10	19.60	2.986	3.931	87.43	27.789	0.427
4.90	11	20.54	3.282	3.835	87.16	27.776	0.429
4.90	12	21.50	3.597	3.739	86.89	27.762	0.432
4.90	13	22.49	3.933	3.642	86.61	27.747	0.434
4.90	14	23.50	4.288	3.546	86.33	27.730	0.438
4.90	15	24.53	4.663	3.451	86.04	27.711	0.441
4.90	16	25.59	5.058	3.356	85.75	27.691	0.444
4.90	17	26.66	5.473	3.261	85.45	27.668	0.448
4.90	18	27.76	5.908	3.167	85.14	27.644	0.453
4.90	19	28.87	6.363	3.074	84.83	27.617	0.457
4.90	20	30.00	6.837	2.982	84.51	27.588	0.462
4.90	21	31.16	7.331	2.890	84.18	27.556	0.467
4.90	22	32.33	7.845	2.800	83.84	27.522	0.473
4.90	23	33.53	8.378	2.711	83.48	27.484	0.479
4.90	24	34.74	8.930	2.622	83.12	27.442	0.486
4.90	25	35.98	9.501	2.535	82.74	27.397	0.494
4.90	26	37.24	10.091	2.449	82.34	27.347	0.501
4.90	27	38.53	10.700	2.363	81.93	27.292	0.510
4.90	28	39.84	11.329	2.279	81.49	27.231	0.519
4.90	29	41.18	11.976	2.195	81.03	27.164	0.530
4.90	30	42.55	12.644	2.112	80.55	27.089	0.541
4.90	31	43.96	13.332	2.029	80.03	27.005	0.553
4.90	32	45.42	14.042	1.946	79.48	26.910	0.567
4.90	33	46.92	14.775	1.864	78.88	26.803	0.582
4.90	34	48.47	15.533	1.780	78.23	26.679	0.600
4.90	35	50.10	16.320	1.696	77.51	26.534	0.619
4.90	36	51.82	17.143	1.611	76.70	26.363	0.642
4.90	37	53.66	18.009	1.522	75.78	26.155	0.669
4.90	38	55.67	18.936	1.429	74.69	25.892	0.702
4.90	39	57.95	19.957	1.327	73.33	25.540	0.745
4.90	40	60.77	21.166	1.207	71.44	25.005	0.807
5.00	0	11.52	0.996	5.003	90.00	29.000	0.415
5.00	1	12.18	1.131	4.895	89.75	28.999	0.415

(continued)

**Table A.3** (Continued)

$M_1$	Weak solution			Strong solution			
	$\theta$	$\beta$	$p_2/p_1$	$M_2$	$\beta$	$p_2/p_1$	$M_2$
5.00	2	12.85	1.277	4.792	89.50	28.998	0.416
5.00	3	13.56	1.437	4.692	89.25	28.995	0.416
5.00	4	14.30	1.613	4.592	89.00	28.991	0.417
5.00	5	15.07	1.806	4.493	88.75	28.986	0.418
5.00	6	15.88	2.016	4.395	88.49	28.980	0.419
5.00	7	16.71	2.244	4.296	88.24	28.972	0.420
5.00	8	17.57	2.491	4.197	87.98	28.964	0.422
5.00	9	18.46	2.757	4.098	87.72	28.954	0.423
5.00	10	19.38	3.043	3.999	87.45	28.942	0.425
5.00	11	20.32	3.350	3.900	87.19	28.930	0.428
5.00	12	21.28	3.676	3.801	86.91	28.915	0.430
5.00	13	22.27	4.024	3.702	86.64	28.900	0.433
5.00	14	23.29	4.392	3.603	86.36	28.882	0.436
5.00	15	24.32	4.781	3.504	86.08	28.863	0.439
5.00	16	25.38	5.190	3.406	85.79	28.842	0.443
5.00	17	26.45	5.621	3.309	85.49	28.819	0.447
5.00	18	27.55	6.072	3.212	85.19	28.794	0.451
5.00	19	28.67	6.544	3.117	84.88	28.767	0.455
5.00	20	29.80	7.037	3.022	84.56	28.737	0.460
5.00	21	30.96	7.550	2.929	84.23	28.705	0.465
5.00	22	32.13	8.083	2.836	83.89	28.670	0.471
5.00	23	33.33	8.637	2.745	83.54	28.631	0.477
5.00	24	34.54	9.210	2.655	83.18	28.589	0.484
5.00	25	35.78	9.803	2.566	82.81	28.542	0.491
5.00	26	37.04	10.416	2.478	82.41	28.591	0.499
5.00	27	38.32	11.048	2.391	82.01	28.435	0.508
5.00	28	39.63	11.701	2.305	81.58	28.374	0.517
5.00	29	40.97	12.373	2.220	81.12	28.305	0.527
5.00	30	42.34	13.066	2.136	80.65	28.229	0.538
5.00	31	43.75	13.780	2.052	80.14	28.144	0.550
5.00	32	45.20	14.516	1.968	79.59	28.048	0.564
5.00	33	46.69	15.276	1.885	79.00	27.938	0.579
5.00	34	48.24	16.061	1.801	78.37	27.813	0.596
5.00	35	49.86	16.876	1.716	77.66	27.668	0.615
5.00	36	51.56	17.725	1.630	76.88	27.496	0.638
5.00	37	53.37	18.618	1.542	75.98	27.287	0.664
5.00	38	55.35	19.570	1.449	74.93	27.027	0.695
5.00	39	57.57	20.612	1.349	73.63	26.683	0.736
5.00	40	60.26	21.821	1.233	71.87	26.175	0.794
5.00	41	64.65	23.652	1.055	68.40	25.048	0.914

Note: In Table A.3  $\theta$  and  $\beta$  values are in degrees.

## SOLUTIONS

### P.1 ■ Solution

The pressure behind the wave is given by

$$\frac{p_2}{p_1} = 1 + \frac{2\gamma}{\gamma+1} (M_1^2 - 1) \rightarrow \frac{p_2}{0.6} = 1 + \frac{2 \times 1.4}{1.4+1} \times (2.4^2 - 1)$$

$$\therefore p_2 = 0.6 \times 6.55 = \boxed{3.93 \text{ atm}}$$

The temperature behind the wave is given by

$$\frac{T_2}{T_1} = \left[ 1 + \frac{2\gamma}{\gamma+1} (M_1^2 - 1) \right] \left[ \frac{2 + (\gamma-1)M_1^2}{(\gamma+1)M_1^2} \right] \rightarrow \frac{T_2}{300} = \left[ 1 + \frac{2 \times 1.4}{1.4+1} \times (2.4^2 - 1) \right] \times \left[ \frac{2 + (1.4-1) \times 2.4^2}{(1.4+1) \times 2.4^2} \right]$$

$$T_2 = 2.04 \times 300 = \boxed{612 \text{ K}}$$

The Mach number behind the wave is given by

$$M_2^2 = \frac{1 + [(\gamma-1)/2]M_1^2}{\gamma M_1^2 - (\gamma-1)/2} \rightarrow M_2 = \sqrt{\frac{1 + [(\gamma-1)/2] \times 2.4^2}{1.4 \times 2.4^2 - (\gamma-1)/2}}$$

$$\therefore M_2 = \boxed{0.523}$$

For  $T_2 = 612 \text{ K}$ , the speed of sound is

$$a_2 = \sqrt{\gamma R T_2} = \sqrt{1.4 \times 287 \times 612} = 496 \text{ m/s}$$

so that

$$V_2 = M_2 a_2 = 0.523 \times 496 = \boxed{259 \text{ m/s}}$$

★ Statements **1** and **2** are true, whereas statements **3** and **4** are false.

### P.2 ■ Solution

First of all, the speed of sound upstream of the wave is

$$a_1 = \sqrt{\gamma R T_1} = \sqrt{1.4 \times 287 \times 310} = 353 \text{ m/s}$$

As illustrated below, the ratio of the speed upstream of the shock to the speed downstream of the shock is such that

$$\frac{u_1}{u_2} = \frac{C_s}{C_s - u_p} = \frac{1 + [(\gamma-1)/2]M_1^2}{\gamma M_1^2 + (\gamma-1)/2}$$

where  $C_s$  is the wave speed and  $u_p$  is the piston speed. Also,  $M_1 = C_s/a_1$ . Note that the rightmost side of the equation above can be restated as

$$\frac{1 + [(\gamma-1)/2]M_1^2}{\gamma M_1^2 + (\gamma-1)/2} = \frac{(\gamma+1)M_1^2}{(\gamma-1)M_1^2 + 2}$$

so that

$$\frac{C_s}{C_s - u_p} = \frac{(\gamma+1) \left( \frac{C_s}{a_1} \right)^2}{(\gamma-1) \left( \frac{C_s}{a_1} \right)^2 + 2}$$

$$\therefore (\gamma-1) \frac{C_s^2}{a_1^2} + 2 = (\gamma+1) \frac{C_s}{a_1^2} (C_s - u_p)$$

$$\therefore (\gamma-1) C_s^2 + 2a_1^2 = (\gamma+1) C_s^2 - (\gamma+1) u_p C_s$$

$$\therefore 2C_s^2 - (\gamma + 1)u_p C_s - 2a_1^2 = 0$$

$$\therefore C_s^2 - \left(\frac{\gamma + 1}{2}\right)u_p C_s - a_1^2 = 0$$

$$\therefore M_1^2 - \left(\frac{\gamma + 1}{2}\right)\frac{u_p}{a_1} M_1 - 1 = 0$$

Note that we have arrived at a second-degree equation in  $M_1$ . The solutions are

$$M_1 = \frac{1}{2} \left\{ \frac{(\gamma + 1)}{2} \frac{u_p}{a_1} \pm \sqrt{\left[ \frac{(\gamma + 1)}{2} \frac{u_p}{a_1} \right]^2 + 4} \right\}$$

$$\therefore M_1 = \frac{1}{2} \left\{ \frac{(1.4 + 1)}{2} \times \frac{130}{353} + \sqrt{\left[ \frac{(1.4 + 1)}{2} \times \frac{130}{353} \right]^2 + 4} \right\} = 1.25$$

Here we have neglected the solution with the negative sign, which is a physically meaningless negative number. It follows that the wave speed is

$$C_s = M_1 a_1 = 1.25 \times 353 = \boxed{441 \text{ m/s}}$$

The pressure on the face of the piston, in turn, is

$$\frac{p_2}{p_1} = 1 + \frac{2\gamma}{\gamma + 1} (M_1^2 - 1) \rightarrow \frac{p_2}{1.2} = 1 + \frac{2 \times 1.4}{1.4 + 1} \times (1.25^2 - 1)$$

$$\therefore p_2 = 1.2 \times 1.66 = 1.99 \text{ atm} = \boxed{202 \text{ kPa}}$$

★ The correct answer is **A**.

### P.3 ■ Solution

With the piston moving to the left, the pressure ratio can be straightforwardly determined with the isentropic flow relation

$$\frac{p_2}{p_1} = \left( 1 - \frac{\gamma - 1}{2} \frac{|u_p|}{a_1} \right)^{\frac{2\gamma}{\gamma - 1}}$$

where

$$a_1 = \sqrt{\gamma RT_1} = \sqrt{1.4 \times 287 \times 340} = 370 \text{ m/s}$$

so that

$$\frac{p_2}{p_1} = \left( 1 - \frac{1.4 - 1}{2} \times \frac{150}{370} \right)^{\frac{2 \times 1.4}{1.4 - 1}} = 0.553$$

$$\therefore p_2 = 0.553 \times p_1 = 0.553 \times 1.25 = \boxed{0.691 \text{ atm}}$$

Now, modelling the piston moving to the right, we apply the same reasoning as we did in the previous problem and arrive at the similar (identical, actually) equation

$$\frac{u_1}{u_2} = \frac{C_s}{C_s - u_p} = \frac{(\gamma + 1)M_1^2}{(\gamma - 1)M_1^2 + 2}$$

$$\therefore (\gamma - 1)M_1^2 + 2 = (\gamma + 1)M_1^2 - (\gamma + 1) \frac{u_p}{a_1} M_1$$

$$\therefore M_1^2 - \frac{\gamma + 1}{2} \frac{u_p}{a_1} M_1 - 1 = 0$$

Here,

$$\frac{\gamma+1}{2} \frac{u_p}{a_1} = \frac{1.4+1}{2} \times \frac{150}{370} = 0.486$$

so that

$$M_1^2 - \frac{\gamma+1}{2} \frac{u_p}{a_1} M_1 - 1 = 0 \rightarrow M_1^2 - 0.486 M_1 - 1 = 0$$

$$\therefore M_1 = 1.27$$

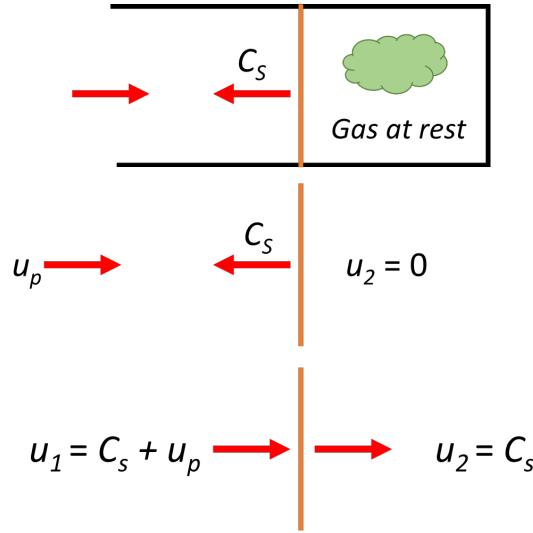
The pressure ratio across the shock is then

$$\frac{p_2}{p_1} = 1 + \frac{2\gamma}{\gamma+1} (M_1^2 - 1) = 1 + \frac{2 \times 1.4}{1.4+1} \times (1.27^2 - 1) = 1.72$$

$$\therefore p_2 = 1.72 p_1 = 1.72 \times 1.25 = \boxed{2.15 \text{ atm}}$$

★ The correct answer is C.

#### P.4 ■ Solution



The velocity of the wave relative to the pipe is  $C_s$ , and the velocity of the piston is  $u_p$ . The velocity of air entering the normal shock wave relative to the shock wave is  $u_1$ , namely

$$u_1 = C_s + u_p \rightarrow M_1 = \frac{C_s + u_p}{a_1}$$

The ratio of velocities is (equation 4)

$$\frac{u_1}{u_2} = \frac{1 + [(\gamma-1)/2] M_1^2}{\gamma M_1^2 + (\gamma-1)/2} = \frac{C_s + u_p}{C_s}$$

Noting that  $C_s = M_1 a_1 - u_p$ , the relation above becomes

$$\frac{u_1}{u_2} = \frac{1 + [(\gamma-1)/2] M_1^2}{\gamma M_1^2 + (\gamma-1)/2} = \frac{(M_1 a_1 - u_p) + u_p}{M_1 a_1 - u_p} \rightarrow \frac{(\gamma+1) M_1^2}{2 + (\gamma-1) M_1^2} = \frac{M_1 a_1}{M_1 a_1 - u_p}$$

$$\therefore \frac{(\gamma+1) M_1^2}{2 + (\gamma-1) M_1^2} = \frac{M_1}{M_1 - \frac{u_p}{u_1}}$$

$$\therefore M_1^2 - \frac{\gamma+1}{2} \left( \frac{u_p}{a_1} \right) M_1 - 1 = 0$$

The speed of sound  $a_1$  is given by

$$a_1 = \sqrt{\gamma R T_1} = \sqrt{1.4 \times 287 \times 350} = 375 \text{ m/s}$$

Substituting in the second-degree equation for  $M_1$ ,

$$M_1^2 - \frac{1.4+1}{2} \times \left( \frac{150}{375} \right) M_1 - 1 = 0 \rightarrow M_1^2 - 0.48 M_1 - 1 = 0$$

$$\therefore M_1 = -0.788, 1.27$$

Naturally, we ignore the solution with the negative sign. The wave speed follows as

$$C_s = M_1 a_1 - u_p = 1.27 \times 375 - 150 = 326 \text{ m/s}$$

The pressure downstream of the shock is given by

$$\frac{p_2}{p_1} = 1 + \frac{2\gamma}{\gamma+1} (M_1^2 - 1) \rightarrow \frac{p_2}{160,000} = 1 + \frac{2 \times 1.4}{1.4+1} \times (1.27^2 - 1)$$

$$\therefore p_2 = 1.72 \times 160,000 = 275 \text{ kPa}$$

Since the gas is at rest, its pressure  $p_{02} = p_2 = 275 \text{ kPa}$ .

The temperature downstream of the shock is given by

$$\frac{T_2}{T_1} = \left[ 1 + \frac{2\gamma}{\gamma+1} (M_1^2 - 1) \right] \left[ \frac{2 + (\gamma-1)M_1^2}{(\gamma+1)M_1^2} \right] \rightarrow \frac{T_2}{350} = \left[ 1 + \frac{2 \times 1.4}{1.4+1} \times (1.27^2 - 1) \right] \times \left[ \frac{2 + (1.4-1) \times 1.27^2}{(1.4+1) \times 1.27^2} \right]$$

$$\therefore T_2 = 1.17 \times 350 = 410 \text{ K}$$

Since the gas is at rest, its temperature  $T_{02} = T_2 = 410 \text{ K}$ .

★ Statements **1** and **2** are true, whereas statement **3** is false.

## P.5 ■ Solution

**Part 1:** Let subscripts 1 and 2 refer to conditions upstream and downstream of the normal shock, respectively. From the normal shock table, with  $M_1 = 2.5$ , we read  $p_2/p_1 = 7.1250$ , which can be used to determine pressure ratio  $p_{02}/p_1 = 8.5261$ . Equation 7 works just as well,

$$\frac{p_{02}}{p_1} = \frac{\left( \frac{\gamma+1}{2} M_1^2 \right)^{\gamma/(\gamma-1)}}{\left( \frac{2\gamma}{\gamma+1} M_1^2 - \frac{\gamma-1}{\gamma+1} \right)^{1/(\gamma-1)}} = \frac{\left( \frac{1.4+1}{2} \times 2.5^2 \right)^{1.4/(1.4-1)}}{\left( \frac{2 \times 1.4}{1.4+1} \times 2.5^2 - \frac{1.4-1}{1.4+1} \right)^{1/(1.4-1)}} = 8.526$$

The static pressure just downstream of the shock is

$$p_2 = 7.13 \times 1.0 = 7.13 \text{ atm}$$

If a normal shock has to be positioned at the nozzle exit, the back pressure to which the nozzle discharges has to be equal to the pressure downstream of the shock. The total pressure downstream of the shock is then

$$p_{02} = 8.53 \times 1.0 = 8.53 \text{ atm} = 864 \text{ kPa}$$

That is, the back pressure has to be 864 kPa to position a normal shock at the nozzle exit.

★ The correct answer is **C**.

**Part 2:** Referring to the isentropic table, for  $M_e = 2.5$ , we read

$$\frac{p_e}{p_{0e}} = 0.0585 ; \frac{A_e}{A^*} = 2.637$$

where subscripts 0, e and \* denote stagnation, exit, and throat, respectively. The throat area is

$$A^* = \frac{A_e}{2.637} = \frac{5.0}{2.637} = 1.90 \text{ cm}^2$$

and

$$p_{0e} = \frac{1}{0.0585} = 17.1 \text{ atm}$$

For the normal shock, the upstream Mach number is 1.5. From the isentropic table, we read

$$\frac{A_1}{A^*} = 1.176 ; \frac{p_1}{p_{01}} = 0.2724 ; \frac{T_1}{T_{01}} = 0.6897$$

so that

$$A_1 = 1.176 \times 1.90 = 2.23 \text{ cm}^2$$

$$p_1 = 0.2724 \times 17.1 = 4.66 \text{ atm}$$

$$T_1 = 0.6897 \times 480 = 331 \text{ K}$$

From a normal shock table, for  $M_1 = 1.5$ , we see that

$$\frac{p_2}{p_1} = 2.4583 ; \frac{T_2}{T_1} = 1.3202 ; \frac{p_{02}}{p_1} = 3.4133 ; M_2 = 0.7011$$

The back pressure required is  $p_{02}$ , namely

$$p_{02} = 4.66 \times 3.4133 = \boxed{15.9 \text{ atm}}$$

Downstream of the shock, the flow is isentropic up to the nozzle exit. However, for this flow the effective throat area is not the same as  $A^*$ , since  $p_{02} < p_{01}$ . Let the effective throat area downstream of the shock be  $A_2^*$ . From the isentropic table, with  $M_2 \approx 0.70$ ,

$$\frac{A_2}{A_2^*} = 1.094$$

where  $A_2$  is the area of the shock location, which is the same as  $A_1$ . Accordingly,

$$A_2^* = \frac{2.23}{1.094} = 2.04 \text{ cm}^2$$

and

$$\frac{A_e}{A_2^*} = \frac{4}{2.04} = 1.96$$

Mapping this area ratio onto the isentropic table, we see that the corresponding temperature ratio and exit Mach number are, by interpolation,

$$\frac{T_2}{T_e} = 0.9343 ; M_e = 0.313$$

Thus,

$$T_2 = 0.9343 \times 480 = \boxed{448 \text{ K}}$$

The speed of sound is

$$a_e = \sqrt{1.4 \times 287 \times 448} = 424 \text{ m/s}$$

The flow velocity, in turn, is

$$V_e = M_e a_e = 0.313 \times 424 = \boxed{133 \text{ m/s}}$$

★ Statement 1 is true, whereas statements 2 and 3 are false.

## P.6 ■ Solution

Since the Mach number upstream of the shock is 2.30, the area ratio corresponding to this Mach number will give the area at the shock location. For  $M_1 = 2.30$ , we have, from the isentropic table,

$$\frac{A_1}{A^*} = 2.193$$

Thus, the area at the shock location is  $A_1 = 8 \times 2.193 = 17.5 \text{ cm}^2$ .

Referring to the normal shock table, the Mach number  $M_2$  downstream of the shock is found to be 0.5344. In turn, we enter this Mach number into the isentropic flow table and find that it corresponds to an area ratio

$$\frac{A_2}{A^*} = 1.286$$

Since  $A_1 = A_2 = \text{area at the shock location}$ , we have

$$A_2^* = \frac{A_2}{1.286} = \frac{17.5}{1.286} = 13.6 \text{ cm}^2$$

Therefore,

$$\frac{A_e}{A_2^*} = \frac{19.0}{13.6} = 1.40$$

Entering this area ratio into the isentropic table, we find that the exit Mach number is  $M_e \approx 0.47$ .

For the given nozzle, the area ratio  $A_e/A_{th}$  is

$$\frac{A_e}{A_{th}} = \frac{A_e}{A^*} = \frac{19}{8} = 2.38$$

Appealing to the isentropic table, we see that this area ratio corresponds to a Mach number and a pressure ratio

$$M_e = 2.39 ; \frac{p_2}{p_{02}} = 0.0695$$

For complete isentropic flow,  $p_{02} = p_0 = 820 \text{ kPa}$ . Thus,  $p_2 = 0.0695 \times 820 = 57.0 \text{ kPa}$ . That is, in order for the flow to be completely isentropic the back pressure must be lower than 57 kPa; a back pressure of 60 kPa would not allow flow throughout the duct to be fully isentropic.

★ Statement 2 is true, whereas statements 1 and 3 are false.

### P.7 ■ Solution

The shock must be located in the divergent portion of the nozzle, since only after the throat does the flow become supersonic. The Mach number  $M_1$  just upstream of the shock can be determined by entering area ratio  $A_{shock}/A_{th} = 2000/1000 = 2$  into the isentropic table; doing so, we find

$$M_1 = 2.2 ; \frac{p_1}{p_{01}} = 0.0935$$

Up to the location of the shock, the stagnation pressure does not change, hence

$$p_1 = 0.0935 \times 250 = 23.4 \text{ kPa}$$

Now, entering  $M_1 = 2.2$  onto the normal shock table, we read

$$M_2 = 0.5471 ; \frac{p_2}{p_1} = 5.4800 ; \frac{p_{02}}{p_{01}} = 0.6281$$

Here, subscript 2 denotes conditions just downstream of the shock. Thus,

$$p_2 = 5.48 \times 23.4 = 128 \text{ kPa}$$

$$p_{02} = 0.6281 \times 250 = 157 \text{ kPa}$$

Next, entering  $M_2 = 0.55$  onto the isentropic table, we get the ratio

$$\frac{A_2}{A_{th,2}} = 1.255$$

where  $A_{th,2}$  is the throat area required for the flow downstream of the shock to choke,

$$A_{th,2} = \frac{A_2}{1.255}$$

But  $A_2 = A_1 = A_{shock} = 2000 \text{ mm}^2$ , hence

$$A_{th,2} = \frac{2000}{1.255} = 1590 \text{ mm}^2$$

and

$$\frac{A_e}{A_{th,2}} = \frac{3000}{1590} = 1.89$$

For this area ratio, we appeal to the subsonic part of the isentropic table and read  $M_e = 0.325$ . It remains to determine the exit pressure  $p_e$ ,

$$\frac{p_{02}}{p_e} = \left(1 + \frac{\gamma - 1}{2} M_e^2\right)^{\gamma/(\gamma-1)} \rightarrow \frac{157}{p_e} = \left(1 + \frac{1.4 - 1}{2} \times 0.325^2\right)^{1.4/(1.4-1)}$$

$$\therefore p_e = \frac{157}{1.08} = \boxed{145 \text{ kPa}}$$

Note that this corresponds to a pressure loss  $\Delta p_0 = p_{01} - p_{02} = 250 - 157 = 93 \text{ kPa}$  across the nozzle.

★ The correct answer is C.

### P.8 ■ Solution

The Pitot tube will read the actual total pressure in a subsonic stream. But in a supersonic flow, the pressure measured by a Pitot probe is the total pressure downstream of a detached shock which stands at the nose of the Pitot tube. Therefore, it is essential to find whether the flow is subsonic or supersonic.

Referring to the isentropic table, we see that for  $M = 1$  the pressure ratio  $p/p_0 = 0.528$ . Hence,

$$p_0 = \frac{p}{0.528} = \frac{0.95}{0.528} = 1.799 \approx 1.80 \text{ atm}$$

Thus, when  $p_0 < 1.8 \text{ atm}$ , the flow is subsonic, and when  $p_0 > 1.8 \text{ atm}$ , the flow is supersonic.

With  $p = 1.3 \text{ atm}$ , the flow is subsonic and hence the Pitot tube is measuring the actual total pressure of the flow. The pressure ratio is

$$\frac{p}{p_0} = \frac{1.3}{1.80} = 0.722$$

Entering this ratio into the isentropic table, we interpolate and read a Mach number  $M \approx 0.698$ .

With  $p = 2.6 \text{ atm}$ , the flow is supersonic and the Pitot tube measures  $p_{02}$  behind a normal (detached) shock. Mathematically,

$$\frac{p_{02}}{p_1} = \frac{2.5}{0.95} = 2.63$$

Entering this pressure ratio into the normal shock table, we get  $M \approx 1.28$ .

With  $p = 10 \text{ atm}$ , the flow is supersonic and the Pitot tube measures  $p_{02}$  behind a normal (detached) shock. In mathematical terms,

$$\frac{p_{02}}{p_1} = \frac{10}{0.95} = 10.5$$

Entering this pressure ratio into the normal shock table, we obtain  $M \approx 2.79$ .

★ Statement **1** is true, whereas statements **2** and **3** are false.

### P.9 ■ Solution

Let subscripts 1 and 2 denote conditions ahead of and behind the shock, respectively, and  $e$  denote the nozzle exit. Given

$$\frac{p_{01} - p_{02}}{p_{01}} \times 100\% = 12.4 \rightarrow 1 - \frac{p_{02}}{p_{01}} = 0.124$$

$$\therefore \frac{p_{02}}{p_{01}} = 0.876$$

Entering this stagnation pressure ratio into the normal shock table, we read

$$M_1 = 1.65 ; M_2 = 0.654 ; \frac{T_2}{T_1} = 1.4228$$

Now, entering  $M_1 = 1.65$  into the isentropic table, we get for temperature

$$\frac{T_1}{T_{01}} = 0.6475 \rightarrow T_1 = 0.6475 \times 345 = 223 \text{ K}$$

and

$$T_2 = 1.4228 \times 223 = 317 \text{ K}$$

At this point, we evoke the energy equation,

$$h_e + \frac{V_e^2}{2} = h_{0e} \rightarrow c_p T_e + \frac{V_e^2}{2} = c_p T_{0e}$$

Since the flow across the shock is adiabatic, we must have  $T_{01} = T_{02} = T_{0e}$ . It follows that the flow speed at the nozzle exit becomes

$$V_e = \sqrt{2c_p(T_{0e} - T_e)} = \sqrt{2 \times 1000 \times (345 - 295)} = 316 \text{ m/s}$$

The flow speed behind the shock is

$$V_2 = M_2 a_2 = 0.654 \times \sqrt{1.4 \times 287 \times 317} = 233 \text{ m/s}$$

The mass flow rate is given by (equation 2)

$$\dot{m} = \frac{0.6847 p_{01} A_{th}}{\sqrt{RT_{01}}} = \frac{0.6847 \times (6 \times 101,325) \times (7 \times 10^{-4})}{\sqrt{287 \times 345}} = 0.926 \text{ kg/s}$$

★ Statements **2** and **3** are true, whereas statements **1** and **4** are false.

### P.10 ■ Solution

Given  $M_1 = 2.4$  and  $\beta = 42^\circ$ , the normal Mach number is  $M_{1n} = 2.4 \times \sin 42^\circ = 1.61$ . Referring to the normal shock table, we see that

$$\frac{p_2}{p_1} = 2.8574 ; \frac{T_2}{T_1} = 1.3949 ; M_2 = 0.6655$$

It follows that, for pressure,

$$p_2 = 2.8574 \times 52 = 149 \text{ kPa}$$

and, for temperature,

$$T_2 = 1.3949 \times (273 + 2) = 384 \text{ K}$$

For an adiabatic process, the total temperatures behind and after the shock are equal, i.e.  $T_{t1} = T_{t2}$ . From the isentropic table, for Mach 2.4, we read  $T_1/T_t = 0.4647$ . Thus,

$$T_{1t} = T_{2t} = \frac{275}{0.4647} = 592 \text{ K}$$

Next, we enter  $T_2/T_{2t} = 384/592 = 0.649$  into the isentropic table to obtain

$$M_2 = 1.65$$

It remains to find the wedge angle  $2\theta$ ,

$$\begin{aligned}\sin(\beta - \theta) &= \frac{u_2}{w_2} \rightarrow \beta - \theta = \arcsin\left(\frac{0.6655}{1.65}\right) \\ \therefore \beta - \theta &= 23.79^\circ \\ \therefore \theta &= 42^\circ - 23.79^\circ = 18.21^\circ \\ \therefore 2\theta &= 36.42^\circ\end{aligned}$$

★ Statements **2** and **4** are true, whereas statements **1** and **3** are false.

### P.11 ■ Solution

The flow deflection angle is related to Mach number  $M_1$  and shock angle  $\beta$  by equation 8,

$$\tan \theta = 2 \cot \beta \frac{M_1^2 \sin^2 \beta - 1}{M_1^2 (\gamma + \cos 2\beta) + 2}$$

Substituting  $\theta = 13^\circ$ ,  $M_1 = 2.1$ , and  $\gamma = 1.4$  brings to

$$\tan 13^\circ = \frac{2 \cot \beta (2.2^2 \times \sin^2 \beta - 1)}{2.2^2 \times (1.4 + \cos 2\beta) + 2}$$

This transcendental equation can be easily solved with Mathematica's *Solve* command,

$$\text{In[137]:= } \text{Solve}[\text{Tan}[13^\circ] == \frac{2 * \text{Cot}[\text{b}] * (2.2^2 * \text{Sin}[\text{b}]^2 - 1)}{2.2^2 * (1.4 + \text{Cos}[2 * \text{b}]) + 2}, \text{b}]$$

... Solve: Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information.

Out[137]= { {b \rightarrow -2.46156}, {b \rightarrow -1.6954}, {b \rightarrow -0.328531}, {b \rightarrow 0.680028}, {b \rightarrow 1.44619}, {b \rightarrow 2.81306} }

We are interested only in the first two positive solutions,  $\beta_w = 0.680 \text{ rad} = 39.0^\circ$ , which is the weak shock solution, and  $\beta_s = 1.45 \text{ rad} = 83.1^\circ$ , which is the strong shock solution. For the strong shock, the pressure ratio is

$$\frac{p_2}{p_1} = \frac{2\gamma}{\gamma+1} M_1^2 \sin^2 \beta - \frac{\gamma-1}{\gamma+1} = \frac{2 \times 1.4}{1.4+1} \times 2.2^2 \times \sin^2 83.1^\circ - \frac{1.4-1}{1.4+1} = 5.40$$

Given angles  $\beta = 83.1^\circ$  and  $\theta = 13^\circ$ , the density ratio is calculated as

$$\frac{\rho_2}{\rho_1} = \frac{\tan \beta}{\tan(\beta - \theta)} = \frac{\tan 83.1^\circ}{\tan(83.1^\circ - 13^\circ)} = 2.99$$

Alternatively, we can use equation 10,

$$\frac{\rho_2}{\rho_1} = \frac{(\gamma+1)M_1^2 \sin^2 \beta}{(\gamma-1)M_1^2 \sin^2 \beta + 2} = \frac{(1.4+1) \times 2.2^2 \times \sin^2 83.1^\circ}{(1.4-1) \times 2.2^2 \times \sin^2 83.1^\circ + 2} = 2.93$$

Equipped with the preceding results, determining the temperature ratio is straightforward,

$$\frac{T_2}{T_1} = \frac{p_2/p_1}{\rho_2/\rho_1} = \frac{5.40}{2.99} = 1.81$$

We could just as well apply equation 11,

$$\frac{T_2}{T_1} = 1 + \frac{2(\gamma-1)}{(\gamma+1)^2} \frac{M_1^2 \sin^2 \beta - 1}{M_1^2 \sin^2 \beta} (\gamma M_1^2 \sin^2 \beta + 1)$$

$$\therefore \frac{T_2}{T_1} = 1 + \frac{2 \times (1.4 - 1)}{(1.4 + 1)^2} \times \frac{2.2^2 \times \sin^2 83.1^\circ - 1}{2.2^2 \times \sin^2 83.1^\circ} \times (1.4 \times 2.2^2 \times \sin^2 83.1^\circ + 1) = 1.84$$

The Mach number behind the shock is given by

$$M_2 = \frac{M_{n2}}{\sin(\beta - \theta)}$$

where normal component  $M_{n2}$  is expressed as

$$M_{n2}^2 = \frac{M_1^2 \sin^2 \beta + \frac{2}{\gamma - 1}}{\frac{2\gamma}{\gamma - 1} M_1^2 \sin^2 \beta - 1} = \frac{2.2^2 \times \sin^2 83.1^\circ + \frac{2}{1.4 - 1}}{\frac{2 \times 1.4}{1.4 - 1} \times 2.2^2 \times \sin^2 83.1^\circ - 1} = 0.302$$

so that

$$M_2^2 = \frac{0.302}{\sin^2(83.1^\circ - 13^\circ)} = \frac{0.302}{0.884} = 0.362$$

$$\therefore M_2 = \sqrt{0.362} = \boxed{0.602}$$

For the weak shock solution, let us employ the oblique shock table. Entering  $M_1 = 2.2$  and  $\beta = 39^\circ$ , we read

$$\boxed{\frac{p_2}{p_1} = 2.066}$$

As for the density ratio, with  $\theta = 13^\circ$  and  $\beta = 39.0^\circ$ , we find that

$$\frac{\rho_2}{\rho_1} = \frac{\tan \beta}{\tan(\beta - \theta)} = \frac{\tan 39.0^\circ}{\tan(39.0^\circ - 13^\circ)} = \boxed{1.66}$$

The temperature ratio is determined next,

$$\frac{T_2}{T_1} = \frac{p_2/p_1}{\rho_2/\rho_1} = \frac{2.066}{1.66} = \boxed{1.24}$$

From the oblique shock table, we see that the Mach number downstream of the weak shock equals about 1.706.

★ Statements **1, 4, 5**, and **7** are true, whereas statements **2, 3, 6**, and **8** are false.

## P.12 ■ Solution

Entering  $\theta = 18^\circ$  and  $M_1 = 3.2$  into the oblique shock table, we read  $\beta = 34.07^\circ$ . The normal component of the upstream Mach number is

$$M_{1n} = M_1 \sin \beta = 3.2 \times \sin 34.07^\circ = 1.79$$

and can be used to determine  $M_{2n}$ ,

$$M_{2n}^2 = \frac{M_1^2 \sin^2 \beta + \frac{2}{\gamma - 1}}{\frac{2\gamma}{\gamma - 1} M_1^2 \sin^2 \beta - 1} = \frac{1.79^2 + \frac{2}{1.4 - 1}}{\frac{2 \times 1.4}{1.4 - 1} \times 1.79^2 - 1} = 0.383$$

$$\therefore M_{2n} = 0.619$$

Further, we have the pressure ratio

$$\boxed{\frac{p_2}{p_1} = 3.583}$$

the density ratio

$$\frac{\rho_2}{\rho_1} = \frac{\tan \beta}{\tan(\beta - \theta)} = \frac{\tan 34.07^\circ}{\tan(34.07^\circ - 18^\circ)} = 2.35$$

the temperature ratio

$$\frac{T_2}{T_1} = \frac{p_2/p_1}{\rho_2/\rho_1} = \frac{3.583}{2.35} = 1.52$$

and the stagnation pressure ratio

$$\frac{p_{02}}{p_{01}} = 0.8171$$

From the isentropic table, for  $M_1 = 3.2$ , we have

$$\frac{p_1}{p_{01}} = 0.0202 ; \quad \frac{T_1}{T_{01}} = 0.3281$$

Substituting the stagnation parameters we were given,  $p_{01} = 1050$  kPa and  $T_{01} = 320$  K, we find that

$$p_1 = 0.0202 \times 1050 = 21.2 \text{ kPa}$$

$$T_1 = 0.3281 \times 320 = 105 \text{ K}$$

From the ideal gas law,

$$\rho_1 = \frac{p_1}{RT_1} = \frac{21,200}{287 \times 105} = 0.704 \text{ kg/m}^3$$

Using the ratios obtained above, we can determine conditions on the face of the wedge. For pressure,

$$p_2 = 3.583 \times 21.2 = \boxed{76.0 \text{ kPa}}$$

For density,

$$\rho_2 = 2.35 \times 0.704 = \boxed{1.65 \text{ kg/m}^3}$$

For temperature,

$$T_2 = 1.52 \times 105 = \boxed{160 \text{ K}}$$

For stagnation pressure,

$$p_{02} = 0.8171 \times 1050 = \boxed{858 \text{ kPa}}$$

For Mach number,

$$M_2 = \frac{M_{2n}}{\sin(\beta - \theta)} = \frac{0.619}{\sin(34.07^\circ - 18^\circ)} = \boxed{2.24}$$

Lastly, to determine flow speed we require speed of sound  $a_2$ ,

$$a_2 = \sqrt{\gamma RT_2} = \sqrt{1.4 \times 287 \times 160} = 254 \text{ m/s}$$

so that

$$V_2 = M_2 a_2 = 2.24 \times 254 = \boxed{569 \text{ m/s}}$$

★ Statements **1, 5, and 6** are true, whereas statements **2, 3, and 4** are false.

### P.13 ■ Solution

Inspecting the oblique shock table, we see that, for  $M_1 = 3.0$ , the maximum deflection angle for which the shock remains attached to the wedge is about  $34^\circ$ :

Weak solution				Strong solution			
$M_1$	$\theta$	$\beta$	$p_2/p_1$	$M_2$	$\beta$	$p_2/p_1$	$M_2$
3.00	33	58.91	7.533	1.160	70.71	9.188	0.794
3.00	34	63.67	8.267	1.003	66.75	8.697	0.908
3.10	0	18.80	0.998	3.102	89.99	11.045	0.470
3.10	1	19.50	1.083	3.047	89.66	11.045	0.470
3.10	2	20.20	1.171	2.994	89.32	11.043	0.470

To evaluate the second statement, begin at the first row and keep scrolling down until you see a deflection angle  $\theta = 40^\circ$  for the first time; the upstream Mach number listed in the same row as this value is the minimum  $M_1$  for which the oblique shock remains attached to the wedge. In doing so, we find  $M_1 = 4.50$ :

Weak solution				Strong solution			
$M_1$	$\theta$	$\beta$	$p_2/p_1$	$M_2$	$\beta$	$p_2/p_1$	$M_2$
4.50	34	49.60	13.534	1.689	77.56	22.361	0.618
4.50	35	51.30	14.220	1.606	76.76	22.219	0.640
4.50	36	53.10	14.943	1.521	75.85	22.046	0.665
4.50	37	55.07	15.714	1.432	74.79	21.831	0.697
4.50	38	57.29	16.559	1.335	73.47	21.546	0.736
4.50	39	59.98	17.543	1.223	71.70	21.129	0.793
4.50	40	64.33	19.026	1.052	68.25	20.214	0.909

★ The correct answer is **B**.

### P.14 ■ Solution

The first step is to enter  $M_1 = 2.5$  and  $\theta = 12^\circ$  into the oblique shock table, giving

$$M_2 = 2.002$$

Then, we enter  $M_2 \approx 2.0$  into the same table and scroll down to the highest value of  $\theta$  for which there is an attached weak shock solution; in doing so, we find  $\theta_{\max} = 22^\circ$ .

Weak solution				Strong solution			
$M_1$	$\theta$	$\beta$	$p_2/p_1$	$M_2$	$\beta$	$p_2/p_1$	$M_2$
2.00	13	42.78	1.986	1.526	81.49	4.398	0.625
2.00	14	44.03	2.087	1.487	80.69	4.378	0.634
2.00	15	45.34	2.195	1.446	79.83	4.355	0.644
2.00	16	46.73	2.307	1.403	78.92	4.328	0.656
2.00	17	48.20	2.427	1.359	77.94	4.296	0.669
2.00	18	49.79	2.554	1.313	76.86	4.259	0.685
2.00	19	51.51	2.692	1.264	75.66	4.214	0.704
2.00	20	53.42	2.843	1.210	74.27	4.157	0.728
2.00	21	55.64	3.014	1.150	72.59	4.082	0.758
2.00	22	58.46	3.223	1.076	70.33	3.971	0.802
2.10	0	28.42	0.998	2.101	89.99	4.978	0.561

★ The correct answer is **C**.

### P.15 ■ Solution

To find the ratio of stagnation pressures in arrangement 1, all we have to do is enter  $M_1 = 4.0$  into the normal shock table and read

$$\frac{p_{02}}{p_{01}} = 0.1388$$

Accordingly, the pressure loss in arrangement 1 is  $1 - 0.1388 = 86.1\%$ . Moving on to the wedge-shaped diffuser, we map values  $M_1 = 4.0$  and  $\theta = 7^\circ$  onto the oblique shock table and read

$$\beta = 19.63^\circ ; M_2 = 3.498 \approx 3.50$$

Next, we plug  $M_2 = 3.50$  and  $\theta = 7^\circ$  into the same table to obtain

$$\beta = 21.79^\circ ; M_3 = 3.081$$

Accordingly,

$$M_{1n} = M_1 \sin \beta = 4.0 \times \sin 19.63^\circ = 1.34$$

$$M_{2n} = 3.50 \times \sin 21.79^\circ = 1.30$$

From the normal shock table, we have, for  $M_{1n} = 1.34$ ,

$$\frac{p_{02}}{p_{01}} = 0.9718$$

For  $M_{2n} = 1.30$ , in turn,

$$\frac{p_{03}}{p_{01}} = 0.9794$$

and for  $M_3 = 3.081$ ,

$$\frac{p_{04}}{p_{03}} = 0.3065$$

The overall stagnation pressure ratio is

$$\frac{p_{04}}{p_{01}} = 0.9718 \times 0.9794 \times 0.3065 = 0.292$$

Therefore, the pressure loss in arrangement 2 is  $1 - 0.292 = 70.8\%$ .

★ The correct answer is **B**.

### P.16 ■ Solution

**Part 1:** Consider first the oblique shock in the transition from region 1 to region 2. Entering  $M_1 = 2.0$  and  $\theta = 6^\circ$  into the oblique shock table, we read  $\beta = 35.24^\circ$ . Normal Mach number  $M_{1n}$  is then

$$M_{1n} = M_1 \sin \beta = 2.0 \times \sin 35.24^\circ = 1.15$$

Next, we enter  $M_{1n}$  into the normal shock table to glean

$$\frac{p_2}{p_1} = 1.3762 ; \frac{T_2}{T_1} = 1.0966 ; \frac{a_2}{a_1} = 1.0472 ; \frac{p_{02}}{p_{01}} = 0.9967 ; M_{2n} = 0.8750$$

Thus,

$$M_2 = \frac{M_{2n}}{\sin(\beta - \theta)} = \frac{0.8750}{\sin(35.24^\circ - 6^\circ)} = 1.79$$

Also,

$$p_2 = 1.3762 \times p_1 = 1.3762 \times 72 = 99.1 \text{ kPa}$$

$$T_2 = 1.0966 \times T_1 = 1.0966 \times 285 = 313 \text{ K}$$

Now, mapping  $M_2 = 1.79$  onto the normal shock table, we read

$$\frac{p_3}{p_2} = 3.5714 ; \frac{T_3}{T_2} = 1.5241 ; \frac{a_3}{a_2} = 1.2346 ; \frac{p_{03}}{p_{02}} = 0.8171$$

Also,  $M_3 = 0.6188$ . We proceed to determine  $p_3$  and  $T_3$ ,

$$p_3 = 3.5714 \times 99.1 = 354 \text{ kPa}$$

$$T_3 = 1.5241 \times T_2 = 1.5241 \times 313 = 477 \text{ K}$$

To determine the diffuser inlet area, we adjust the mass flow rate equation

$$\dot{m} = \rho_3 A_3 V_3 \rightarrow A_3 = A_i = \frac{\dot{m}}{\rho_3 V_3}$$

Before proceeding, we need density  $\rho_3$ , speed  $V_3$ , and mass flow rate  $\dot{m}$ . Density  $\rho_3$  follows from the ideal gas law,

$$\rho_3 = \frac{p_3}{RT_3} = \frac{354,000}{287 \times 477} = 2.59 \text{ kg/m}^3$$

Velocity  $V_3$  is given by

$$V_3 = M_3 a_3$$

We have  $M_3$ , but  $a_3$  is missing. At section 1,  $M_1 = 2.0$ ,  $T_1 = 285 \text{ K}$ , and  $a_2 = 1.0472 \times a_1$ , or

$$a_2 = 1.0472 \times \sqrt{\gamma R T_1} = 1.0472 \times \sqrt{1.4 \times 287 \times 285} = 354 \text{ m/s}$$

In the transition from 2 to 3, in turn, we have

$$\frac{a_3}{a_2} = 1.2346 \rightarrow a_3 = 1.2346 \times a_2$$

$$\therefore a_3 = 1.2346 \times 354 = 437 \text{ m/s}$$

Finally,

$$V_3 = M_3 a_3 = 0.6188 \times 437 = 270 \text{ m/s}$$

and

$$A_i = \frac{18}{2.59 \times 270} = 0.0257 \text{ m}^2 = \boxed{257 \text{ cm}^2}$$

★ The correct answer is **B**.

**Part 2:** At the exit,  $M_e = V_e/a_e$ . Appealing to the energy equation, we may write

$$\frac{V_e^2}{2} + \frac{a_e^2}{\gamma - 1} = \frac{a_{0e}^2}{\gamma - 1}$$

This can be arranged to yield

$$a_e = \sqrt{a_{0e}^2 - 0.2V_e^2}$$

We know that  $V_e = 40 \text{ m/s}$ , but it remains to determine  $a_{0e}$ . Mathematically,

$$a_{0e} = a_{03} = a_{01} = \sqrt{1.4 \times 287 \times T_{01}} = 20.04 \sqrt{T_{01}}$$

Entering  $M_1 = 2.0$  into the isentropic table, we read

$$\frac{T_1}{T_{01}} = 0.5556$$

so that

$$T_{01} = \frac{285}{0.5556} = 513 \text{ K}$$

and

$$a_{0e} = 20.04 \times \sqrt{513} = 454 \text{ m/s}$$

Then, we can determine speed of sound  $a_e$ ,

$$a_e = \sqrt{454^2 - 0.2 \times 35^2} \approx 454 \text{ m/s}$$

and Mach number  $M_e$ ,

$$M_e = \frac{35}{454} = 0.0771$$

Mapping  $M_e = 0.0771$  onto the isentropic table, we find that

$$\frac{p_e}{p_{03}} = 0.9958 ; \frac{T_e}{T_{03}} = 0.9989$$

Similarly, entering  $M_3 = 0.6188 \approx 0.62$  into the same table,

$$\frac{p_3}{p_{03}} = 0.7716 ; \frac{T_3}{T_{03}} = 0.9286$$

Pressure  $p_e$  is determined next,

$$p_e = \frac{p_e}{p_{03}} \times \frac{p_{03}}{p_3} \times p_3 = 0.9958 \times \frac{1}{0.7716} \times 354 = \boxed{457 \text{ kPa}}$$

It remains to determine the area  $A_e$  at the compressor inlet. We first determine density  $\rho_e$ , which in turn requires temperature  $T_e$ ,

$$\frac{T_3}{T_{03}} = 0.9286 \rightarrow T_{03} = \frac{477}{0.9286} = 514 \text{ K}$$

$$\therefore \frac{T_e}{T_{03}} = 0.9989 \rightarrow T_e = 0.9989 \times 514$$

$$\therefore T_e = 513 \text{ K}$$

Thus,

$$\rho_e = \frac{p_e}{RT_e} = \frac{457,000}{287 \times 513} = 3.10 \text{ kg/m}^3$$

Lastly,

$$A_e = \frac{\dot{m}}{\rho_e V_3} = \frac{18}{3.10 \times 35} = \boxed{0.166 \text{ m}^2}$$

★ The correct answer is **C.**

### P.17 ■ Solution

Consider first arrangement 1. Entering  $M_1 = 2.1$  and  $\theta = 7^\circ$  into the oblique shock table, we read  $\beta = 34.45^\circ$ . Therefore, the Mach number normal to the shock is

$$M_{1n} = M_1 \sin \beta = 2.1 \times \sin 34.45^\circ = 1.19$$

From the normal shock table, for  $M_{1n} = 1.19$ , we have

$$\frac{p_{02}}{p_{01}} = 0.9937 ; M_2 = 0.8485$$

Accordingly,

$$M_2 = \frac{M_{2n}}{\sin(\beta - \theta)} = \frac{0.8485}{\sin(34.45^\circ - 7^\circ)} = 1.84$$

From the normal shock table, for  $M_2 = 1.84$ , we have

$$\frac{p_{03}}{p_{02}} = 0.7948$$

The pressure ratio we aim for is

$$\frac{p_{03}}{p_{02}} = \frac{p_{03}}{p_{02}} \times \frac{p_{02}}{p_{01}} = 0.7948 \times 0.8485 = 0.674$$

Thus, the pressure loss in arrangement 1 is  $1 - 0.674 = 32.6\%$ . Consider now arrangement 2. Entering  $M_2 = 1.84 \approx 1.85$  and  $\theta = 7^\circ$  into the oblique shock table, we read  $\beta = 39.34^\circ$ . Therefore, the Mach number normal to the shock is

$$M_{2n} = M_2 \sin \beta = 1.84 \times \sin 39.34^\circ = 1.17$$

From the normal shock table, with  $M_{2n} = 1.17$ , we have

$$\frac{p_{03}}{p_{02}} = 0.9953 ; M_{3n} = 0.8615$$

Accordingly,

$$M_3 = \frac{M_{3n}}{\sin(\beta - \theta)} = \frac{0.8615}{\sin(39.34^\circ - 7^\circ)} = 1.61$$

From the normal shock table, for  $M_3 = 1.61$ , we read

$$\frac{p_{04}}{p_{03}} = 0.8915$$

Finally,

$$\frac{p_{04}}{p_{01}} = \frac{p_{04}}{p_{03}} \times \frac{p_{03}}{p_{02}} \times \frac{p_{02}}{p_{01}} = 0.8915 \times 0.9953 \times 0.9937 = 0.882$$

and the pressure loss in arrangement 2 is calculated to be  $1 - 0.882 = 11.8\%$ .

★ The correct answer is **A.**

### P.18 ■ Solution

For  $M_1 = 2.5$  and  $\theta = 15^\circ$ , we read  $\beta = 36.95^\circ$  from the oblique shock table. The normal Mach number  $M_{1n}$  follows as

$$M_{1n} = M_1 \sin \beta = 2.5 \times \sin 36.95^\circ = 1.50$$

Entering  $M_{1n} = 1.50$  into the normal shock table, we read

$$M_{2n} = 0.7011 ; \frac{T_2}{T_1} = 1.3202$$

It follows that

$$M_2 = \frac{M_{2n}}{\sin(\beta - \theta)} = \frac{0.7011}{\sin(36.95^\circ - 15^\circ)} = \boxed{1.88}$$

$$T_2 = 1.3202 \times 450 = \boxed{594 \text{ K}}$$

For  $M_2 = 1.88$ , from the isentropic table,  $v_2 = 23.019^\circ$ . Also,  $|\theta| = 15 + 15 = 30^\circ$ . It follows that

$$v_3 = v_2 + |\theta| = 23.019^\circ + 30^\circ = 53.019^\circ$$

Mapping  $v_3 \approx 53.0^\circ$  onto the isentropic table, we see that

$$\boxed{M_3 = 3.17} ; \frac{T_3}{T_0} = 0.3323$$

Therefore,

$$\frac{T_3}{T_2} = \frac{T_3}{T_{02}} \frac{T_{02}}{T_2}$$

But  $T_{02} = T_{03}$ , thus

$$\frac{T_3}{T_2} = \frac{T_3}{T_{02}} \frac{T_{02}}{T_2} = 0.3323 \times \frac{1}{0.5859} = 0.567$$

$$\therefore T_3 = 0.567 \times 594 = \boxed{337 \text{ K}}$$

For  $M_3 = 3.17$  and  $\theta = 15^\circ$ , the oblique shock table gives  $\beta = 31.08^\circ$ , so that

$$M_{3n} = M_3 \sin \beta = 3.17 \times \sin 31.08^\circ = 1.64$$

Entering  $M_{3n}$  into the normal shock table, we read

$$M_{4n} = 0.6568 ; \frac{T_4}{T_3} = 1.4158$$

The final step is to determine Mach number  $M_4$  and temperature  $T_4$ ,

$$M_4 = \frac{0.6568}{\sin(31.08^\circ - 15^\circ)} = \boxed{2.37}$$

$$T_4 = 1.4158 \times 337 = \boxed{477 \text{ K}}$$

★ Statements **3, 4, 6** and **7** are true, whereas statements **1, 2, 5**, and **8** are false.

### P.19 ■ Solution

Consider first the upper surface. For  $M_1 = 2.5$ , from the isentropic table, we have  $v_1 = 39.124^\circ$ . It follows that

$$v_2 = 39.124^\circ + 10^\circ = 49.124^\circ$$

Entering  $v_2 = 49.1^\circ$  into the isentropic table, we see that

$$M_2 = 2.97$$

The flow Mach number normal to the oblique shock, we have  $M_{2n} = M_2 \sin \beta$ . The shock angle can be determined by entering  $M_2 = 2.97 \approx 3.0$  and  $\theta = 10^\circ$  into the oblique shock table, giving  $\beta = 27.38^\circ$ . Accordingly,

$$M_{2n} = M_2 \sin \beta = 2.97 \times \sin 27.38^\circ = 1.37$$

From the normal shock table, for  $M_{2n} = 1.37$ , we read  $M_{3n} = 0.7527$ . It follows that

$$M_3 = \frac{M_{3n}}{\sin(\beta - \theta)} = \frac{0.7527}{\sin(27.38^\circ - 10^\circ)} = 2.52$$

Consider now the lower surface. Entering  $M_1 = 2.5$  and  $\theta = 10^\circ$  into the oblique shock table, we read  $\beta = 31.85^\circ$ , so

$$M_{1n} = 2.5 \times \sin 31.85^\circ = 1.32$$

Entering this normal Mach number into the normal shock table, we read  $M_{2'n} = 0.7760$ . Thus,

$$M_{2'} = \frac{0.7760}{\sin(31.85^\circ - 10^\circ)} = 2.09$$

One more Mach number to go. Mapping  $M_{2'} = 2.09$  onto the isentropic table, we read an angle  $v_{2'} = 28.829^\circ$ . Thus,

$$v_{3'} = 28.829^\circ + 10^\circ = 38.829^\circ$$

Entering this angle into the isentropic table, we have

$$M_{3'} = 2.49$$

★ Statements **3** and **4** are true, whereas statements **1** and **2** are false.

### P.20 ■ Solution

This problem follows the same reasoning as Problem 19. For expansion wave 1→2, we enter  $M_1 = 2.8$  into the isentropic table and read  $v_1 = 47.746^\circ$ . With  $\theta_{12} = 5^\circ$ , it follows that

$$v_2 = 47.746^\circ + 5^\circ = 52.746^\circ$$

Entering  $v_2$  into the isentropic table, we read a Mach number

$$M_2 \approx 3.16$$

Next, for expansion wave 2→3, we first determine  $v_3 = v_2 + \theta_{23} = 52.75^\circ + 20^\circ = 72.75^\circ$ . Then, we enter  $v_3$  into the isentropic table and read a Mach number

$$M_3 \approx 4.585$$

Consider now normal shock 3→4. The flow direction is  $\theta_{34} = 25^\circ$ . Entering this angle and  $M_3 \approx 4.60$  into the oblique shock table, we read  $\beta = 36.65^\circ$ . Accordingly,

$$M_{3n} = M_3 \sin \beta = 4.585 \times \sin 36.65^\circ = 2.74$$

Entering this Mach number into the normal shock table, we read  $M_{4n} = 0.4926$ . Then, Mach number  $M_4$  is calculated to be

$$M_4 = \frac{M_{4n}}{\sin(\beta - \theta)} = \frac{0.4926}{\sin(36.65^\circ - 25^\circ)} = \boxed{2.44}$$

Consider now the lower surface. Entering  $M_1 = 2.8$  and  $\theta = 25^\circ$  into the oblique shock table, we read  $\beta = 46.10^\circ$ , so that

$$M_{1'n} = 2.8 \times \sin 46.10^\circ = 2.02$$

Entering  $M_{1'm} \approx 2.0$  into the normal shock table, we read

$$M_{2'n} = 0.5774$$

so that

$$M_{2'} = \frac{0.5774}{\sin(46.10^\circ - 25^\circ)} = \boxed{1.60}$$

From the same table entry,

$$\nu_{2'} = 14.860^\circ$$

so that, given  $\theta_{2',3'} = 20^\circ$ ,

$$\nu_{3'} = 14.860^\circ + 20^\circ = 34.860^\circ$$

Entering  $\nu_{3'}$  into the isentropic table, we read

$$\boxed{M_{3'} \approx 2.325}$$

One Mach number to go. Noting that  $\theta_{3',4'} = 5^\circ$ , we have

$$\nu_{4'} = 34.860^\circ + 5^\circ = 39.860^\circ$$

Entering  $\nu_{4'}$  into the isentropic table, we have

$$\boxed{M_{4'} \approx 2.53}$$

★ Statements **1, 4, and 6** are true, whereas statements **2, 3 and 5** are false.

## P.21 ■ Solution

Entering  $M_1 = 1.8$  in an isentropic table, we read  $p_1/p_0 = 0.1740$ . It follows that

$$\frac{p_2}{p_0} = \frac{p_2}{p_1} \times \frac{p_1}{p_0} = \frac{1}{2} \times 0.1740 = 0.087$$

For  $p_2/p_0 = 0.0870$ , we refer to the isentropic table a second time to get  $M_2 \approx 2.25$ . At this point, we appeal to the Prandtl-Meyer function to obtain

$$\begin{aligned} \nu(M_1) &= \sqrt{\frac{\gamma+1}{\gamma-1}} \arctan \sqrt{\frac{\gamma-1}{\gamma+1} (M_1^2 - 1)} - \arctan \sqrt{M_1^2 - 1} \\ \therefore \nu(M_1) &= \sqrt{\frac{1.4+1}{1.4-1}} \arctan \sqrt{\frac{1.4-1}{1.4+1} \times (1.8^2 - 1)} - \arctan \sqrt{1.8^2 - 1} = 20.7^\circ \\ \therefore \nu(M_2) &= \sqrt{\frac{1.4+1}{1.4-1}} \arctan \sqrt{\frac{1.4-1}{1.4+1} \times (2.25^2 - 1)} - \arctan \sqrt{2.25^2 - 1} = 33.0^\circ \end{aligned}$$

Finally, the deflection angle is calculated to be

$$\theta_2 = \nu(M_2) - \nu(M_1) = 33.0^\circ - 20.7^\circ = \boxed{12.3^\circ}$$

★ The correct answer is **A**.

## P.22 ■ Solution

Entering  $M_1 = 1.7$  into the isentropic table, we read

$$\frac{p_1}{p_0} = 0.2026 ; \frac{T_1}{T_{01}} = 0.6337$$

Noting that  $p_1 = 4 \text{ kPa}$  and  $p_2 = 1 \text{ kPa}$ , we may write

$$\frac{p_2}{p_{01}} = \frac{p_2}{p_1} \times \frac{p_1}{p_{01}} = \frac{1.0}{4.0} \times 0.2026 = 0.0507$$

Entering this pressure ratio into the isentropic table, we have

$$M_2 = 2.59 ; \frac{T_2}{T_{02}} = 0.4271$$

We proceed to determine the Prandtl-Meyer functions,

$$\nu_{M_1} = \sqrt{\frac{1.4+1}{1.4-1}} \arctan \sqrt{\frac{1.4-1}{1.4+1} \times (1.7^2 - 1)} - \arctan \sqrt{1.7^2 - 1} = 17.8^\circ$$

$$\nu_{M_2} = \sqrt{\frac{1.4+1}{1.4-1}} \arctan \sqrt{\frac{1.4-1}{1.4+1} \times (2.59^2 - 1)} - \arctan \sqrt{2.59^2 - 1} = 41.2^\circ$$

The deflection angle is calculated to be

$$\theta = \nu_{M_1} - \nu_{M_2} = 17.8^\circ - 41.2^\circ = -23.4^\circ$$

Lastly, the temperature after the shock is

$$T_2 = \frac{T_2}{T_{02}} \times \frac{T_{01}}{T_1} \times T_1 = 0.4271 \times \frac{1}{0.6337} \times 400 = 270 \text{ K}$$

★ Statement 2 is true, whereas statements 1 and 3 are false.

## ANSWER SUMMARY

<b>Problem 1</b>	T/F				
<b>Problem 2</b>	A				
<b>Problem 3</b>	C				
<b>Problem 4</b>	T/F				
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5.1	C				
5.2	T/F				
<b>Problem 6</b>	T/F				
<b>Problem 7</b>	C				
<b>Problem 8</b>	T/F				
<b>Problem 9</b>	T/F				
<b>Problem 10</b>	T/F				
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<b>Problem 13</b>	B				
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16.2	C				
<b>Problem 17</b>	A				
<b>Problem 18</b>	T/F				
<b>Problem 19</b>	T/F				
<b>Problem 20</b>	T/F				
<b>Problem 21</b>	A				
<b>Problem 22</b>	T/F				

## REFERENCE

- ETHIRAJAN, R. (2019). *Applied Gas Dynamics*. 2nd edition. Hoboken: John Wiley and Sons.



Got any questions related to this quiz? We can help!  
Send a message to [contact@montogue.com](mailto:contact@montogue.com) and we'll  
answer your question as soon as possible.