Quiz HT101

One-Dimensional Conduction and Thermal Resistance

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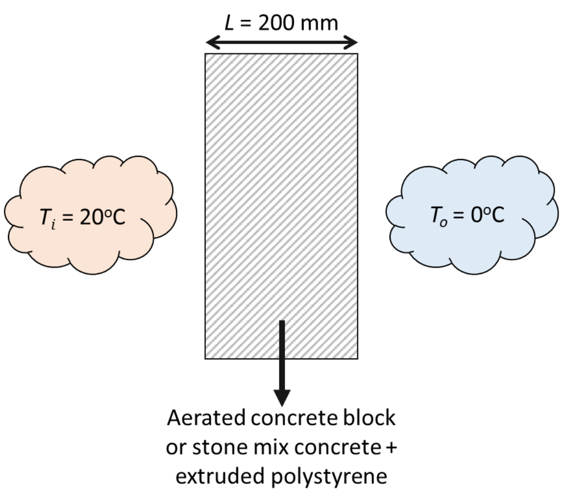
Problems

Problem 1 (Bergman et al., 2011, w/ permission)

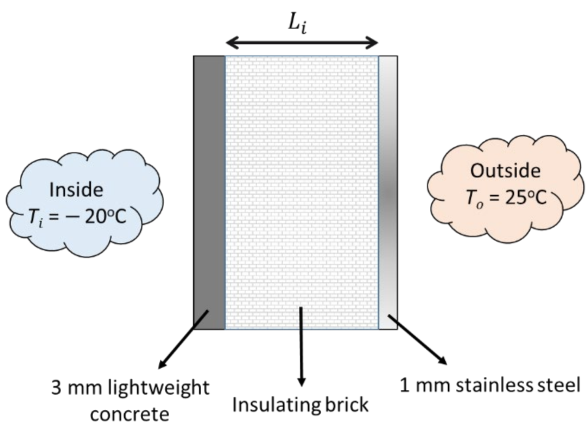
A new building to be located in a cold climate is being designed with a basement that has a L = 200-mm thick wall. The inner and outer basement temperatures are T_i = 20°C and T_o = 0°C, respectively. The architect can specify the wall material to be either aerated concrete block with k_{ac} = 0.15 W/mK or stone mix concrete. To reduce the conduction heat flux through the stone mix wall to a level equivalent to that of the aerated concrete wall, what thickness of extruded polystyrene sheet must be applied onto the inner surface of the stone mix concrete wall?



A) t = 23 mm
B) t = 32 mm
C) t = 43 mm
D) t = 52 mm

Problem 2

An industrial freezer is designed to operate with an external air temperature of −20°C when the external air temperature is 25°C and the internal and external heat transfer coefficients are 12 W/m^{2}K and 8 W/m^{2}K, respectively. The walls of the freezer are of composite construction, consisting of an inner layer of lightweight concrete with 3 mm thickness and an outer layer of stainless steel. Sandwiched between these two layers is a layer of insulation bricks. Determine the width of the brick layer required to reduce the convective heat loss to 15 W/m^2.



A) L_i = 26.7 cm
B) L_i = 35.5 cm
C) L_i = 41.8 cm
D) L_i = 51.0 cm

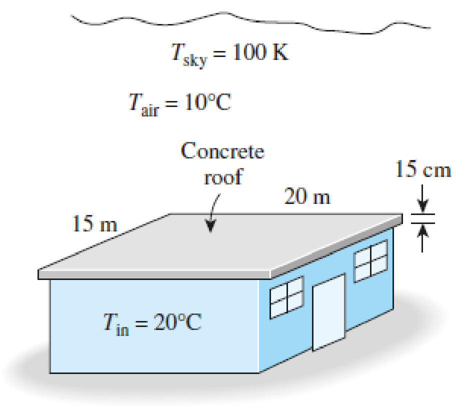
Problem 3 (Welty et al., 2008, w/ permission)

A composite wall is to be constructed of 1/4 in. of stainless steel with k = 10 Btu/h·ft·°F, 3 in. of corkboard with k = 0.025 Btu/h·ft·°F and 1/2 in. of plastic with k = 1.5 Btu/h·ft·°F. Determine the thermal resistance of this wall if it is bolted together by 1/2-in.-diameter bolts on 6-in. centers made of stainless steel.

A) R_{total} = 1.46 Btu/h·°F
B) R_{total} = 2.51 Btu/h·°F
C) R_{total} = 3.65 Btu/h·°F
D) R_{total} = 4.70 Btu/h·°F

Problem 4A (Çengel & Ghajar, 2015, w/ permission)

The roof of a house consists of a 15-cm-thick dense concrete slab that is 15 m wide and 20 m long. The convection heat transfer coefficients on the inner and outer surfaces of the roof are 5 and 12 W/m^{2}K, respectively. On a clear winter night, the ambient air is reported to be at 10°C, while the night sky temperature is 100 K. The house and interior surfaces of the wall are maintained at a constant temperature of 20°C. The emissivity of both surfaces of the concrete roof is 0.9. Considering both radiation and convection heat transfers, determine the heat transfer through the roof.



A) $\dot{q}$ = 35,700 W
B) $\dot{q}$ = 43,900 W
C) $\dot{q}$ = 51,500 W
D) $\dot{q}$ = 60,100 W

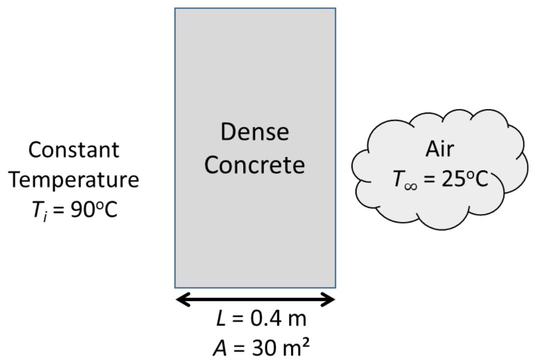
Problem 4B

If the house is heated by a furnace burning natural gas with an efficiency of 80 percent, and the price of natural gas is $1.20/therm (1 therm = 105,500 kJ of energy content), determine the money lost through the roof that night during a 14-h period.

A) Money lost = $19,35
B) Money lost = $25,20
C) Money lost = $31,40
D) Money lost = $37,80

Problem 5A

Consider a large plane wall of dense concrete with thickness L = 0.4 m and surface area A = 30 m^2. The left side of the wall is maintained at a constant temperature T_i = 90°C while the right side loses heat by convection to the surrounding air at T_∞ = 25°C with a heat transfer coefficient of h = 24 W/m^{2}K. Assuming constant thermal conductivity and no heat generation in the wall, obtain an expression for the variation of temperature of the wall with the horizontal coordinate x (where x is measured from the left wall and given in meters).



A) $T(x)$ = 90 – 73.5x °C
B) $T(x)$ = 90 – 101.2x °C
C) $T(x)$ = 90 – 138.1x °C
D) $T(x)$ = 90 – 156.4x °C

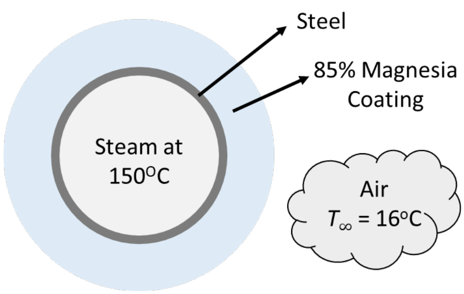
Problem 5B

Evaluate the rate of heat transfer through the wall considered in the previous problem.

A) $\dot{q}$ = 1527.8 W
B) $\dot{q}$ = 3622.5 W
C) $\dot{q}$ = 5461.1 W
D) $\dot{q}$ = 7040.7 W

Problem 6

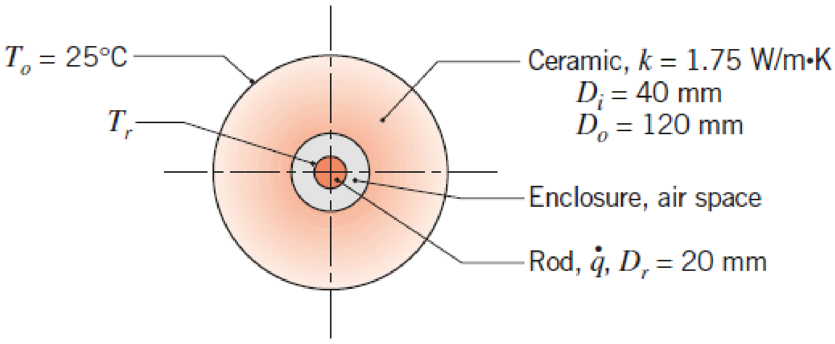
Calculate the rate of heat loss per foot for a 15 cm schedule 40 steel pipe covered with an 8-cm thick layer of 85% magnesia. Superheated steam at 150°C flows inside the pipe (h_c = 170 W/m^{2}K) and still air at 16°C is outside (h_c = 30 W/m^{2}K). The inside diameter of the pipe is D_i = 15.16 cm and the outside diameter is D_o = 16.56 cm.



A) $\dot{q}'$ = 114.9 W/m
B) $\dot{q}'$ = 234.4 W/m
C) $\dot{q}'$ = 325.5 W/m
D) $\dot{q}'$ = 441.6 W/m

■ Problem 7A (Bergman et al., 2011, w/ permission)

Electric current flows through a long rod generating thermal energy at a uniform volumetric rate of $\dot{q}$ = 2 × 10^6 W/m^3. The rod is concentric with a hollow ceramic cylinder, creating an enclosure that is filled with air. The thermal resistance per unit length due to radiation between the enclosure surfaces is R_{rod} = 0.30 m·K/W, and the coefficient associated with free convection in the enclosure is h = 20 W/m^{2}K. Compute the total thermal resistance of this system.



A) R_{tot} = 0.24 m·K/W
B) R_{tot} = 0.34 m·K/W
C) R_{tot} = 0.43 m·K/W
D) R_{tot} = 0.52 m·K/W

■ Problem 7B

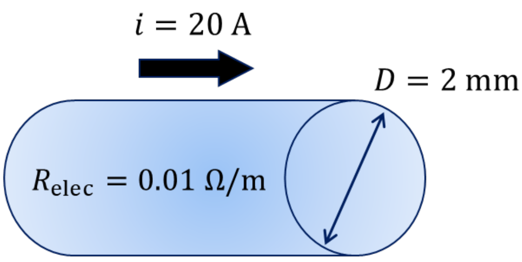
Calculate the surface temperature of the rod considered in the previous problem.
A) T_s = 239°C
B) T_s = 263°C
C) T_s = 297°C
D) T_s = 322°C

Problem 8 (Welty et al., 2008, w/ permission)

A 2 in. schedule 40 steel pipe carries saturated steam at 292°F through a laboratory that is 60 ft long. The pipe is insulated with 1.5 in. of 85% magnesia that costs $0.75 per foot. How long must the steam line be in service to justify the insulation cost if the heating cost is $0.68 per 10^5 Btu? The outside surface convective heat transfer coefficient may be taken as 5 Btu/h·ft^2·°F. The conductivity of steel is k_s = 24.8 Btu/h·ft·°F, whereas that of 85% magnesia is k_m = 0.041 Btu/h·ft·°F. The room temperature for the lab is 70°F. For the schedule 40 pipe in question, consider outside diameter = 2.375 in., thickness = 0.154 in., and inside diameter = 2.067 in.
A) Time = 393 h
B) Time = 456 h
C) Time = 569 h
D) Time = 678 h

Problem 9A (Bergman et al., 2011, w/ permission)

A wire of diameter D = 2 mm and uniform temperature T has an electrical resistance of 0.01 Ω//m and a current flow of 2 A. What is the heat dissipation per unit volume within the wire?



A) $q = 1.27 \times 10^6$ W/m^3
B) $q = 1.76 \times 10^6$ W/m^3
C) $q = 2.25 \times 10^6$ W/m^3
D) $q = 2.78 \times 10^6$ W/m^3

Problem 9B

If the wire is not insulated and is in ambient air and large surroundings for which $T_\infty = T_{sur}$ = 20°C, what is the temperature T of the wire? The wire has an emissivity of 0.3, and the coefficient associated with heat transfer by natural convection may be approximated by an expression of the form $h = C[(T - T_\infty)/D]^{(1/4)}$, where C = 1.25 W/m$^{7/4}K^{5/4}$.

A) T_w = 45°C
B) T_w = 58°C
C) T_w = 67°C
D) T_w = 76°C

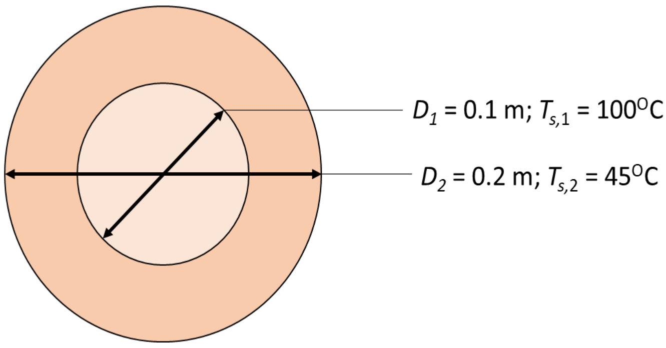
Problem 9C

If the wire is coated with plastic insulation of 2 mm thickness and a thermal conductivity of 0.25 W/mK, what is the *inner* temperature of the insulation? The insulation has an emissivity of 0.9, and the convection coefficient is given by the expression proposed in Part B.

A) $T_{s,i}$ = 31.1°C
B) $T_{s,i}$ = 34.8°C
C) $T_{s,i}$ = 37.6°C
D) $T_{s,i}$ = 40.5°C

Problem 10A (Bergman et al., 2011, w/ permission)

A spherical Pyrex glass shell has inside and outside diameters of D_1 = 0.1 m and D_2 = 0.2 m, respectively. The inner surface is at $T_{s,1}$ = 100°C while the outer surface is at $T_{s,2}$ = 45°C. Determine the temperature at the midpoint of the shell thickness.



A) $T(r_m)$ = 52.5°C
B) $T(r_m)$ = 63.3°C
C) $T(r_m)$ = 74.1°C
D) $T(r_m)$ = 82.4°C

Problem 10B

For the same surface temperatures and dimensions as in the previous problem, what would be the midpoint temperature if the shell material were aluminum?

A) $T(r_m)$ = 52.5°C

B) $T(r_m)$ = 63.3°C

C) $T(r_m)$ = 74.1°C

D) $T(r_m)$ = 82.4°C

Additional Information

Table 1 Thermal conductivity for selected materials at room temperature

Material	k (W/m·K)
1% carbon steel	43
85% magnesia	0.06
Aluminum	205
Dense concrete	1.7
General ceramic	1.75
Insulation brick	0.15
Lightweight concrete	0.3
Pyrex glass	1.0
Rigid extruded polystyrene sheet	0.027
Stainless steel	16
Stone mix concrete	1.4

Solutions

P.1 Solution

The heat flux through the aerated concrete can be easily obtained from Fourier's law,

$$\dot{q}''_{\text{ac}} = \frac{k_{\text{ac}}\left(T_i - T_o\right)}{L} = \frac{0.15\times\left(20-0\right)}{0.2} = 15\text{ W/m}^2$$

The heat flux across the stone mix concrete and polystyrene composite wall, in turn, can be obtained from the equation

$$\dot{q}''_{\text{comp}} = \frac{\left(T_i - T_o\right)}{\frac{t}{k_{ps}} + \frac{L}{k_{sm}}} = \frac{(20-0)}{\frac{t}{0.027} + \frac{0.2}{1.4}}$$

If the composite wall is to produce the same benefit as the aerated concrete wall, their heat fluxes must be the same. Hence, $q''_{\text{ac}} = q''_{\text{comp}}$ = 15 W/m². Then, the equation above can be solved for the thickness t, giving

$$\dot{q}''_{\text{comp}} = 15 = \frac{20}{\frac{t}{0.027} + \frac{0.2}{1.4}} \rightarrow t = 0.032 = \boxed{32\text{ mm}}$$

♦ The correct answer is **B**.

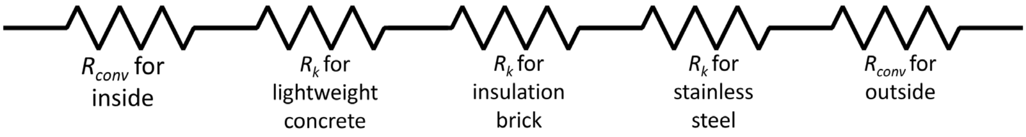
P.2 Solution

The overall heat transfer coefficient, U, is given by

$$q = U\Delta T \rightarrow U = \frac{q}{\Delta T}$$

$$\therefore U = \frac{15}{25-(-20)} = 0.333\text{ W/m}^2\text{K}$$

The thermal circuit for this problem is shown below. We have five thermal resistances, three of which are conduction resistances, two of which are convection resistances.



Coefficient U is given by

$$U=\left(\frac{1}{h_i}+\frac{L_p}{k_p}+\frac{L_i}{k_i}+\frac{L_s}{k_s}+\frac{1}{h_o}\right)^{-1}=0.333\rightarrow\left(\frac{1}{h_i}+\frac{L_p}{k_p}+\frac{L_i}{k_i}+\frac{L_s}{k_s}+\frac{1}{h_o}\right)=\frac{1}{0.333}$$

The latter equation can be easily solved for L_i, i.e., the thickness of the insulation brick layer. Accordingly,

$$L_i=k_i\left[\frac{1}{0.333}-\left(\frac{1}{h_i}+\frac{L_p}{k_p}+\frac{L_s}{k_s}+\frac{1}{h_o}\right)\right]=0.15\times\left[\frac{1}{0.333}-\left(\frac{1}{12}+\frac{0.003}{0.3}+\frac{0.001}{16}+\frac{1}{8}\right)\right]$$

$$\therefore L_i=0.418\text{ m}=\boxed{41.8\text{ cm}}$$

♦ The correct answer is **C**.

P.3 Solution

We begin by computing the resistance R_s offered by the stainless steel, $R_s = L_s/k_sA$, assuming a unit cross-sectional area,

$$R_s=\frac{L_s}{k_sA}=\frac{\left(\frac{0.25}{12}\right)}{10\times 1}=0.00208\text{ h°F/Btu}$$

Next, we compute the resistance offered by the corkboard, noting that A = 1 ft² as before,

$$R_c=\frac{L_c}{k_cA}=\frac{\left(\frac{3}{12}\right)}{0.025\times 1}=10\text{ h°F/Btu}$$

Next, we compute the resistance offered by the plastic,

$$R_p=\frac{L_p}{k_pA}=\frac{\left(\frac{0.5}{12}\right)}{1.5\times 1}=0.0278\text{ h°F/Btu}$$

Because these layers of material are superposed one over the other, we have a series thermal circuit pattern and the total resistance of the system is

$$R_t=R_s+R_c+R_p=0.00208+10+0.0278=10.03\text{ h°F/Btu}$$

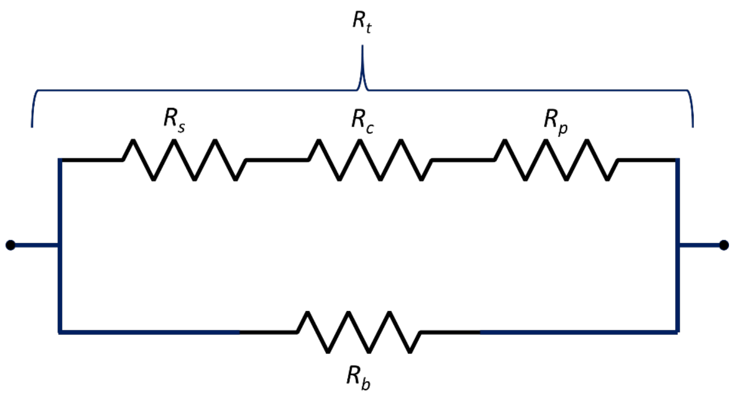
Now, let us account for the presence of the stainless steel bolts. The area of each bolt is $A_b = \pi \times (0.5/12)^2/4 = 1.36 \times 10^{-3}$ ft^2. The resistance R_b provided by the bolts is

$$R_b=\frac{L_b}{k_{\text{st}}A_t}$$

where $L_b = L_s + L_c + L_p = (0.25+3+0.5)/12 = 0.313$ ft, $k_{st} = 10$ Btu/h ft°F, and $A_t = 4A_b$. Substituting these data, we obtain

$$R_b=\frac{L_b}{k_sA_t}=\frac{L_b}{k_s\times 4A_b}=\frac{0.313}{10\times 4\left(1.36\times 10^{-3}\right)}=5.75\text{ h°F/Btu}$$

The bolts could be interpreted as being placed connected in parallel to the series circuit we have proposed. This situation is illustrated below.



The overall thermal resistance R_{total} is then

$$R_{\text{total}} = \frac{1}{\left(\frac{1}{R_t}+\frac{1}{R_b}\right)} = \frac{1}{\left(\frac{1}{10.03}+\frac{1}{5.75}\right)} = \boxed{3.65\text{ Btu/h°F}}$$

♦ The correct answer is **C**.

P.4 Solution

Part A: We assume that steady operating conditions exist, emissivity and thermal conductivity are constant, and that heat transfer occurs in steady fashion. Accordingly, the heat transfer from a room to the roof (by convection and radiation) must be equal to the heat transfer from the roof to the surroundings (by convection and radiation), which must in turn be equal to the heat transfer through the roof by conduction. In mathematical terms,

$$\dot{q}_{\text{roof,cond}} = \dot{q}_{\text{room to roof, conv}} + \dot{q}_{\text{rad,1}} = \dot{q}_{\text{roof to surroundings, conv}} + \dot{q}_{\text{rad,2}}$$

The inner and outer surface temperatures of the roof are $T_{s,\text{in}}$ and $T_{s,\text{out}}$, respectively. We can write

$$\dot{q}_{\text{room to roof, conv}} + \dot{q}_{\text{rad,1}} = h_i A\left(T_{\text{room}} - T_{s,\text{in}}\right) + \varepsilon A\sigma\left(T_{\text{room}}^4 - T_{s,\text{in}}^4\right)$$

$$= 5\times(15\times 20)\times\left(293 - T_{s,\text{in}}\right) + 0.9\times(15\times 20)\times\left(5.67\times 10^{-8}\right)\left(293^4 - T_{s,\text{in}}^4\right)$$

$$= 1500\left(293 - T_{s,\text{in}}\right) + 1.531\times 10^{-5}\left(293^4 - T_{s,\text{in}}^4\right)\ \text{(I)}$$

The heat transfer by conduction in the roof is

$$\dot{q}_{\text{roof, cond}} = kA\frac{T_{s,\text{in}} - T_{s,\text{out}}}{L} = 1.7\times 300\times\frac{\left(T_{s,\text{in}} - T_{s,\text{out}}\right)}{0.15} = 3400\left(T_{s,\text{in}} - T_{s,\text{out}}\right)\ \text{(II)}$$

We then consider the rate of heat transfer from the roof to the surroundings,

$$\dot{q}_{\text{roof to surroundings, conv}} + \dot{q}_{\text{rad,2}} = h_o A\left(T_{s,\text{out}} - T_{\text{air}}\right) + \varepsilon A\sigma\left(T_{s,\text{out}}^4 - T_{\text{surr}}^4\right)$$

$$= 12\times 300\left(T_{s,\text{out}} - 283\right) + 0.9\times 300\times\left(5.67\times 10^{-8}\right)\left(T_{s,\text{out}}^4 - 100^4\right)$$

$$= 3600\left(T_{s,\text{out}} - 283\right) + 1.531\times 10^{-5}\left(T_{s,\text{out}}^4 - 100^4\right)\ \text{(III)}$$

In accordance with our initial energy balance, equation (I) must be equal to equation (II); that is,

$$1500\left(293 - T_{s,\text{in}}\right) + 1.531\times 10^{-5}\left(293^4 - T_{s,\text{in}}^4\right) = 3400\left(T_{s,\text{in}} - T_{s,\text{out}}\right)$$

and equation (II) must be equal to equation (III), i.e.,

$$3400\left(T_{s,\text{in}}-T_{s,\text{out}}\right)=3600\left(T_{s,\text{out}}-283\right)+1.531\times10^{-5}\left(T_{s,\text{out}}^{4}-100^{4}\right)$$

Observe that we have obtained a system of two nonlinear equations with two unknowns. The system can be solved with a CAS such as Mathematica, in which case we could apply the command *NSolve* with the *Reals* option activated,

$$\begin{aligned}&\text{NSolve}[\{1500(293-T_{\text{in}})+1.531*10^{-5}(293^4-T_{\text{in}}{}^4)=\\&=3400(T_{\text{in}}-T_{\text{out}}),3400(T_{\text{in}}-T_{\text{out}})=\\&=3600(T_{\text{out}}-283)+1.531*10^{-5}(T_{\text{out}}{}^4-100^4)\},\{T_{\text{in}},T_{\text{out}}\},\text{Reals}]\end{aligned}$$

This returns the solution pair ($T_{s,\text{in}}=-672.059,\ T_{s,\text{out}}=-212.407$), which is preposterous, and ($T_{s,\text{in}}=280.921, T_{s,\text{out}}=270.449$), which is feasible. Hence, $T_{s,\text{in}}=7.9^{\circ}\text{C}$ and $T_{s,\text{out}}=-2.6^{\circ}\text{C}$. We can now insert these quantities into equation (II) (Fourier's law) to compute the heat transfer rate,

$$\dot{q}=kA\frac{\left(T_{s,\text{in}}-T_{s,\text{out}}\right)}{L}=1.7\times\left(15\times20\right)\times\frac{\left[7.9-\left(-2.6\right)\right]}{0.15}=\boxed{35,700\text{ W}}$$

♦ The correct answer is **A**.

Part B: We first calculate the total amount of energy consumed during the period,

$$Q_{\text{gas}}=\frac{Q_{\text{total}}}{0.80}\rightarrow Q_{\text{gas}}=\frac{\dot{Q}\Delta t}{0.80}$$

$$\therefore Q_{\text{gas}}=\frac{35.2\ \frac{\text{kJ}}{\cancel{s}}\times14\ \cancel{h}\times3600\ \frac{\cancel{s}}{\cancel{h}}}{0.8}=2.218\times10^{6}\text{ kJ}$$

The number of therms is

$$n=2.218\times10^{6}\ \cancel{\text{kJ}}\times\frac{1\text{ therm}}{105,500\ \cancel{\text{kJ}}}=21.02\text{ therms}$$

Lastly, the money lost due to energy dissipation in the roof is

$$\text{Money lost}=21.02\text{ therms}\times\frac{\$1.20}{\text{therm}}=\boxed{\$25,20}$$

♦ The correct answer is **B**.

P.5 Solution

Part A: The equation for one-dimensional steady state conduction with no internal heat generation is simply

$$\frac{d^2T}{dx^2}=0$$

At the left side of the wall, the temperature is maintained constant, which is to say that at x = 0 the temperature of the surface is T_1 = 90°C. At the right end of the wall, the amount of heat conduction is equal to the amount of heat convection, q_{cond} = q_{conv}. Mathematically,

$$-k\left(\frac{dT}{dx}\right)_{x=L}=h\left(T_{x=L}-T_{\infty}\right)$$

Summarizing, the heat conduction equation in this case is $d^2T/dx^2=0$ and the two boundary conditions are

$$T\left(0\right)=T_i=90^{\circ}\text{C}\ ;\ -k\left(\frac{dT}{dx}\right)_{x=L}=h\left[T\left(L\right)-T_{\infty}\right]$$

Integrating the heat equation twice, we obtain

$$T(x) = C_1 x + C_2$$

where C_1 and C_2 are constants. Let us apply the first boundary condition by substituting $x = 0$ and $T(0) = T_i$,

$$T(0) = T_i \;\rightarrow\; T_i = C_1 \times 0 + C_2$$
$$\therefore C_2 = T_i$$

Then, we apply the second boundary condition, obtaining

$$-k\left(\frac{dT}{dx}\right)_{x=L} = h\left(T(L) - T_\infty\right)$$
$$\therefore -kC_1 = h\left(C_1 L + C_2 - T_\infty\right)$$
$$\therefore -kC_1 = h\left(C_1 L + T_i - T_\infty\right)$$
$$\therefore -kC_1 = hC_1 L + hT_i - hT_\infty$$
$$\therefore hC_1 L + kC_1 = hT_\infty - hT_i$$
$$\therefore C_1\left(hL + k\right) = hT_\infty - hT_i = h\left(T_\infty - T_i\right)$$
$$\therefore C_1 = \frac{h\left(T_\infty - T_i\right)}{hL + k}$$

Using the values of C_1 and C_2 we have obtained, *T(x)* can be written as

$$T(x) = C_1 x + C_2 = \frac{h\left(T_\infty - T_i\right)}{hL + k} x + T_i$$

Substituting the pertaining variables, it follows that

$$T(x) = \frac{h\left(T_\infty - T_i\right)}{hL + k} x + T_i = \frac{24(25 - 90)}{24 \times 0.4 + 1.7} x + 90 = \boxed{90 - 138.1x\ ^\circ\text{C}}$$

♦ The correct answer is **C**.

Part B: The rate of heat conduction through the wall can be obtained from the following form of Fourier's law,

$$\dot{q} = -kA\frac{dT}{dx}$$

The derivative of *T* with respect to *x* can be obtained from the expression for the temperature field *T(x)*, which, in the current case, is

$$T(x) = C_1 x + C_2 \rightarrow \frac{dT}{dx} = C_1$$

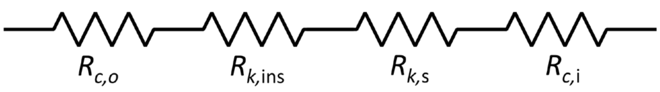
Thus, replacing the temperature gradient in the equation for $\dot{q}$, we have

$$\dot{q} = -kA\frac{dT}{dx} = -kAC_1 = -kAh\frac{\left(T_\infty - T_i\right)}{hL + k}$$
$$\therefore \dot{q} = -1.7 \times 30 \times 24 \frac{(25 - 90)}{24 \times 0.4 + 1.7} = \boxed{7040.7\ \text{W}}$$

♦ The correct answer is **D**.

P.6 Solution

The thermal circuit for the insulated pipe is shown below.



To begin, R_{co} is the resistance for convection outside of the pipe,

$$R_{c,o}=\frac{1}{h_{co}A_o}=\frac{1}{h_{co}\times(\pi D_o L)}=\frac{1}{L}\times\frac{1}{30\times\pi\times0.1656}=\frac{0.0641}{L}$$

Next, $R_{k,ins}$ is the thermal resistance due to the insulation layer,

$$R_{k,\text{ins}}=\frac{\ln\left(\frac{r_l}{r_o}\right)}{2\pi Lk_{\text{ins}}}=\frac{\ln\left(\frac{0.1656+0.08}{0.1656}\right)}{2\pi L\times0.06}=\frac{1.045}{L}$$

Next, $R_{k,s}$ is the thermal resistance of the steel,

$$R_{k,s}=\frac{\ln\left(\frac{r_o}{r_i}\right)}{2\pi Lk_s}=\frac{\ln\left(\frac{0.1656}{0.1516}\right)}{2\pi L\times43}=\frac{3.27\times10^{-4}}{L}$$

Lastly, R_{ci} is the resistance for convection inside of the pipe,

$$R_{c,i}=\frac{1}{h_{c,i}A_i}=\frac{1}{h_{c,i}\times(\pi D_i L)}=\frac{1}{L}\times\frac{1}{170\times\pi\times0.1516}=\frac{0.0124}{L}$$

The total resistance R_{total} is then

$$R_{\text{total}}=R_{c,o}+R_{k,\text{ins}}+R_{k,s}+R_{c,i}=\frac{0.0641}{L}+\frac{1.045}{L}+\frac{3.27\times10^{-4}}{L}+\frac{0.0124}{L}=\frac{1.122}{L}$$

Finally, the rate of heat transfer is such that

$$\dot{q}=\frac{\Delta T}{R_{\text{total}}}\rightarrow\dot{q}=\frac{(150-16)}{\frac{1.122}{L}}$$

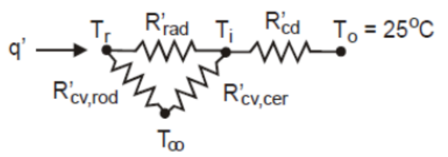
$$\therefore\dot{q}'=\frac{\dot{q}}{L}=\boxed{119.4\text{ W/m}}$$

Observe that almost all of the thermal resistance is due to the insulation and that the thermal resistance of the steel pipe is essentially negligible.

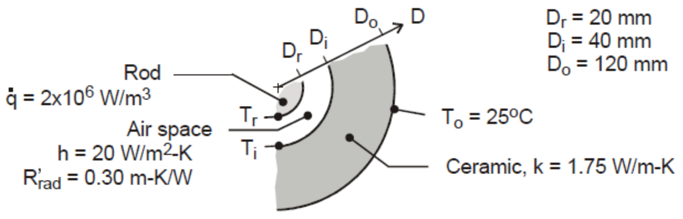
♦ The correct answer is **A**.

P.7 Solution

Part A: The thermal circuit for this situation is illustrated below.



A schematic of the problem is provided in continuation.



The resistance due to radiation exchange for the enclosure is R_{rad} = 0.30 m·K/W. The resistance for free convection in the rod is given by

$$R_{\text{cv,rod}}=\frac{1}{h\pi D_r}=\frac{1}{20\times\pi\times0.02}=0.8\text{ m}\cdot\text{K/W}$$

Next, the resistance for free convection in the ceramic is obtained with the equation

$$R_{\text{cv,cer}} = \frac{1}{h\pi D_i} = \frac{1}{20\times\pi\times 0.04} = 0.4\ \text{m}\cdot\text{K/W}$$

Next, the resistance for conduction in the ceramic cylinder is

$$R_{\text{cyl}} = \frac{\ln(D_o/D_i)}{2\pi k} = \frac{\ln(0.12/0.04)}{2\pi\times 1.75} = 0.1\ \text{m}\cdot\text{K/W}$$

The thermal resistance between the enclosure surfaces due to convection and radiation exchange is

$$\frac{1}{R_{\text{enc}}} = \frac{1}{R_{\text{rad}}} + \frac{1}{R_{\text{cv,rod}} + R_{\text{cv,cer}}}$$

$$\therefore R_{\text{enc}} = \left(\frac{1}{0.30} + \frac{1}{0.80+0.40}\right)^{-1} = 0.24\ \text{m}\cdot\text{K/W}$$

Ultimately, the total resistance between the rod surface and the outer surface of the cylinder is

$$R_{\text{tot}} = R_{\text{enc}} + R_{\text{cyl}} = 0.24 + 0.10 = \boxed{0.34\ \text{m}\cdot\text{K/W}}$$

♦ The correct answer is **B**.

Part B: The surface temperature T_r can be determined from an energy balance on the rod. Accordingly,

$$E_{\text{in}} - E_{\text{out}} + E_{\text{gen}} = 0$$
$$\therefore -q + \dot{q}A = 0$$

Noting that $q = -(T_r - T_i)/R_{\text{tot}}$ and $\dot{q}A = \dot{q}(\pi D_r^2/4)$ with $\dot{q} = 2\times 10^6\ \text{W/m}^3$, it follows that

$$-\frac{(T_r - T_i)}{R_{\text{tot}}} + \dot{q}\left(\pi D_r^2/4\right) = 0$$

$$\therefore -\frac{(T_r - 25)}{0.34} + 2\times 10^6\left(\pi\times 0.02^2/4\right) = 0$$

$$\therefore \boxed{T_r = 239^\circ\text{C}}$$

♦ The correct answer is **A**.

P.8 Solution

As given in the text, the geometric properties of the pipe are D_o = 2.375 in., t = 0.154 in., and D_i = 2.067 in. The inner radius of the pipe is $r_i = 2.067/2 = 1.034$ in. = 0.086 ft. The outer diameter of the pipe in feet is $D_o = 2.375/12 = 0.198$ ft. The outer radius of the pipe is $r_o = D_o/2 = 0.198/2 = 0.099$ ft. The temperature difference between the inside and the outside of the pipe is $\Delta T = 292.5 - 70 = 222.5^\circ$F. Using $k_{\text{steel}} = 24.8$ Btu/h ft °F and other pertaining variables, we can obtain the resistance of the steel pipe,

$$R_s = \frac{\ln(r_o/r_i)}{2\pi k_s L} = \frac{\ln(0.099/0.086)}{2\pi\times 24.8\times 60} = 1.51\times 10^{-5}\ \text{h}^\circ\text{F/Btu}$$

Next, we have the resistance offered to convection heat transfer from the steel surface to the surroundings,

$$\left(R_{\text{out}}\right)_{\text{steel}} = \frac{1}{hA_o} = \frac{1}{h(\pi\times r_o\times L)} = \frac{1}{5(\pi\times 0.099\times 60)} = 0.0107\ \text{h}^\circ\text{F/Btu}$$

Next, we compute the outer radius after the insulation has been implemented as $r_{o,ins} = r_o + t = 0.099 + 1.5/12 = 0.224$ in. The resistance offered by the insulation follows as

$$R_{\text{ins}} = \frac{\ln\left(r_{o,\text{ins}}/r_o\right)}{2\pi k_{\text{m}} L} = \frac{\ln(0.224/0.099)}{2\pi \times 0.041 \times 60} = 0.0528 \text{ h°F/Btu}$$

The outer area of the insulation is $A_o = \pi r_{o,\text{ins}} L = \pi \times 0.224 \times 60 = 42.22 \text{ ft}^2$. We then compute the resistance offered to convection heat transfer from the insulated surface to the surroundings,

$$\left(R_{\text{out}}\right)_{\text{with ins}} = \frac{1}{hA_o} = \frac{1}{5 \times 42.22} = 0.00474 \text{ h°F/Btu}$$

Finally, the net resistance offered by the whole system with insulation is

$$\Sigma R_{\text{ins}} = R_s + R_{\text{ins}} + \left(R_{\text{out}}\right)_{\text{with ins}} = 1.51\times10^{-5} + 0.0528 + 0.00474 = 0.05756 \text{ h°F/Btu}$$

The net resistance of the system without insulation, in turn, is

$$\Sigma R_{\text{no ins}} = R_s + \left(R_{\text{out}}\right)_{\text{steel}} = 1.51\times10^{-5} + 0.0107 = 0.01072 \text{ h°F/Btu}$$

The heat transfer with insulation is then

$$q_{\text{with ins}} = \frac{\Delta T}{\Sigma R_{\text{ins}}} = \frac{222}{0.05756} = 3856.9 \text{ Btu/h}$$

Similarly, the heat transfer without insulation is determined as

$$q_{\text{no ins}} = \frac{\Delta T}{\Sigma R_{\text{no ins}}} = \frac{222}{0.01072} = 20{,}709.0 \text{ Btu/h}$$

The amount of heat saved by insulating the pipe is

$$q_{\text{saved}} = q_{\text{no ins}} - q_{\text{with ins}} = 20709.0 - 3856.9 = 16{,}851.1 \text{ Btu/h}$$

Finally, the money saved per hour due to the pipe insulation is given by

$$\text{Cost} = q_{\text{saved}} \times R$$

where R is the cost required to generate energy. Inserting q_{saved} = 16,851.1 Btu/h and R = \$0.68/10^5 Btu, we obtain

$$\text{Cost} = 16{,}851.1 \frac{\text{Btu}}{\text{h}} \times 0.68 \frac{\$}{10^5 \text{ Btu}} = \$0.1146 \text{ per h}$$

It remains to calculate the time required for insulation to be in service to recover the cost of insulation.

$$\text{Time} = 60 \text{ ft} \times \frac{\$0.75/\text{ft}}{\$0.1146/\text{h}} = \boxed{393 \text{ h}}$$

♦ The correct answer is **A**.

P.9 Solution

Part A: The rate of energy dissipation per unit length is given by

$$\dot{E}_g = R_{\text{elec}} i^2 = 0.01 \times 20^2 = 4 \text{ W/m}$$

The rate of energy dissipation per unit volume is given by $\dot{q} = \dot{E}_g/A_c$, where A_c is the cross-sectional area of the wire; that is,

$$\dot{q} = \frac{E_g}{A_c} = \frac{4}{\left(\pi \times 0.002^2/4\right)} = \boxed{1.27\times10^6 \text{ W/m}^3}$$

♦ The correct answer is **A**.

Part B: Without the insulation, an energy balance of the wire yields

$$\dot{E}_g = \dot{q}_{\text{conv}} + \dot{q}_{\text{rad}} = \pi D h\left(T - T_\infty\right) + \pi D \varepsilon_w \sigma\left(T^4 - T_{\text{sur}}^4\right)$$

where $h = 1.25[(T - T_\infty)/D]^{(1/4)}$. Substituting this expression and the pertaining data, we obtain

$$4 = 1.25\pi \times 0.002^{3/4} \times \left(T - 293\right)^{5/4} + \pi \times 0.002 \times 0.3 \times 5.67 \times 10^{-8}\left(T^4 - 293^4\right)$$

The foregoing equation cannot be solved by ordinary means and thus requires a trial-and-error procedure. Using Mathematica, one could apply the *FindRoot* command with an initial guess of 293 K,

$$\text{FindRoot}[4 - 1.25\pi * 0.002^{\frac{3}{4}}(T - 293)^{5/4} - \pi * 0.002 * 0.3 * 5.67 * 10^{-8}(T^4 - 293^4), \{T, 293\}]$$

This returns T = 331 K, which corresponds to T_w = 331 – 273 = 58°C.

♦ The correct answer is **B**.

Part C: The inner surface temperature can be obtained with the equation for heat transfer through the insulation,

$$\dot{q} = \frac{T_{s,i} - T_{s,o}}{R_{\text{cond}}} = \frac{T_{s,i} - T_{s,o}}{\left[\ln\left(r_2/r_1\right)\big/2\pi k\right]}$$

To obtain the inner temperature, however, we require the *outer* temperature of the insulation, $T_{s,o}$. In order to determine this quantity, we perform an energy balance at the outer surface,

$$\dot{E}_g = \dot{q}_{\text{conv}} + \dot{q}_{\text{rad}} = \pi D h\left(T_{s,o} - T_\infty\right) + \pi D \varepsilon_i \sigma\left(T_{s,o}^4 - T_{\text{sur}}^4\right)$$

$$\therefore 4 = 1.25\pi \times 0.006^{3/4} \times \left(T_{s,o} - 293\right)^{\frac{5}{4}} + \pi \times 0.006 \times 0.9 \times \left(5.67 \times 10^{-8}\right) \times \left(T_{s,o}^4 - 293^4\right)$$

As before, this equation does not lend itself to simple solution methods and a trial-and-error procedure is required. Once again, we could apply the *FindRoot* command with an initial guess of 293 K.

$$\text{FindRoot}[4 - 1.25\pi(0.006)^{\frac{3}{4}}(T - 293)^{\frac{5}{4}} - \pi * 0.006 * 0.9 * 5.67 * 10^{-8}(T^4 - 293^4), \{T, 293\}]$$

The command returns $T_{s,i} = 307.8\ \text{K} = 34.8^\circ\text{C}$. We are now able to backsubstitute in our initial equation and establish the inner temperature of the insulation,

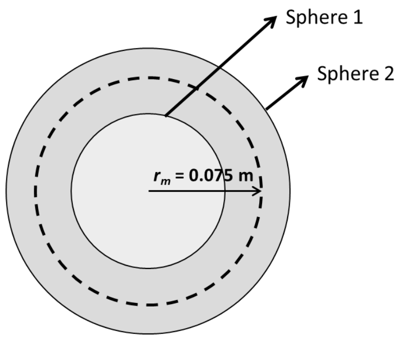
$$\dot{q} = \frac{T_{s,i} - T_{s,o}}{\left(\ln\left(r_2/r_1\right)\big/2\pi k\right)} \rightarrow 4 = \frac{\left(T_{s,i} - 307.8\right)}{\dfrac{\ln\left(0.006/0.002\right)}{2\pi \times 0.25}}$$

$$\therefore T_{s,i} = 310.6\ \text{K} = \boxed{37.6^\circ\text{C}}$$

♦ The correct answer is **C**.

P.10 Solution

Part A: We want to compute the temperature at the midpoint of the shell thickness, $T(r_m)$, where r_m = (0.05 + 0.10)/2 = 0.075 m. This region is underscored by the dashed line in the illustration below.



The conduction heat rate into the dashed control surface must equal the conduction heat rate out of the control surface. Thus, we can propose the equality

$$q_r = \frac{4\pi k\left(T_{s,1} - T(r_m)\right)}{(1/r_1)-(1/r_m)} = \frac{4\pi k\left(T(r_m) - T_{s,2}\right)}{(1/r_m)-(1/r_2)}$$

This can be solved for the temperature $T(r_m)$ to yield

$$T(r_m) = \frac{1}{\left[\frac{1}{(1/r_m)-(1/r_2)}\right]+\left[\frac{1}{(1/r_1)-(1/r_m)}\right]}\left[\frac{T_{s,1}}{(1/r_1)-(1/r_m)}+\frac{T_{s,2}}{(1/r_m)-(1/r_2)}\right]$$

$$\therefore T(r_m) = \frac{1}{\left[\frac{1}{(1/0.075)-(1/0.1)}\right]+\left[\frac{1}{(1/0.05)-(1/0.075)}\right]}$$

$$\times\left[\frac{100}{(1/0.05)-(1/0.075)}+\frac{45}{(1/0.075)-(1/0.1)}\right] = \boxed{63.3^{\circ}\mathrm{C}}$$

♦ The correct answer is **B**.

Part B: Observe that, from the expression we have devised for $T(r_m)$ – and, by extension, to every radial distance r – the temperature distribution depends only on the spatial coordinates of the shells and on the temperature of the shells. The thermal conductivity does not appear in the equation, which is to say that the temperature distribution is independent of the shell material. The midpoint temperature would be the same regardless of whether the shell material were Pyrex glass or aluminum.

♦ The correct answer is **B**.

Answer Summary

Problem 1		B
Problem 2		C
Problem 3		C
Problem 4	4A	A
	4B	B
Problem 5	5A	C
	5B	D
Problem 6		A
Problem 7	7A	B
	7B	A
Problem 8		A
Problem 9	9A	A
	9B	B
	9C	C
Problem 10	10A	B
	10B	B

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- WELTY, J., WICKS, C., WILSON, R., and RORRER, G. (2008). *Fundamentals of Momentum, Heat and Mass Transfer.* 5th edition. Hoboken: John Wiley and Sons.

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answer your question as soon as possible.