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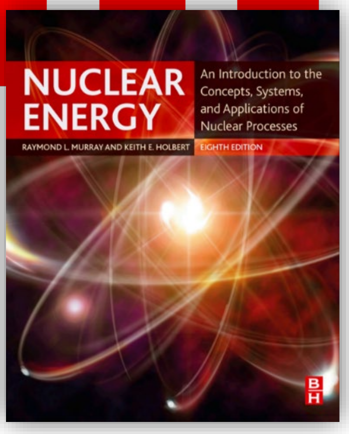

Quiz NUC03

Reviewed Solutions to *Nuclear Energy*, 8th Ed., by Murray and Holbert

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Problem Distribution

Chapter	Problems Covered
5	5.3, 5.6, 5.7, 5.8, 5.11, 5.13, 5.19
6	6.2, 6.3, 6.4(a,b,e), 6.5, 6.9(a,c,e)
10	10.1, 10.2, 10.4, 10.5, 10.8, 10.10, 10.11, 10.13
11	11.1(a,b), 11.2, 11.5, 11.7, 11.9, 11.10, 11.11
16	16.2, 16.8, 16.9, 16.10, 16.11, 16.12
17	17.3, 17.4, 17.7, 17.9, 17.14
19	19.4, 19.8, 19.9, 19.12
20	20.3, 20.5, 20.6, 20.8, 20.9, 20.11, 20.14, 20.19

Problems

■ Chapter 5 – Radiation and Materials

Problem 5.3

For 180° scattering of gamma or X-rays by electrons of rest mass E_0, **(a)** verify that the final energy of the photon is

$$E' = \left(\frac{1}{E} + \frac{2}{E_0}\right)^{-1}$$

(b) What is the final photon energy for the 6 MeV gamma ray of Exercise 5.2? **(c)** Verify that if $E \gg E_0$, then $E' \approx E_0/2$ and if $E \ll E_0$, $(E - E')/E \approx 2E/E_0$. **(d)** Which approximation should be used for a 6-MeV gamma ray? Verify numerically. **(e)** Show that the final (maximum) electron energy is

$$E^{\circ}_{K,\max} = \frac{E}{1 + E_0/(2E)}$$

Problem 5.6

The element lead, M = 207.2, has a density of 11.35 g/cm^3. **(a)** Find the number of atoms per cubic centimeter. If the total gamma ray cross-section at 3 MeV is 14.6 barns, what are **(b)** the linear attenuation coefficient and **(c)** the half-thickness?

Problem 5.7

The range of beta particles of maximum energy between 0.8 and 3.0 MeV is given roughly by the Feather (1938) relation

$$R[\text{cm}] = \frac{0.543E[\text{MeV}] - 0.160}{\rho\left[\text{g/cm}^3\right]}$$

(a) Using this formula, find what thickness of aluminum sheet (density 2.7 g/cm^3) is enough to stop the betas from phosphorus-32 (see Table 3.2). **(b)** Repeat the calculation using the Katz-Penfold relation of Eq. (5.2).

Problem 5.8
A radiation worker's hands are exposed for 5 s to a $3\times10^8/(cm^2\cdot s)$ beam of 1 MeV beta particles. Find the range in tissue of density 1.0 g/cm^3 and calculate the amounts of charge and energy deposition in C/cm^3 and J/g. Note that the charge on the electron is 1.60×10^{-19} C. For tissue, use the equation given in Exercise 5.7.

Problem 5.11
Find the percentage reduction through 1.5 cm of lead from a gamma-ray flux produced by **(a)** ^{137}Cs, **(b)** ^{40}K, and **(c)** ^{99}Mo.

Problem 5.13
Compare the percent energy change of 10 keV and 10 MeV photons scattered at 90°. What conclusion do these results suggest?

Problem 5.19
Compute the half-thickness of gamma rays from ^{137}Cs for shielding composed of **(a)** lead, **(b)** iron, **(c)** concrete, and **(d)** water.

■ Chapter 6 – Fission

Problem 6.2
If three neutrons are produced when a neutron bombards a U-235 atom, determine the second fission product isotope when the first fission fragment is **(a)** xenon-133, **(b)** barium-144, **(c)** cesium-143, **(d)** tellurium-137, and **(e)** lanthanum-146.

Problem 6.3
The total kinetic energy of two fission fragments is 166 MeV. **(a)** Assuming that the momenta of the fission neutrons are negligible, what are the energies of each fragment if the mass ratio of each fragment is 3/2? **(b)** What are the two mass numbers if three neutrons were released in U-235 fission? **(c)** What are the velocities of the fragments? **(d)** What is the ratio of the momentum of a fission fragment to that of a fission neutron?

Problem 6.4 **[Parts (a), (b) and (e) only]**
Calculate the energy yield from the following fission reactions.

(a) $^{235}_{92}\mathrm{U} + ^{1}_{0}n \rightarrow ^{92}_{37}\mathrm{Rb} + ^{140}_{55}\mathrm{Cs} + 4^{1}_{0}n + Q$

(b) $^{235}_{92}\mathrm{U} + ^{1}_{0}n \rightarrow ^{94}_{38}\mathrm{Sr} + ^{140}_{54}\mathrm{Xe} + 2^{1}_{0}n + Q$

(e) $^{235}_{94}\mathrm{Pu} + ^{1}_{0}n \rightarrow ^{103}_{40}\mathrm{Zr} + ^{134}_{54}\mathrm{Xe} + 3^{1}_{0}n + Q$

Problem 6.5
Compute the reproduction factor for **(a)** U-233, **(b)** Pu-239, and **(c)** Am-241.

Problem 6.9 **[Parts (a), (c) and (e) only]**
Given the spontaneous fission half-lives and neutron yields per fission in Table 6.1, determine the spontaneous fission yield (n/(g·s)) for **(a)** U-232, **(c)** Pu-238, and **(e)** Am-241.

■ Chapter 10 – Biological Effects of Radiation

Problem 10.1
A beam of 2-MeV alpha particles with current density 10^6 /(cm^{2}s) is stopped in a distance of 1 cm in air, number density 2.7×10^{19} cm^{-3}. How many ion pairs per cm^3 are formed each second? What fraction of the targets experience ionization per second?

Problem 10.2
If the chance of fatality from a radiation dose is taken as 0.5 for 400 rem, by what factor would the chance at 2 rem be overestimated if the effect varied as the square of the dose rather than linearly?

Problem 10.4
A person receives the following exposures in millirem in a year: one medical X-ray, 100; drinking water, 50; cosmic rays, 30; radon in house, 150; K-40 and other isotopes, 25; airplane flights, 10. Find the percentage increase in exposure that would be experienced if he also lived at a reactor site boundary, assuming that the maximum NRC radiation level existed there.

Problem 10.5

A plant worker accidentally breathes some stored gaseous tritium, a beta emitter with maximum particle energy 0.0186 MeV. The energy absorbed by the lungs, of total weight 1 kg, is 4×10^{-3} J. How many millirem dose equivalent was received? How many millisieverts?

Problem 10.8

An employee is seeking hazard pay as compensation for chronic occupational radiation exposure. Data compiled by Cohen (1991) indicate that a person's lifespan is reduced by 51 days if exposed to 1 rem/y from age 18 to 65. If the person's time is valued at $100/h, estimate the reparation per mrem.

Problem 10.10

(a) Dose rate estimates for a mission to Mars consist of 1.9 mSv/d during each 180-day outbound and return flight, and 0.7 mSv/d while on Mars for nearly 2 years. What fraction or dose of the annual NRC dose limits do the astronauts receive during total flight time and while exploring the planet each year? **(b)** How many days before the occupational dose limit is reached on the International Space Station where the dose rate is approximately 0.25 mSv/d?

Problem 10.11

Calculate the temperature rise due to a dose of 4 Gy (400 rad) in water.

Problem 10.13

If a cell is 10 μm thick, how many cells do **(a)** a 3-MeV and **(b)** a 6-MeV alpha particle pass through fully before being stopped?

■ Chapter 11 – Radiation Protection

Problem 11.1 [Parts (a) and (b) only]

What is the dose rate in mrem/y corresponding to a continuous **(a)** 0.5 MeV, and **(b)** 2 MeV gamma-ray flux of $100/(\text{cm}^2{\cdot}\text{s})$? What dose equivalent would be received for a person who worked 40 h/wk throughout the year in such a flux?

Problem 11.2

A Co-60 source is to be selected to test radiation detectors for operability. Assuming that the source can be kept at least 1 m from the body, what is the largest strength acceptable (in μCi) to assure an exposure rate of less than 500 mrem/y? (Recall that ^{60}Co emits two gammas of energy 1.17 and 1.33 MeV.)

Problem 11.5

Find the uncollided gamma ray flux at the surface of a spherical lead shield of radius 12 cm surrounding a very small source of 200 mCi of 1 MeV gammas.

Problem 11.7

Water discharged from a nuclear plant contains in solution traces of strontium-90, cesium-144, and cesium-137. Assuming that the concentrations of each isotope are proportional to their fission yields, find the allowed activities per mL of each. Note the following data:

Isotope	Half-life	Yield	Limit (μCi/mL)
^{90}Sr	29.1 yr	0.0575	5×10^{-7}
^{144}Ce	284.6 d	0.0545	3×10^{-6}
^{137}Cs	30.1 y	0.0611	1×10^{-6}

Problem 11.9

The activities of U-238, Ra-226, and Rn-222 in a closed system are approximately equal, in accord with the principle of secular equilibrium. Assuming that the natural uranium content of soil is 10 ppm (parts per million), calculate the specific activities of the isotopes in microcuries per gram of soil (Table 3.2 gives the needed half-lives).

Problem 11.10

For the conditions in Example 11.11, determine the annual dose to **(a)** an infant's kidney, **(b)** a child's bones, **(c)** a teenager's liver, and **(d)** an adult's GI tract.

Problem 11.11

Over the course of six 10-h working days, an employee receives an external dose of 200 mrem and inhales air consisting of 1.3×10^{-8} μCi/mL of I-131 and 7.3×10^{-8} μCi/mL of Cs-137. Compute the ratio of the dose received to the annual effective dose equivalent limit.

Chapter 16 – Neutron Chain Reactions

Problem 16.2

If a power of 100 W is developed by **(a)** the Godiva-type reactor of mass 50 kg (93.9 w/o enrichment) or **(b)** the Jezebel-type reactor of mass 17 kg, what is the average flux within each respective assembly?

Problem 16.8

Constants for a spherical fast uranium-235 metal assembly are: diffusion coefficient D = 1.02 cm; macroscopic absorption cross-section Σ_a = 0.0795 cm^{-1}; effective radius R = 10 cm. Calculate **(a)** the diffusion length L, **(b)** the geometric buckling B^2, and **(c)** the nonleakage factor $\mathcal{L}$.

Problem 16.9

Using the Lady Godiva specifications in Table 16.1, determine **(a)** the diffusion area and **(b)** the nonleakage probability.

Problem 16.10

Compute the geometric buckling for **(a)** a parallelepiped 25 cm × 35 cm × 40 cm, and **(b)** a cylinder with radius 25 cm and height 50 cm. If these cores are bare metal reactors of fully enriched uranium with density 19 g/cm^3, what is the nonleakage probability for each?

Problem 16.11

Calculate the resonance escape probability for a homogeneous uranium-graphite mixture in which the ratio of moderator-to-fuel atoms is 450. The resonance integral of the uranium is 277 b, and the epithermal scattering cross-section of carbon is 4.66 b.

Problem 16.12

Show that $\sigma_{tr} = \sigma_t - \bar{\mu}\sigma_s \approx \sigma_t$ for heavy nuclides.

Chapter 17 – Nuclear Heat Energy

Problem 17.3

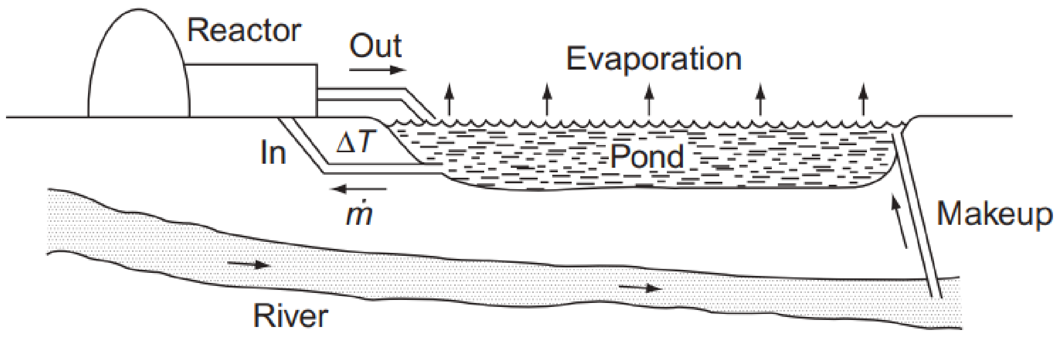
If the power density of a uranium oxide fuel pin, of radius 0.6 cm, is 500 W/cm^3, what is the heat flux across the fuel rod surface? If the temperatures of rod surface and coolant are 300°C and 250°C, respectively, what must the heat transfer coefficient h be?

Problem 17.4

A PWR operates at thermal power of 2500 MW, with water coolant mass flow rate of 15,000 kg/s. **(a)** If the coolant inlet temperature is 275°C, what is the outlet temperature? **(b)** If bulk boiling occurs at 336°C, at what reactor power would this take place?

Problem 17.7

As shown here, water is drawn from a cooling pond and returned by a temperature 14°C higher to extract 1500 MW of waste heat. All the heat is dissipated by water evaporation from the pond with an absorption of 2258 J/g. **(a)** How many kilograms per second of makeup water must be supplied from an adjacent river? **(b)** What percentage is this of the circulating flow to the condenser?



Problem 17.9

How many gallons of water must be evaporated each day to dissipate 100% of the waste thermal power of approximately 2030 MWt from a reactor?

Problem 17.14

Calculate the volumetric heat generation rate in a thermal reactor fueled with naturally enriched UO_2 of density 10 g/cm^3 and which is exposed to a flux of 10^{14} n/(cm^2·s).

■ Chapter 19 – Reactor Theory Introduction

Problem 19.4

In a simple core such as a bare uranium metal sphere of radius *R*, the neutron flux varies with position, as given in Table 19.1. Calculate and plot the flux distribution for a core with *R* = 10 cm, *d* = 0 and central flux ϕ_C = 5×10^{11}/(cm^2·s).

Problem 19.8

For the graphite-uranium mixture of Example 19.3, determine the minimum physical size and the minimum fuel mass for criticality for **(a)** a cube, **(b)** a sphere, and **(c)** a cylinder. Assume that the extrapolation distance is negligible.

Problem 19.9

While neglecting the extrapolation distance(s), compute the maximum-to-average volumetric heat generation rate $q'''_{\max}/q'''_{\text{avg}}$ in **(a)** a rectangular parallelepiped and **(b)** a sphere. Note that the differential volume in spherical coordinates is $4\pi r^2 dr$.

Problem 19.12

Compare the nonleakage probabilities for **(a)** a small 550 MWt core and **(b)** a large 3500 MWt power reactor with power densities of 80 kW/L and 100 kW/L, respectively. Each cylindrical reactor has the optimal *H*/*D* ratio and L^2 = 1.9 cm^2.

■ Chapter 20 – Time Dependent Reactor Behavior

Problem 20.3

A U-235 fueled reactor is operating at a power level of 250 MWe. Control rods are removed to give a reactivity of 0.0005. Noting that this is much less than β_T, calculate the time required to go to a power of 300 MWe, neglecting any temperature feedback.

Problem 20.5

During a critical experiment in which fuel is initially loaded into a reactor, a fuel element of reactivity worth 0.0036 is suddenly dropped into a core that is already critical. If the temperature coefficient is –9×10^{-5}/°C, how high will the temperature of the system go above room temperature before the positive reactivity is canceled out?

Problem 20.6

A reactivity of —0.0025 caused by the Doppler effect results when the thermal power goes from 2500 to 2800 MW. Estimate the contribution of this effect on the power coefficient for the reactor.

Problem 20.8

The initial concentration of boron in a 10,000-ft^3 reactor coolant system is 1500 ppm. What volume of solution of concentration 8000 ppm should be added to achieve a new value of 1600 ppm?

Problem 20.9

An adjustment of boron content from 1500 to 1400 ppm is made in the reactor described in Exercise 20.8. Pure water is pumped in, and then mixed coolant and poison are pumped out in two separate steps. How long should the 500 ft^3/min pump operate in each of the operations?

Problem 20.11

When a control rod is withdrawn 4 cm from its position with its tip at the center of a critical reactor, the power rises on a period of 200 s. **(a)** With a value β_T = 0.008 and τ = 13 s, estimate the δk produced by the rod shift and the slope of the calibration curve Δ*k*/ Δ*z*. **(b)** Estimate the rod worth if the core height is 300 cm.

Problem 20.14

(a) Show that a megawatt per tonne is the same as a watt per gram. **(b)** Because most nuclear power plants are base loaded, the power is constant and the product of the flux and fissile atom concentration is likewise. Show that the burnup is given by the formulas

$$B = \frac{w\sigma_f N_{235}(0)\phi t}{\rho_U} = \frac{w\sigma_f N_{235}(0) N_A}{N_U M_U}$$

(c) Calculate *B* for a three-year fuel cycle having an initial atomic enrichment $N_{235}(0)/N_U$ = 0.03 and flux ϕ_0 of 2×10^{13}/(cm^2·s).

Problem 20.19

A safety rod having a worth of 75 cents is positioned 110 cm into a 1.5-m tall core. Determine the reactivity change and subsequent reactor period if the rod is **(a)** withdrawn 15 cm or **(b)** inserted 15 cm from its original depth in the critical reactor.

SOLUTIONS

■ P5.3

Part (a): Substituting θ = 180° into equation (5.9) brings to

$$\frac{1}{E'}-\frac{1}{E}=\frac{1}{m_e c^2}\left[1-\cos\left(180^\circ\right)\right]=\frac{2}{m_e c^2}$$

Letting $m_e c^2 = E_0$ and solving for E',

$$\frac{1}{E'}-\frac{1}{E}=\frac{2}{E_0}$$

$$\therefore \frac{1}{E'}=\frac{1}{E}+\frac{2}{E_0}$$

$$\therefore E'=\left(\frac{1}{E}+\frac{2}{E_0}\right)^{-1}$$

Part (b): Noting that the mass-energy of an electron is 0.511 MeV, we can use the foregoing formula to obtain

$$E'=\left(\frac{1}{6}+\frac{2}{0.511}\right)^{-1}=\boxed{0.245\text{ MeV}}$$

Part (c): With $E \gg E_0$, E' becomes

$$E'=\left(\underbrace{\frac{1}{E}}_{\to 0}+\frac{2}{E_0}\right)^{-1}$$

$$\therefore E'=\left(\frac{2}{E_0}\right)^{-1}$$

$$\therefore E'=\frac{E_0}{2}$$

Now, we can write $E - E'$ as

$$E-E'=E-1\big/\left(1/E+2/E_0\right)$$

$$\therefore E-E'=\frac{2E^2}{2E+E_0}$$

$$\therefore \frac{\left(E-E'\right)}{E}=\frac{2E}{2E+E_0}$$

$$\therefore \frac{\left(E-E'\right)}{E}=\frac{2E/E_0}{\left(1+\frac{2E}{E_0}\right)}$$

If $E \ll E_0$, the equation reduces to

$$\frac{\left(E-E'\right)}{E}=\frac{2E/E_0}{\left(1+\underbrace{\frac{2E}{E_0}}_{\to 0}\right)}\approx\frac{2E}{E_0}$$

as we intended to show.

Part (d): For 6 MeV gamma rays, we have E = 6 MeV $\gg E_0$ = 0.511 MeV, and the approximation to use is

$$E' = \frac{E_0}{2} = \frac{0.511}{2} = \boxed{0.256\text{ MeV}}$$

Part (e): To show the relationship in question, we manipulate to obtain

$$E_K^{\text{o}} = E - E' = E - \frac{1}{\frac{1}{E} + \frac{2}{E_0}} = E - \frac{EE_0}{E_0 + 2E}$$

$$\therefore E_K^{\text{o}} = \frac{E(E_0 + 2E)}{E_0 + 2E} - \frac{EE_0}{E_0 + 2E}$$

$$\therefore E_K^{\text{o}} = \frac{\cancel{EE_0} + 2E^2 - \cancel{EE_0}}{E_0 + 2E}$$

$$\therefore E_K^{\text{o}} = \frac{2E^2}{E_0 + 2E}$$

$$\therefore E_K^{\text{o}} = \frac{\frac{2E^2}{2E}}{\frac{E_0 + 2E}{2E}}$$

$$\therefore \boxed{E_K^{\text{o}} = \frac{E}{1 + E_0/(2E)}}$$

■ P5.6

Part (a): The number density is

$$N = \frac{11.32 \times (6.02 \times 10^{23})}{207.2} = \boxed{3.29 \times 10^{22}\text{ cm}^{-3}}$$

Part (b): The linear attenuation coefficient is given by the product of number density and micro cross-section:

$$\mu = N\sigma = (3.29 \times 10^{22}) \times (14.6 \times 10^{-24}) = \boxed{0.480\text{ cm}^{-1}}$$

Part (c): The half-thickness is

$$t_{1/2} = \frac{\ln 2}{\mu} = \frac{0.693}{0.480} = \boxed{1.44\text{ cm}}$$

■ P5.7

Part (a): With reference to Table 3.2, we glean that the principal radiation of P-32 is a β^- particle at 1.711 MeV. Substituting into the Feather correlation gives

$$R = \frac{0.543 \times 1.711 - 0.16}{2.7} = \boxed{0.285\text{ cm}}$$

Part (b): Applying the form of equation (5.2) for $0.01 \le E \le 2.5$ MeV, we obtain

$$R_{\max} = 0.412E^{1.265 - 0.0954\ln(E)} = 0.412 \times 1.711^{1.265 - 0.0954\ln(1.711)}$$

$$\therefore R_{\max} = 0.791\text{ cm}$$

The corresponding range is $R = 0.791/2.70 = 0.293$ cm.

■ P5.8

To find the range of 1-MeV particles, we use the relationship given in Problem 5.7:

$$R[\text{cm}] = \frac{0.543E[\text{MeV}] - 0.160}{\rho[\text{g/cm}^3]} = \frac{0.543 \times 1.0 - 0.16}{1.0} = 0.383\text{ cm}$$

If 34 eV are needed to produce an ion-pair, the ion-pair yield obtained with β radiation at 1 MeV is

$$\text{No. of ion pairs per } \beta \text{ particle} = 10^6 \frac{\text{eV}}{\beta} \times \frac{1}{34} \frac{\text{ion pair}}{\text{eV}} = 29,400 \text{ ip/β}$$

This corresponds to a charge such that

$$\text{Charge} = 29,400 \frac{\text{ip}}{\beta} \times \left(1.6 \times 10^{-19}\right) \frac{\text{C}}{\text{ip}} = 4.70 \times 10^{-15} \text{ C/β}$$

The number of β-particles deposited over the course of 5 sec of exposure is

$$\text{No. of beta particles deposited} = \phi t = \left(3 \times 10^8\right) \frac{\beta}{\text{cm}^2 \cdot \text{s}} \times 5 \text{ s} = 1.5 \times 10^9 \text{ β/cm}^2$$

The total charge deposited is

$$\begin{bmatrix} \text{Total charge} \\ \text{deposited} \end{bmatrix} = Q = \left(1.5 \times 10^9 \frac{\beta}{\text{cm}^2}\right) \times \left(4.70 \times 10^{-15} \frac{\text{C}}{\beta}\right) = 7.05 \times 10^{-6} \frac{\text{C}}{\text{cm}^2}$$

The charge density is

$$\text{Charge density} = \frac{Q}{R} = \frac{7.05 \times 10^{-6} \text{ C/cm}^2}{0.383 \text{ cm}} = 1.84 \times 10^{-5} = \boxed{18.4 \text{ μC/cm}^3}$$

The energy deposited per cm^2 is

$$\begin{bmatrix} \text{Energy deposited} \\ \text{per cm}^2 \end{bmatrix} = E = \left(1.5 \times 10^9 \frac{\beta}{\text{cm}^2}\right) \times \left(1 \frac{\text{MeV}}{\beta}\right) \times \left(1.6 \times 10^{-13} \frac{\text{J}}{\text{MeV}}\right)$$

$$\therefore E = 2.4 \times 10^{-4} \text{ J/cm}^2$$

The mass of tissue per cm^2 is

$$m = \rho R = 1.0 \frac{\text{g}}{\text{cm}^3} \times 0.383 \text{ cm} = 0.383 \text{ g/cm}^2$$

The energy per gram of tissue is determined to be

$$e = \frac{E}{m} = \frac{2.4 \times 10^{-4} \text{ J/cm}^2}{0.383 \text{ g/cm}^2} = 6.27 \times 10^{-4} = \boxed{0.627 \text{ mJ/g}}$$

■ P5.11

Part (a): γ-decay of ^{137}Cs yields radiation of energy content equal to 0.662 MeV. With reference to the rightmost column on Table A.6, we interpolate between (μ/ρ) = 0.1248 for 0.6 MeV and (μ/ρ) = 0.08870 for 0.8 MeV to obtain (μ/ρ) = 0.114. The flux ratio $\phi(x)/\phi_0$ follows as

$$\phi(x) = \phi_0 \exp\left[-(\mu/\rho)\rho x\right] \;\rightarrow\; \frac{\phi}{\phi_0} = \exp\left[-(\mu/\rho)\rho x\right]$$

$$\therefore \frac{\phi}{\phi_0} = \exp\left(-0.114 \times 11.35 \times 1.5\right) = 0.144$$

The percentage reduction is 1 – 0.144 = 0.856 = 85.6%.

Part (b): γ-decay of ^{40}K yields radiation of energy content equal to 1.461 MeV. Referring to the rightmost column on Table A.6, we interpolate between (μ/ρ) = 0.05876 for 1.25 MeV and (μ/ρ) = 0.052 for 1.5 MeV to obtain (μ/ρ) = 0.0532. The flux ratio $\phi(x)/\phi_0$ follows as

$$\phi(x) = \phi_0 \exp\left[-(\mu/\rho)\rho x\right] \;\rightarrow\; \frac{\phi}{\phi_0} = \exp\left[-(\mu/\rho)\rho x\right]$$

$$\therefore \frac{\phi}{\phi_0} = \exp\left(-0.0532 \times 11.35 \times 1.5\right) = 0.404$$

The percentage reduction is 1 – 0.404 = 0.596 = 59.6%.

Part (c): γ-decay of ^{99}Mo yields radiation of energy content equal to 0.740 MeV. Referring to the rightmost column on Table A.6, we interpolate between (μ/ρ) = 0.1248 for 0.6 MeV and (μ/ρ) = 0.08870 for 0.8 MeV to obtain (μ/ρ) = 0.0995. The flux ratio $\phi(x)/\phi_0$ follows as

$$\phi(x) = \phi_0 \exp\left[-(\mu/\rho)\rho x\right] \rightarrow \frac{\phi}{\phi_0} = \exp\left[-(\mu/\rho)\rho x\right]$$

$$\therefore \frac{\phi}{\phi_0} = \exp(-0.0995 \times 11.35 \times 1.5) = 0.184$$

The percentage reduction is 1 – 0.184 = 0.816 = 81.6%.

■ P5.13

Appealing to equation (5.9) with θ = 90°, we may write

$$\frac{1}{E'} - \frac{1}{E} = \frac{1}{m_e c^2}\left[1 - \underbrace{\cos(90^\circ)}_{=0}\right]$$

$$\therefore \frac{1}{E'} - \frac{1}{E} = \frac{1}{m_e c^2}$$

$$\therefore \frac{1}{E'} - \frac{1}{E} = \frac{1}{0.511}$$

$$\therefore \frac{1}{E} = \frac{1}{E'} - \frac{1}{0.511}$$

$$\therefore E' = \left(\frac{1}{E} + \frac{1}{0.511}\right)^{-1}$$

Substituting E = 10 keV = 0.01 MeV brings to

$$E' = \left(\frac{1}{0.01} + \frac{1}{0.511}\right)^{-1} = 0.00981 \text{ MeV} = \boxed{9.81 \text{ keV}}$$

The percentage change in energy is (10 – 9.81)/10 × 100% = 1.9%. Proceeding similarly with E = 10 MeV, we have

$$E' = \left(\frac{1}{10} + \frac{1}{0.511}\right)^{-1} = \boxed{0.486 \text{ MeV}}$$

The percentage change in energy is (10 – 0.486)/10 × 100% = 95.1%. Clearly, higher energy photons lose a greater amount of energy.

■ P5.19

Part (a): Cesium-137 emits γ-rays with energy content equal to 0.662 MeV. Following Problem 5.11(a), we have (μ/ρ) = 0.114. The density of lead may be taken as 11.35 g/cm^3. The half-thickness $t_{1/2}$ follows as

$$x_{1/2} = \frac{\ln 2}{\mu} = \frac{\ln 2}{(\mu/\rho)\rho} = \frac{0.693}{0.114 \times 11.35} = \boxed{0.536 \text{ cm}}$$

Part (b): Referring to the second-to-last column on Table A.6, we interpolate between (μ/ρ) = 0.07704 for 0.6 MeV and (μ/ρ) = 0.06699 for 0.8 MeV to obtain (μ/ρ) = 0.0739. The density of iron may be taken as 7.874 g/cm^3. The half-thickness $t_{1/2}$ follows as

$$x_{1/2} = \frac{\ln 2}{\mu} = \frac{\ln 2}{(\mu/\rho)\rho} = \frac{0.693}{0.0739 \times 7.874} = \boxed{1.19 \text{ cm}}$$

Part (c): Referring to the third-to-last column on Table A.6, we interpolate between (μ/ρ) = 0.08236 for 0.6 MeV and (μ/ρ) = 0.07227 for 0.8 MeV to obtain (μ/ρ) = 0.0792. The density of concrete may be taken as 2.30 g/cm^3. The half-thickness $t_{1/2}$ follows as

$$x_{1/2} = \frac{\ln 2}{\mu} = \frac{\ln 2}{(\mu/\rho)\rho} = \frac{0.693}{0.0792 \times 2.30} = \boxed{3.80 \text{ cm}}$$

Part (d): Referring to the fourth column on Table A.6, we interpolate between (μ/ρ) = 0.08956 for 0.6 MeV and (μ/ρ) = 0.07865 for 0.8 MeV to obtain (μ/ρ) = 0.0862. The density of water may be taken as 1.0 g/cm^3. The half-thickness $t_{1/2}$ follows as

$$x_{1/2}=\frac{\ln 2}{\mu}=\frac{\ln 2}{(\mu/\rho)\rho}=\frac{0.693}{0.0862\times1.0}=\boxed{8.04\text{ cm}}$$

■ P6.2

Part (a): In view of the problem statement, an uranium-235 atom is fissioned by a neutron, yielding xenon-133, three neutrons, and an unknown isotope:

$${}^{235}_{92}\text{U}+{}^{1}_{0}n\rightarrow{}^{133}_{54}\text{Xe}+3{}^{1}_{0}\text{n}+{}^{A}_{Z}?$$

The mass number of the unknown isotope is

$$A=235+1-133-3\times1=100$$

while the atomic number is

$$Z=92+0-54-3\times0=38$$

Referring to the periodic table, we see that Z = 38 corresponds to strontium; therefore, the missing isotope in the nuclear reaction is strontium-100, ${}^{100}_{38}\text{Sr}$.

Part (b): The nuclear reaction in this case is

$${}^{235}_{92}\text{U}+{}^{1}_{0}n\rightarrow{}^{144}_{56}\text{Ba}+3{}^{1}_{0}\text{n}+{}^{A}_{Z}?$$

The mass number of the unknown isotope is

$$A=235+1-144-3\times1=89$$

while the atomic number is

$$Z=92+0-56-3\times0=36$$

Referring to the periodic table, we see that Z = 36 corresponds to krypton; therefore, the missing isotope in the nuclear reaction is krypton-89, ${}^{89}_{36}\text{Kr}$.

Part (c): The nuclear reaction in this case is

$${}^{235}_{92}\text{U}+{}^{1}_{0}n\rightarrow{}^{143}_{55}\text{Cs}+3{}^{1}_{0}\text{n}+{}^{A}_{Z}?$$

The mass number of the unknown isotope is

$$A=235+1-143-3\times1=90$$

while the atomic number is

$$Z=92+0-55-3\times0=37$$

Referring to the periodic table, we see that Z = 37 corresponds to rubidium; therefore, the missing isotope in the nuclear reaction is rubidium-90, ${}^{90}_{37}\text{Rb}$.

Part (d): The nuclear reaction in this case is

$${}^{235}_{92}\text{U}+{}^{1}_{0}n\rightarrow{}^{137}_{52}\text{Te}+3{}^{1}_{0}\text{n}+{}^{A}_{Z}?$$

The mass number of the unknown isotope is

$$A=235+1-137-3\times1=96$$

while the atomic number is

$$Z=92+0-52-3\times0=40$$

Referring to the periodic table, we see that Z = 40 corresponds to zirconium; therefore, the missing isotope in the nuclear reaction is zirconium-96, ${}^{96}_{40}\text{Zr}$.

Part (e): The nuclear reaction in this case is

$${}^{235}_{92}\text{U}+{}^{1}_{0}n\rightarrow{}^{146}_{57}\text{La}+3{}^{1}_{0}\text{n}+{}^{A}_{Z}?$$

The mass number of the unknown isotope is

$$A=235+1-146-3\times1=87$$

while the atomic number is

$$Z=92+0-57-3\times0=35$$

Referring to the periodic table, we see that Z = 35 corresponds to bromine; therefore, the missing isotope in the nuclear reaction is bromine-87, ${}^{87}_{35}\text{Br}$.

■ **P6.3**

Part (a): Let subscripts 1 and 2 denote the heavier and lighter fragment, respectively. From conservation of momentum, we may write

$$m_1 v_1 = m_2 v_2 \rightarrow \frac{v_2}{v_1} = \frac{m_1}{m_2} = \frac{3}{2}$$

$$\therefore m_2 = \left(\frac{2}{3}\right) m_1$$

or $v_2 = (3/2)v_1$. The kinetic energies must add up to 166 MeV:

$$E_1 + E_2 = 166 \text{ MeV} \rightarrow \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = 166$$

$$\therefore \frac{1}{2} m_1 v_1^2 + \frac{1}{2} \times \left(\frac{2}{3} m_1\right) \times \left[\left(\frac{3}{2} v_1\right)^2\right] = 166$$

$$\therefore \frac{1}{2} m_1 v_1^2 + \left(\frac{1}{3} m_1\right) \times \frac{9}{4} v_1^2 = 166$$

$$\therefore \frac{1}{2} m_1 v_1^2 + \frac{3}{4} m_1 v_1^2 = 166$$

$$\therefore \frac{1}{2} m_1 v_1^2 + \frac{6}{4} \times \frac{1}{2} m_1 v_1^2 = 166$$

$$\therefore \left(1 + \frac{3}{2}\right) \times \underbrace{\frac{1}{2} m_1 v_1^2}_{=E_1} = 166$$

$$\therefore \frac{5}{2} \times E_1 = 166$$

$$\therefore E_1 = 166 \times \frac{2}{5} = \boxed{66.4 \text{ MeV}}$$

The kinetic energy of the second fragment easily follows:

$$E_1 + E_2 = 166 \text{ MeV} \rightarrow E_2 = 166 - E_1$$

$$\therefore E_2 = 166 - 66.4 = \boxed{99.6 \text{ MeV}}$$

Part (b): The mass numbers A_1 and A_2 must add up to $A = 235 + 1 - 3 = 233$ (where we add 1 to account for the fission neutron, then subtract 3 to account for the three neutrons released in the fission reaction, as mentioned in the problem statement). Thus,

$$A_1 + A_2 = 233$$

But $A_1/A_2 = 3/2$, giving

$$A_1 + A_2 = 233 \rightarrow \frac{3}{2} A_2 + A_2 = 233$$

$$\therefore \frac{5}{2} A_2 = 233$$

$$\therefore A_2 = \frac{2}{5} \times 233 = 93.2 \approx \boxed{93}$$

In turn,

$$A_1 = 233 - A_2 = 233 - 93 = \boxed{140}$$

Part (c): We first use the results of part (b) to compute the masses of the nuclei in kilograms, noting that 1 amu ≈ 1.66×10^{-27} kg:

$$m_1 = 140 \text{ amu} \times 1.66 \times 10^{-27} \frac{\text{kg}}{\text{amu}} = 2.32 \times 10^{-25} \text{ kg}$$

$$m_2 = 93 \text{ amu} \times 1.66 \times 10^{-27} \frac{\text{kg}}{\text{amu}} = 1.54 \times 10^{-25} \text{ kg}$$

In turn, we convert the energies obtained in part (a) to J, noting that 1 MeV ≈ 1.60×10^{-13} J:

$$E_1 = 66.4 \text{ MeV} \times 1.60 \times 10^{-13} \frac{\text{J}}{\text{MeV}} = 1.06 \times 10^{-11} \text{ J}$$

$$E_2 = 99.6 \text{ MeV} \times 1.60 \times 10^{-13} \frac{\text{J}}{\text{MeV}} = 1.59 \times 10^{-11} \text{ J}$$

It remains to compute the velocities attained by the two fragments:

$$E_1 = \frac{1}{2} m_1 v_1^2 \rightarrow v_1 = \sqrt{\frac{2E_1}{m_1}}$$

$$\therefore v_1 = \sqrt{\frac{2 \times \left(1.06 \times 10^{-11}\right)}{2.32 \times 10^{-25}}} = \boxed{9.56 \times 10^6 \text{ m/s}}$$

$$\therefore v_2 = \sqrt{\frac{2 \times \left(1.59 \times 10^{-11}\right)}{1.54 \times 10^{-25}}} = \boxed{1.44 \times 10^7 \text{ m/s}}$$

Part (d): The linear momentum is the same for both fission fragments, hence we can compute it on the basis of data from either fragment; using results from fragment 1, we may write

$$p = m_1 v_1 = \left(2.32 \times 10^{-25}\right) \times \left(9.56 \times 10^6\right) = 2.22 \times 10^{-18} \text{ kg} \cdot \text{m/s}$$

Fission neutrons have an average energy of E_n = 2 MeV. Further, the rest mass of a fission neutron may be taken as 1.675×10^{-27} kg. The corresponding velocity is

$$E_n = \frac{1}{2} m_n v_n^2 \rightarrow v_n = \sqrt{\frac{2E_n}{m_n}}$$

$$\therefore v_n = \sqrt{\frac{2 \times \left[2 \times \left(1.60 \times 10^{-13}\right)\right]}{1.675 \times 10^{-27}}} = 1.95 \times 10^7 \text{ m/s}$$

and the linear momentum is

$$p_n = m_n v_n = \left(1.675 \times 10^{-27}\right) \times \left(1.95 \times 10^7\right) = 3.27 \times 10^{-20} \text{ kg} \cdot \text{m/s}$$

Lastly, the momentum ratio is

$$\frac{p}{p_n} = \frac{2.22 \times 10^{-18}}{3.27 \times 10^{-20}} \approx \boxed{67.9}$$

■ P6.4

Part (a): We first compute the mass defect Δm,

$$\Delta m = \left(M_{U-235} + m_n\right) - \left(M_{\text{Kr-92}} + M_{\text{Ba-141}} + 3m_n\right)$$

$$\therefore \Delta m = \left(235.043930 + 1.008665\right) - \left(91.926173 + 140.914403 + 3 \times 1.008665\right)$$

$$\therefore \Delta m = 0.186024 \text{ u}$$

Then, the energy yield Q becomes

$$Q = \Delta mc^2 = \left(0.186024 \text{ u}\right) \times \left(931.49 \frac{\text{MeV}}{\text{u}}\right) = \boxed{173.28 \text{ MeV}}$$

Part (b): The mass defect Δm is

$$\Delta m = \left(M_{\text{U-235}} + m_n\right) - \left(M_{\text{Sr-94}} + M_{\text{Xe-140}} + 3m_n\right)$$

$$\therefore \Delta m = \left(235.043930 + 1.008665\right) - \left(93.915356 + 139.921646 + 2 \times 1.008665\right)$$

$$\therefore \Delta m = 0.198263 \text{ u}$$

The energy yield Q follows as

$$Q = \Delta mc^2 = \left(0.198263 \text{ u}\right) \times \left(931.49 \frac{\text{MeV}}{\text{u}}\right) = \boxed{184.68 \text{ MeV}}$$

Part (e): The mass defect Δm is

$$\Delta m = \left(M_{\text{Pu-239}} + m_n\right) - \left(M_{\text{Zr-103}} + M_{\text{Xe-134}} + 3m_n\right)$$

$$\therefore \Delta m = (239.052164 + 1.008665) - (102.927191 + 133.905395 + 3 \times 1.008665)$$

$$\therefore \Delta m = 0.202248 \text{ u}$$

The energy yield *Q* follows as

$$Q = \Delta mc^2 = (0.202248 \text{ u}) \times \left(931.49 \frac{\text{MeV}}{\text{u}}\right) = \boxed{188.39 \text{ MeV}}$$

■ P6.5

Part (a): With reference to Table 6.2, the total neutron yield ν of U-233 is read as 2.4968. Appealing to Table 6.4, the fission (σ_f) and capture (σ_γ) cross sections of this same element are 529.1 and 45.5 b, respectively. The reproduction factor is given by

$$\eta = \frac{\nu\sigma_f}{\sigma_a} = \frac{\nu\sigma_f}{\sigma_f + \sigma_\gamma} = \frac{2.4968 \times 529.1}{529.1 + 45.5} = \boxed{2.299}$$

Part (b): Taking ν = 2.8836 (Table 6.2), σ_f = 748.1 b (Table 6.4) and σ_γ = 269.3 b (Table 5.4), the reproduction factor for Pu-239 is determined as

$$\eta = \frac{\nu\sigma_f}{\sigma_f + \sigma_\gamma} = \frac{2.8836 \times 748.1}{748.1 + 269.3} = \boxed{2.120}$$

Part (c): Taking ν = 3.239 (Table 6.2), σ_f = 3.20 b (Table 6.4) and σ_γ = 587 b (Table 5.4), the reproduction factor for Am-241 is determined as

$$\eta = \frac{\nu\sigma_f}{\sigma_f + \sigma_\gamma} = \frac{3.239 \times 3.20}{3.20 + 587} = \boxed{0.0176}$$

■ P6.9

Part (a): We first estimate the specific activity for spontaneous fission of U-232; note that $t_{1/2} = 8\times10^{13}$ y = 2.53×10^{21} sec (Table 6.1):

$$SA = \frac{\lambda N_A}{M} = \frac{\ln 2 \times N_A}{t_{1/2}M} = \frac{\ln 2 \times (6.02 \times 10^{23})}{(2.53 \times 10^{21}) \times 232} = 0.711 \text{ fissions/}(\text{g}\cdot\text{s})$$

Multiplying this by the spontaneous neutron yield per fission ν = 1.71 (Table 6.1), we obtain

$$Y = \nu \times (SA) = 1.71 \times 0.711 = \boxed{1.22 \text{ n/}(\text{g}\cdot\text{s})}$$

Part (c): Noting that the spontaneous fission half-life of Pu-238 is $t_{1/2} = 4.77\times10^{10}$ y = 1.51×10^{18} sec (Table 6.1), we first compute *SA*:

$$SA = \frac{\lambda N_A}{M} = \frac{\ln 2 \times N_A}{t_{1/2}M} = \frac{\ln 2 \times (6.02 \times 10^{23})}{(1.51 \times 10^{18}) \times 238} = 1160 \text{ fissions/}(\text{g}\cdot\text{s})$$

Multiplying this by the spontaneous neutron yield per fission ν = 2.21 (Table 6.1), we obtain

$$Y = \nu \times (SA) = 2.21 \times 1160 = \boxed{2560 \text{ n/}(\text{g}\cdot\text{s})}$$

Part (e): Knowing that the spontaneous fission half-life of americium-241 is $t_{1/2}$ = 1.05×10^{14} y = 3.31×10^{21} sec (Table 6.1), we may write

$$SA = \frac{\lambda N_A}{M} = \frac{\ln 2 \times N_A}{t_{1/2}M} = \frac{\ln 2 \times (6.02 \times 10^{23})}{(3.31 \times 10^{21}) \times 241} = 0.523 \text{ fissions/}(\text{g}\cdot\text{s})$$

Multiplying this by the spontaneous neutron yield per fission ν = 3.22 (Table 6.1), we obtain

$$Y = \nu \times (SA) = 3.22 \times 0.523 = \boxed{1.68 \text{ n/}(\text{g}\cdot\text{s})}$$

■ P10.1

According to Section 10.1, 34 eV of energy are needed to create an ion pair. Accordingly, the number of ion pairs per cm^2 formed in each second is

$$\text{No. of ion pairs} = \frac{10^6 \dfrac{\alpha}{\text{cm}^2\cdot\text{s}} \times \left(2.0\times10^6\right)\dfrac{\text{eV}}{\alpha}}{34\dfrac{\text{eV}}{\text{ion pair}}\times 1\text{ cm}} = \boxed{5.88\times10^{10}\ \dfrac{\text{ip}}{\text{cm}^3\cdot\text{s}}}$$

The fraction χ of the targets that experience ionization each second is

$$\chi = \frac{5.88\times10^{10}\ \text{cm}^{-3}\cdot\text{s}^{-1}}{2.7\times10^{19}\ \text{cm}^{-3}} = \boxed{2.18\times10^{-9}\ \text{s}^{-1}}$$

■ P10.2

In linear terms, we may write the relationship between chance of fatality C_L and dose H as $C_L = aH$, where a is a constant such that

$$C_L = a\times H \quad\rightarrow\quad 0.5 = a\times 400$$

$$\therefore a = \frac{0.5}{400} = 1.25\times10^{-3}\ \text{rem}^{-1}$$

In quadratic terms, we may write the relationship between chance of fatality C_Q and dose H as $C_Q = bH^2$, where b is a constant such that

$$C_Q = b\times H^2 \quad\rightarrow\quad 0.5 = b\times 400^2$$

$$\therefore b = \frac{0.5}{400^2} = 3.13\times10^{-6}\ \text{rem}^{-2}$$

Gleaning our results, we have, for the linear approximation,

$$C_L = \left(1.25\times10^{-3}\right)\times H$$

whereas for the quadratic approximation,

$$C_Q = \left(3.13\times10^{-3}\right)\times H^2$$

At H = 2 rem, the ratio C_L/C_S becomes

$$\frac{C_L}{C_Q} = \frac{\left(1.25\times10^{-3}\right)\times 2}{\left(3.13\times10^{-6}\right)\times 2^2} \approx 200$$

Therefore, for a dose of 2 rem, using the linear approximation instead of the square model would overpredict the chance of fatality 200-fold.

■ P10.4

The total exposure experienced by the person is 100 + 50 + 30 + 150 + 25 + 10 = 365 mrems/y. The NRC limit is 3 mrems/y, which in the present case amounts to an increase of 3/365 × 100% = 0.82% relatively to the original array of exposures.

■ P10.5

The dose afforded by the tritium sample can be easily converted to rads:

$$D = 4\times10^{-3}\ \frac{\text{J}}{\text{kg}}\times\frac{1}{0.01}\ \frac{\text{rad}}{\text{J/kg}} = 0.40\ \text{rads}$$

Noting that the quality factor QF = 1 for beta radiation, the dose equivalent H becomes

$$H = D\times\left(QF\right) = 0.40\times1.0 = 0.4\ \text{rem} = \boxed{400\ \text{mrem}}$$

Alternatively, noting that 1 J/kg = 1 Gy and 1 Gy = 1 Sv, we may write the dose equivalent as

$$H = D\times\left(QF\right) = \left(4\times10^{-3}\ \text{Gy}\right)\times1.0 = 0.004\ \text{Sv} = \boxed{4.0\ \text{mSv}}$$

■ P10.8

The reparation R per mrem of exposure is

$$R = \frac{\left(51\ \text{days}\right)\times\left(24\ \text{hr/day}\right)\times\left(\$100/\text{yr}\right)}{\left(1000\ \text{mrem/yr}\right)\times\left(65-18+1\ \text{yr}\right)} = \boxed{\$2.55/\text{mrem}}$$

■ P10.10

Part (a): Including a factor of 2 to account for outbound and return flights, the flight dose is

$$D = 1.9\ \frac{\text{mSv}}{\text{d}} \times (2\times 180)\ \text{day} \times 100\ \frac{\text{mrem}}{\text{mSv}} = 68{,}400\ \text{mrem}$$

Comparing this with the annual NRC limit of 5000 mrem gives

$$\text{Ratio} = \frac{68{,}400}{5000} = 13.68 \approx \boxed{13.7}$$

Proceeding similarly with the planetary dose, we have

$$D = 0.7\ \frac{\text{mSv}}{\text{d}} \times (2\times 365)\ \text{day} \times 100\ \frac{\text{mrem}}{\text{mSv}} = 51{,}100\ \text{mrem}$$

The corresponding ratio is

$$\text{Ratio} = \frac{51{,}100}{5000} = 10.22 \approx \boxed{10.2}$$

Part (b): Noting that the annual NRC limit can be converted as 5000 mrem = 50 mSv, the time before continuous exposure at 0.25 mSv/d can exceed this limit is 50/0.25 = 200 days.

■ P10.11

Taking c_p = 4.186 J/g·°C for water and noting that1 Gy = 0.001 J/g, we may write

$$Q = mc\Delta T \quad \to \quad \Delta T = \frac{Q}{mc}$$

$$\therefore \Delta T = \frac{4\times 0.001}{1.0\times 4.186} = \boxed{9.56\times 10^{-4}\ ^{\circ}\text{C}}$$

■ P10.13

Part (a): We shall use water to approximate the cell material. Further, we make use of equation (5.4) to estimate the range of the α-particle in air and then employ the relation developed in Example 5.3 (page 88) to adapt this result to liquid water. Proceeding accordingly, we refer to equation (5.4) to obtain

$$R_{\text{air}} = 0.56E_{\alpha} = 0.56\times 3.0 = 1.68\ \text{cm}$$

Then, using (5.7) with $M_{water} \approx$ 18 g/mol and ρ = 1.0 g/cm^3,

$$R = 3.2\times 10^{-4}\frac{\sqrt{M}}{\rho}R_{\text{air}} = 3.2\times 10^{-4}\times\frac{\sqrt{18}}{1.0}\times 1.68 = 0.00228\ \text{cm} = 22.8\ \mu\text{m}$$

This result indicates that the particle will pass fully through two 10-μm cells before being brought to a halt.

Part (b): With reference to equation (5.4), we have

$$R_{\text{air}} = 1.24E_{\alpha} - 2.62 = 1.24\times 6.0 - 2.62 = 4.82\ \text{cm}$$

Using (5.7) as in part (a),

$$R = 3.2\times 10^{-4}\times\frac{\sqrt{18}}{1.0}\times 4.82 = 0.00654\ \text{cm} = 65.4\ \mu\text{m}$$

This result indicates that the particle will pass fully through six 10-μm cells before being brought to a halt.

■ P11.1

Part (a): Using water to approximate tissue, we have μ_{en}/ρ = 0.03299 cm^2/g at 0.5 MeV (see the fifth column of Table A.6 on page 575). Noting that 0.5 MeV = 8.0×10^{-14} J = 8.0×10^{-9} rad·g, we proceed to write

$$\dot{D} = \phi E_{\gamma}\left(\frac{\mu_{\text{en}}}{\rho}\right) = 100\times\left(8.0\times 10^{-9}\right)\times 0.03299 = 2.64\times 10^{-8}\ \text{rad/s}$$

$$\therefore \dot{D} = 2.64\times 10^{-8}\ \frac{\text{rad}}{\text{s}}\times 3.16\times 10^{7}\ \frac{\text{s}}{\text{yr}}\times 1000\ \frac{\text{mrad}}{\text{rad}}\times 1\ \frac{\text{mrem}}{\text{mrad}} = \boxed{834\ \text{mrem/yr}}$$

For a person working 40 hours per week in such a flux, the dose equivalent becomes

$$\text{Dose for working 40 hours/week} = \frac{40\text{ h}}{7\text{ d}\times 24\text{ h/d}}\times 834\text{ mrem/yr} = \boxed{199\ \frac{\text{mrem}}{\text{y}}}$$

Part (b): Using water to represent tissue, we read μ_{en}/ρ = 0.02608 at 2 MeV (see the fifth column of Table A.6 on page 576). Noting that 2 MeV = 3.2×10^{-13} J = 3.2×10^{-8} rad·g, we may write

$$\dot{D} = \phi E_\gamma \left(\frac{\mu_{en}}{\rho}\right) = 100\times\left(3.2\times10^{-8}\right)\times 0.02608 = 8.35\times10^{-8}\ \text{rad/s}$$

$$\therefore \dot{D} = 8.35\times10^{-8}\ \frac{\text{rad}}{\text{s}}\times 3.16\times10^{7}\ \frac{\text{s}}{\text{yr}}\times 1000\ \frac{\text{mrad}}{\text{rad}}\times 1\ \frac{\text{mrem}}{\text{mrad}} = \boxed{2640\text{ mrem/yr}}$$

For a person working 40 hours per week in such a flux, the dose equivalent becomes

$$\text{Dose for working 40 hours/week} = \frac{40\text{ h}}{7\text{ d}\times 24\text{ h/d}}\times 2640\text{ mrem/yr} = \boxed{629\ \frac{\text{mrem}}{\text{y}}}$$

■ P11.2

Since Co-60 emits gammas of energy 1.17 MeV and 1.33 MeV, it seems appropriate to use information for radiation at (1.17 + 1.33)/2 = 1.25 MeV. Entering this energy level into Table A.6 on page 576, we read μ_{en}/ρ = 0.02965 cm^2/g. Then, the flux for a limiting exposure rate of 500 mrem/y becomes

$$\phi = \frac{\dot{H}/QF}{\left(\mu_{en}/\gamma\right)E_\gamma} = \frac{\left[(500\text{ mrem/y})/(1000\text{ mrem/rad})\right]\times\left(10^{-5}\text{ J/g/rad}\right)}{\left(0.02965\text{ cm}^2/\text{g}\right)\times\left(1.25\text{ MeV}/\gamma\right)\times\left(1.60\times10^{-13}\text{J/MeV}\right)\times\left(3.16\times10^{7}\text{ s/yr}\right)}$$

$$\therefore \phi = 26.7\ \gamma/\left(\text{cm}^2\cdot\text{g}\right)$$

Next, we solve the inverse square law for the number S of emissions per second:

$$\phi = \frac{S}{4\pi r^2}\ \rightarrow\ S = 4\pi r^2\phi$$

$$\therefore S = 4\pi\times 100^2\times 26.7 = 3.36\times10^{6}\ \gamma/\text{s}$$

Since two gammas are emitted per decay of Co-60, the activity A becomes

$$A_{max} = \frac{S}{(2\text{ gammas/decay})} = \frac{3.36\times10^{6}}{2} = 1.68\times10^{6}\ \text{Bq}$$

Converting this result to μCi:

$$A_{max} = \frac{1.68\times10^{6}\ \text{Bq}}{3.7\times10^{4}\ \text{Bq/μCi}} = \boxed{45.4\ \text{μCi}}$$

In order for the source not to exceed an exposure rate of 500 mrem/year, its activity must be no greater than ≈45.4 microcuries.

■ P11.5

Referring to Table A.4, the density of lead is read as ρ = 11.35 g/cm^3. From Table A.6, we read that μ/ρ = 0.07102 cm^2/g for 1-MeV gammas. We use these data to compute the linear attenuation coefficient μ:

$$\mu = \left(\frac{\mu}{\rho}\right)\rho = 0.07102\times 11.35 = 0.806\ \text{cm}^{-1}$$

The source strength is converted as

$$S = 200\text{ mCi}\times 3.7\times10^{7}\ \frac{\text{Bq}}{\text{mCi}} = 7.4\times10^{9}\ \text{Bq}$$

Lastly, the gamma ray flux is given by equation (11.9),

$$\phi = \frac{S\exp(-\mu r)}{4\pi r^2} = \frac{\left(7.4\times10^{9}\right)\times\exp(-0.806\times 12)}{4\pi\times 12^2} = \boxed{258\ \text{cm}^{-2}\text{s}^{-1}}$$

■ P11.7

The allowed activity is $C = ay_i$, where y_i is yield and a is a constant given by

$$a = \left[\sum_i \frac{y_i}{(MPC)_i}\right]^{-1} = \left[\frac{0.0575}{(5\times10^{-7})} + \frac{0.0545}{(3\times10^{-6})} + \frac{0.0611}{(1\times10^{-6})}\right]^{-1} = 5.15\times10^{-6}\ \mu\text{Ci/mL}$$

so that

$$C_{\text{Sr}} = (5.15\times10^{-6})\times0.0575 = \boxed{2.96\times10^{-7}\ \mu\text{Ci/mL}}$$

$$C_{\text{Sr}} = (5.15\times10^{-6})\times0.0545 = \boxed{2.81\times10^{-7}\ \mu\text{Ci/mL}}$$

$$C_{\text{Sr}} = (5.15\times10^{-6})\times0.0611 = \boxed{3.15\times10^{-7}\ \mu\text{Ci/mL}}$$

As a final check, note that the sum of ratios $C_i/(MPC_i)$ should equal unity:

$$\sum_i \frac{C_i}{(MPC)_i} = \frac{2.96\times10^{-7}}{5\times10^{-7}} + \frac{2.81\times10^{-7}}{3\times10^{-6}} + \frac{3.15\times10^{-7}}{1\times10^{-6}} = 1.00067 \approx 1\ \text{(Check!)}$$

■ P11.9

The half-life of U-238 is 4.468×10^9 yr, or 1.41×10^{17} sec. The corresponding decay constant is

$$\lambda = \frac{\ln 2}{t_{1/2}} = \frac{0.693}{1.41\times10^{17}} = 4.91\times10^{-18}\ \text{sec}^{-1}$$

Noting that 10 ppm = 10×10^{-6} g/g, the number density N on the basis of unit mass becomes

$$N = \frac{10^{-5}\times(6.02\times10^{23})}{238} = 2.53\times10^{16}\ \text{at/g}$$

The specific activity follows as

$$A = N\lambda = (2.53\times10^{16})\times(4.91\times10^{-18}) = 0.124\ \text{Bq/g}$$

or

$$A = 0.124\ \frac{\cancel{\text{Bq}}}{\text{g}}\times\left(\frac{1}{3.7\times10^4}\ \frac{\mu\text{Ci}}{\cancel{\text{Bq}}}\right) = \boxed{3.35\times10^{-6}\ \mu\text{Ci/g}}$$

■ P11.10

Part (a): Using a bioaccumulation factor of 2000 as discussed in the solution to Example 11.11, the corrected water consumption for the infant is

$$\dot{V} = 330 + 0\times2000 = 330\ \text{L/yr}$$

From Table 11.5, the ingestion dose conversion factor for an infant's kidney is 16.4×10^{-5} mrem/pCi. The corresponding dose is

$$D = 3.47\ \frac{\text{pCi}}{\text{L}}\times330\ \frac{\text{L}}{\text{yr}}\times(16.4\times10^{-5})\ \frac{\text{mrem}}{\text{pCi}} = \boxed{0.188\ \text{mrem}}$$

Part (b): With reference to Table 11.4, the corrected water consumption for the child is

$$\dot{V} = 510 + 6.9\times2000 = 14{,}300\ \text{L/yr}$$

From Table 11.5, the ingestion dose conversion factor for a child's bones is 32.7×10^{-5} mrem/pCi. The corresponding dose is

$$D = 3.47\times14{,}300\times(32.7\times10^{-5}) = \boxed{16.2\ \text{mrem}}$$

Part (c): Referring to Table 11.4, the corrected water consumption for the teenager is

$$\dot{V} = 510 + 16\times2000 = 32{,}500\ \text{L/yr}$$

From Table 11.5, the ingestion dose conversion factor for a teenager's liver is 14.9×10^{-5} mrem/pCi. The corresponding dose is

$$D = 3.47 \times 32{,}500 \times \left(14.9 \times 10^{-5}\right) = \boxed{16.8 \text{ mrem}}$$

Part (d): Appealing to Table 11.4, the corrected water consumption for the adult is

$$\dot{V} = 730 + 21 \times 2000 = 42{,}700 \text{ L/yr}$$

From Table 11.5, the ingestion dose conversion factor for an adult's GI tract is 0.211×10^{-5} mrem/pCi. The corresponding dose is

$$D = 3.47 \times 42{,}700 \times \left(0.211 \times 10^{-5}\right) = \boxed{0.313 \text{ mrem}}$$

■ P11.11

With an air intake rate of 1.2 m^3/h, the total volume respired over the course of six 10-h working days is 1.2 × 6 × 10 = 72 m^3. The remaining calculations are identical to those of Example 11.13 (page 195). The annual exposure limit is 5000 mrem (or 5 rem). The annual limits of intake of I-131 and Cs-137 by inhalation are 50 μCi and 200 μCi (Table 11.7), respectively. The amounts of radiation from I-131 and Cs-137 to which the worker was exposed are, respectively,

$$\text{I-131}: \left(1.3 \times 10^{-8} \frac{\mu\text{Ci}}{\text{mL}}\right) \times \left(72 \times 10^{6} \text{ mL}\right) = 0.936\ \mu\text{Ci}$$

$$\text{Cs-137}: \left(7.3 \times 10^{-8} \frac{\mu\text{Ci}}{\text{mL}}\right) \times \left(72 \times 10^{6} \text{ mL}\right) = 5.26\ \mu\text{Ci}$$

The ratio we're looking for is

$$\text{Ratio} = \frac{200}{5000} + \frac{0.936}{50} + \frac{5.26}{200} = \boxed{0.085}$$

■ P16.2

Part (a): For an enrichment of 93.9 w/o, the mass of U-235 is 0.939 × 50 = 46.95 kg. The fission cross-section of U-235 is 1.4 b. The number of fuel atoms is determined as

$$n_F = \frac{m_{235} N_A}{M_{235}} = \frac{46{,}950 \times \left(6.02 \times 10^{23}\right)}{235} = 1.20 \times 10^{26} \text{ atoms}$$

The average flux ϕ_{avg} follows as

$$\phi_{\text{avg}} = \frac{P}{n_F \sigma_f w} = \frac{100}{\left(1.20 \times 10^{26}\right) \times \left(1.4 \times 10^{-24}\right) \times \left[1/\left(3.29 \times 10^{10}\right)\right]} = \boxed{1.96 \times 10^{10} \text{ cm}^{-2}\text{s}^{-1}}$$

It is worth noting that inclusion of U-238 fast fission has little effect on the calculated flux. The mass of U-238 for the present reactor is 50 – 46.95 = 3.050 kg = 3050 g. The fission cross-section of U-238 is 0.095 b. The number of U-238 atoms is calculated as

$$n_{238} = \frac{m_{238} N_A}{M_{238}} = \frac{3050 \times \left(6.02 \times 10^{23}\right)}{238} = 7.71 \times 10^{24} \text{ atom}$$

and the updated flux becomes

$$\phi_{\text{avg}} = \frac{100}{\left[\left(1.20 \times 10^{26}\right) \times \left(1.4 \times 10^{-24}\right) + \left(7.71 \times 10^{24}\right) \times \left(0.095 \times 10^{-24}\right)\right] \times \left[1/\left(3.29 \times 10^{10}\right)\right]}$$

$$\therefore \phi_{\text{avg}} \approx 1.95 \times 10^{10}$$

Clearly, the value of ϕ_{avg} obtained while accounting for U-238 fast fission is almost identical to the flux obtained without this contribution.

Part (b): The mass of plutonium-239 that powers this Jezebel-type reactor is 17 kg. The fission cross-section of Pu-239 is σ_f = 1.85 b. The number of fuel atoms then becomes

$$n_F = \frac{m_{239} N_A}{M_{239}} = \frac{17{,}000 \times \left(6.02 \times 10^{23}\right)}{239} = 4.28 \times 10^{25} \text{ atoms}$$

To find the average flux ϕ_{avg}, we write

$$\phi_{\text{avg}} = \frac{P}{n_F \sigma_f w} = \frac{100}{\left(4.28\times10^{25}\right)\times\left(1.85\times10^{-24}\right)\times\left[1/\left(3.29\times10^{10}\right)\right]} = \boxed{4.16\times10^{10}\ \text{cm}^{-2}\text{s}^{-1}}$$

■ P16.8

Part (a): The diffusion length L is determined as

$$L^2 = \frac{D}{\Sigma_a} \rightarrow L = \sqrt{\frac{D}{\Sigma_a}}$$

$$\therefore L = \sqrt{\frac{1.02}{0.0795}} = \boxed{3.58\ \text{cm}}$$

Part (b): The geometric buckling for a spherical reactor such as the present one is

$$B^2 = \left(\frac{\pi}{R}\right)^2 = \left(\frac{\pi}{10}\right)^2 = \boxed{0.0987\ \text{cm}^{-2}}$$

Part (c): The nonleakage factor is

$$\ell = \frac{1}{1+L^2B^2} = \frac{1}{1+3.58^2\times0.0987} = \boxed{0.442}$$

■ P16.9

Part (a): Using ω_{235} = 0.939 and ρ_U = 18.75 g/cm^3 as specified on Table 16.1, the number density of U-235 atoms is

$$N_{235} = \frac{\omega_{235}\rho_U N_A}{M_{235}} = \frac{0.939\times18.75\times\left(6.02\times10^{23}\right)}{235} = 4.51\times10^{22}\ \text{cm}^{-3}$$

In turn, with ω_{238} = 1 – 0.939 = 0.061, the number density of U-238 is

$$N_{235} = \frac{\omega_{235}\rho_U N_A}{M_{235}} = \frac{0.061\times18.75\times\left(6.02\times10^{23}\right)}{238} = 2.89\times10^{21}\ \text{cm}^{-3}$$

Using the appropriate constants from Table 16.3, we have the macro absorption cross-section

$$\Sigma_a = \Sigma_{a,235} + \Sigma_{a,238} = N_{235}\sigma_{a,235} + N_{238}\sigma_{a,238}$$

$$\therefore \Sigma_a = \left(4.51\times10^{22}\right)\times\left[\left(1.4+0.25\right)\times10^{-24}\right] + \left(2.89\times10^{21}\right)\times\left[\left(0.095+0.16\right)\times10^{-24}\right]$$

$$\therefore \Sigma_a = 0.0752\ \text{cm}^{-1}$$

and the macro transport cross-section

$$\Sigma_{\text{tr}} = \Sigma_{\text{tr},235} + \Sigma_{\text{tr},238} = N_{235}\sigma_{\text{tr},235} + N_{238}\sigma_{\text{tr},238}$$

$$\therefore \Sigma_{\text{tr}} = \left(4.51\times10^{22}\right)\times\left(6.8\times10^{-24}\right) + \left(2.89\times10^{21}\right)\times\left(6.9\times10^{-24}\right)$$

$$\therefore \Sigma_{\text{tr}} = 0.327\ \text{cm}^{-1}$$

The diffusion coefficient is then

$$D = \frac{1}{3\Sigma_{tr}} = \frac{1}{3\times0.327} = 1.02\ \text{cm}$$

and the diffusion area follows as

$$L^2 = \frac{D}{\Sigma_a} = \frac{1.02}{0.0752} = \boxed{13.6\ \text{cm}^2}$$

Part (b): Per Table 16.1, the critical mass of a Godiva assembly is 48.8 kg. The corresponding mass of uranium-235 is

$$m_U = \frac{m_{235}}{\omega_{235}} = \frac{48.8}{0.939} = 51.97\ \text{kg}$$

or, in volumetric terms,

$$V_U = \frac{m_U}{\rho_U} = \frac{51,970}{18.75} = 2772 \text{ cm}^3$$

The corresponding radius is

$$V_U = \frac{4}{3}\pi R^3 \rightarrow R = \left(\frac{3V_U}{4\pi}\right)^{\frac{1}{3}}$$

$$\therefore R = \left(\frac{3\times 2772}{4\pi}\right)^{\frac{1}{3}} = 8.71 \text{ cm}$$

and the buckling is

$$B^2 = \left(\frac{\pi}{R}\right)^2 = \left(\frac{\pi}{8.71}\right)^2 = 0.130 \text{ cm}^{-2}$$

Lastly, the nonleakage probability is

$$\ell = \frac{1}{1+L^2B^2} = \frac{1}{1+13.6\times 0.130} = \boxed{0.361}$$

■ P16.10

Part (a): The buckling of a parallelepiped reactor of dimensions $L \times W \times H$ is expressed as

$$B_g^2 = \left(\frac{\pi}{L}\right)^2 + \left(\frac{\pi}{W}\right)^2 + \left(\frac{\pi}{H}\right)^2 = \left(\frac{\pi}{25}\right)^2 + \left(\frac{\pi}{35}\right)^2 + \left(\frac{\pi}{40}\right)^2 = \boxed{0.0300 \text{ cm}^{-2}}$$

In Example 16.3, we learned that a Godiva assembly composed of U-235 with a density of 19 g/cm^3 has diffusion area equal to 12.5 cm^2. Using this result, the nonleakage probability is

$$\ell = \frac{1}{1+L^2B_g^2} = \frac{1}{1+12.5\times 0.03} = \boxed{0.727}$$

Part (b): The buckling of a cylindrical reactor of radius R and height H is given by

$$B_g^2 = \left(\frac{\pi}{H}\right)^2 + \left(\frac{2.405}{R}\right)^2 = \left(\frac{\pi}{50}\right)^2 + \left(\frac{2.405}{25}\right)^2 = \boxed{0.0132 \text{ cm}^{-2}}$$

while the NLP is

$$\ell = \frac{1}{1+L^2B_g^2} = \frac{1}{1+12.5\times 0.0132} = \boxed{0.858}$$

■ P16.11

The logarithmic energy decrement for carbon was calculated in Example 4.9 and equals approximately 0.158. Substituting the pertaining data into equation (16.11), we obtain

$$p = \exp\left(-\frac{N_F I_F}{\xi_M \Sigma_{\text{s,epi}}^M}\right) = \exp\left(-\frac{N_U I_U}{\xi_C N_C \sigma_{\text{s,epi}}^C}\right)$$

$$\therefore p = \exp\left(-\frac{277}{0.158\times 450\times 4.66}\right) = \boxed{0.433}$$

■ P16.12

Replacing $\bar{\mu}$ with equation (4.50), we may write

$$\sigma_{\text{tr}} = \sigma_t - \bar{\mu}\sigma_s = \sigma_t - \frac{2}{3A}\sigma_s$$

For heavy nuclides such as uranium or plutonium, A is large and the rightmost term tends to zero, so that the approximation $\sigma_{\text{tr}} \approx \sigma_t$ holds.

■ P17.3

Different heat transfer rate terms in a cylindrical fuel pin are such that

$$q = q'H = q''\times 2\pi RH = q'''\times \pi R^2 H$$

Hence, the heat flux q'' and the power density q''' are related as

$$q'' \times 2\pi RH = q''' \times \pi R^2 H \quad \rightarrow \quad q'' = \frac{\pi R^2 H}{2\pi RH} q'''$$

$$\therefore q'' = \frac{R}{2} q'''$$

$$\therefore q'' = \frac{0.6}{2} \times 500 = \boxed{150 \text{ W/cm}^2}$$

For a temperature difference $\Delta T_s = 300 - 250 = 50°\text{C}$, the heat transfer coefficient is

$$h = \frac{q''}{\Delta T_S} = \frac{150}{300 - 250} = \boxed{3.0 \text{ W/cm}^2{\cdot}°\text{C}}$$

■ P17.4

Part (a): Taking $c_p = 6.06 \times 10^3$ J/(kg·°C) for PWR conditions, the temperature difference ΔT_C from coolant inlet to outlet is

$$Q_R = \dot{m} c_p \Delta T_C \quad \rightarrow \quad \Delta T_C = \frac{Q_R}{\dot{m} c_p}$$

$$\therefore \Delta T_C = \frac{2500 \times 10^6}{15{,}000 \times \left(6.06 \times 10^3\right)} = 27.5°\text{C}$$

The outlet temperature then becomes

$$T_o = T_i + \Delta T_C = 275 + 27.5 = \boxed{302.5°\text{C}}$$

Part (b): In this case, the temperature difference ΔT_C is

$$\Delta T_C = T_{c,\text{out}} - T_{c,\text{in}} = 336 - 275 = 61°\text{C}$$

so that

$$Q_R = \dot{m} c_p \Delta T_C = 15{,}000 \times \left(6.06 \times 10^3\right) \times 61 = 5.55 \times 10^9 = \boxed{5550 \text{ MW}}$$

Note that this is more than twice the power rating used in part (a).

■ P17.7

Part (a): Taking 2260 J/g as the latent heat of vaporization of water, the makeup water mass flow becomes

$$\dot{m}_{MU} = \frac{1500 \times 10^6}{2260} = 664{,}000 \text{ g/s} = \boxed{664 \text{ kg/s}}$$

Part (b): Taking 4180 J/kg·°C as the specific heat capacity of water, the condenser mass flow is calculated as

$$\dot{m}_C = \frac{Q_W}{c_p \Delta T} = \frac{1500 \times 10^6}{4180 \times 14} = 25{,}600 \text{ kg/s}$$

Therefore, the makeup water mass flow obtained in part (a) accounts for a fraction of $\dot{m}_C$ such that

$$\frac{\dot{m}_{MU}}{\dot{m}_C} = \frac{664}{25{,}600} \times 100\% = \boxed{2.59\%}$$

■ P17.9

Taking 2260 J/g as the latent heat of evaporation of water, the mass of water evaporated per day is

$$\dot{m} = \frac{Qt}{L} = \frac{\left(2030 \times 10^6\right) \times 86{,}400}{2260} = 7.76 \times 10^{10} \text{ g/day}$$

Converting this result to a volume flow rate,

$$\dot{V} = \frac{\dot{m}}{\rho} = \frac{7.76 \times 10^{10} \text{ g/day}}{1.0 \text{ g/cm}^3} = 7.76 \times 10^{10} \text{ cm}^3\text{/day}$$

Finally, noting that 1 gal = 3785.4 cm^3, we have

$$\dot{V} = 7.76 \times 10^{10} \frac{\text{cm}^3}{\text{day}} \times \frac{1}{3785.4} \frac{\text{gal}}{\text{cm}^3} = \boxed{2.05 \times 10^7 \text{ gal/day}}$$

Over 20 million gallons of water are required each day to dissipate all the waste thermal power of the reactor in question.

■ P17.14

Since one molecule of UO_2 contains one atom of U, the number density of uranium atoms becomes

$$N_{\text{U}} = N_{\text{UO}_2} = \rho_{\text{UO}_2} \frac{N_A}{M_U + 2M_O}$$

$$\therefore N_{\text{U}} = 10 \times \frac{(6.02 \times 10^{23})}{238 + 2 \times 16} = 2.23 \times 10^{22} \text{ at/cm}^3$$

For naturally enriched uranium, γ_{235} = 0.0072, so that

$$N_{235} = \gamma_{235} N_U = 0.0072 \times (2.23 \times 10^{22}) = 1.61 \times 10^{20} \text{ at/cm}^3$$

Noting that the micro cross-section for U-235 is $\sigma_{f,235}$ = 582.6 b, we have

$$\Sigma_f = N_{235} \sigma_{f,235} = (1.61 \times 10^{20}) \times (582.6 \times 10^{-24}) = 0.0938 \text{ cm}^{-1}$$

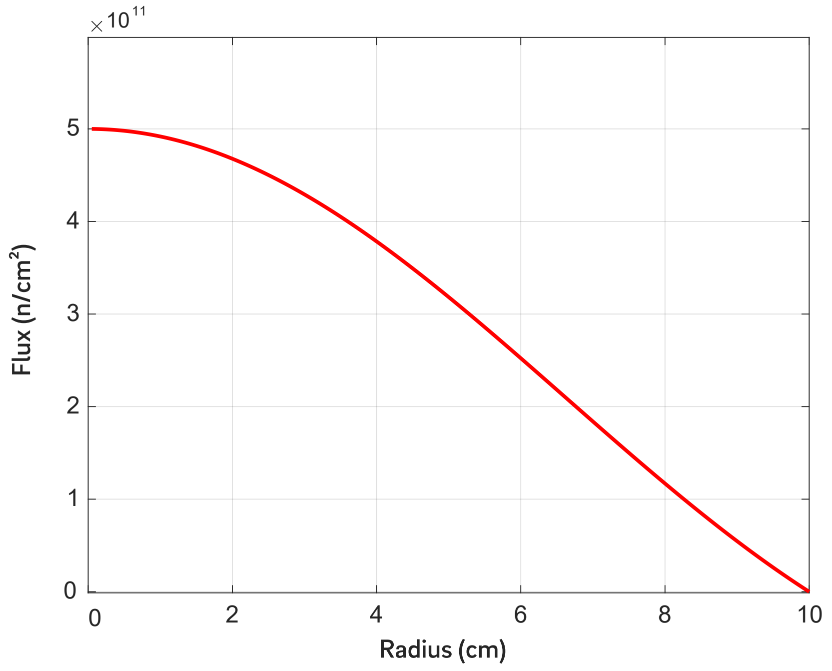
Noting that approximately 3.2×10^{10} fissions are required to yield 1 J of energy, the volumetric heat generation rate is

$$q''' = \frac{\Sigma_f \phi}{E_R} = \frac{0.0938 \times 10^{14}}{(3.2 \times 10^{10})} = \boxed{293 \text{ W/cm}^3}$$

■ P19.4

Referring to Table 19.1, we see that the neutron flux distribution $\phi(x)$ for a spherical reactor is given by $\phi(x) = \phi_C \sin(x)/x$, where $x = \pi r/R$ and in the present case R = 10 cm, $\phi_C = 5\times10^{11}/(\text{cm}^2\cdot\text{s})$. The following MATLAB code can be used to plot the neutron flux as a function of radial distance from the center of the spherical reactor:

```
phiC = 5e11;
R = 10;
r = 0:0.05:10;
x = pi.*r./R;
phi = phiC.*sin(x)./x;
plot(r, phi, 'LineWidth', 2, 'Color', 'red')
ylim([0 6e11])
grid on
```



■ P19.8

Part (a): Example 19.6 used the composition given in Example 19.3 to find that B_m^2 = 0.0028 cm^{-2}. The fuel atomic density from Example 19.3 was 8.3×10^{19} atoms/cm^3, therefore the mass density is

$$\rho_F = \frac{N_F M_F}{N_A} = \frac{(8.3 \times 10^{19}) \times 235}{6.02 \times 10^{23}} = 0.0324 \text{ g/cm}^3$$

For a cube of side a, the geometric buckling is

$$B = \left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{a}\right)^2 = 3\left(\frac{\pi}{a}\right)^2$$

Solving for *a* brings to

$$B = 3\left(\frac{\pi}{a}\right)^2 \rightarrow a = \pi\sqrt{\frac{3}{B}}$$

$$\therefore a = \pi\sqrt{\frac{3}{0.0028}} = \boxed{103 \text{ cm}}$$

The minimum cube volume is

$$V_{\text{cube}} = a^3 = 103^3 = 1.09 \times 10^6 \text{ cm}^3 = \boxed{1.09 \text{ m}^3}$$

The minimum fuel mass is

$$m_{\text{cube}} = \rho_F V_{\text{cube}} = 0.0324 \times \left(1.09 \times 10^6\right) = 35{,}300 \text{ g} = \boxed{35.3 \text{ kg}}$$

Part (b): For a sphere of radius R, the geometric buckling is $B^2 = (\pi/R)^2$; solving for radius, we get

$$B^2 = \left(\frac{\pi}{R}\right)^2 \rightarrow R = \frac{\pi}{\sqrt{B^2}}$$

$$\therefore R = \frac{\pi}{\sqrt{0.0028}} = \boxed{59.4 \text{ cm}}$$

The minimum sphere volume is

$$V_{\text{sphere}} = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi \times 59.4^3 = 878{,}000 \text{ cm}^3 = \boxed{0.878 \text{ m}^3}$$

The minimum fuel mass is

$$m_{\text{sphere}} = \rho_F V_{\text{sphere}} = 0.0324 \times \left(8.78 \times 10^5\right) = 28{,}400 \text{ g} = \boxed{28.4 \text{ kg}}$$

Part (c): For a cylinder, the optimal height-to-diameter ratio is $H/D \approx 0.924$, so that

$$B_g^2 = \left(\frac{\pi}{H}\right)^2 + \left(\frac{2.405}{R}\right)^2$$

$$\therefore B_g^2 = \left(\frac{\pi}{0.924D}\right)^2 + \left(\frac{2.405}{R}\right)^2$$

$$\therefore B_g^2 = \left(\frac{\pi}{0.924 \times 2R}\right)^2 + \left(\frac{2.405}{R}\right)^2$$

$$\therefore B_g^2 = \frac{1}{R^2}\left[\left(\frac{\pi}{1.848}\right)^2 + 2.405^2\right]$$

$$\therefore B_g^2 = \frac{8.674}{R^2}$$

$$\therefore R = \frac{\sqrt{8.674}}{\sqrt{B^2}}$$

$$\therefore R = \frac{\sqrt{8.674}}{\sqrt{0.0028}} = 55.7 \text{ cm}$$

$$\therefore D = 2 \times 55.7 = 111 \text{ cm}$$

Also, $H = 0.924 \times 111 = 103$ cm. The minimum cylinder volume is

$$V_{\text{cylinder}} = \pi R^2 H = \pi \times 55.7^2 \times 103 = 1.0 \times 10^6 \text{ cm} = \boxed{1.0 \text{ m}^3}$$

The minimum fuel mass is

$$m_{\text{cylinder}} = \rho_F V_{\text{cylinder}} = 0.0324\times\left(1.0\times10^6\right) = 32{,}400\text{ g} = \boxed{32.4\text{ kg}}$$

The results obtained in parts (a) through (c) are summarized below.

Reactor Geometry	Minimum volume	Minimum mass
Cube	1.09 m^3	35.3 kg
Sphere	0.878 m^3	28.4 kg
Cylinder	1.0 m^3	32.4 kg

■ P19.9

Part (a): The maximum-to-average linear power density of a sinusoidal distribution is expressed as

$$q'_{\text{avg}} = \frac{1}{H}\int_{-H/2}^{H/2} q'_{\max}\cos\left(\pi z/H\right)dz = \frac{2}{\pi}q'_{\max}$$

$$\therefore \frac{q'_{\max}}{q'_{\text{avg}}} = \frac{\pi}{2}$$

For a 3-D parallelepiped of dimensions $a \times b \times c$, the average and maximum heat generation rate are related as

$$q'''_{\text{avg}} = \frac{q'''_{\max}}{abc}\int_{-a/2}^{a/2}\cos\left(\pi x/a\right)dx\int_{-a/2}^{a/2}\cos\left(\pi y/b\right)dy\int_{-a/2}^{a/2}\cos\left(\pi z/c\right)dz$$

$$\therefore q'''_{\text{avg}} = \frac{q'''_{\max}}{abc}\times\frac{2a}{\pi}\times\frac{2b}{\pi}\times\frac{2c}{\pi} = q'''_{\max}\left(\frac{2}{\pi}\right)^3$$

$$\therefore \frac{q'''_{\max}}{q'''_{\text{avg}}} = \left(\frac{\pi}{2}\right)^3 = \boxed{3.876}$$

Part (b): The average heat generation rate for a spherical reactor is

$$q'''_{\text{avg}} = \frac{q'''_{\max}}{V_{\text{sphere}}}\int_V \frac{R}{\pi r}\sin\left(\frac{\pi r}{R}\right)dV$$

$$\therefore q'''_{\text{avg}} = \frac{q'''_{\max}}{\frac{4}{3}\pi R^3}\int_0^R \frac{R}{\pi r}\sin\left(\frac{\pi r}{R}\right)\times 4\pi r^2 dr$$

$$\therefore q'''_{\text{avg}} = \frac{3q'''_{\max}}{\pi^3}\int_0^R \sin\left(\frac{\pi r}{R}\right)\times\frac{\pi r}{R}\times\frac{\pi}{R}dr$$

$$\therefore q'''_{\text{avg}} = \frac{3q'''_{\max}}{\pi^3}\int_0^\pi \sin\left(\xi\right)\xi$$

where $\xi = \pi r/R$ is such that $d\xi = (\pi/R)dr$. A table of integrals reveals that $\int \xi\sin(\xi)\,d\xi = \sin(\xi) - \xi\cos(\xi)$, thus

$$q'''_{\text{avg}} = \frac{3q'''_{\max}}{\pi^3}\int_0^\pi \sin\left(\xi\right)\xi = \frac{3q'''_{\max}}{\pi^3}\left\{\left[\sin\left(\pi\right) - \pi\times\cos\left(\pi\right)\right] - \left[\sin\left(0\right) - 0\times\cos\left(0\right)\right]\right\}$$

$$\therefore q'''_{\text{avg}} = \frac{3q'''_{\max}}{\pi^3}\left\{\left[0 - \pi\times\left(-1\right)\right] - \left[0 - 0\times1\right]\right\}$$

$$\therefore q'''_{\text{avg}} = \frac{3q'''_{\max}}{\pi^2}$$

$$\therefore \frac{q'''_{\max}}{q'''_{\text{avg}}} = \frac{\pi^2}{3} = \boxed{3.290}$$

■ P19.12

Parts (a) and (b): The volume of the small reactor is

$$V_S = \frac{550{,}000\text{ kW}}{80\text{ kW/L}} = 6875\text{ L}$$

The volume of the large reactor is

$$V_L = \frac{3500\times10^3\text{ kW}}{100\text{ kW/L}} = 35{,}000\text{ L}$$

Working under optimal conditions, the height-to-diameter ratio $H/D \approx 0.924$ and the cylinder volume becomes

$$V_{\text{cylinder}} = \pi R^2 H = \pi R^2 \times 0.924 D = \pi R^2 \times 0.924 \times (2R)$$

$$\therefore V_{\text{cylinder}} = 5.806 R^3$$

Noting that 1 L = 10^3 cm^3, we can solve the relation above for radius and obtain, for the small reactor,

$$V_{\text{cylinder}} = 5.806 R^3 \rightarrow R = \sqrt[3]{\frac{V_{\text{cylinder}}}{5.806}}$$

$$\therefore R_S = \sqrt[3]{\frac{6875 \times 10^3}{5.806}} = 106 \text{ cm}$$

and for the large one,

$$R_L = \sqrt[3]{\frac{35{,}000 \times 10^3}{5.806}} = 182 \text{ cm}$$

Now, the buckling of a cylindrical reactor can be expressed in terms of radius as

$$B_g^2 = \left(\frac{\pi}{H}\right)^2 + \left(\frac{2.405}{R}\right)^2 = \left(\frac{\pi}{0.924 \times D}\right)^2 + \left(\frac{2.405}{R}\right)^2$$

$$\therefore B_g^2 = \left(\frac{\pi}{0.924 \times 2R}\right)^2 + \left(\frac{2.405}{R}\right)^2$$

$$\therefore B_g^2 = \frac{8.674}{R^2}$$

so that, for the small reactor,

$$B_{g,S}^2 = \frac{8.674}{106^2} = 7.72 \times 10^{-4} \text{ cm}^{-2}$$

whereas for the large one,

$$B_{g,L}^2 = \frac{8.674}{182^2} = 2.62 \times 10^{-4} \text{ cm}^{-2}$$

With a diffusion area L^2 = 1.9 cm^2, the nonleakage probability of the small reactor becomes

$$\ell_S = \frac{1}{1 + L^2 B_S^2} = \frac{1}{1 + 1.9 \times (7.72 \times 10^{-4})} = \boxed{0.9985}$$

whereas that of the large one is

$$\ell_L = \frac{1}{1 + L^2 B_S^2} = \frac{1}{1 + 1.9 \times (2.62 \times 10^{-4})} = \boxed{0.9995}$$

■ P20.3

Using an effective lifetime $\beta\tau \approx 0.083$, the reactor period upon removal of the control rods is

$$T = \frac{\beta\tau}{\rho} = \frac{0.083}{0.0005} = 166 \text{ s}$$

The reactor power evolves exponentially; setting *P(0)* = 250 MWe, *P(t)* = 300 MWe, *T* = 166 s, and solving for time, we obtain

$$P(t) = P(0)\exp(t/T) \rightarrow t = T \times \ln\left[\frac{P(t)}{P(0)}\right]$$

$$\therefore t = 166 \times \ln\left(\frac{300}{250}\right) = \boxed{30.3 \text{ s}}$$

■ P20.5

For the reactivity ρ to be canceled out, we must have

$$\rho + \alpha \Delta T = 0$$

Solving for the change in temperature ΔT, we get

$$\Delta T = -\frac{\rho}{\alpha} = -\frac{0.0036}{\left(-9\times10^{-5}\right)} = 40^\circ\text{C}$$

Thus, the temperature will rise from 20°C to 60°C before the positive reactivity in question is cancelled out.

■ P20.6

Given $\rho = -0.0025$, $P = 2500$ MW, and $\Delta P = (2800 - 2500) = 300$ MW, the contribution α_P to the power coefficient of the reactor is

$$\rho = \alpha_P \frac{\Delta P}{P} \quad \rightarrow \quad \alpha_P = \rho \frac{P}{\Delta P}$$

$$\therefore \alpha_P = -0.0025 \times \frac{2500}{300} = \boxed{-0.0208}$$

■ P20.8

The volume V of solution to be added is

$$(1500 \text{ ppm}) \times \left(10{,}000 \text{ ft}^3\right) + (8000 \text{ ppm}) \times V = (1600 \text{ ppm}) \times \left(10{,}000 \text{ ft}^3 + V\right)$$

$$\therefore 1.5\times10^7 + 8000V = 1.6\times10^7 + 1600V$$

$$\therefore 6400V = 1.0\times10^6$$

$$\therefore V = \frac{1.0\times10^6}{6400} = \boxed{156 \text{ ft}^3}$$

A volume of 156 ft^3 of 8000-ppm solution should be added to increase the boron concentration in the coolant solution from 1500 to 1600 ppm.

■ P20.9

The initial boron content is 1500 ppm × 10,000 ft^3 = 1.5×10^7 ppm·ft^3. The volume V of 1400-ppm solution that must be added while maintaining the same B content is

$$1.5\times10^7 \text{ ppm}\cdot\text{ft}^3 = (1400 \text{ ppm}) \times \left(10{,}000 \text{ ft}^3 + V\right)$$

$$\therefore 1.5\times10^7 = 1.4\times10^7 + 1400V$$

$$\therefore 1.0\times10^6 = 1400V$$

$$\therefore V = \frac{1.0\times10^6}{1400} = 714 \text{ ft}^3$$

Assuming the volumes of water and solution are the same, the pump supplying a flow rate of 500 ft^3/min must be active for a time t such that

$$t = \frac{714 \text{ ft}^3}{500 \text{ ft}^3/\text{min}} = 1.428 \approx \boxed{1.43 \text{ min}}$$

■ P20.11

Part (a): We have all the information needed to compute δk:

$$\delta k = \frac{\beta\tau}{T} = \frac{0.008\times13}{200} = \boxed{0.00052}$$

If the rod is withdrawn 4 cm from its position, the center slope becomes

$$\left(\frac{\delta k}{\delta z}\right)_{\text{center}} = \frac{0.00052}{4} = \boxed{0.00013 \text{ cm}^{-1}}$$

Part (b): The average slope is

$$\left(\frac{\delta k}{\delta z}\right)_{\text{avg}} = \frac{\left(\delta k/\delta z\right)_{\text{center}}}{2} = \frac{0.00013}{2} = 6.5\times10^{-5} \text{ cm}^{-1}$$

Given the core height $H = 300$ cm, the rod worth becomes

$$\delta k = \left(\frac{\delta k}{\delta z}\right)_{\text{avg}} H = \left(6.5\times10^{-5}\right)\times 300 = \boxed{0.0195}$$

Alternatively, we can solve for ρ_T using the differential worth equation:

$$\frac{d\rho}{dz} = \frac{\rho_T}{H}\left[1-\cos\left(\frac{2\pi z}{H}\right)\right]$$

$$\therefore \frac{0.00052}{4} = \frac{\rho_T}{300}\times\left[1-\cos\left(\frac{2\pi\times150}{300}\right)\right]$$

$$\therefore 0.00013 = \frac{\rho_T}{300}\times 2.0$$

$$\therefore \rho_T = \frac{0.00013\times300}{2} = \boxed{0.0195}$$

■ P20.14

Part (a): Noting that 1 MW = 10^6 W and 1 metric tonne = 10^6 g, the unit conversion is straightforward:

$$1\frac{\cancel{\text{MW}}}{\cancel{\text{tonne}}}\times10^6\frac{\text{W}}{\cancel{\text{MW}}}\times\frac{1}{10^6}\frac{\cancel{\text{tonne}}}{\text{g}} = 1\frac{\text{W}}{\text{g}}$$

Part (b): The thermal energy afforded by the reactor is given by the product of power rating Q_R and time t, that is, $E = Q_R t$. The power rating Q_R can be stated as

$$Q_R = w\sigma_f N_{235}(0)\phi_0 V$$

where V is the fuel volume. The burnup B then becomes

$$B = \frac{E}{m_U} = \frac{Q_R t}{\rho_U V} = \frac{w\sigma_f N_{235}(0)\phi_0 \cancel{V} t}{\rho_U \cancel{V}} = \frac{w\sigma_f N_{235}(0)\phi_0 t}{\rho_U}$$

as we intended to show. Since $N_U = \rho_U N_A/M_U$, the equation can be restated as

$$B = \frac{w\sigma_f N_{235}(0) N_A \phi_0 t}{N_U M_U}$$

Part (c): We have $w = 3.04\times10^{-11}$ J/fission; all other data are known. Substituting in the formula obtained in the previous part, we obtain

$$B = \frac{\left(3.04\times10^{-11}\right)\times\left(582.6\times10^{-24}\right)\times0.03\times\left(6.02\times10^{23}\right)\times\left(2\times10^{13}\right)\times\left(3\times365\right)}{238}$$

$$\therefore \boxed{B = 29{,}400\ \text{MW}\cdot\text{d/tonne}}$$

■ P20.19

Part (a): For a position change from 110 cm to 110 – 15 = 95 cm, we may refer to equation (20.16) to obtain

$$\Delta\rho = \rho\left(z_{\text{initial}}\right) - \rho\left(z_{\text{final}}\right) = \rho(110\ \text{cm}) - \rho(95\ \text{cm})$$

$$\therefore \rho(110\ \text{cm}) = \left\{\rho_T\left[\frac{z}{H} - \frac{1}{2\pi}\sin\left(\frac{2\pi z}{H}\right)\right]\right\}\Bigg|_{z=110\ \text{cm}}$$

$$\therefore \rho(110\ \text{cm}) = \rho_T\times\left[\frac{110}{150} - \frac{1}{2\pi}\sin\left(\frac{2\pi\times110}{150}\right)\right] = 0.8916\rho_T$$

$$\therefore \rho(95\ \text{cm}) = \left\{\rho_T\left[\frac{z}{H} - \frac{1}{2\pi}\sin\left(\frac{2\pi z}{H}\right)\right]\right\}\Bigg|_{z=95\ \text{cm}}$$

$$\therefore \rho(95\ \text{cm}) = \rho_T\times\left[\frac{95}{150} - \frac{1}{2\pi}\sin\left(\frac{2\pi\times95}{150}\right)\right] = 0.7516\rho_T$$

$$\Delta\rho = \rho(110\text{ cm}) - \rho(95\text{ cm}) = \$0.75 \times (0.8916 - 0.7516) = \boxed{\$0.105}$$

Thus, the reactivity change is about 10.5 cents. Using τ = 12.7 s (page 368), the reactor period is

$$T = \frac{\beta_T \tau}{\delta k} = \frac{\beta_T \times 12.7}{0.105\beta_T} = \boxed{121\text{ s}}$$

Part (b): For a position change from 110 cm to 110 + 15 = 125 cm, the change in reactivity is found as

$$\Delta\rho = \rho(z_{\text{initial}}) - \rho(z_{\text{final}}) = \rho(110\text{ cm}) - \rho(125\text{ cm})$$

$$\therefore \rho(110\text{ cm}) = \left\{\rho_T\left[\frac{z}{H} - \frac{1}{2\pi}\sin\left(\frac{2\pi z}{H}\right)\right]\right\}\Bigg|_{z=110\text{ cm}} = 0.8916\rho_T$$

$$\therefore \rho(125\text{ cm}) = \rho_T \times \left[\frac{125}{150} - \frac{1}{2\pi}\sin\left(\frac{2\pi \times 125}{150}\right)\right] = 0.9712\rho_T$$

$$\Delta\rho = \rho(110\text{ cm}) - \rho(95\text{ cm}) = \$0.75 \times (0.8916 - 0.9712) = \boxed{-\$0.0597}$$

Thus, the reactivity change is about −6 cents. Using τ = 12.7 s (page 368), the reactor period becomes

$$T = \frac{\beta_T \tau}{\delta k} = \frac{\beta_T \times 12.7}{-0.0597 \times \beta_T} = \boxed{-213\text{ s}}$$