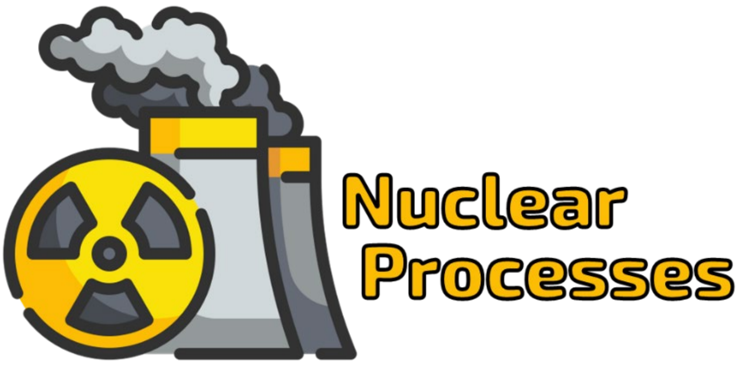
Montogue

Quiz NC1



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Problems	Subject
1 – 9	Basic nuclear processes
10 – 14	Semiempirical mass formula and applications
15 – 20	Scattering and electrostatic energy
21 – 34	Radioactive decay

PROBLEMS

I. Basic Nuclear Processes

Problem 1

Consider the following nuclear reaction equations. True or false?

$$(a)\ {}_0^1n + {}_7^{14}\mathrm{N} \to (?) + {}_1^1\mathrm{H}$$
$$(b)\ {}_1^2\mathrm{H} + {}_4^9\mathrm{Be} \to (?) + {}_0^1n$$
$$(c)\ {}_0^1n + {}_5^{10}\mathrm{B} \to (?) + {}_2^4\mathrm{He}$$

1.( ) In reaction (a), the missing product is carbon-14.
2.( ) In reaction (b), the missing product is carbon-12.
3.( ) In reaction (c), the missing product is lithium-6.

Problem 2.1

Find the energy release in the reaction $^6\mathrm{Li}(n,\alpha)^3\mathrm{H}$.
Given: $M({}_3^6\mathrm{Li})$ = 6.015123 u, $M({}_2^4\mathrm{He})$ = 4.002603 u, $M({}_1^3\mathrm{H})$ = 3.016049 u and m_n = 1.008665 u.

Problem 2.2

Calculate the energy yield from the following reaction.

$${}_{92}^{235}\mathrm{U} + {}_0^1n \to {}_{37}^{92}\mathrm{Rb} + {}_{55}^{140}\mathrm{Cs} + 4{}_0^1n + Q$$

Given: $M({}_{92}^{235}\mathrm{U})$ = 235.04392992 u, $M({}_{37}^{92}\mathrm{Rb})$ = 91.9197289 u, $M({}_{55}^{140}\mathrm{Cs})$ = 139.91728235 u and m_n = 1.008665 u.

Problem 3

Using data from the following table, determine the reproduction factor of U-233 and Pu-239.

Element	Total neutron yield	Thermal microscopic cross-sections	
		Capture, σ_γ	Fission, σ_f
U-233	2.4968	45.5	529.1
Pu-239	2.8836	269.3	748.1

A) $\eta_{U,233}$ = 2.214 and $\eta_{\mathrm{Pu},239}$ = 2.121
B) $\eta_{U,233}$ = 2.214 and $\eta_{\mathrm{Pu},239}$ = 2.183
C) $\eta_{U,233}$ = 2.299 and $\eta_{\mathrm{Pu},239}$ = 2.121
D) $\eta_{U,233}$ = 2.299 and $\eta_{\mathrm{Pu},239}$ = 2.183

■ Problem 4.1

Calculate the macroscopic cross-section Σ for scattering of 1 eV neutrons in water, using 0.0334×10^{24} cm^{-3} as the number density N of water, along with scattering cross-sections of 20 barns for hydrogen and 3.8 barns for oxygen.

A) $\Sigma = 1.21$ cm^{-1}
B) $\Sigma = 1.46$ cm^{-1}
C) $\Sigma = 1.60$ cm^{-1}
D) $\Sigma = 1.82$ cm^{-1}

■ Problem 4.2

What is the scattering mean free path λ_s for the situation introduced in the previous problem?

▶ Problem 5

Find the speed v and the density of neutrons of energy 1.5 MeV in a flux of 7×10^{13} $cm^{-2}\cdot s^{-1}$.

A) $v = 1.69\times10^7$ m/s and $n = 4.14\times10^4$ cm^{-1}
B) $v = 1.69\times10^7$ m/s and $n = 8.28\times10^4$ cm^{-1}
C) $v = 3.38\times10^7$ m/s and $n = 4.14\times10^4$ cm^{-1}
D) $v = 3.38\times10^7$ m/s and $n = 8.28\times10^4$ cm^{-1}

▶ Problem 6

Compute the flux, macroscopic cross-section, and reaction rate for the following data. True or false?

Neutron density, n	2×10^5 cm^{-3}
Neutron speed, v	3×10^8 cm/s
Number density, N	0.04×10^{24} cm^{-3}
Microscopic cross-section, σ	0.5×10^{-24} cm^2

1.( ) The flux is greater than 7.0×10^{13} $cm^{-2}s^{-1}$.
2.( ) The macroscopic cross-section is greater than 0.015 cm^{-1}.
3.( ) The reaction rate is greater than 10^{12} $cm^{-3}s^{-1}$.

▶ Problem 7

What are the values of the average logarithmic energy change ξ and the average cosine of the scattering angle $\bar{\mu}$ for neutrons in beryllium, $A = 9$? How many collisions are needed to slow neutrons from 2 MeV to 0.025 eV in Be-9? What is the value of the diffusion coefficient D for 0.025-eV neutrons if the macroscopic cross-section Σ is 0.90 cm^{-1}? True or false?

1.( ) The average logarithmic energy change for Be-9 is greater than 0.2.
2.( ) The average cosine of the scattering angle for Be-9 is greater than 0.08.
3.( ) The No. of collisions needed to reduce the energy of neutrons from 2 MeV to 0.025 eV is greater than 95.
4.( ) The diffusion coefficient is greater than 0.35 cm.

▶ Problem 8

Verify that neutrons of speed 2200 m/s have an energy of 0.0253 eV. If the neutron absorption cross-section of boron-10 at 0.0253 eV is 3842 barns, what would it be at 0.1 eV?

▶ Problem 9

Calculate the absorption cross-section of the element zirconium using the isotopic data in the following table.

Mass number	Abundance (atom %)	Cross-section (barns)
90	51.45	0.014
91	11.22	1.2
92	17.15	0.2
94	17.38	0.049
96	2.80	0.020

A) $\sigma = 0.1467$ b
B) $\sigma = 0.1611$ b
C) $\sigma = 0.1852$ b
D) $\sigma = 0.2015$ b

II. Semi-Empirical Mass Formula and Applications

Problem 10.1

The nuclear binding energy may be approximated by the empirical expression

$$B.E. = a_1 A - a_2 A^{2/3} - a_3 Z^2 A^{-1/3} - a_4 \left(A - 2Z\right)^2 A^{-1}$$

Name the four terms in this expression.

Problem 10.2

Considering a set of isobaric nuclei, derive a relationship between A and Z for naturally occurring nuclei.

Problem 10.3

Use a Fermi gas model to estimate the magnitude of a_4. You may assume $A \neq 2Z$ and that the nuclear radius is $R = R_0 A^{1/3}$.

Problem 11

Among the A = 197 isobars, the nucleus ${}^{197}_{79}\mathrm{Au}$ is stable. What are the expected radioactive decay types for ${}^{197}_{78}\mathrm{Pt}$ and ${}^{197}_{80}\mathrm{Hg}$ to ${}^{197}_{79}\mathrm{Au}$?

Problem 12

The sun is a copious source of neutrinos (*solar neutrinos*). The first observation of these particles was achieved in 1978 by R. Davis. Davis used a large detector filled with C_2Cl_4; the reaction used for the detection was

$$\nu_e + {}^{37}_{17}\mathrm{Cl} \rightarrow {}^{37}_{18}\mathrm{Ar} + e^-$$

Using the data below, calculate the threshold energy for this reaction.

Data: Assume both nuclei to be in their ground states. The difference between rest mass of a proton and a neutron can be taken as $m_p - m_n = -1.293$ MeV/c^2; the rest mass of an electron can be taken as $m_e = 0.511$ MeV/c^2. The Coulomb and asymmetry coefficients in the semi-empirical mass formula can be taken as $a_C = 0.697$ MeV and $a_A = 23.3$ MeV, respectively.

A) E_{th} = 0.114 MeV
B) E_{th} = 0.693 MeV
C) E_{th} = 1.50 MeV
D) E_{th} = 2.11 MeV

Problem 13

Use the semi-empirical mass formula to establish if the nucleus ${}^{64}_{29}\mathrm{Cu}$ can undergo β^- decay to yield ${}^{64}_{30}\mathrm{Zn}$ or β^+ decay to yield ${}^{64}_{28}\mathrm{Ni}$. Also calculate the maximum energies of the emitted electrons/positrons. Use $m_p = 938.272$ MeV/c^2, $m_n = 939.565$ MeV/c^2, and $m_e = 0.511$ MeV/c^2 as the rest masses of a proton, a neutron, and an electron, respectively.

A) Both β^- and β^+ decays are allowed.
B) Only β^- decay is allowed.
C) Only β^+ decay is allowed.
D) Neither β^- nor β^+ decay is allowed.

Problem 14.1

A nuclear reactor produces a total power of 1.88 GW. The fission reaction involved in energy production is

$${}^{235}_{92}\mathrm{U} \rightarrow \left(A_1, Z\right) + \left(A_2, 92 - Z\right) + k\,n + 200\,\mathrm{MeV}$$

where k is an integer with value 2 or 3. Calculate the number of fission reactions per second.

A) $r = 3.45\times10^{18}$ s^{-1}
B) $r = 8.17\times10^{18}$ s^{-1}
C) $r = 1.34\times10^{19}$ s^{-1}
D) $r = 5.88\times10^{19}$ s^{-1}

Problem 14.2

Knowing that the fissile nucleus, ${}^{235}_{92}\mathrm{U}$, constitutes about 33% of the total fuel mass, estimate how much fuel is consumed in a year.

A) M = 723 kg
B) M = 985 kg
C) M = 1810 kg
D) M = 2190 kg

Problem 14.3

Nuclear reactors are the main source of human-generated neutrinos. Neutrinos are produced in the β^- decay of neutron-rich uranium fission fragments, which implies that they may actually be antineutrinos, $\bar{\nu}_e$. Assuming for simplicity that all neutrinos originate from the decay of $^{145}_{57}$La (as representative of all possible β^- decays) and the total decay rate corresponds to about 22% of the total fission rate, calculate the maximum neutrino energy.

A) Q_- = 2.45 MeV
B) Q_- = 4.81 MeV
C) Q_- = 6.76 MeV
D) Q_- = 8.09 MeV

Problem 14.4

Calculate the neutrino flux at a distance of 400 m from the reactor core.

A) $\Phi = 8.75\times10^{-11}$ m^{-2}s^{-1}
B) $\Phi = 2.44\times10^{12}$ m^{-2}s^{-1}
C) $\Phi = 6.42\times10^{12}$ m^{-2}s^{-1}
D) $\Phi = 1.15\times10^{13}$ m^{-2}s^{-1}

Problem 14.5

Knowing that the cross-section for the inverse beta decay reaction $\bar{\nu}_e + p \rightarrow n + e^+$ at reactor energies is about 6×10^{-44} cm^2, with an ideal detector of 1 ton active mass placed at 400 m from the core, how many neutrinos per year are detected? In the following alternatives, A denotes the atomic mass of the material that constitutes the detector.

A) 454/A neutrinos will be detected per year.
B) 732/A neutrinos will be detected per year.
C) 1080/A neutrinos will be detected per year.
D) 1340/A neutrinos will be detected per year.

III. Scattering and Electrostatic Energy

Problem 15.1

A gamma ray from neutron capture has an energy of 7 MeV. Find its frequency and wavelength.

A) $f = 1.69\times10^{21}$ Hz and λ = 0.178 pm
B) $f = 1.69\times10^{21}$ Hz and λ = 0.356 pm
C) $f = 3.38\times10^{21}$ Hz and λ = 0.178 pm
D) $f = 3.38\times10^{21}$ Hz and λ = 0.356 pm

Problem 15.2

For 180° scattering of gamma or X-rays by electrons, the final energy of the photon is

$$E' = \left(\frac{1}{E} + \frac{2}{E_0}\right)^{-1}$$

where E is the energy of an incident photon and $E_0 \approx 0.511$ MeV is the rest mass energy of an electron. What is the final photon energy for the 7-MeV gamma ray of the previous problem?

A) E' = 0.113 MeV
B) E' = 0.247 MeV
C) E' = 0.378 MeV
D) E' = 0.525 MeV

Problem 15.3

Verify that if $E \gg E_0$, then $E' \approx E_0/2$ and if $E \ll E_0$, then $(E - E') \approx 2E/E_0$. Which approximation should be used for a 7-MeV gamma ray? Verify numerically.

Problems 16.1 - 16.5

Referring to Figure 1 in the Additional Information section, indicate which of the three major interaction processes (photoelectric absorption, Compton scattering or pair production) is dominant in the situations 16.1 to 16.5.

		A. Photoelectric absorption	**B.** Compton scattering	**C.** Pair production
16.1	1 MeV gamma rays in aluminum			
16.2	100 keV gamma rays in hydrogen			
16.3	100 keV gamma rays in germanium			
16.4	10 MeV gamma rays in carbon			
16.5	10 MeV gamma rays in lead			

Problem 17.1

Calculate the electrostatic energy W of a charge Q distributed uniformly throughout a sphere of radius R.

A) $W = 2Q^2/5R$
B) $W = Q^2/2R$
C) $W = 3Q^2/5R$
D) $W = 3Q^2/4R$

Problem 17.2

Since ${}^{27}_{14}\mathrm{Si}$ and ${}^{27}_{13}\mathrm{Al}$ are "mirror nuclei," their ground states are identical except for charge. If their mass difference is 6 MeV, estimate their radius. Neglect the proton-neutron mass difference.

A) R = 3.08 fm
B) R = 3.48 fm
C) R = 3.88 fm
D) R = 4.28 fm

Problem 18

The binding energy of ${}^{90}_{40}\mathrm{Zr}_{50}$ is 783.916 MeV. The binding energy of ${}^{90}_{39}\mathrm{Y}_{51}$ is 782.410 MeV. Estimate the excitation energy of the lowest $T = 6$ isospin state in ^{90}Zr.

A) E = 12.98 MeV
B) E = 13.40 MeV
C) E = 13.84 MeV
D) E = 14.15 MeV

Problem 19.1

A convenient model for the potential energy V of a particle of charge q scattering from an atom of nuclear charge Q is

$$V = \frac{qQe^{-\alpha r}}{r}$$

where α^{-1} represents the screening length of the atomic electrons.
Use the Born approximation,

$$f = -\frac{1}{4\pi}\int e^{-i\Delta\mathbf{k}\cdot\mathbf{r}}\frac{2m}{\hbar^2}V(r)\,d^3\mathbf{r}$$

to calculate the scattering cross-section σ.

Problem 19.2

How should α depend on the nuclear charge Z?

Problem 20.1

Consider the scattering of a 1-keV proton by a hydrogen atom. What do you expect the angular distribution to look like? Sketch a graph and comment on its shape.

Problem 20.2

Estimate the total cross-section and choose the option that best approximates your result.

A) $\sigma = 1.8\times10^{-20}$ cm^2
B) $\sigma = 1.8\times10^{-18}$ cm^2
C) $\sigma = 1.8\times10^{-16}$ cm^2
D) $\sigma = 1.8\times10^{-14}$ cm^2

IV. Radioactive Decay

Problem 21.1

If a reactor part contains 5000 curies of cobalt-60 (half-life = 5.27 years), how much activity will remain after 45 years?

A) A = 13.4 Ci
B) A = 17.7 Ci
C) A = 26.5 Ci
D) A = 33.0 Ci

Problem 21.2

Assume that a source of ^{137}Cs (half-life = 30.07 years) can be safely managed if it contains less than 0.01 Ci. How long would it take for a source containing 4 Ci of ^{137}Cs to reach this value?

A) t = 114 yr
B) t = 260 yr
C) t = 355 yr
D) t = 481 yr

Problem 21.3

Consider 12% of technetium-99m-DTPA (diethylene-triamine-pentaacetate) is eliminated from the body of a patient by renal excretion, 22% by fecal excretion, and 3% by perspiration within six hours. What is the effective half-life of this radiopharmaceutical, given that the physical half-life for Tc-99m is six hours?

A) T_e = 3.45 hr
B) T_e = 4.59 hr
C) T_e = 5.35 hr
D) T_e = 6.04 hr

Problem 22.1

The data in the following table show the measured activity A in millicuries (mCi) as a function of time t for an unknown radionuclide F that decays into a stable daughter G. Estimate the half-life of unknown radionuclide F.

Time, t (min)	0	1	2	3	4	5	6	8	10	12
Activity, A (mCi)	3.6	3.23	2.78	2.41	2.09	1.88	1.62	1.31	0.98	0.73

A) $t_{1/2}$ = 3.49 min
B) $t_{1/2}$ = 4.12 min
C) $t_{1/2}$ = 5.29 min
D) $t_{1/2}$ = 6.88 min

Problem 22.2

In its original (1911) form the Geiger-Nuttall law expresses the general relationship between α-particle range (R_α) and decay constant (λ) in natural α-radioactivity as a linear relation between $\log \lambda$ and $\log R$. Subsequently, this was modified to an approximate linear relationship between $\log \lambda$ and some power of the α-particle energy, $E^x(\alpha)$.

Explain how this relationship between decay constant and energy is explained quantum-mechanically. Show also how the known general features of the atomic nucleus make it possible to explain the extremely rapid dependence of λ on $E(\alpha)$. (For example, from $E(\alpha)$ = 5.3 MeV for Po-210 to $E(\alpha)$ = 7.7 MeV for Po-214, λ increases by a factor of some 10^{10}, from a half-life of about 140 days to one of 1.6×10^{-4} sec.)

Problem 22.3

Natural gold $^{197}_{79}Au$ is radioactive since it is unstable against α-decay with an energy of 3.3 MeV. Estimate the lifetime of $^{197}_{79}Au$ to explain why gold does not burn a hole in your pocket.

Problems 23.1 - 23.3

Determine the specific activity a for the following radionuclides.

23.1. Cobalt-60 (half-life = 5.26 yr)

23.2. Iodine-131 (half-life = 8.02 d)

23.3. Cesium-137 (half-life = 30 yr)

23.1	23.2	23.3
A) a = 104 Ci/g	**A)** a = 78.8 Ci/g	**A)** a = 86.9 Ci/g
B) a = 1130 Ci/g	**B)** a = 1670 Ci/g	**B)** a = 185 Ci/g
C) a = 8400 Ci/g	**C)** a = 45,200 Ci/g	**C)** a = 512 Ci/g
D) a = 54,500 Ci/g	**D)** a = 124,000 Ci/g	**D)** a = 2430 Ci/g

Problem 24

How long does it take for a sample of 400 MBq of iodine-123 ($t_{1/2}$ = 13.2 h) and a sample of 1780 MBq of technetium-99m ($t_{1/2}$ = 6 h) to reach the same activity?

A) t = 6.82 h

B) t = 12.2 h

C) t = 16.5 h

D) t = 23.8 h

Problem 25

Carbon is often used in dating of organic specimens. Carbon-14 is a radioactive isotope of carbon produced by the action of cosmic rays in the atmosphere. If the flux of cosmic rays remains roughly constant over time, the ratio of ${}^{14}_{6}C$ to the stable most abundant isotope ${}^{12}_{6}C$ tends to an equilibrium value of about 1.3×10^{-12}. ${}^{14}_{6}C$ decays by β^- with a half-life of 5700 years. Measuring the activity of a fossil of 6-g mass, earth scientists measured 5400 decays in 3 hours. The age of the fossil is, most nearly:

A) T = 5250 yr

B) T = 6400 yr

C) T = 7850 yr

D) T = 9090 yr

Problem 26.1

A solution contains an unknown amount of gold-198 (Au-198) and iodine-131 (I-131) beta-emitters. If the total activity $A(t)$ of the solution at time t = 0 is 0.25 µCi (9.25 kBq) and drops to half of its initial value in 3 days, calculate the initial activities $A_{Au}(0)$ and $A_I(0)$ of Au-198 and I-131, respectively, in the solution. The half-lives of Au-198 and I-131 are 2.70 days and 8.05 days, respectively.

Problem 26.2

Calculate the total activity $A(t)$ of the solution at time t = 6 days.

A) $A(6)$ = 0.0325 µCi

B) $A(6)$ = 0.0650 µCi

C) $A(6)$ = 0.0934 µCi

D) $A(6)$ = 0.141 µCi

Problem 26.3

Calculate the time T at which the activities of Au-198 and I-131 in the solution are equal. What is the common activity at this time T?

A) The activities of Au-198 and I-131 both equal 0.0109 µCi at T = 5.85 days.

B) The activities of Au-198 and I-131 both equal 0.0109 µCi at T = 11.7 days.

C) The activities of Au-198 and I-131 both equal 0.0218 µCi at T = 5.85 days.

D) The activities of Au-198 and I-131 both equal 0.0218 µCi at T = 11.7 days.

Problem 27.1

Radioactive decay through a series of radioactive transformations is much more common than the simple radioactive decay from a radioactive parent into a stable daughter. The radioactive decay series forms a decay chain starting with the parent radionuclide and moves through several generations to eventually end with a stable nuclide.

Consider the simple chain $P \to D \to G$ where both the parent P and daughter D are radioactive and the granddaughter G is stable. The parent decays with a decay constant λ_P while the daughter decays with a decay constant λ_D. For this simple decay series, state the differential equations governing the kinetics of the radioactive parent and radioactive daughter.

Problem 27.2

Solve the differential equations in Part 1 with the following initial conditions.

(I) Initial number of parent nuclei $N_P(t)$ at time t = 0 is $N_P(0)$.

(II) Initial number of daughter nuclei $N_D(t)$ at time t = 0 is $N_D(0)$ = 0.

■ Problem 27.3

Using the results of Part 2, obtain an expression for the activity of the daughter $A_D(t)$.

■ Problem 27.4

The expression for the daughter activity $A_D(t)$ derived in the previous part should predict $A_D(t) = 0$ (in accord with the initial condition $N_D(0) = 0$) and $A_D(t \to \infty) = 0$ (because at infinite time all daughter nuclei will have decayed). It follows that $A_D(t)$ must reach a maximum value $(A_D)_{max}$ at a characteristic time $(t_{max})_D$ somewhere between the two extremes, $(t_{max})_D \in (0, \infty)$. Derive an expression for the characteristic time $(t_{max})_D$.

■ Problem 27.5

Show that for $\lambda_P \gtrsim \lambda_D$ (but not for $\lambda_P = \lambda_D$) and for $\lambda_P \lesssim \lambda_D$ (but not for $\lambda_P = \lambda_D$) the characteristic time $(t_{max})_D$ can be approximated by

$$\left(t_{\max}\right)_D \approx \frac{1}{\sqrt{\lambda_P \lambda_D}}$$

To verify this approximation calculate $(t_{max})_D$ with this approximation and compare results with the expression derived in 27.4 for the following two radioactive series decays: (1) Series decay with $\lambda_P = 1.08$ yr and $\lambda_D = 1.0$ yr and (2) Series decay with $\lambda_P = 3.1$ yr and $\lambda_D = 3.5$ yr.

■ Problem 28.1

The molybdenum-99 (Mo-99) → technetium-99m (Tc-99m) → technetium-99 (Tc-99) decay series plays an important role in nuclear medicine because it serves as a source of Tc-99m, the most widely used radionuclide for nuclear imaging tests. The series parent radionuclide Mo-99 decays through β^- decay with a half-life $(t_{1/2})_{\text{Mo-99}} = 66.0$ hours into daughter radionuclide Tc-99m. Subsequently, the daughter Tc-99m decays through gamma emission with a half-life $(t_{1/2})_{\text{Tc-99m}} = 6.02$ hours to the granddaughter radionuclide Tc-99. The Tc-99 radionuclide has a much longer half-life [$(t_{1/2})_{\text{Tc-99}} = 2.1\times10^5$ yr] in comparison with Mo-99 and Tc-99m and decays through β^- decay to ruthenium-99 (Ru-99). Starting with a pure 20 mCi (0.74 GBq) Mo-99 source, state or derive equations for activities of the Mo-99 parent and Tc-99m daughter as a function of time.

■ Problem 28.2

Calculate the characteristic time $(t_{max})_{\text{Tc-99m}}$ at which the Tc-99m daughter radionuclide attains its maximum activity.

A) $(t_{max})_{\text{Tc-99m}} = 20.3$ h
B) $(t_{max})_{\text{Tc-99m}} = 22.9$ h
C) $(t_{max})_{\text{Tc-99m}} = 25.2$ h
D) $(t_{max})_{\text{Tc-99m}} = 26.7$ h

■ Problem 28.3

Calculate the maximum activity of the Tc-99m radionuclide.

A) $A_{D,max} = 9.67$ mCi
B) $A_{D,max} = 11.4$ mCi
C) $A_{D,max} = 15.7$ mCi
D) $A_{D,max} = 19.4$ mCi

■ Problem 28.4

Sketch the activities of the Mo-99 parent and the Tc-99m daughter as a function of time and highlight the salient features of the two radioactive decay curves.

■ Problem 29.1

Consider the simplest radioactive decay series $P \to D \to G$, where both the parent P and daughter D are radioactive and the granddaughter G is stable. State or derive expressions for $N_P(t)$, $N_D(t)$, and $N_G(t)$, where $N_P(t)$ is the number of parent nuclei, $N_D(t)$ is the number of daughter nuclei, and $N_G(t)$ is the number of grand-daughter nuclei, all as a function of time t, such that $0 \le t < \infty$. Use as initial conditions $N_P(t=0) = N_P(0) > 0$, $N_D(t=0) = 0$, and $N_G(t=0) = 0$.

Problem 29.2

Validate the expression for $N_G(t)$ obtained in Part 1 by showing that the following limits are true:

$$\text{(I)}\ \lim_{t\to 0} N_G(t) = 0$$

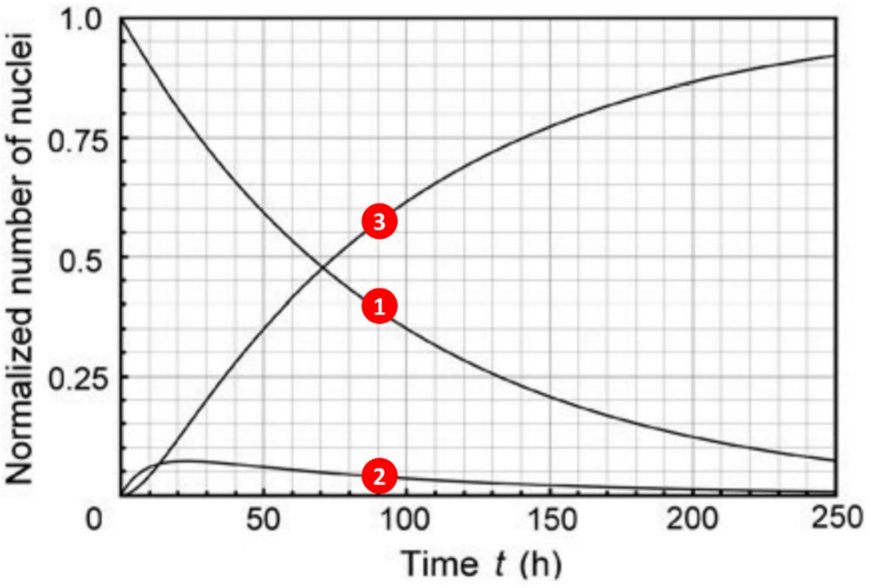
$$\text{(II)}\ \lim_{t\to \infty} N_G(t) = N_P(0)$$

Problem 29.3

Calculate the sum $N_p(t) + N_D(t) + N_G(t)$ using expressions for $N_P(t)$, $N_D(t)$, and $N_G(t)$ obtained in Part 1. Did you get the result you expected?

Problem 29.4

The following figure shows three curves representing $N_P(t)$, $N_D(t)$, and $N_G(t)$ normalized such that $N_P(t=0) = 1$ and plotted against time t for the decay series molybdenum-99 (Mo-99) → Technetium-99m (Tc-99m) → Technetium-99 (Tc-99), for which we have decay constants λ_P = 0.0105 h^{-1}, λ_D = 0.115 h^{-1}, and $\lambda_G \approx 0$. Identify the 3 curves.



Problem 29.5

Of the 3 curves in the Figure given, curve 1 decreases from 1 exponentially with time. Curve 2 starts at zero, increases with time, exhibits a peak and then decreases with time. Curve 3 increases with time from zero and approaches unity asymptotically. For curve 2, calculate the time t_{max} at which the curve attains its peak value and determine the normalized peak value.

A) t_{max} = 11.5 h; the normalized peak activity is 0.0359.
B) t_{max} = 11.5 h; the normalized peak activity is 0.0718.
C) t_{max} = 22.9 h; the normalized peak activity is 0.0359.
D) t_{max} = 22.9 h; the normalized peak activity is 0.0718.

Problem 30

A radioactive source (^{210}At) with unknown activity is positioned in a vacuum chamber with volume equal to 12 dm^3. The activity is determined by measuring the amount of He (helium) that is obtained when the emitted α particles attract electrons and produce stable He atoms. Assume that all α particles are transformed to He gas. After one month (30 days) of exposure, the mass of He was measured to be 2.5 ng. Calculate the initial activity of ^{210}Al. Recall that ^{210}At decays to ^{210}Po by electron capture; the half-lives of ^{210}At and ^{210}Po are 8.10 h and 138.4 days, respectively.

A) $A_{At}(0)$ = 12.1 GBq
B) $A_{At}(0)$ = 25.4 GBq
C) $A_{At}(0)$ = 51.1 GBq
D) $A_{At}(0)$ = 74.4 GBq

▶ Problem 31

At time t = 0, a radioactive source of pure ^{210}Bi is placed in a container of lead that absorbs all emitted radiation, including the produced bremsstrahlung, emanated when electrons are absorbed. All energy absorbed by the radioactive source and the container is converted to heat; the heat power is measured with a calorimeter. If at t = 0 the ^{210}Bi activity is 105 GBq, calculate the time it takes for the heat power read by the calorimeter to equal 4 mW. Assume that in every decay the bismuth atoms are converted to the ground state of the daughter nuclide, and that only one particle is emitted in each decay.

Data: → The decay process in question is ^{210}BiI → ^{210}Po → ^{206}Pb (stable). → For ^{210}Bi, the half-life is $T_{1/2}$ = 5.01 days, the *maximum* energy of β-decay is $E_{\beta,\text{max}}$ = 1.16 MeV, and the *mean* energy of β-decay is $E_{\beta,\text{mean}}$ = 0.344 MeV. → For ^{210}Po, the half-life is $T_{1/2}$ = 138.4 days and the energy of α-decay is E_α = 5.2497 MeV.

A) t = 1.76 days
B) t = 4.21 days
C) t = 8.40 days
D) t = 10.2 days

■ Problem 32.1

A gold foil 0.03 cm thick is irradiated by a beam of thermal neutrons with a flux of 2×10^{12} neutrons/cm^2/s. The nuclide ^{198}Au with a half-life of 2.7 days is produced by the reaction ^{197}Au(n,γ)^{198}Au. The density of gold is 19.3 g/cm^3 and the cross-section for the reaction is 91.5×10^{-24} cm^2. ^{197}Au is 100% naturally abundant. If the foil is irradiated for 6 minutes, what is the ^{198}Au activity of the foil in decays/cm^2/s?

A) A(t = 6 min) = 1.73×10^8 cm^{-2}s^{-1}
B) A(t = 6 min) = 3.46×10^8 cm^{-2}s^{-1}
C) A(t = 6 min) = 5.19×10^8 cm^{-2}s^{-1}
D) A(t = 6 min) = 7.79×10^8 cm^{-2}s^{-1}

■ Problem 32.2

What is the maximum amount of ^{198}Au/cm^2 that can be produced in the foil?

■ Problem 32.3

How long must the foil be irradiated if it is to have 2/3 of its maximum activity?

▶ Problem 33

A foil of ^{7}Li of mass 0.08 gram is irradiated with thermal neutrons (capture cross-section 37 millibarns) and forms ^{8}Li, which decays by β^- decay with a half-life of 0.85 s. Find the time it takes to reach equilibrium and the equilibrium activity (number of β decays per second) when the foil is exposed to a steady neutron flux of 4×10^{12} neutrons/sec·cm^2.

A) t_{eq} = 18.0 s and A_{eq} = 27.7 mCi
B) t_{eq} = 18.0 s and A_{eq} = 55.4 mCi
C) t_{eq} = 36.0 s and A_{eq} = 27.7 mCi
D) t_{eq} = 36.0 s and A_{eq} = 55.4 mCi

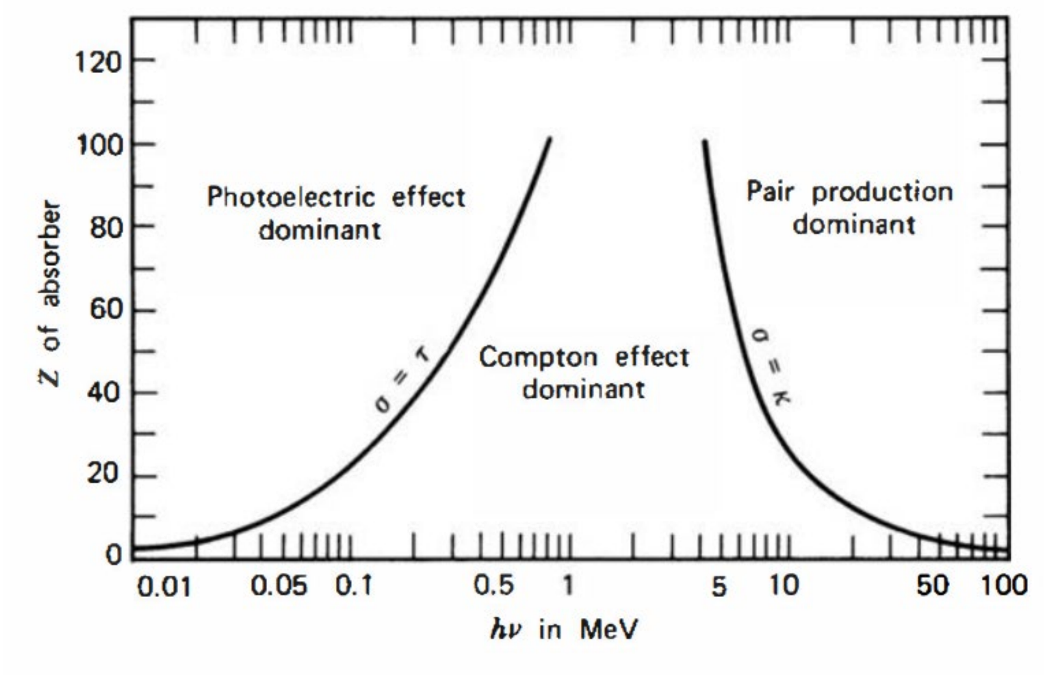
▶ Problem 34

In a neutron-activation experiment, a flux of 2×10^8 neutrons/cm^2·s is incident normally on a foil of area 2 cm^2, density 10^{22} atoms/cm^3, and thickness 0.018 cm. The target nuclei have a total cross-section for neutron capture of 1 barn, and the capture leads uniquely to a nuclear state which β-decays with a lifetime of 10^4 sec. At the end of 120 sec of neutron irradiation, at what rate will the foil be emitting β-rays?

A) A = 859 Bq
B) A = 1.72 kBq
C) A = 4.05 kBq
D) A = 8.10 kBq

ADDITIONAL INFORMATION

Figure 1. *Relative importance of the three major types of gamma-ray interaction. The solid lines show the values of Z and hv for which the two neighboring effects are just equal.*



SOLUTIONS

P.1 ➡ *Solution*

1. True. Firstly, from the conservation of atomic mass number, denoting by *A* the mass number of the missing species, we find that

$$1+14=A+1\rightarrow A=14$$

Next, from the conservation of atomic number, denoting by *Z* the atomic No. of the missing species, we find that

$$0+7=Z+1\rightarrow Z=6$$

Thus, ${}^{14}_{6}C$ is the missing species.

2. False. Conservation of atomic mass number implies that the missing species should have an *A* value equal to *A* = (2 + 9) – 1 = 10. Further, conservation of atomic number implies that the missing species should have a *Z* value equal to *Z* = (1 + 4) – 0 = 5. This combination of *A* and *Z* corresponds to boron, ${}^{10}_{5}B$, not carbon-12.

3. False. In this case, the missing species has *A* = (1 + 10) – 4 = 7 and *Z* = (0 + 5) – 2 = 3. The missing species is lithium-7, not lithium-6.

P.2 ➡ *Solution*

Problem 2.1: The reaction in question is

$${}^{6}_{3}\text{Li}+{}^{1}_{0}n\rightarrow{}^{4}_{2}\text{He}+{}^{3}_{1}\text{H}$$

The variation in mass is Δm = 6.015123 + 1.008665 – 4.002603 – 3.016049 = 0.005136, and the energy release is calculated to be

$$Q=\Delta mc^2=0.005136\text{ u}\times\frac{931.5\,\text{MeV}/c^2}{\text{u}}=\boxed{4.78\,\text{MeV}}$$

Problem 2.2: The variation in mass is Δm = 235.04392992 + 1.008665 – 91.9197289 – 139.91728235 – 4 × 1.008665 = 0.18092367, and the energy release is found as

$$Q=\Delta mc^2=0.18092367\text{ u}\times\frac{931.5\,\text{MeV}/c^2}{\text{u}}=\boxed{168.5\,\text{MeV}}$$

P.3 ➡ *Solution*

The reproduction factor is given by

$$\eta = \frac{\nu\sigma_f}{\sigma_a}$$

where ν is the number of neutrons produced per fission, σ_f is the fission cross-section, and σ_a is the absorption cross-section. For uranium-233, we have ν = 2.4968, σ_f = 529.1 and $\sigma_a = \sigma_f + \sigma_\gamma$ = 529.1 + 45.5 = 574.6, giving a reproduction factor such that

$$\eta_{U,233} = 2.4968 \times \frac{529.1}{574.6} = \boxed{2.299}$$

Proceeding similarly with plutonium-239, we have ν = 2.8836, σ_f = 748.1, and $\sigma_a = \sigma_f + \sigma_\gamma$ = 748.1 + 269.3 = 1017, so that

$$\eta_{\text{Pu},239} = \frac{\nu\sigma_f}{\sigma_a} = 2.8836 \times \frac{748.1}{1017} = \boxed{2.121}$$

▶ The correct answer is **C**.

P.4 ➡ *Solution*

Part 1: The macroscopic cross-section can be estimated as

$$\Sigma = N\left(2\sigma_H + \sigma_O\right)$$

where $N = 0.0334\times10^{24}$ is the number density of water and σ_H = 20 b and σ_O = 3.8 b are the scattering cross-sections for hydrogen and oxygen, respectively. Substituting, we obtain

$$\Sigma = \left(0.0334\times10^{24}\right)\times\left(2\times20+3.8\right)\times\frac{10^{-24}\text{ cm}^2}{1\text{b}} = \boxed{1.46\text{ cm}^{-1}}$$

▶ The correct answer is **B**.

Part 2: The scattering mean free path is simply the reciprocal of Σ,

$$\lambda_s = \frac{1}{\Sigma} = \frac{1}{1.46} = \boxed{0.685\,\text{cm}}$$

P.5 ➡ *Solution*

As the student surely knows, kinetic energy, velocity and mass are related by $KE = mv^2/2$. Solving for v yields

$$KE = \frac{mv^2}{2} \rightarrow v = \sqrt{\frac{2(KE)}{m}}$$

Noting that 1.5 MeV = 1.6×10^{-13} J and recalling that 1 u = 1.66×10^{-27} kg, we obtain

$$v = \sqrt{\frac{2\times\left[1.5\times\left(1.6\times10^{-13}\right)\right]}{1.008665\times\left(1.66\times10^{-27}\right)}} = \boxed{1.69\times10^7\text{ m/s}}$$

The neutron density can be established by dividing the given flux by the velocity obtained above,

$$n = \frac{\phi}{v} = \frac{7\times10^{13}\text{ cm}^{-2}\cdot\cancel{\text{sec}}^{-1}}{1.69\times10^7\,\cancel{\text{m}}/\cancel{\text{sec}}\times100\,\text{cm}/\cancel{\text{m}}} = \boxed{4.14\times10^4\,\text{cm}^{-3}}$$

▶ The correct answer is **A**.

P.6 ➡ *Solution*

1.False. The flux is the product of neutron density and neutron speed,

$$\phi = nv = \left(2\times10^5\right)\times\left(3\times10^8\right) = \boxed{6.0\times10^{13}\text{ cm}^{-2}\text{s}^{-1}}$$

2.True. The macroscopic cross-section is given by the product of number density N and micro cross-section σ,

$$\Sigma = N\sigma = \left(0.04\times10^{24}\right)\times\left(0.5\times10^{-24}\right) = \boxed{0.02\,\text{cm}^{-1}}$$

3.True. The reaction rate is simply the product of ϕ and Σ,

$$R = \Sigma\phi = 0.02\times\left(6.0\times10^{13}\right) = \boxed{1.2\times10^{12}\ \text{cm}^{-3}\text{s}^{-1}}$$

P.7 ➡ *Solution*

1.True. The average logarithmic energy change is given by

$$\xi = 1+\frac{\alpha\ln\alpha}{1-\alpha}$$

where α is given by

$$\alpha = \frac{(A-1)^2}{(A+1)^2} = \frac{(9-1)^2}{(9+1)^2} = 0.64$$

so that

$$\xi = 1+\frac{0.64\times\ln 0.64}{1-0.64} = \boxed{0.207}$$

2.False. The average cosine of the scattering angle is given by

$$\bar{\mu} = \frac{2}{3A} = \frac{2}{3\times9} = \boxed{0.0741}$$

3.False. The number of collisions C required to reduce the energy of neutrons from some initial value E_0 = 2 MeV to a final value E_F = 0.025 eV is determined as

$$C = \frac{\ln\left(E_0/E_F\right)}{\xi} = \frac{\ln\left(2\times10^6/0.025\right)}{0.207} = 87.9 \approx \boxed{88}$$

About 88 collisions are necessary to reduce the energy of neutrons from 2 MeV to 0.025 eV.

4.True. Given the macroscopic cross-section Σ = 0.90 cm^{-1} and the scattering angle cosine $\bar{\mu}$ = 0.0741 calculated above, the diffusion coefficient follows as

$$D = \frac{1}{3\Sigma\left(1-\bar{\mu}\right)} = \frac{1}{3\times0.90\times\left(1-0.0741\right)} = \boxed{0.400\,\text{cm}}$$

P.8 ➡ *Solution*

The mass of a neutron is 1.008665 u, and 1 u = 1.66×10^{-27} kg. Thus, the kinetic energy of a neutron at 2200 m/s is

$$KE = \frac{mv^2}{2} = \frac{\left(1.008665\times1.66\times10^{-27}\right)\times2200^2}{2} = 4.05\times10^{-21}\ \text{J}$$

Noting that 1 eV = 1.6×10^{-19} J,

$$KE = 4.05\times10^{-21}\ \text{J}\times\frac{1\,\text{eV}}{1.6\times10^{-19}\ \text{J}} = 0.0253\,\text{eV}$$

as we intended to show. Now, cross-sections and kinetic energies are related as

$$\sigma_0\sqrt{E_0} = \sigma_1\sqrt{E_1} \quad\rightarrow\quad \sigma_1 = \sigma_0\sqrt{\frac{E_0}{E_1}}$$

$$\therefore \sigma_1 = 3842\times\sqrt{\frac{0.0253}{0.1}} = \boxed{1932\,\text{b}}$$

The absorption cross-section of the neutron at 0.1 eV is close to 1930 barns.

P.9 ➡ *Solution*

The absorption cross-section of zirconium is given by the general formula

$$\sigma = \sum_i \gamma_i \sigma_i$$

where γ_i is the abundance of a given isotope and σ_i is its cross-section. Substituting the data from the table given, we obtain

$$\therefore \sigma_i = 0.5145 \times 0.014 + 0.1122 \times 1.2$$
$$+0.1715 \times 0.2 + 0.1738 \times 0.049 + 0.028 \times 0.02 = \boxed{0.1852\,\text{b}}$$

▶ The correct answer is **C**.

P.10 ➡ *Solution*

Part 1: Term a_1 is the volume term; a_2 is the surface term; a_3 is the Coulomb term; and a_4 is the asymmetry term. Explanations of each term can be found in any introductory nuclear physics text, e.g. Krane (1988), chapter 3.

Part 2: For isobaric nuclei, that is, atoms of the same A but different Z, the stable nuclides should satisfy

$$\frac{\partial(B.E.)}{\partial Z} = -2A^{-1/3}a_3 Z + 4a_4 A^{-1}(A - 2Z) = 0$$
$$\therefore -2A^{-1/3}a_3 Z + 4a_4 - 8a_4 A^{-1} Z = 0$$
$$\therefore \left(-2A^{-1/3}a_3 - 8a_4 A^{-1}\right)Z + 4a_4 = 0$$
$$\left(-2A^{2/3}a_3 - 8a_4\right)Z + 4a_4 A = 0$$
$$\therefore \left(2A^{2/3}a_3 + 8a_4\right)Z = 4a_4 A$$
$$\therefore \left(A^{2/3}a_3 + 4a_4\right)Z = 2a_4 A$$
$$\therefore Z = \frac{2a_4 A}{a_3 A^{2/3} + 4a_4}$$
$$\therefore Z = \frac{A}{\dfrac{a_3 A^{2/3}}{2a_4} + \dfrac{4a_4}{2a_4}}$$
$$\therefore \boxed{Z = \frac{A}{2 + \dfrac{a_3 A^{2/3}}{2a_4}}}$$

Substituting a_3 = 0.714 MeV and a_4 = 23.20 MeV, we obtain

$$Z = \frac{A}{2 + \dfrac{0.714 A^{2/3}}{2 \times 23.20}} = \frac{A}{2 + 0.0154 A^{2/3}}$$

Part 3: A Fermi gas of volume V and absolute temperature T = 0 has energy

$$E = \frac{2V}{\hbar^2} \times \frac{4\pi}{5} \times \frac{p_0^5}{2m}$$

and particle number

$$N = \frac{2V}{h^3} \times \frac{4\pi}{3} \times p_0^3$$

where we have assumed that each phase cell can accommodate two particles (neutrons or protons) of opposite signs. The limiting momentum is then

$$p_0 = h\left(\frac{3}{8\pi} \times \frac{N}{V}\right)^{1/3}$$

and the corresponding energy is

$$E=\frac{3}{40}\left(\frac{3}{\pi}\right)^{\frac{2}{3}}\frac{h^2}{m}V^{-\frac{2}{3}}N^{\frac{5}{3}}$$

For nucleus (A,Z) consider the neutrons and protons as independent gases in the nuclear volume V. Then the energy of the lowest state is

$$E=\frac{3}{40}\left(\frac{3}{\pi}\right)^{\frac{2}{3}}\frac{h^2}{m}\frac{N^{5/3}+Z^{5/3}}{V^{2/3}}$$

$$\therefore E=\frac{3}{40}\left(\frac{9}{4\pi^2}\right)^{\frac{2}{3}}\frac{h^2}{mR_0^2}\frac{N^{5/3}+Z^{5/3}}{A^{2/3}}$$

$$\therefore E=C\frac{N^{5/3}+Z^{5/3}}{A^{2/3}}$$

In these passages, $V=(4\pi/3)R_0^3A$, $R_0\approx 1.2$ fm, and C is the rather complicated constant

$$C=\frac{3}{40}\left(\frac{9}{4\pi^2}\right)^{\frac{2}{3}}\frac{1}{mc^2}\left(\frac{hc}{R_0}\right)^2$$

$$\therefore C=\frac{3}{40}\left(\frac{9}{4\pi^2}\right)^{\frac{2}{3}}\frac{1}{940}\left(\frac{1238}{1.2}\right)^2=31.69\approx 31.7\,\text{MeV}$$

For stable nuclei, $N+Z=A$, $N\approx Z$. Now, let

$$N=\frac{1}{2}A\left(1+\frac{\varepsilon}{A}\right)$$

and

$$Z=\frac{1}{2}A\left(1-\frac{\varepsilon}{A}\right)$$

where $\varepsilon/A\ll 1$. Given the smallness of this ratio, we can employ the series expansions

$$\left(1+\frac{\varepsilon}{A}\right)^{5/3}=1+\frac{5\varepsilon}{3A}+\frac{5\varepsilon^2}{9A^2}+\ldots$$

$$\left(1-\frac{\varepsilon}{A}\right)^{5/3}=1-\frac{5\varepsilon}{3A}+\frac{5\varepsilon^2}{9A^2}-\ldots$$

so that

$$N^{\frac{5}{3}}+Z^{\frac{5}{3}}\approx 2\left(\frac{A}{2}\right)^{\frac{5}{3}}\left(1+\frac{5\varepsilon^2}{9A^2}\right)$$

and

$$E\approx 2^{-2/3}CA\left(1+\frac{5\varepsilon^2}{9A^2}\right)=2^{-2/3}CA+\frac{5}{9}\times 2^{-2/3}C\frac{(N-Z)^2}{A}$$

The second term has the form $a_4(N-Z)^2/A$, leading us to conclude that

$$a_4=\frac{5}{9}\times 2^{-2/3}C$$

That is,

$$a_4 = \frac{5}{9} \times 2^{-2/3} \times 31.7 = \boxed{11.1\ \text{MeV}}$$

P.11 ➡ *Solution*

Decays among isobars are beta decays in nature. In the present case the mass number, A = 197, is odd, so there is only one stable nucleus. Indeed, using the semi-empirical mass formula the atomic mass $M(A, Z)$ as a function of Z is a single curve, because the *pairing* term is null for all isobars. The stable nucleus has Z_s = 79, and can be achieved by β^- decay of the nucleus with atomic number Z_s – 1 = 78 or via β^+ decay or electron capture of the nucleus with atomic number Z_s + 1 = 80. The atomic mass of the A = 197 nuclei can be described by the general expression

$$M(197,Z) = Zm_p + (197 - Z)m_n - \frac{B(197,Z)}{c^2} + Zm_e$$

where m_p is the mass of a proton, m_n is the mass of a neutron, m_e is the mass of an electron, and $B(197, Z)$ is the nuclear binding energy. Writing explicitly only the terms depending on Z, we have

$$M(197,Z)c^2 = \text{const.} + Z(m_p - m_n + m_e)c^2 + a_C \frac{Z^2}{197^{1/3}} + a_A \frac{(107-2Z)^2}{197}$$

Noting that a_A = 23.3 eV (asymmetry coefficient) and a_C = 0.697 eV (Coulomb coefficient), we have

$$M(197,Z)c^2 = \text{const.} - 0.782Z + 0.120Z^2 + 0.118(197-2Z)^2$$

For the β^- transition from $^{197}_{78}\text{Pt}$, we get

$$M(197,78)c^2 - M(197,79)c^2 = \text{const.} - 0.782\times 78 + 0.120\times 78^2 + 0.118(197-2\times 78)^2$$
$$-\left[\text{const.} - 0.782\times 79 + 0.120\times 79^2 + 0.118(197-2\times 79)^2\right] \approx 0.822\ \text{MeV}$$

Thus, the β^- transmutation of $^{197}_{78}\text{Pt}$ is allowed. Now, the β^+ transition from $^{197}_{80}\text{Hg}$ is allowed if $M(197, 80) - M(197, 79) > 2m_e$, otherwise only electron capture is possible. Evaluating the difference in question gives

$$M(197,80) - M(197,79) = \text{const.} + 0.782\times 80 + 0.120\times 80^2 + 0.118\times(197-2\times 80)^2$$
$$-\left[\text{const.} + 0.782\times 79 + 0.120\times 79^2 + 0.118\times(197-2\times 79)^2\right] \approx 0.362\,\text{MeV/c}^2$$

Lastly, we conclude that the possible decay types in the situation at hand are β^- decay and electron capture, in accord with the following reactions.

$$\beta^- : {}^{197}_{78}\text{Pt} \to {}^{197}_{79}\text{Au} + e^- + \overline{\nu}_e$$

$$EC : e^- + {}^{197}_{80}\text{Hg} \to {}^{197}_{79}\text{Au} + \nu_e$$

P.12 ➡ *Solution*

The reaction given belongs to the general class of reactions

$$\nu_e + (A,Z) \to (A,Z+1) + e^-$$

The threshold energy is given by

$$E_{\text{th}} = \frac{(m_e + M')^2 - M}{2M} \quad \text{(I)}$$

where M and M' are the masses of (A, Z) and $(A, Z+1)$ nuclei, respectively. The former can be expressed as

$$M = Zm_p + (A-Z)m_n - B(A,Z)/c^2$$

and the latter as

$$M' = (Z+1)m_p + (A-Z-1)m_n - B(A,Z+1)/c^2 = M + \Delta M$$

where

$$\Delta M = (m_p - m_n) + \Delta B/c^2 \quad \text{(II)}$$

with

$$\Delta B = B(A,Z) - B(A,Z+1)$$

ΔB can be determined with the binding energy formula, noting all the while that only the Coulomb and asymmetry terms are needed because (1) the volume and surface terms depend only on A and they cancel out in the difference, and (2) for odd A the pairing term is null for both initial and final nuclei. Thus, we write

$$\Delta B = -a_C\left[\frac{Z^2}{A^{1/3}} - \frac{(Z+1)^2}{A^{1/3}}\right] - a_A\left\{\frac{(A-2Z)^2}{A} - \frac{[A-2(Z+1)]^2}{A}\right\}$$

$$\therefore \Delta B = a_C\frac{2Z+1}{A^{1/3}} - 4a_A\frac{A-2Z-1}{A}$$

For the reaction considered in the text, Z = 17 and A = 37, giving

$$\Delta B = 0.697\times\frac{2\times17+1}{37^{1/3}} - 4\times23.3\times\frac{37-2\times17-1}{37} = 2.28\text{ MeV}$$

Substituting ΔB into equation (II) gives

$$\Delta M = -1.293 + 2.28 = 0.987\,\text{MeV}$$

Lastly, we insert this result into equation (I) to obtain

$$E_{\text{th}} = \frac{[m_e + (M+\Delta M)]^2 - M^2}{2M} = \frac{m_e(m_e+2M) + \Delta M(2m_e + \Delta M + 2M)}{2M}$$

$$\therefore E_{\text{th}} \approx m_e + \Delta M = 0.511 + 0.987 = 1.498 \approx \boxed{1.50\text{ MeV}}$$

The threshold energy for the reaction is approximately 1.5 MeV.

▶ The correct answer is **C**.

P.13 ➡ *Solution*

Denoting by Q^- the Q-factor for the β^- decay

$${}^{64}_{29}\text{Cu} \to {}^{64}_{30}\text{Zn} + e^- + \bar{\nu}_e$$

and by Q^+ the one for the β_+ decay

$${}^{64}_{29}\text{Cu} \to {}^{64}_{28}\text{Ni} + e^+ + \nu_e$$

we have (omitting the factor c^2 in the mass terms)

$$Q_- = 29m_p + (64-29)m_n - B(64,29) - 30m_p - (64-30)m_n + B(64,30) - m_e$$

$$\therefore Q_- = m_n - m_p - m_e + B(64,30) - B(64,29)$$

$$\therefore Q_- = 939.565 - 938.272 - 0.511 + B(64,30) - B(64,29)$$

$$\therefore Q_- = 0.782 + B(64,30) - B(64,29)$$

and, similarly,

$$Q_+ = m_p - m_n - m_e + B(64,28) - B(64,29)$$

$$\therefore Q_+ = 938.272 - 939.565 - 0.511 + B(64,28) - B(64,29)$$

$$\therefore Q_+ = -1.804 + B(64,28) - B(64,29)$$

The difference in blue can be evaluated with recourse to the semi-empirical formula,

$$B(64,30)-B(64,29)=-0.697\times\frac{30^2-29^2}{64^{1/3}}-23.3\times\frac{(64-60)^2-(64-58)^2}{64}+\frac{12+12}{\sqrt{64}}=5\times10^{-4}\text{ MeV}$$

The same applies to the difference in red,

$$B(64,28)-B(64,29)=-0.697\times\frac{28^2-29^2}{64^{1/3}}-23.3\times\frac{(64-56)^2-(64-58)^2}{64}+\frac{12+12}{\sqrt{64}}=2.7385\approx2.74\text{ MeV}$$

Finally,

$$Q_-=0.782+0.0005=0.783\,\text{MeV}$$

and

$$Q_+=-1.80+2.74=0.940\text{ MeV}$$

Both decays are allowed. The maximum kinetic energy of the electron yielded when $^{64}_{29}$Cu decays to $^{64}_{30}$Zn is about 0.78 MeV. The maximum kinetic energy of the positron produced when $^{64}_{29}$Cu decays to $^{64}_{28}$Ni is about 0.94 MeV.

▶ The correct answer is **A**.

P.14 ➡ *Solution*

Part 1: Each fission reaction yields 200 MeV = 2×10^8 eV × 1.6×10^{-19} J/eV = 3.2×10^{-11} J. For the reaction to produce 1.88 GJ per second, the number of fission reactions in each second must equal

$$r=\frac{P}{E_{\text{fiss}}}=\frac{1.88\times10^9}{3.2\times10^{-11}}=\boxed{5.88\times10^{19}\text{ s}^{-1}}$$

▶ The correct answer is **D**.

Part 2: One gram of ^{235}U releases an energy

$$E_{\text{fiss}}\times\frac{N_A}{A}=\left(3.2\times10^{-11}\right)\times\frac{6.02\times10^{23}}{235}=8.20\times10^{10}\text{ J/g}$$

The total energy produced by the reactor in one year is

$$1.88\times10^9\,\frac{\text{J}}{\text{s}}\times86{,}400\frac{\text{s}}{\text{day}}\times365\frac{\text{day}}{\text{year}}=5.93\times10^{16}\text{ J/yr}$$

The mass of uranium consumed over the course of a year is calculated as

$$m=\frac{5.93\times10^{16}\text{ J/yr}}{8.20\times10^{10}\text{ J/g}}=723{,}000\,\text{g}=723\,\text{kg}$$

Since 33% of the fuel is constituted of uranium-235, the mass of solid fuel required to feed the reactor for a full year equals 723/0.33 = 2190 kg = 2.19 metric tons.

▶ The correct answer is **D**.

Part 3: The maximum neutrino energy equals the Q-factor of the β^- decay. Denoting this factor as Q_-, we have, for β^- decay of $^{145}_{57}$La,

$$Q_-=m_n-m_p-m_e-B(145,57)+B(145,58)$$

$$\therefore Q_-=0.782-\Delta B$$

where ΔB is the difference in binding energy between parent and daughter nuclei. Referring to the semi-empirical formula, we have, for odd-atomic mass nuclei,

$$\Delta B_- = B(A,Z) - B(A,Z+1)$$

$$\therefore \Delta B_- = -a_C \frac{Z^2-(Z+1)^2}{A^{1/3}} - a_A \frac{(A-2Z)^2 - [A-2(Z+1)]^2}{A}$$

so that, in the present case,

$$\therefore \Delta B_- = -0.697 \times \frac{57^2-58^2}{145^{1/3}} - 23.3 \times \frac{(145-114)^2-(145-116)^2}{145} = -4.0256 \approx -4.03\,\text{MeV}$$

The maximum neutrino energy is calculated to be

$$Q_- = 0.782 - (-4.03) = \boxed{4.81\text{ MeV}}$$

▶ The correct answer is **B**.

Part 4: The neutrino intensity is 22% of the fission rate, or I_ν = 0.22 × 5.88×10[19] = 1.29×10[19] s[-1]. At 400-m distance the neutrino flux is determined as

$$\Phi = \frac{I_\nu}{4\pi R^2} = \frac{1.29\times10^{19}}{4\pi\times400^2} = \boxed{6.42\times10^{12}\text{ m}^{-2}\text{s}^{-1}}$$

▶ The correct answer is **C**.

Part 5: For a detector having length ℓ (along the neutrino direction), a section S, constituted of material of density ρ and atomic mass A, the interaction rate is

$$r = \Phi\times\sigma\times\frac{N_A}{A}\times\rho\ell S \approx \Phi\times\sigma\times\frac{N_A}{A}\times M$$

Such proportionality between rate and mass holds whenever the detector length is much smaller than the interaction length. Inserting our values brings to

$$r = (6.42\times10^{12})\times(6\times10^{-48})\times\frac{6.02\times10^{23}}{A}\times10^6 = \frac{2.32\times10^{-5}}{A}\text{s}^{-1}$$

$$\therefore \boxed{r = \frac{732}{A}\text{ yr}^{-1}}$$

▶ The correct answer is **B**.

P.15 ➡ *Solution*

Part 1: Noting that Planck's constant h = 4.14×10[-15] eV·s, the frequency of the radiation in question is

$$E = hf \;\rightarrow\; f = \frac{E}{h}$$

$$\therefore f = \frac{7\times10^6}{4.14\times10^{-15}} = \boxed{1.69\times10^{21}\text{ Hz}}$$

The wavelength, in turn, is

$$\lambda = \frac{c}{f} = \frac{3\times10^8}{1.69\times10^{21}} = 1.78\times10^{-13}\text{ m} = \boxed{0.178\text{ pm}}$$

▶ The correct answer is **A**.

Part 2: Substituting E = 7 MeV and E_0 = 0.511 MeV into the equation in focus, we get

$$E' = \left(\frac{1}{E}+\frac{2}{E_0}\right)^{-1} \rightarrow E' = \left(\frac{1}{7}+\frac{2}{0.511}\right)^{-1}$$

$$\therefore E' = \boxed{0.247\text{ MeV}}$$

▶ The correct answer is **B**.

Part 3: If $E \gg E_0$, the term 1/E can be neglected in the equation for E', giving

$$E' = \left(\frac{1}{E} + \frac{2}{E_0}\right)^{-1} \approx \left(\frac{2}{E_0}\right)^{-1}$$

$$\therefore \boxed{E' = \frac{E_0}{2}}$$

The first approximation has been proved. Suppose now that $E \ll E_0$. We first rearrange the equation given to obtain

$$E' = \left(\frac{1}{E} + \frac{2}{E_0}\right)^{-1} \rightarrow E' = \frac{1}{\frac{1}{E} + \frac{2}{E_0}}$$

$$\therefore E' = \frac{1}{\frac{E_0 + 2E}{EE_0}}$$

$$\therefore E' = \frac{EE_0}{E_0 + 2E}$$

It follows that

$$\frac{E - E'}{E} = \frac{E - \frac{EE_0}{E_0 + 2E}}{E}$$

$$\therefore \frac{E - E'}{E} = \frac{\frac{E(E_0 + 2E)}{E_0 + 2E} - \frac{EE_0}{E_0 + 2E}}{E}$$

$$\therefore \frac{E - E'}{E} = \frac{EE_0 + 2E^2 - EE_0}{E(E_0 + 2E)}$$

$$\therefore \frac{E - E'}{E} = \frac{2E}{E_0 + 2E}$$

Neglecting 2E in the denominator, we ultimately obtain

$$\boxed{\frac{E - E'}{E} \approx \frac{2E}{E_0}}$$

as we intended to show. The energy of a 7-MeV gamma ray is substantially greater than the rest mass energy of an electron, so the approximation to use is $E' \approx E_0/2$; that is,

$$E' = \frac{E_0}{2} = \frac{0.511}{2} = 0.256 \text{ MeV}$$

This result is within 3.5% of the actual final photon energy of 0.247 MeV; a decent approximation, indeed.

P.16 ➡ *Solution*

Part 1: The atomic number of aluminum is Z = 13. Entering this value of Z and $h\nu$ = 1 MeV into Figure 1, we see that the dominant interaction process in this case is Compton scattering (blue dot).

▶ The correct answer is **B**.

Part 2: The atomic number of hydrogen is Z = 1. Entering this value of Z and $h\nu$ = 0.1 MeV into Figure 1, we see that the dominant interaction process in this case is Compton scattering (red dot).

▶ The correct answer is **B**.

Part 3: The atomic number of germanium is Z = 32. Entering this value of Z and $h\nu$ = 0.1 MeV into Figure 1, we see that the dominant interaction process in this case is photoelectric absorption (green dot).

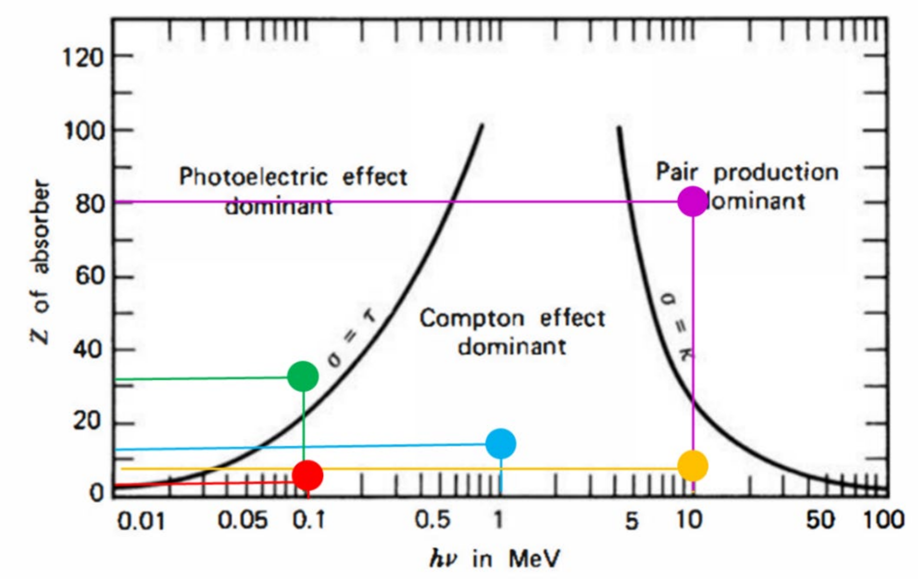
▶ The correct answer is **A**.

Part 4: The atomic number of carbon is Z = 6. Entering this value of Z and $h\nu$ = 10 MeV into Figure 1, we see that the dominant interaction process in this case is Compton scattering (yellow dot).

▶ The correct answer is **B**.

Part 5: The atomic number of carbon is Z = 6. Entering this value of Z and $h\nu$ = 10 MeV into Figure 1, we see that the dominant interaction process in this case is pair production (purple dot).

▶ The correct answer is **C**.



P.17 ➡ *Solution*

Part 1: The electric field intensity at a point distance r from the center of a uniformly charged sphere of radius R is given by

$$E(r)=\begin{cases}\dfrac{Qr}{R^3} & ;\ r<R\\[2mm] \dfrac{Q}{r^2} & ;\ r<R\end{cases}$$

The electrostatic energy is obtained from the integral

$$W=\int_0^\infty \frac{E^2}{8\pi}dr=\frac{Q^2}{8\pi}\left[\int_0^R\left(\frac{r}{R^3}\right)^2 4\pi r^2dr+\int_R^\infty \frac{1}{r^4}\times 4\pi r^2 dr\right]$$

$$\therefore W=\frac{Q^2}{8\pi}\left(\int_0^R \frac{4\pi r^4}{R^6}dr+\int_R^\infty \frac{4\pi}{r^2}dr\right)$$

$$\therefore W=\frac{Q^2}{8\pi}\left(\frac{4\pi r^5}{5R^6}\bigg|_0^R-\frac{4\pi}{r}\bigg|_R^\infty\right)$$

$$\therefore W=\frac{Q^2}{8\pi}\left(\frac{4\pi R^5}{5R^6}-0-0+\frac{4\pi}{R}\right)=\boxed{\frac{3Q^2}{5R}}$$

▶ The correct answer is **C**.

Part 2: The mass difference between mirror nuclei $^{27}_{14}$Si and $^{27}_{13}$Al can be attributed to a difference in electrostatic energy, giving

$$\Delta W=\frac{3e^2}{5R}\left(Z_1^2-Z_2^2\right)$$

Solving for radius and manipulating, we obtain

$$\Delta W = \frac{3e^2}{5R}\left(Z_1^2 - Z_2^2\right) \rightarrow R = \frac{3e^2}{5\Delta W}\left(Z_1^2 - Z_2^2\right)$$

$$\therefore R = \frac{3\times \underbrace{hc}_{=1.97\times 10^{-11}}}{5\Delta W}\underbrace{\left(\frac{e^2}{hc}\right)}_{=1/137}\left(14^2 - 13^2\right)$$

$$\therefore R = \frac{3\times\left(1.97\times 10^{-11}\right)}{5\times 6}\times\frac{1}{137}\times\left(14^2-13^2\right) = 3.88\times 10^{-13}\text{ m}$$

$$\therefore \boxed{R = 3.88\text{ fm}}$$

▶ The correct answer is **C**.

P.18 ➡ *Solution*

The energy difference between two members of the same isospin multiplet is determined by the Coulomb energies and the neutron-proton mass difference. Mathematically,

$$\Delta E = E\left(A,Z+1\right) - E\left(A,Z\right) = \Delta E_e - \left(m_n - m_p\right)c^2$$

$$\therefore \Delta E = \frac{3e^2}{5R}\left(2Z+1\right) - 0.78 = \frac{3\left(2Z+1\right)ch\alpha}{5R} - 0.78$$

Using $R \approx 1.2A^{1/3}$ fm, we find that

$$\Delta E = \frac{3\times\left(2\times 39+1\right)\times 197}{5\times\left(1.2\times 90^{1/3}\right)\times 137} - 0.78 = 11.89\text{ MeV}$$

Hence the excitation energy of the $T = 6$ state of ^{90}Zr is calculated to be

$$E = -782.410 + 11.89 + 783.916 = \boxed{13.40\text{ MeV}}$$

▶ The correct answer is **B**.

P.19 ➡ *Solution*

Part 1: Appealing to the Born approximation, we may write

$$f = -\frac{m}{2\pi\hbar^2}\int V\left(\mathbf{r}\right)e^{-i\mathbf{q}\cdot\mathbf{r}}d^3\mathbf{r}$$

where $\boldsymbol{q} = \boldsymbol{k} - \boldsymbol{k_0}$ is the momentum transferred from the incident particle to the outgoing particle. Here, $|\boldsymbol{q}| = 2k_0 sin(\theta/2)$, where θ is the angle between the incident and outgoing particles. As $V(r)$ is spherically symmetric, we may write

$$f\left(\theta\right) = -\frac{m}{2\pi\hbar^2}\int_0^\infty\int_0^{2\pi}\int_0^\pi V\left(r\right)e^{-i\Delta kr\cos\theta}\sin\left(\theta\right)r^2drd\varphi d\theta$$

$$\therefore f\left(\theta\right) = -\frac{2m}{\hbar^2\Delta k}\int_0^\infty V\left(r\right)\sin\left(\Delta kr\right)rdr$$

$$\therefore f\left(\theta\right) = -\frac{2mQq}{\hbar^2}\frac{1}{\alpha^2+\left(\Delta k\right)^2}$$

The differential cross-section is

$$d\sigma = \left|f\left(\theta\right)\right|^2 d\Omega = \frac{4m^2Q^2q^2}{\hbar^4}\frac{d\Omega}{\left[\alpha^2+\left(\Delta k\right)^2\right]^2}$$

$$\therefore d\sigma = \frac{m^2Q^2q^2}{4\hbar^4k_0^4}\frac{d\Omega}{\left(\frac{\alpha^2}{4k_0^2}+\sin^2\frac{\theta}{2}\right)^2}$$

The total cross-section is then

$$\sigma = \int d\sigma = \frac{m^2Q^2q^2}{4\hbar^4k_0^4}\int_0^{2\pi}\int_0^{\pi}\frac{\sin\theta d\theta d\varphi}{\left(\frac{\alpha^2}{4k_0^2}+\sin^2\frac{\theta}{2}\right)^2}$$

$$\therefore \boxed{\sigma = \frac{16\pi m^2Q^2q^2}{\hbar^4\alpha^2\left(4k_0^2+\alpha^2\right)}}$$

Part 2: α^{-1} is a measure of the size of atoms. As Z increases, the number of electrons outside the nucleus as well as their probability of finding them near the nucleus will increase, enhancing the screening effect. Hence, α is an increasing function of Z.

P.20 ➡ *Solution*

Part 1: The differential cross-section for scattering is (see also Problem 19.1),

$$\frac{d\sigma}{d\Omega} = \frac{m^2q^2Q^2}{4\hbar^4k_0^4}\frac{1}{\left(\frac{\alpha^2}{4k_0^4}+\sin\frac{\theta}{2}\right)^2}$$

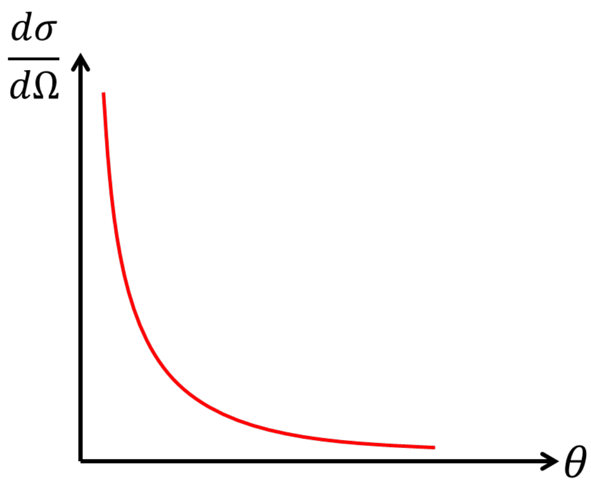
For a proton and a hydrogen nucleus, $Q = q = e$, where e is the elementary charge. The screening length can be taken to be $\alpha^{-1} \approx R_0$, where R_0 is the Bohr radius of the hydrogen atom. For an incident proton of 1-keV energy, the wavelength can be estimated to be

$$\lambda_0 = \frac{\hbar}{\sqrt{2\mu E}} = \frac{c\hbar}{\sqrt{2\mu c^2 E}} = \frac{197}{\sqrt{1\times\left(938\times10^{-3}\right)}} = 203.4\text{ fm}$$

With $\alpha^{-1} \approx R_0 = 5.4\times10^{-11}$ m = 53,000 fm as the Bohr radius of hydrogen, we see that $\alpha^2/4k_0^2 = (\lambda_0/2\alpha^{-1})^2 \ll 1$, enabling us to simplify the equation for $d\sigma/d\Omega$ and obtain

$$\frac{d\sigma}{d\Omega} \approx \frac{m^2e^4}{4\hbar^2k_0^2\sin^4(\theta/2)}$$

which is the Rutherford scattering formula. A sketch of $d\sigma/d\Omega$ versus θ is provided below. The scattering of 1-keV protons from hydrogen atoms occurs mainly at small angles. The probability associated with large-angle scattering is very small, as the hydrogen atom has a very small nucleus.



Part 2: To estimate the total cross-section, we appeal to the equation derived in Part 2 of the previous problem,

$$\sigma = \frac{16\pi m^2e^4}{\hbar^4\alpha^2\left(4k_0^2+\alpha^2\right)} \approx \frac{16\pi m^2e^4}{\hbar^4\alpha^2 4k_0^2} = 4\pi\left[\frac{mc^2R_0\lambda_0}{\hbar c}\left(\frac{e^2}{\hbar c}\right)\right]^2$$

$$\therefore \sigma = 4\pi \times \left[\frac{938\times\left(5.3\times10^{4}\right)\times 203}{197\times 137}\right]^2 = 1.757\times10^{12}\ \text{fm}^2$$

$$\therefore \boxed{\sigma = 1.76\times10^{-14}\ \text{cm}^2}$$

▶ The correct answer is **D**.

P.21 ➡ *Solution*

Problem 21.1: The activity *A(t)* of cobalt-60 at any given time is $A(n) = A_0/2^n$, where *n* is the number of half-lives elapsed over the course of time *t*; in the case in focus, A_0 = 5000 Ci, *n* = 45/5.27 = 8.54 half-lives, and

$$A(8.54) = \frac{5000}{2^{8.54}} = \boxed{13.4\ \text{Ci}}$$

▶ The correct answer is **A**.

Problem 21.2: In this case, we are looking for the number *n* of half-lives that need to elapse for the cesium sample to transition from an initial activity A_0 = 4 Ci to a final activity *A* = 0.01 Ci; referring to the decay equation, we have

$$A(n) = \frac{A_0}{2^n} \rightarrow 0.01 = \frac{4}{2^n}$$

$$\therefore 2^n = \frac{4}{0.01}$$

$$\therefore 2^n = 400$$

$$\therefore n\times\log_2 2 = \log_2 400$$

$$\therefore n\times 1 = 8.64$$

$$\therefore n = 8.64\ \text{HL}$$

This quantity of half-lives amounts to a time *t* such that

$$t = 8.64\ \cancel{\text{HL}}\times 30.07\frac{\text{yr}}{\cancel{\text{HL}}} = \boxed{260\ \text{yr}}$$

▶ The correct answer is **B**.

Problem 21.3: The total biological elimination is 12 + 22 + 3 = 37%. The time required to achieve 50% elimination is the biological half-life T_b, namely

$$T_b = \frac{50}{37}\times 6.0\,\text{h} = 8.11\,\text{h}$$

The effective half-life T_e is given by

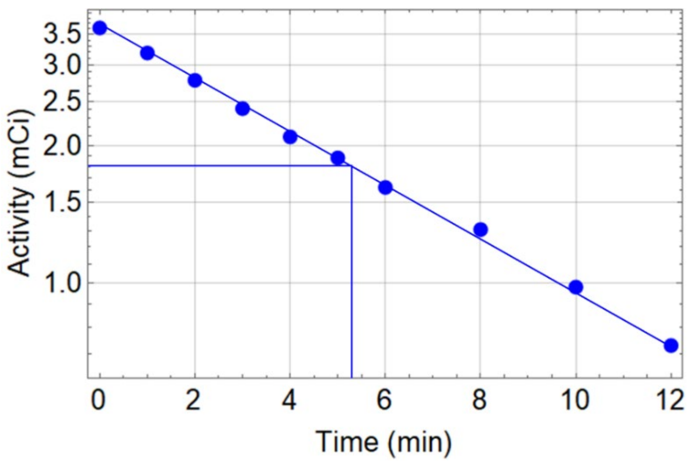
$$\frac{1}{T_e} = \frac{1}{T_p} + \frac{1}{T_b} \rightarrow T_e = \frac{T_b T_p}{T_b + T_p}$$

$$\therefore T_e = \frac{8.11\times 6.0}{8.11+6.0} = \boxed{3.45\,\text{h}}$$

▶ The correct answer is **A**.

P.22 ➡ *Solution*

Problem 22.1: The activity *A(t)* of the radionuclide in question is plotted against time in semilogarithmic paper, as shown to the side.



The half-life is the time required to reduce the initial activity by half. In the present case, A_0 = 3.6 mCi, so the half-life is the time required to reduce the activity to 1.8 millicuries. Entering this ordinate into the graph, we read $t_{1/2} \approx$ 5.3 min. Instead of eyeballing the plot above, we could use the fact that the activity can be modelled by the usual exponential law

$$A(t) = A_0 \exp(-\lambda t)$$

where A_0 = 3.6 mCi and λ is the decay constant. We can fit our data to an equation of the form 3.6exp($-\lambda t$) by dint of Mathematica's *FindFit* command,

```
In[2032]:= data = {{0, 3.62}, {1, 3.19}, {2, 2.78}, {3, 2.41}, {4, 2.09}, {5, 1.88}, {6, 1.62},
    {8, 1.31}, {10, 0.98}, {12, 0.73}}
In[2042]:= FindFit[data, 3.6*Exp[-λ*t], λ, t]
Out[2042]=
    {λ → 0.131072}
```

That is, the program returns a decay constant of 0.131. The half-life easily follows,

$$\lambda = \frac{\ln 2}{t_{1/2}} \;\rightarrow\; t_{1/2} = \frac{\ln 2}{\lambda}$$

$$\therefore t_{1/2} = \frac{\ln 2}{0.131} = \boxed{5.29\,\text{min}}$$

▶ The correct answer is **C.**

Problem 22.2: α-decay can be viewed as the transmission of an α-particle through the potential barrier of the daughter nucleus. Suppose R denotes the nuclear radius and r_1 is the point where the Coulomb repulsive potential $V(r) = Zze^2/r$ equals the α-particle energy E. Using a three-dimensional potential and neglecting angular momentum, we can appeal to the WKB method and write the following expression for transmission coefficient T,

$$T = e^{-2G}$$

where

$$G = \frac{1}{\hbar}\int_R^{r_1} \left(2M\left|E - V\right|\right)^{1/2} dr$$

with $V = zZe^2/r$, $E = zZe^2/r_1$, z = 2, Ze being the charge of the daughter nucleus. Integration gives

$$G = \frac{1}{\hbar}\left(2mzZe^2 r_1\right)^{1/2}\left[\arccos\left(\frac{R}{r_1}\right) - \left(\frac{R}{r_1} - \frac{R^2}{r_1^2}\right)^{1/2}\right]$$

so that, with $R/r_1 \rightarrow 0$,

$$G = \frac{1}{\hbar}\left(2mzZe^2 r_1\right)^{1/2}\left[\frac{\pi}{2} - \left(\frac{R}{r_1}\right)^{1/2}\right]$$

Suppose the α-particle has velocity v_0 in the potential well. Then it collides with the walls v_0/R times per unit time and the probability of decay per unit time is $\lambda = v_0 T/R$. Accordingly,

$$\ln \lambda = -\frac{\sqrt{2m}BR\pi}{\hbar}\left(E^{-\frac{1}{2}} - \frac{2}{\pi}B^{-\frac{1}{2}}\right) + \ln\frac{v_0}{R}$$

where $B = zZe^2/R$. As can be seen, there is a linear relationship between *log* λ and $E^{-1/2}$ for α-emitters of the same radioactive series. Considering the polonium isotopes specifically, we have the ratio

$$\log_{10}\frac{T\left({}^{210}\text{Po}\right)}{T\left({}^{214}\text{Po}\right)} = 0.434\left[\ln\lambda\left({}^{214}\text{Po}\right) - \ln\left({}^{210}\text{Po}\right)\right]$$

$$\therefore \log_{10}\frac{T\left({}^{210}\text{Po}\right)}{T\left({}^{214}\text{Po}\right)} = 0.434\sqrt{2mc^2}\,zZ\left(\frac{e^2}{\hbar c}\right)\left(\frac{1}{\sqrt{E_{210}}}-\frac{1}{\sqrt{E_{214}}}\right)\approx 10$$

That is, the lifetimes differ by 10 orders of magnitude.

Problem 22.3: Let us apply the Geiger-Nuttall law,

$$\log_{10}\lambda = C - DE_\alpha^{-1/2}$$

where λ is the decay constant, E_α is the α-particle energy, and C and D are constants. For a rough estimate, use the values of C and D for Pb, namely $C \approx 52$ and $D \approx 140$ MeV$^{1/2}$,

$$\log_{10}\lambda = C - DE_\alpha^{-1/2} \quad \rightarrow \quad \lambda = 10^{C-DE_\alpha^{-1/2}}$$

$$\therefore \lambda = 10^{52-140\times 3.3^{-1/2}} = 8.56\times 10^{-26}\ \text{s}^{-1}$$

The corresponding half-life is

$$t_{1/2} = \frac{\ln 2}{\lambda} = \frac{\ln 2}{8.56\times 10^{-26}} = 8.10\times 10^{24}\ \text{s} = 2.57\times 10^{17}\ \text{yr}$$

Thus the number of decays in a human's lifetime is too small to worry about.

P.23 ➡ *Solution*

The specific activity a of a radionuclide is defined as the activity $\mathcal{A}$ per unit mass m,

$$a = \frac{\mathcal{A}}{m}$$

However, the activity can be replaced by the product of decay constant λ and No. of atoms N,

$$a = \frac{\lambda N}{m}$$

and the decay constant can be replaced by the ratio of $\ln 2$ to half-life $t_{1/2}$,

$$a = \frac{\ln 2 N}{t_{1/2} m}$$

Lastly, we replace N/m with the ratio of Avogadro's number N_A to atomic mass A,

$$a = \frac{\ln 2 N_A}{t_{1/2} A}$$

This equation enables us to determine the specific activity of any isotope from its atomic number and half-life. For cobalt-60, A = 60 g/mol and $t_{1/2}$ = 5.26 yr = 5.26 × 86,400 × 365 = 1.66×10^8 sec, giving

$$a = \frac{\ln 2\times\left(6.02\times 10^{23}\ \text{mol}^{-1}\right)}{\left(1.66\times 10^{8}\text{sec}\right)\times 60\dfrac{\text{g}}{\text{mol}}} = 4.19\times 10^{13}\ \text{s}^{-1}/\text{g} = 4.19\times 10^{13}\ \text{Bq/g}$$

Lastly, since 1 Bq = 3.70×10^{-11} Ci, we find that

$$a = 4.19\times 10^{13}\frac{\text{Bq}}{\text{g}}\times 3.70\times 10^{-11}\frac{\text{Ci}}{\text{Bq}} = \boxed{1130\ \text{Ci/g}}$$

For iodine-131, A = 131 g/mol and $t_{1/2}$ = 8.02 days = 8.02 × 86,400 = 693,000 sec, so that

$$a = \frac{\ln 2 \times \left(6.02 \times 10^{23}\ \text{mol}^{-1}\right)}{\left(693{,}000\ \text{sec}\right) \times 131 \dfrac{\text{g}}{\text{mol}}} = 4.60 \times 10^{15}\ \text{Bq/g}$$

Converting to curies,

$$a = 4.60 \times 10^{15} \frac{\text{Bq}}{\text{g}} \times 3.70 \times 10^{-11} \frac{\text{Ci}}{\text{Bq}} = \boxed{124{,}000\ \text{Ci/g}}$$

For cesium-137, A = 137 g/mol and $t_{1/2}$ = 30 yrs = 30 × 86,400 × 365 = 9.46×10^{8} sec, so that

$$a = \frac{\ln 2 \times \left(6.02 \times 10^{23}\ \text{mol}^{-1}\right)}{\left(9.46 \times 10^{8}\ \text{sec}\right) \times 137 \dfrac{\text{g}}{\text{mol}}} = 3.22 \times 10^{12}\ \text{Bq/g}$$

Converting to curies,

$$a = 3.22 \times 10^{12} \frac{\text{Bq}}{\text{g}} \times 3.70 \times 10^{-11} \frac{\text{Ci}}{\text{Bq}} = \boxed{86.9\ \text{Ci/g}}$$

▶ The correct answers to **23.1, 23.2,** and **23.3** are **B, D,** and **A,** respectively.

P.24 ➡ *Solution*

The decay constant of iodine-123 is λ_{I} = *ln* 2/13.2 = 0.0525 h^{-1}, and the radioactive decay of the iodine sample is described by the exponential law

$$A_I(t) = 400e^{-0.0525t}$$

Similarly, the decay constant of technetium-99m is λ_{Tc} = *ln* 2/6 = 0.115 h^{-1}, and the decay of the Tc sample is described by

$$A_{\text{Tc}}(t) = 1780e^{-0.115t}$$

Equating the two expressions and manipulating,

$$A_I(t) = A_{\text{Tc}}(t) \rightarrow 400e^{-0.0525t} = 1780e^{-0.115t}$$

$$\therefore \frac{e^{-0.0525t}}{e^{-0.115t}} = \frac{1780}{400}$$

$$\therefore e^{0.0625t} = 4.45$$

$$\therefore 0.0625t = \ln 4.45$$

$$\therefore 0.0625t = 1.49$$

$$\therefore t = \frac{1.49}{0.0625} = \boxed{23.8\text{h}}$$

The two samples will have equal activities within approximately 24 hours.

▶ The correct answer is **D**.

P.25 ➡ *Solution*

β^- decay of carbon-14 is described by the reaction

$${}^{14}_{6}\text{C} \rightarrow {}^{14}_{7}\text{N} + e^- + \overline{\nu}_e$$

The specimen activity is given by

$$A = \left|\frac{dN}{dt}\right| = \frac{N\left({}^{14}\text{C}\right)}{\tau\left({}^{14}\text{C}\right)} \quad \text{(I)}$$

The number of ${}^{14}_{6}\text{C}$ nuclei present in the specimen when it was still a living organism is

$$N_0\left({}^{14}\mathrm{C}\right) = f \times N_0(\mathrm{C}) = f \times m \times \frac{N_A}{\langle A(\mathrm{C})\rangle}$$

Here, f = 1.3×10^{-12} is the fraction of ${}^{14}_{6}\mathrm{C}$ nuclei in a living organism, m = 6 g is the mass of the organism, N_A = 6.02×10^{23} is Avogadro's number, and $\langle A(C)\rangle$ = 12.001 is the atomic mass of natural carbon, giving

$$N_0\left({}^{14}\mathrm{C}\right) = \frac{fmN_A}{\langle A(C)\rangle} = \frac{\left(1.3\times10^{-12}\right)\times 6\times\left(6.02\times10^{23}\right)}{12.001} = 3.91\times10^{11}$$

The mean lifetime of carbon-14 equals the ratio of its half-life to *ln* 2,

$$\tau\left({}^{14}\mathrm{C}\right) = \frac{5700}{\ln 2} = 8220 \text{ yrs}$$

Substituting the two previous results into (I) gives the activity when the organism died,

$$A_0 = \frac{N_0\left({}^{14}\mathrm{C}\right)}{\tau\left({}^{14}\mathrm{C}\right)} \approx \frac{3.91\times10^{11}}{8220\times(365\times86{,}400)} = 1.51\mathrm{s}^{-1}$$

The present activity, in turn, is

$$A(T) = \frac{5400}{3\times3600} = 0.5\,\text{decays/s}$$

The present and past activities are related by the equation

$$A(t) = A_0\exp(-t/\tau)$$

Solving for the age of the fossil T,

$$A(t) = A_0\exp(-T/\tau) \rightarrow T = -\tau\ln\frac{A(T)}{A_0}$$

$$\therefore T = -8220\times\ln\frac{0.5}{1.51} = \boxed{9090 \text{ yr}}$$

The fossil is over nine thousand years old.

▶ The correct answer is **D**.

P.26 ➡ *Solution*

Problem 26.1: We express the variation in activity of the gold isotope with the exponential law

$$A_{\mathrm{Au}}(t) = A_{\mathrm{Au}}(0)\exp(-\lambda_{\mathrm{Au}}t) = A_{\mathrm{Au}}(0)\exp\left(-\frac{\ln 2}{\left(t_{1/2}\right)_{\mathrm{Au}}}t\right)$$

Likewise, for iodine-131,

$$A_I(t) = A_{\mathrm{I}}(0)\exp(-\lambda_I t) = A_{\mathrm{I}}(0)\exp\left(-\frac{\ln 2}{\left(t_{1/2}\right)_I}t\right)$$

The data given stipulates an initial activity of 0.25 μCi for the solution; in mathematical terms,

$$A_{\mathrm{Au}}(0) + A_I(0) = 0.25 \quad (\mathrm{I})$$

Further, we know that the total activity dropped by half within three days, so

$$A_{\mathrm{Au}}(0)\exp\left(-\frac{\ln 2}{\left(t_{1/2}\right)_{\mathrm{Au}}}\times 3\right) + A_{\mathrm{I}}(0)\exp\left(-\frac{\ln 2}{\left(t_{1/2}\right)_I}\times 3\right) = 0.125$$

$$\therefore A_{Au}(0)\exp\left(-\frac{\ln 2}{2.70}\times 3\right)+A_I(0)\exp\left(-\frac{\ln 2}{8.05}\times 3\right)=0.125$$

$$\therefore 0.463A_{Au}(0)+0.772A_I(0)=0.125\ \text{(II)}$$

Equations (I) and (II) constitute a system of linear equations with variables $A_{Au}(0)$ and $A_I(0)$,

$$\begin{cases} A_{Au}(0)+A_I(0)=0.25 \\ 0.463A_{Au}(0)+0.772A_I(0)=0.125 \end{cases}$$

Solving the system above gives the initial activities $A_{Au}(0)$ = 0.220 μCi and $A_I(0)$ = 0.0299 μCi.

Problem 26.2: Equipped with the half-lives given and the initial activities determined in Part 1, we can easily establish the total activity at any time *t*,

$$A(6)=0.220\times\exp\left(-\frac{\ln 2}{2.70}\times 6\right)+0.0299\times\exp\left(-\frac{\ln 2}{8.05}\times 6\right)=\boxed{0.0650\ \mu\text{Ci}}$$

▶ The correct answer is **B**.

Problem 26.3: To determine the time *t* at which the activities of the gold and iodine isotopes are equal, simply equate the exponential laws that yield the activity of each element and solve for time *T*,

$$A_{Au}(t)=A_I(t)\ \rightarrow\ A_{Au}(0)\exp\left(-\frac{\ln 2}{(t_{1/2})_{Au}}T\right)=A_I(0)\exp\left(-\frac{\ln 2}{(t_{1/2})_I}T\right)$$

$$\therefore T=\frac{(t_{1/2})_{Au}(t_{1/2})_I\ln\left[A_{Au}(0)/A_I(0)\right]}{\left[(t_{1/2})_I-(t_{1/2})_{Au}\right]\ln 2}$$

The half-lives were given in the problem statement, and the initial activities $A_{Au}(0)$ and $A_I(0)$ were determined in Part 1; substituting,

$$T=\frac{2.70\times 8.05\times\ln(0.220/0.0299)}{(8.05-2.70)\times\ln 2}=\boxed{11.7\text{ days}}$$

To calculate the activity at this time, simply substitute *T* into either of the two exponential laws,

$$A_{Au}(11.7)=0.220\times\exp\left(-\frac{\ln 2}{2.70}\times 11.7\right)=\boxed{0.0109\,\mu\text{Ci}}$$

$$A_I(11.7)=0.0299\times\exp\left(-\frac{\ln 2}{8.05}\times 11.7\right)=0.0109\,\mu\text{Ci}$$

Notice that $A_{Au}(11.7)=A_I(11.7)$, as expected.

▶ The correct answer is **B**.

P.27 ➡ *Solution*

Part 1: We aim to describe the rate of change in the number of parent nuclei, N_P, and in the number of daughter nuclei, N_D. For the parent, the rate of change in the number of parent nuclei is given by the usual nuclear decay ODE,

$$\frac{dN_P(t)}{dt}=-\lambda_P N_P(t)\ \text{(i)}$$

The negative sign on the right-hand side indicates a decrease in the number of parent nuclei $N_P(t)$ with increasing time *t*.

Now, the rate of change $dN_D(t)/dt$ in the number of daughter nuclei *D* is equal to the supply of new daughter nuclei *D* through the decay of *P* (given as

$\lambda_P N_P(t)$) and the loss of daughter nuclei D from the decay of D to G (formulated as $-\lambda_D N_D(t)$). The rate of change dN_D/dt then becomes

$$\frac{dN_D(t)}{dt} = \lambda_P N_P(t) - \lambda_D N_D(t) \text{ (ii)}$$

Part 2: The governing equation we proposed for the rate of change in parent nuclei, combined with the initial condition $N_P(t = 0) = N_p(0)$, suggests that the solution has the form

$$N_P(t) = N_P(0)e^{-\lambda_P t} \text{ (iii)}$$

The solution that describes the evolution in number of daughter nuclei is more complicated and can be determined after inserting $N_P(t)$ into the differential equation

$$\frac{dN_D(t)}{dt} = \lambda_P N_P(0)e^{-\lambda_P t} - \lambda_D N_D(t) \text{ (iv)}$$

The general solution to this differential equation is

$$N_D(t) = N_P(0)\left(a \times e^{-\lambda_P t} + b \times e^{-\lambda_D t}\right) \text{ (v)}$$

Here, a and b are constants to be determined as follows.
Step 1: Differentiate the equation with respect to time to obtain

$$\frac{dN_D(t)}{dt} = N_P(0)\left(-a\lambda_p e^{-\lambda_P t} - b\lambda_D e^{-\lambda_D t}\right) \text{ (vi)}$$

Then, insert (*v*) and (*vi*) into (*iv*) and rearrange terms to obtain

$$e^{-\lambda_P t}\left(-a\lambda_P - \lambda_P + a\lambda_D\right) = 0 \text{ (vii)}$$

The factor in parentheses must be equal to zero to satisfy the equation for all values of t, yielding the following expression for constant a,

$$a = \frac{\lambda_P}{\lambda_D - \lambda_P} \text{ (viii)}$$

Coefficient b, in turn, depends on the initial condition $N_D(0) = 0$, which can be applied to (*v*) to yield, ultimately,

$$a + b = 0 \text{ (ix)}$$

Inserting (*viii*) yields

$$b = -a = -\frac{\lambda_P}{\lambda_D - \lambda_P} = \frac{\lambda_P}{\lambda_P - \lambda_D} \text{ (x)}$$

Finally, we can insert (*viii*) and (*x*) into (*vi*) to obtain the following equation for the number of daughter nuclei as a function of time t,

$$N_D(t) = N_P(0)\frac{\lambda_P}{\lambda_D - \lambda_P}\left(e^{-\lambda_P t} - e^{-\lambda_D t}\right) \text{ (xi)}$$

Part 3: The simple $P \rightarrow D \rightarrow G$ radioactive series decay with radioactive parent P decaying through radioactive daughter D into stable grand-daughter G is characterized by equations describing the number of parent nuclei $N_P(t)$ and number of daughter nuclei $N_D(t)$ given by eqs. (*iii*) and (*xi*), respectively. Activities $A_p(t)$ and $A_D(t)$ of the parent and daughter, respectively, are also of interest and can be determined by recalling that, in general, the activity $A(t)$ of a radionuclide is the product of its decay constant λ and the number $N(t)$ of radioactive nuclei present in the sample. It follows that the activity of the parent nuclei is given by

$$A_p(t) = \lambda_P N_P(t) = \lambda_P N_P(0)e^{-\lambda_P t} = A_P(0)e^{-\lambda_P t} \text{ (xii)}$$

The activity of the daughter nuclei, in turn, is given by

$$A_D(t) = \lambda_D N_D(t) = \lambda_D N_P(0) \frac{\lambda_P}{\lambda_D - \lambda_P}\left(e^{-\lambda_P t} - e^{-\lambda_D t}\right) \text{ (xiii)}$$

The product of the terms in blue is actually $A_P(0)$; finally,

$$A_D(t) = \lambda_D N_P(0) \frac{\lambda_P}{\lambda_D - \lambda_P}\left(e^{-\lambda_P t} - e^{-\lambda_D t}\right) = A_P(0) \frac{\lambda_D}{\lambda_D - \lambda_P}\left(e^{-\lambda_P t} - e^{-\lambda_D t}\right) \text{ (xiv)}$$

Note that $A_D(t)$ yields zero for t = 0, in accord with the initial condition $N_D(0)$ = 0, and for $t \to \infty$, because by then all daughter nuclei will have decayed.

Part 4: The characteristic time $(t_{max})_D$ at which the daughter activity $A_D(t)$ attains its maximum $(A_D)_{max}$ is determined by differentiating $A_D(t)$, setting the ensuing expression to zero, and solving for $(t_D)_{max}$.

$$\left.\frac{dA_D(t)}{dt}\right|_{t=(t_{\max})_D} = N_P(0) \frac{\lambda_D \lambda_P}{\lambda_D - \lambda_P} \frac{d}{dt}\left[e^{-\lambda_P t} - e^{-\lambda_D t}\right]_{t=(t_{\max})_D}$$

$$\therefore \left.\frac{dA_D(t)}{dt}\right|_{t=(t_{\max})_D} = N_P(0) \frac{\lambda_D \lambda_P}{\lambda_D - \lambda_P}\left[-\lambda_P e^{-\lambda_P (t_{\max})_D} + \lambda_D e^{-\lambda_D (t_{\max})_D}\right] = 0 \ (xv)$$

Setting the equation in brackets to zero, we find that

$$\lambda_P e^{-\lambda_P (t_{\max})_D} + \lambda_D e^{-\lambda_D (t_{\max})_D} = 0$$

$$\therefore \lambda_P e^{-\lambda_P (t_{\max})_D} = -\lambda_D e^{-\lambda_D (t_{\max})_D}$$

$$\therefore \frac{\lambda_P}{\lambda_D} = e^{(\lambda_P - \lambda_D)(t_{\max})_D} \ (xvi)$$

Applying logarithms and solving for $(t_{max})_D$,

$$\frac{\lambda_P}{\lambda_D} = e^{(\lambda_P - \lambda_D)(t_{\max})_D} \to \boxed{(t_{\max})_D = \frac{\ln(\lambda_P/\lambda_D)}{\lambda_P - \lambda_D}} \ (xvii)$$

The equation above gives the characteristic time at which the daughter activity will have reached a maximum.

Part 5: For $\lambda_P \gtrsim \lambda_D$ and $0 < \varepsilon \ll 1$, we may write the following relation between decay constants λ_P and λ_D of the parent and daughter,

$$\lambda_P = \lambda_D(1+\varepsilon) \ (xviii)$$
$$\lambda_P(1-\varepsilon) \approx \lambda_D \ (xix)$$

Plugging (*xviii*) and (*xix*) into the numerator and denominator of (*xvii*), respectively, we obtain

$$(t_{\max})_D = \frac{\ln(\lambda_P/\lambda_D)}{\lambda_P - \lambda_D} = \frac{\ln(1+\varepsilon)}{\varepsilon\lambda_D} \ (xx)$$

ln$(1+\varepsilon)$ can be expanded as a Taylor series to give

$$\ln(1+\varepsilon) \approx \varepsilon - \frac{1}{2}\varepsilon^2 + \frac{1}{3}\varepsilon^3 - \ldots \ (xxi)$$

Retaining the first two terms only and substituting into (*xx*), we obtain

$$(t_{\max})_D \approx \frac{\varepsilon - \frac{1}{2}\varepsilon^2}{\varepsilon\lambda_D} = \frac{1 - \frac{1}{2}\varepsilon}{\lambda_D} \approx \frac{\sqrt{1-\varepsilon}}{\lambda_D} \ (xxii)$$

From (*xix*), $\sqrt{1-\varepsilon} = (\lambda_D/\lambda_P)^{1/2}$, so that

$$(t_{\max})_D \approx \frac{(\lambda_D/\lambda_P)^{1/2}}{\lambda_D} = \frac{1}{\sqrt{\lambda_D \lambda_P}} \ (xxiii)$$

as we intended to show. Suppose now that $\lambda_P \lesssim \lambda_D$ and $0 < \varepsilon \ll 1$ (which needn't have the same value as the foregoing approximation for $\lambda_P \gtrsim \lambda_D$). We may write

$$\lambda_P = \lambda_D(1-\varepsilon) \quad (xxiv)$$

$$\lambda_P(1+\varepsilon) = \lambda_D \quad (xxv)$$

Inserting (*xxiv*) and (*xxv*) into (*xvii*) brings to

$$(t_{\max})_D = \frac{\ln(\lambda_P/\lambda_D)}{\lambda_P-\lambda_D} = \frac{\ln(1-\varepsilon)}{-\varepsilon\lambda_D} \quad (xxvi)$$

ln(1 − ε) can be expanded as a Taylor series,

$$\ln(1-\varepsilon) \approx -\left(\varepsilon+\frac{1}{2}\varepsilon^2+\frac{1}{3}\varepsilon^3+\ldots\right) \quad (xxvii)$$

As before, we take only the first two terms and substitute into (*xxvii*), giving

$$(t_{\max})_D \approx \frac{-\left(\varepsilon+\frac{1}{2}\varepsilon^2\right)}{-\varepsilon\lambda_D} = \frac{1+\frac{1}{2}\varepsilon}{\lambda_D} \approx \frac{\sqrt{1+\varepsilon}}{\lambda_D} = \frac{1}{\sqrt{\lambda_D\lambda_P}} \quad (xxviii)$$

as expected. We can now proceed to compute the characteristic times for the combinations of decay constants we were given. For decay 1, λ_P = 1.08 yr^{-1} and λ_D = 1.0 yr^{-1}; the exact characteristic time is given by equation (*xvii*),

$$(t_{\max})_D = \frac{\ln(\lambda_P/\lambda_D)}{\lambda_P-\lambda_D} = \frac{\ln(1.08/1.0)}{1.08-1.0} = \boxed{0.962 \text{ yr}}$$

That is, the daughter nuclide will reach maximum activity within 0.96 years or so. Applying the same data to approximation (*xxiii*), we obtain

$$(t_{\max})_D \approx \frac{1}{\sqrt{\lambda_D\lambda_P}} = \frac{1}{\sqrt{1.0\times1.08}} = 0.962 \text{ yr}$$

Clearly, the approximate formula conveys the exact same characteristic time within three decimal figures.

Proceeding similarly with decay 2, we have

$$(t_{\max})_D = \frac{\ln(\lambda_P/\lambda_D)}{\lambda_P-\lambda_D} = \frac{\ln(3.1/3.5)}{3.1-3.5} = \boxed{0.303 \text{ yr}}$$

Applying the same data to approximation (*xxiii*), we obtain

$$(t_{\max})_D \approx \frac{1}{\sqrt{\lambda_D\lambda_P}} = \frac{1}{\sqrt{3.5\times3.1}} = 0.304 \text{ yr}$$

As in the previous situation, the approximate formula provides a good estimate of the characteristic time.

P.28 ➡ *Solution*

Part 1: General equations for activity of a parent and daughter radionuclide in a $P \to D \to G$ decay have been derived in the previous problem. Equation (*xii*) gives the activity of the parent nuclei,

$$A_P(t) = A_P(0)e^{-\lambda_P t}$$

Equation (*xiv*), in turn, yields the activity of the daughter nuclei,

$$A_D(t) = A_P(0)\frac{\lambda_D}{\lambda_D-\lambda_P}\left(e^{-\lambda_P t}-e^{-\lambda_D t}\right)$$

Before proceeding, we need decay constants λ_P and λ_D, namely

$$\lambda_P = \lambda_{\text{Mo-99}} = \frac{\ln 2}{t_{1/2}} = \frac{\ln 2}{66.0} = 0.0105 \text{ h}^{-1}$$

$$\lambda_D = \lambda_{\text{Tc-99m}} = \frac{\ln 2}{t_{1/2}} = \frac{\ln 2}{6.02} = 0.115 \text{ h}^{-1}$$

Further, note that the starting activity is $A_P(0)$ = 20 mCi. Substituting into the equations for $A_P(t)$ and $A_D(t)$, we obtain

$$A_P(t) = 20e^{-0.0105t}$$

$$A_D(t) = 20 \times \frac{0.115}{0.115 - 0.0105} \times \left(e^{-0.0105t} - e^{-0.115t}\right)$$

$$\therefore A_D(t) = 22\left(e^{-0.0105t} - e^{-0.115t}\right)$$

Part 2: An expression for the characteristic time has been derived in the previous problem; the equation to use is (*xvii*), namely

$$\left(t_{\max}\right)_D = \frac{\ln\left(\lambda_P/\lambda_D\right)}{\lambda_P - \lambda_D} = \frac{\ln\left(0.0105/0.115\right)}{0.0105 - 0.115} = \boxed{22.9\,\text{h}}$$

▶ The correct answer is **B**.

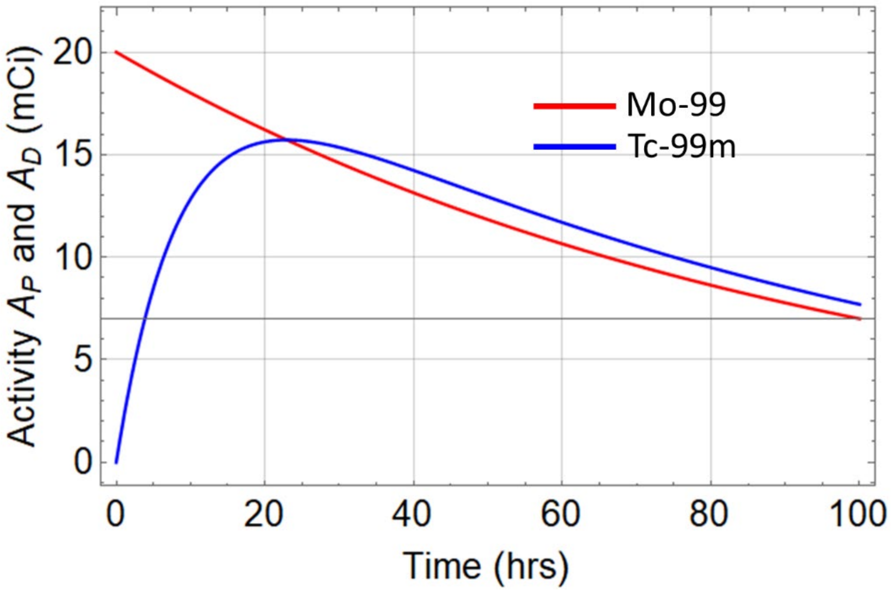
Part 3: To find the maximum activity of the Tc-99m radionuclide, simply substitute $(t_{max})_D$ obtained just now into $A_D(t)$,

$$A_D(t) = 22\left(e^{-0.0105t} - e^{-0.115t}\right)$$

$$\therefore A_D(22.9) = 22\left(e^{-0.0105\times 22.9} - e^{-0.115\times 22.9}\right) = \boxed{15.7\,\text{mCi}}$$

▶ The correct answer is **C**.

Part 4: Activities $A_P(t)$ and $A_D(t)$ are plotted below.



Some findings can be highlighted from the graph:
1. Parent activity $A_P(t)$ follows an exponential decay beginning at initial activity $A_P(0)$ = 20 mCi.
2. Since the half-life of Mo-99 is 66 h, we can tell that $A_p(66) = 0.5A_p(0)$ = 10 mCi, $A_p(132) = 0.25A_p(0)$ = 5 mCi, etc.
3. Daughter activity $A_D(t)$ reaches a maximum $A_{D,\max}$ = 15.7 mCi at t = 22.9 h and then begins to decrease.

P.29 ➡ *Solution*

Part 1: The equations that describe the decay of the parent and daughter nuclei are no different from the ones used in the Problems 27 and 28, namely

$$\frac{dN_P(t)}{dt} = -\lambda_P N_P(t)$$

and

$$\frac{dN_D(t)}{dt} = \lambda_P N_P(t) - \lambda_D N_D(t)$$

The solutions are also unchanged,

$$N_P(t) = N_P(0)e^{-\lambda_P t} \quad (i)$$

$$N_D(t) = N_P(0)\frac{\lambda_P}{\lambda_D - \lambda_P}\left(e^{-\lambda_P t} - e^{-\lambda_D t}\right) \quad (ii)$$

It remains to find the equation that provides the number of granddaughter nuclei, $N_G(t)$; to write a differential equation for $N_G(t)$, we observe that the rate of change of G is governed by the decay of daughters *D*, or, in mathematical terms,

$$\frac{dN_G(t)}{dt} = -\lambda_D N_D(t)$$

Inserting $N_D(t)$ into the equation above and manipulating, we get

$$\frac{dN_G(t)}{dt} = N_P(0)\frac{\lambda_D \lambda_P}{\lambda_D - \lambda_P}\left(e^{-\lambda_P t} - e^{-\lambda_D t}\right)$$

Integrating the relation above from zero to *t*, we get

$$N_G(t) = N_P(0)\frac{\lambda_D \lambda_P}{\lambda_D - \lambda_P}\left(-\frac{e^{-\lambda_P t}}{\lambda_P} + \frac{e^{-\lambda_D t}}{\lambda_D}\right) + C$$

where *C* is an integration constant. Applying the initial condition $N_G(t = 0) = 0$, we obtain

$$N_G(t=0) = N_P(0)\frac{\lambda_D \lambda_P}{\lambda_D - \lambda_P}\left(-\frac{1}{\lambda_D} + \frac{1}{\lambda_P}\right) + C = 0$$

$$\therefore N_G(t=0) = N_P(0)\frac{\lambda_P - \lambda_D}{\lambda_D - \lambda_P} + C = 0$$

$$\therefore C = N_P(0)$$

It follows that

$$N_G(t) = N_P(0)\left[1 - \frac{\lambda_D \lambda_P}{\lambda_D - \lambda_P}\left(\frac{e^{-\lambda_P t}}{\lambda_P} - \frac{e^{-\lambda_D t}}{\lambda_D}\right)\right]$$

$$\therefore N_G(t) = N_P(0)\left(1 - \frac{\lambda_D e^{-\lambda_P t}}{\lambda_D - \lambda_P} + \frac{\lambda_P e^{-\lambda_D t}}{\lambda_D - \lambda_P}\right) \quad (iii)$$

In summary, the number of parent nuclei $N_P(t)$ is given by equation (i), the number of daughter nuclei by (ii), and the number of granddaughter nuclei by (iii).

Part 2: Limit (I) can be obtained via the substitution rule,

$$\lim_{t\to 0} N_G(t) = N_P(0)\left(1 - \frac{\lambda_D e^{-\lambda_P \times 0}}{\lambda_D - \lambda_P} + \frac{\lambda_P e^{-\lambda_D \times 0}}{\lambda_D - \lambda_P}\right)$$

$$\therefore \lim_{t\to 0} N_G(t) = N_P(0)\underbrace{\left(1 - \frac{\lambda_D}{\lambda_D - \lambda_P} + \frac{\lambda_P}{\lambda_D - \lambda_P}\right)}_{=0}$$

$$\therefore \lim_{t\to 0} N_G(t) = 0$$

Limit (II) becomes quite straightforward to show if we note that, as $t \to \infty$, the terms highlighted in red tend to zero,

$$\lim_{t\to\infty} N_G(t) = N_P(0)\left(1 - \frac{\lambda_D e^{-\lambda_P t}}{\lambda_D - \lambda_P} + \frac{\lambda_P e^{-\lambda_D t}}{\lambda_D - \lambda_P}\right) = N_P(0)$$

The limits have been confirmed; limit (I) shows that there are no granddaughter radionuclides at the beginning of the experiment, while limit (II) shows that at very long times the number of granddaughter nuclei approaches the number of parent nuclei.

Part 3: Since the initial conditions for the radioactive sample stipulate that at $t = 0$ we are dealing with a pure parent radioactive source, i.e. such that $N_P(t = 0) = N_P(0)$, $N_D(0) = 0$, and $N_G(0) = 0$, we surmise that at any time $t > 0$ the sum of all nuclei $N_P(t) + N_D(t) + N_D(t)$ must amount to $N_P(0)$. Adding (*i*), (*ii*) and (*iii*) from Part 1, we find that

$$N_P(t) + N_D(t) + N_G(t) = N_P(0)e^{-\lambda_P t} + N_P(0)\frac{\lambda_P}{\lambda_D - \lambda_P}\left(e^{-\lambda_P t} - e^{-\lambda_D t}\right)$$

$$+N_P(0)\left(1 - \frac{\lambda_D e^{-\lambda_P t}}{\lambda_D - \lambda_P} + \frac{\lambda_P e^{-\lambda_D t}}{\lambda_D - \lambda_P}\right)$$

$$\therefore \Sigma N = N_P(0)\left[e^{-\lambda_P t} + \frac{\lambda_P e^{-\lambda_P t}}{\lambda_D - \lambda_P} - \frac{\lambda_P e^{-\lambda_D t}}{\lambda_D - \lambda_P} + 1 - \frac{\lambda_D e^{-\lambda_P t}}{\lambda_D - \lambda_P} + \frac{\lambda_P e^{-\lambda_D t}}{\lambda_D - \lambda_P}\right]$$

$$\therefore \Sigma N = N_P(0)$$

The sum ΣN of nuclides at any given instant t indeed equals $N_P(0)$.

Part 4: Curve 1 depicts the decay of parent radionuclide Mo-99. Curve 2 represents the growth and decay of the daughter radionuclide Tc-99m. Curve 3 illustrates the growth of the granddaughter nuclide Tc-99 under the assumption that, because of its very long half-life, Tc-99 is essentially stable at the time scale of the experiment.

Part 5: As mentioned in the solution to the previous part, curve 2 represents the growth and decay of the daughter radionuclide Tc-99m. We have seen in Problem 27 that the characteristic time at which the daughter radionuclide Tc-99m attains maximum activity is given by the general expression

$$(t_{\max})_D = \frac{\ln(\lambda_P/\lambda_D)}{\lambda_P - \lambda_D} = \frac{\ln(0.0105/0.115)}{0.0105 - 0.115} = \boxed{22.9\,\text{h}}$$

To find the normalized peak value, substitute $(t_{max})_D$ into equation (*xiv*) of Problem 27, giving

$$\frac{N_D(t)}{N_P(0)} = \frac{\lambda_P}{\lambda_D - \lambda_P}\left(e^{-\lambda_P (t_{\max})_D} - e^{-\lambda_D (t_{\max})_D}\right)$$

$$\therefore \frac{N_D(t)}{N_P(0)} = \frac{0.0105}{0.115 - 0.0105}\left(e^{-0.0105\times 22.9} - e^{-0.115\times 22.9}\right) = \boxed{0.0718}$$

The daughter nucleotide reaches a maximum normalized activity of 0.0718 at time $(t_{max})_D = 22.9$ h.

▶ The correct answer is **D**.

P.30 ➡ *Solution*

^{210}At decays to ^{210}Po by electron capture. ^{210}Po is also radioactive and transforms by α-decay to ^{206}Pb, which is stable. The activity of ^{210}Po is given by

$$A_{\text{Po}}(t) = \frac{A_{\text{At}}(0)\,\lambda_{\text{Po}}}{\lambda_{\text{Po}} - \lambda_{\text{At}}}\left(e^{-\lambda_{\text{At}} t} - e^{-\lambda_{\text{Po}} t}\right)$$

The number of ^{210}Po disintegrations from $t = 0$ to some time T is obtained by integration,

$$N=\int_0^T A_{\text{Po}}(t)\,dt=\frac{A_{\text{At}}(0)\lambda_{\text{Po}}}{\lambda_{\text{Po}}-\lambda_{\text{At}}}\left(\frac{e^{-\lambda_{\text{Po}}T}}{\lambda_{\text{Po}}}-\frac{e^{-\lambda_{\text{At}}T}}{\lambda_{\text{At}}}-\frac{1}{\lambda_{\text{Po}}}+\frac{1}{\lambda_{\text{At}}}\right)\ \text{(I)}$$

This is equal to the number of α particles, as there is one α particle per decay. Every α particle is capturing electrons and is transferred to a He nucleus. The relation between the number of He nuclei and mass is

$$N_{\text{He}}=\frac{m_{\text{He}}N_A}{M}$$

Here, N_{He} is the number of helium nuclei, m_{He} = 2.5×10^{-9} g is the mass of helium detected at the end of one month, M = 4 g/mol is the molar mass of helium, and N_A is Avogadro's number. Substituting,

$$N_{\text{He}}=\frac{\left(2.5\times10^{-9}\right)\times\left(6.02\times10^{23}\right)}{4}=3.76\times10^{14}$$

Further, λ_{At} = *ln* 2/$t_{1/2}$ = 0.693/8.10 = 0.856 h^{-1} = 2.05 day^{-1} and λ_{Po} = *ln* 2/138.4 = 0.00501 day^{-1}. Substituting all data into equation (I) and solving for $A_{Po}(t)$,

$$3.76\times10^{14}=\frac{A_{\text{At}}(0)\times0.00502\times30\times3600}{0.0502-2.05}\left(\frac{e^{-0.00502\times30}}{0.00502}-\frac{e^{-2.05\times30}}{2.05}-\frac{1}{0.00501}+\frac{1}{2.05}\right)$$

$$\therefore\ \boxed{A_{\text{At}}(0)=51.1\ \text{GBq}}$$

The initial activity of the ^{210}At source is close to 51 gigabecquerels.

▶ The correct answer is **C**.

P.31 ➡ ***Solution***

The activity of ^{210}Bi, $A_{Bi}(t)$, varies with time according to the relation

$$A_{\text{Bi}}(t)=A_{\text{Bi}}(0)e^{-\lambda_{Bi}t}$$

The activity of ^{210}Po, $A_{Po}(t)$, varies with time according to the relation

$$A_{\text{Po}}(t)=\frac{\lambda_{\text{Po}}A_{\text{Bi}}(0)}{\lambda_{\text{Po}}-\lambda_{\text{Bi}}}\left(e^{-\lambda_{Bi}t}-e^{-\lambda_{\text{Po}}t}\right)$$

The power from the radioactive nuclei equals the product of activity and energy released per decay. In the case at hand, we add the contributions of bismuth and polonium to obtain

$$P=A_{\text{Bi}}(0)\bar{E}_{\beta}e^{-\lambda_{Bi}t}+\frac{\lambda_{\text{Po}}A_{\text{Bi}}(0)Q}{\lambda_{\text{Po}}-\lambda_{\text{Bi}}}\left(e^{-\lambda_{\text{Bi}}t}-e^{-\lambda_{\text{Po}}t}\right)\ \text{(I)}$$

Here, Q is the total energy released in the α decay of polonium. We must account for both the kinetic energy from the α particle, E_α, and the daughter nucleus, E_D; that is,

$$Q=E_D+E_\alpha=\frac{E_\alpha m_\alpha}{m_D}+E_\alpha$$

Here, E_α= 5.2497 MeV, m_α = 4 is the atomic mass of an α particle, and m_D = 206 is the atomic mass of ^{206}Pb, giving

$$Q=\frac{5.2497\times4}{206}+5.2497=5.3516\ \text{MeV}$$

Further, we have decay constants λ_{Bi} = *ln* 2/5.01 = 0.138 d^{-1} and λ_{Po} = *ln* 2/138.4 = 0.00501 d^{-1}. Noting that 1 MeV ≈ 1.60×10^{-13} J, we substitute Q and all else into equation (I) to obtain

$$4.0\times10^{-3}=\left(105\times10^{9}\right)\times\left(0.344\times1.6\times10^{-13}\right)\times e^{-0.138\times t}$$

$$+\frac{0.00501\times\left(105\times10^{9}\right)\times\left(5.3516\times1.6\times10^{-13}\right)}{0.00501-0.138}\times\left(e^{-0.138t}-e^{-0.00501t}\right)$$

$$\therefore 4.0\times10^{-3}=0.00578e^{-0.138t}-0.00339\left(e^{-0.138t}-e^{-0.00501t}\right)$$

Notice that we have arrived at a transcendental equation. One way to solve it is to take all terms to one side and apply Mathematica's *FindRoot* command,

```
In[6]:= FindRoot[4.0 * 10^-3 - 0.00578 * Exp[-0.138 * t] + 0.00339 (Exp[-0.138 * t] - Exp[-0.00501 * t]),
         {t, 1}]
Out[6]= {t → 8.40106}
```

The required time is about 8.4 days.

▶ The correct answer is **C**.

P.32 ➡ *Solution*

Part 1: Initially, the number of ^{197}Au nuclei per unit area of foil is

$$N_1(0)=\frac{0.03\,\text{cm}\times19.3\,\text{g/cm}^3}{197\,\text{g/mol}}\times6.02\times10^{23}\,\frac{1}{\text{mol}}=1.77\times10^{21}\,\text{cm}^{-2}$$

Let the numbers of ^{197}Au and ^{198}Au nuclei at time *t* be N_1 and N_2, respectively; further, σ denotes the cross-section of the (n,γ) reaction, I is the flux of the incident neutron beam, and λ is the decay constant of ^{198}Au. We proceed to write the differential equations

$$\frac{dN_1}{dt}=-\sigma IN_1$$

$$\frac{dN_2}{dt}=\sigma IN_1-\lambda N_2$$

Integrating, we obtain

$$N_1(t)=N_1(0)e^{-\sigma It}$$

$$N_2(t)=\frac{\sigma I}{\lambda-\sigma I}N_1(0)\left(e^{-\sigma It}-e^{-\lambda t}\right)$$

Here,

$$\lambda=\frac{\ln 2}{2.7\times86,400}=2.97\times10^{-6}\,\text{s}^{-1}$$

$$\sigma I=\left(91.5\times10^{-24}\right)\times\left(2\times10^{12}\right)=1.83\times10^{-10}\,\text{s}^{-1}$$

Note that $\sigma I \ll \lambda$. At t = 6 min = 360 s, the activity of ^{198}Au is

$$A(t=360)=\lambda N_2(t=360)=\frac{\lambda\sigma IN_1(0)}{\lambda-\sigma I}\left(e^{-\sigma It}-e^{-\lambda t}\right)$$

$$\therefore A(t=360)\approx\sigma IN_1(0)\left(1-e^{-\lambda t}\right)$$

$$\therefore A(t=360)\approx\left(1.83\times10^{-10}\right)\times\left(1.77\times10^{21}\right)\times\left(1-e^{-\left(2.97\times10^{-6}\right)\times360}\right)=\boxed{3.46\times10^{8}\,\text{cm}^{-2}\text{s}^{-1}}$$

▶ The correct answer is **B**.

Part 2: After equilibrium is reached, the activity of a nuclide, and hence the number of nuclei, remain constant. This is the maximum amount of ^{198}Au that can be produced. As dN_2/dt = 0, we have

$$\lambda N_2=\sigma IN_1\approx\sigma IN_1(0)$$

Solving for N_2 brings to

$$\lambda N_2 = \sigma I N_1(0) \;\rightarrow\; N_2 = \frac{\sigma I N_1(0)}{\lambda}$$

$$\therefore N_2 = \frac{(1.83\times10^{-10})\times(1.77\times10^{21})}{2.97\times10^{-6}} = \boxed{1.09\times10^{17}\ \text{cm}^{-2}}$$

Part 3: Simply set $A = (2/3)A_{\text{max}}$ and solve for time,

$$A = \frac{2}{3}A_{\text{max}} \approx \sigma I N_1(0)\left(1-e^{-\lambda t}\right) \;\rightarrow\; t = -\frac{1}{\lambda}\ln\left(1-\frac{2A_{\text{max}}}{3\sigma I N_1(0)}\right)$$

$$\therefore t = -\frac{1}{2.97\times10^{-6}}\times\ln\left(1-\frac{2}{3}\right) = 370{,}000\,\text{s} = \boxed{4.28\ \text{days}}$$

P.33 ➡ *Solution*

Let the ^{7}Li and ^{8}Li populations be $N_1(t)$ and $N_2(t)$, respectively. Initially, the number of ^{7}Li nuclei is

$$N_1(0) = \frac{0.08}{7}\times6.02\times10^{23} = 6.88\times10^{21}$$

Further, $N_2(0) = 0$. During neutron irradiation, $N_1(t)$ changes according to

$$\frac{dN_1}{dt} = -\sigma\phi N_1$$

where σ is the neutron capture cross-section and ϕ is the neutron flux, or

$$N_1(t) = N_1(0)e^{-\sigma\phi t}$$

$N_2(t)$ changes according to

$$\frac{dN_2}{dt} = -\frac{dN_1}{dt} - \lambda N_2(t) = N_1(0)\sigma\phi e^{-\sigma\phi t} - \lambda N_2(t)$$

where λ is the β-decay constant of ^{8}Li. Integrating, we obtain

$$N_2(t) = \frac{\sigma\phi}{\lambda-\sigma\phi}\left(e^{-\sigma\phi t} - e^{-\lambda t}\right)N_1(0)$$

At equilibrium, $dN_2/dt = 0$, which gives the time t it takes to reach equilibrium,

$$t = \frac{\ln\left[\lambda/(\sigma\phi)\right]}{\lambda-\sigma\phi}$$

In the present case, $\lambda = \ln 2/0.85 = 0.815\ \text{s}^{-1}$ and $\sigma\phi = (3.7\times10^{-26})\times(4\times10^{12}) = 1.48\times10^{-13}\ \text{s}^{-1}$, giving

$$t_{\text{eq}} \approx \frac{\ln\left[\lambda/(\sigma\phi)\right]}{\lambda} = \frac{\ln\left[0.815/(1.48\times10^{-13})\right]}{0.815} = \boxed{36.0\ \text{s}}$$

It remains to determine the equilibrium activity A_{max},

$$A_{\text{eq}} = \lambda N_2(t) \approx \frac{\lambda\sigma\phi N_1(0)}{\lambda-\sigma\phi} \approx \sigma\phi N_1(0)$$

$$\therefore A_{\text{eq}} \approx (1.48\times10^{-13})\times(6.88\times10^{21}) = 1.02\times10^{9}\ \text{Bq} = \boxed{27.7\ \text{mCi}}$$

▶ The correct answer is **C**.

P.34 ➡ *Solution*

Let the number of target nuclei be $N(t)$, and that of the unstable nuclei resulting from neutron irradiation be $N_\beta(t)$. The thickness of the target can be considered thin so that

$$\frac{dN(t)}{dt} = -\sigma\phi N(t)$$

where ϕ is the neutron flux and σ is the total neutron capture cross-section of the target nuclei. Integration gives $N(t) = N(0)e^{-\sigma\phi t}$. As $\sigma\phi = 10^{-24} \times (2\times10^{8}) = 2\times10^{-16}\ \text{s}^{-1}$, the product $\sigma\phi t = 2\times10^{-14} \ll 1$ and we can take $N(t) \approx N(0)$, so that

$$\frac{dN}{dt} \approx -\sigma\phi N(0)$$

That is, the rate of production is approximately constant. Consider the unstable nuclide. In this case, we write

$$\frac{dN_\beta(t)}{dt} \approx \sigma\phi N(0) - \lambda N_\beta(t)$$

where λ is the β decay constant. Integrating,

$$N_\beta(t) = \frac{\sigma\phi N(0)}{\lambda}\left(1 - e^{-\lambda t}\right)$$

and hence

$$A = N_\beta(t)\lambda = \sigma\phi N(0)\left(1 - e^{-\lambda t}\right)$$

Noting that β decay constant $\lambda = 1/10^4 = 10^{-4}\ \text{s}^{-1}$, the activity at the end of 120 sec of neutron irradiation is calculated to be

$$A = \sigma\phi N(0)\left(1 - e^{-\lambda t}\right)$$

$$\therefore A = \left(2\times10^{-16}\right)\times\left(10^{22}\times2\times0.018\right)\times\left(1 - e^{-10^{-4}\times120}\right) = \boxed{859\ \text{Bq}}$$

▶ The correct answer is **A**.

ANSWER SUMMARY

Problem 1		**T/F**
Problem 2	**2.1**	Open-ended pb.
	2.2	Open-ended pb.
Problem 3		**C**
Problem 4	**4.1**	**B**
	4.2	Open-ended pb.
Problem 5		**A**
Problem 6		**T/F**
Problem 7		**T/F**
Problem 8		Open-ended pb.
Problem 9		**C**
Problem 10	**10.1**	Open-ended pbs.
	10.2	
	10.3	
Problem 11		Open-ended pb.
Problem 12		**C**
Problem 13		**A**
Problem 14	**14.1**	**D**
	14.2	**D**
	14.3	**B**
	14.4	**C**
	14.5	**B**
Problem 15	**15.1**	**A**
	15.2	**B**
	15.3	Open-ended pb.
Problem 16	**16.1**	**B**
	16.2	**B**
	16.3	**A**
	16.4	**B**
	16.5	**C**
Problem 17	**17.1**	**C**
	17.2	**C**
Problem 18		**B**

Problem 19	**19.1**	Open-ended pb.
	19.2	Open-ended pb.
Problem 20	**20.1**	Open-ended pb.
	20.2	**D**
Problem 21	**21.1**	**A**
	21.2	**B**
	21.3	**A**
Problem 22	**22.1**	**C**
	22.2	Open-ended pb.
	22.3	Open-ended pb.
Problem 23	**23.1**	**B**
	23.2	**D**
	23.3	**A**
Problem 24		**D**
Problem 25		**D**
Problem 26	**26.1**	Open-ended pb.
	26.2	**B**
	26.3	**B**
Problem 27	**27.1**	Open-ended pbs.
	27.2	
	27.3	
	27.4	
	27.5	
Problem 28	**28.1**	Open-ended pb.
	28.2	**B**
	28.3	**C**
	28.4	Open-ended pb.
Problem 29	**29.1**	Open-ended pbs.
	29.2	
	29.3	
	29.4	
	29.5	**D**
Problem 30		**C**
Problem 31		**C**
Problem 32	**32.1**	**B**
	32.2	Open-ended pb.
	32.3	Open-ended pb.
Problem 33		**C**
Problem 34		**A**

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