Montogue

Optics

30 Practice Questions

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Here's a set of 30 fully solved problems on optical science and related topics. Problems were taken from a carefully researched assortment of textbooks. All problems are solved step by step. Enjoy! ■

→ **Problems 1 – 12 are one-mark problems.**
→ **Problems 13 – 30 are two-mark problems.**

PROBLEMS

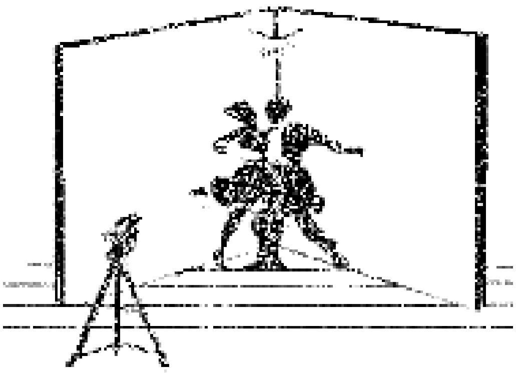
Problem 1. When a light wave propagates from one medium to another, which of the following associated quantities does **not** change?

(A) Velocity
(B) Frequency
(C) Wavelength
(D) Intensity

Problem 2. What is the critical angle at an interface between a glass-like medium with n = 1.6 and air?

(A) 33.7°
(B) 38.7°
(C) 43.7°
(D) 48.7°

Problem 3. A photo is taken of a couple placed within two plane mirrors at an angle of 60°, as illustrated to the side. How many people will appear in the photograph taken by the camera?



(A) 4
(B) 6
(C) 10
(D) 12

Problem 4. Consider an arrow of 8 cm height placed at a distance of 40 cm on the left side of a thin positive lens with refractive index equal to 1.45 and radii of curvature R_1 = 15 cm and R_2 = −15 cm. What is the distance (absolute value) from the lens to the image formed under the given conditions?

(A) 12.7 cm
(B) 18.7 cm
(C) 28.7 cm
(D) 39.7 cm

Problem 5. The most common cause of myopia is:
(A) Increase in the axial length of the eyeball.
(B) Increase in thickness of the lens.
(C) Increase in viscosity of aqueous humor.
(D) Increase in viscosity of vitreous humor.

Problem 6. Which of the following options contains an **incorrect** statement about treatment of ocular pathologies?
(A) Myopia is treated with use of concave spherical lenses.
(B) Hypermetropia is treated with use of convex spherical lenses.
(C) Presbyopia is treated with use of concave spherical lenses.
(D) Astigmatism can be treated with convergent or divergent cylindrical lenses, depending on the situation.

Problem 7. The Fraunhofer diffraction pattern of a circular aperture is observed on a screen placed at the focal plane of a lens. If the aperture is shifted upwards along its plane,
(A) the diffraction pattern will shift upwards.
(B) the diffraction pattern will shift downwards.
(C) the diffraction pattern will not shift.
(D) the diffraction pattern will disappear.

Problem 8. Suppose an electron in a hydrogen atom falls from the shell whose principal quantum number is 4 to the shell whose principal quantum number is 2. Taking 1.10×10^7 m^{-1} as the Rydberg constant, calculate the wavelength of the photon obtained in this transition.
(A) 315 nm
(B) 400 nm
(C) 485 nm
(D) 570 nm

Problem 9. In the 1830s, Cauchy proposed the following approximate formula to describe the wavelength dependence of refractive index in glass for waves in the visible region of the electromagnetic spectrum:

$$n(\lambda) = A + \frac{B}{\lambda_0^2}$$

Here, λ_0 is wavelength and A and B are empirical constants. It is known that, for borosilicate glass, the refractive index that corresponds to a wavelength λ_1 = 600 nm is $n(\lambda_1)$ = 1.508, whereas the refractive index that corresponds to a wavelength λ_2 = 300 nm is $n(\lambda_2)$ = 1.601. With a precision of three decimal places, what is the refractive index of the borosilicate glass at a wavelength of 425 nm?
(A) 1.512
(B) 1.539
(C) 1.556
(D) 1.571

Problem 10. The power of a lens (refractive index n = 1.6) is +6.0 D when immersed in air. The power of the same lens when immersed in water (n = 1.33) equals _______ (◆ diopters, *rounded to two decimal places*).

Problem 11. An optic-fiber cable 3 km long is made up of three 1-km lengths. Each length has a 4-dB loss and each splice contributes a 1-dB loss. If the input power is 12 mW, the output power under the given conditions is _________ (◆ mW, *rounded to three decimal places*).

Problem 12. Assume a 100 W incandescent light bulb is operated with a maximum filament temperature of 2900°C. Assuming the filament has an emissivity of 0.36, the radiant power per square meter produced by the lamp is _______ (◆ MW/m^2, *rounded to two decimal places*). Use $\sigma = 5.67\times10^{-8}$ W/m^{2}K^4 as the Stefan-Boltzmann constant.

Problem 13. In an experiment to measure the quantum yield of a photochemical reaction, the absorbing substance was exposed to 320 nm radiation from an 80-W source for 30 minutes. The intensity of the transmitted radiation was 0.257 times the intensity of the incident radiation. As a result of irradiation, 0.34 mol of the absorbing substance decomposed. Evaluate the quantum yield ϕ.
(A) ϕ = 1.02
(B) ϕ = 1.19
(C) ϕ = 1.29
(D) ϕ = 1.40

Problem 14. A transmission grating having 6000 lines/cm is 6.4 cm wide. Operating in the green at about 550 nm, the resolving power in the third order for this system is **P**. Also, the minimum resolvable wavelength in the second order is **Q**. What are the values of **P** and **Q**?
(A) **P** = 115,200; **Q** = 0.03 Å
(B) **P** = 115,200; **Q** = 0.07 Å
(C) **P** = 153,600; **Q** = 0.03 Å
(D) **P** = 153,600; **Q** = 0.07 Å

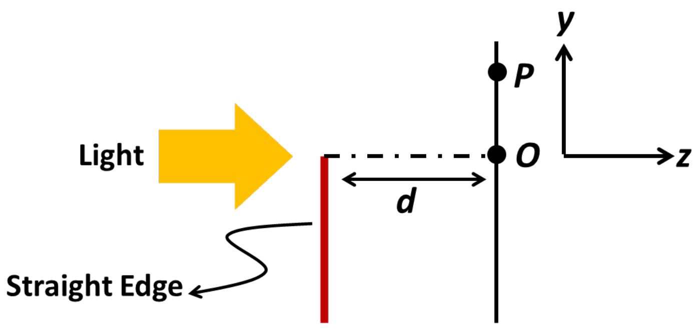
Problem 15. Newton's rings are formed between a spherical lens surface and an optical flat. If the tenth bright ring of green light (546.1 nm) is 7.89 mm in diameter, what is the radius of curvature of the lens surface?
(A) 3.0 m
(B) 4.0 m
(C) 5.0 m
(D) 6.0 m

Problem 16. Calculate the resolving power of a Fabry-Perot interferometer made of reflecting surfaces of reflectivity 0.825 separated by a distance of 1 mm at λ_0 = 4900 Å.
(A) 9310
(B) 27,250
(C) 34,660
(D) 51,920

Problem 17. Plane waves of monochromatic (600-nm) light are incident on an aperture. A detector is situated on axis at a distance of 30 cm from the aperture plane. The radius of the first Fresnel half-period zone relative to the detector is **P**. If the aperture is a circle of radius 1 cm, centered on axis, the number of half-period zones it contains is **Q**. What are the values of **P** and **Q**?
(A) **P** = 0.212 mm; **Q** = 278
(B) **P** = 0.424 mm; **Q** = 278
(C) **P** = 0.212 mm; **Q** = 556
(D) **P** = 0.424 mm; **Q** = 556

Problem 18. In the straight-edge diffraction pattern illustrated below, assume a wavelength λ_0 = 5000 Å and spacing *d* = 100 cm. Point *P* is located at (*y* = 1.0 mm), that is, 1 mm above the edge *O* of the geometrical shadow. The irradiances at *P* and the *O* are I and I_0, respectively. Find the relative irradiance I/I_0 at *P*. Values of Fresnel integrals *C(v)* and *S(v)* are listed below.

v	C(v)	S(v)
1.0 or −1.0	0.7799, −0.7799	0.4383, −0.4383
1.5 or −1.5	0.4453, −0.4453	0.6975, −0.6975
2.0 or −2.0	0.4882, −0.4882	0.3434, −0.3434
3.0 or −3.0	0.6058, −0.6058	0.4963, −0.4963
∞ or −∞	0.5, −0.5	0.5, −0.5



(A) 0.588
(B) 0.652
(C) 0.844
(D) 0.951

Problem 19. The electric field of a uniform plane electromagnetic wave is given by

$$\mathbf{E} = \left[2\vec{\mathbf{a}}_x + j3\vec{\mathbf{a}}_y\right]\exp\left[j\left(2\pi \times 10^6 t - 0.25z\right)\right]$$

The polarization of this wave is:
(A) Right-handed circular.
(B) Right-handed elliptical.
(C) Left-handed circular.
(D) Left-handed elliptical.

Problems 20 and 21

20. A number of dichroic polarizers are available, each of which can be assumed perfect, so that each one passes 50% of the incident unpolarized light. Suppose that we initially have two such polarizers with transmission axes set at 0° and 90°, respectively. What percentage of the incident light energy is transmitted by the pair?
(A) 0%
(B) 15%
(C) 30%
(D) 45%

21. Five additional polarizers of this type are placed between the two just described, with their transmission axes set at 15°, 30°, 45°, 60°, and 75°, in that order, with the 15°-angle polarizer adjacent to the 0° polarizer, and so on. Now, what is the percentage of incident light irradiance transmitted?
(A) 16.5%
(B) 33%
(C) 40%
(D) 46.5%

Problem 22. A cladding mode is excited in an optical fiber whose cladding diameter is 300 μm. The fiber is immersed in a cladding-mode stripper. The reflectance at the surface of the cladding may be taken as 10%. After what distance will the intensity of the cladding modes be reduced by a factor of 10^{-7}? Assume that rays hit the surface at nearly glancing incidence. Take $NA = 0.2$ and $n = 1.5$ as the fiber numerical aperture and refractive index, respectively.
(A) 5.2 mm
(B) 10.4 mm
(C) 15.8 mm
(D) 20.8 mm

Problem 23. A laser cavity is formed by Fresnel reflections between two cleaved facets. The refractive index of the laser cavity is $n = 3.6$ and the absorption parameter is $\alpha = 20$ cm^{-1}. At which cavity length, L, is the mirror loss equal to the absorption loss in the laser cavity?
(A) 112 μm
(B) 196 μm
(C) 285 μm
(D) 400 μm

Problem 24. A given He-Ne laser oscillates in a pure Gaussian TEM_{00} mode at λ = 632.8 nm with an output power of 8 mW. The laser is advertised as having a far-field divergence angle of 1 mrad. Find the spot size **W** and the peak intensity **I**.
(A) **W** = 0.201 mm, **I** = 126 mW/mm^2
(B) **W** = 0.201 mm, **I** = 180 mW/mm^2
(C) **W** = 0.402 mm, **I** = 126 mW/mm^2
(D) **W** = 0.402 mm, **I** = 180 mW/mm^2

Problem 25. A hologram recorded with an Ar^+ laser (λ = 514 nm) and an interbeam angle of 35° is to be illuminated with green light from a high-pressure mercury vapor lamp which has a mean wavelength λ = 546 nm and a spectral bandwidth $\Delta\lambda$ = 6 nm. The lamp can be regarded as a source with an effective diameter of 6 mm and is placed at a distance of 1 m from the hologram. Noting that the resolution of the human eye at a distance of 1 m is about 0.5 mm, what is the maximum depth of image that can be reconstructed with an acceptable value of image blur?

(A) 18.8 mm
(B) 36.5 mm
(C) 58.5 mm
(C) 73.0 mm

Problem 26. Consider a vacuum wavelength λ = 425 nm and an impinging spherical particle of diameter 18 nm. The refractive indices of sphere and background are 1.58 and 1.32, respectively. Assuming Rayleigh theory to be valid, what is the scattering cross-section?

(A) 9.66×10^{-21} m^2
(B) 2.41×10^{-20} m^2
(C) 6.76×10^{-20} m^2
(D) 9.66×10^{-20} m^2

Problem 27. A beam of photons of frequency ν = 3.2×10^{19} Hz is Compton-scattered by electrons of rest mass m_0 = 9.11×10^{-31} kg. After collision, the photons are scattered at 90° from the initial direction. Find the energy loss ΔW for a photon scattered by this process.

(A) ΔW = 8.9 keV
(B) ΔW = 27.3 keV
(C) ΔW = 61.5 keV
(D) ΔW = 93.1 keV

Problem 28. Consider an LED emitting at λ = 1550 nm wavelength window. The internal quantum efficiency is 75%, the external efficiency is 2%, the carrier lifetime is τ = 24 ns, and the injection current is 25 mA. The output optical power of this LED is _______ (◆ mW, *rounded to three decimal places*).

Problem 29. A two-mirror resonator consists of a convex mirror of radius R_1 = −2 m and a concave mirror of radius R_2 = 1.5 m. The maximum possible mirror separation for this system to remain a stable resonator is _______ (◆ m, *rounded to one decimal place*).

Problem 30. Metallic cesium has a photoelectric work function equal to 1.8 eV. The maximum kinetic energy of electrons ejected from a sample of cesium as it is irradiated with 327-nm waves equals _______ (◆ eV, *rounded to one decimal place*).

Answer key is on the next page! Problem solutions also begin on the next page.

ANSWER KEY

Problem	Answer	Problem	Answer
1	B	16	D
2	B	17	D
3	D	18	C
4	C	19	D
5	A	20	A
6	C	21	B
7	C	22	C
8	C	23	C
9	B	24	A
10	2.67	25	B
11	0.478	26	A
12	2.07	27	B
13	B	28	0.301
14	B	29	1.5
15	A	30	2.0

SOLUTIONS

1 ➡ B

As a wave refracts onto a new medium, its frequency remains the same.

2 ➡ B

The critical angle is given by

$$\theta_c = \arcsin\left(\frac{n_1}{n_2}\right) = \arcsin\left(\frac{1.0}{1.6}\right) = \arcsin(0.625)$$

$$\therefore \boxed{\theta_c = 38.7^\circ}$$

3 ➡ D

The number of images formed by two mirrors at an angle α is

$$n = \frac{360^\circ}{\alpha} - 1 = \frac{360^\circ}{60^\circ} - 1 = 5$$

Since each image comprises 2 people, the mirrors will show 2 × 5 = 10 people. We must also include the two 'real' persons, hence the number of people displayed in the photograph is 10 + 2 = 12.

4 ➡ C

The first step is to determine the focal length of the lens:

$$\frac{1}{f} = \frac{(n_2 - n_1)}{n_1}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\therefore \frac{1}{f} = \frac{(1.45-1)}{1} \times \left(\frac{1}{15} - \frac{1}{-15}\right) = 0.06 \text{ cm}^{-1}$$

$$\therefore f = \frac{1}{0.06} = 16.7 \text{ cm}$$

Then, we substitute into the thin lens equation to obtain

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f} \rightarrow \frac{1}{40} + \frac{1}{q} = \frac{1}{16.7}$$

$$\therefore 0.025 + \frac{1}{q} = 0.0599$$

$$\therefore \frac{1}{q} = 0.0599 - 0.025$$

$$\therefore q = \frac{1}{0.0599 - 0.025} = \boxed{28.7 \text{ cm}}$$

5 ➡ A

Myopia is primarily related to an increase in the axial length of the eyeball.

6 ➡ C

Myopia occurs when rays of light come to a focus in front of the retina; this disorder can be corrected by placing a concave lens in front of the eye. Hypermetropia occurs when rays of light come to a focus behind the retina; this disorder can be corrected by placing a convex lens in front of the eye. Pathological astigmatism is a refractive defect associated with meridional asymmetry in the eye's refractive power; it can be treated with cylindrical lenses. Presbyopia is a slow, gradual, age-related decline in amplitude of accommodation; this disorder can be corrected with convex, not concave, spherical lenses.

7 ➡ C

The Fraunhofer diffraction pattern does not depend on the exact position of the aperture.

8 ➡ C

This is a straightforward application of the Rydberg-Ritz formula, namely

$$\lambda = \left[R_H \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right) \right]^{-1}$$

$$\therefore \lambda = \left[\left(1.10 \times 10^7 \right) \times \left(\frac{1}{2^2} - \frac{1}{4^2} \right) \right]^{-1} = 4.85 \times 10^{-7} \text{ m}$$

$$\therefore \boxed{\lambda = 485 \text{ nm}}$$

9 ➡ B

The key to this problem is determining constants *A* and *B*; once these have been established, we can obtain the refractive index for any wavelength in the interval $\lambda \in [300, 600]$ nm. Substituting $n(\lambda_1 = 600 \text{ nm}) = 1.508$ brings to

$$n(\lambda_1 = 600) = A + \frac{B}{600^2} = 1.508 \quad \text{(I)}$$

Substituting $n(\lambda_2 = 300 \text{ nm}) = 1.601$ brings to

$$n(\lambda_2 = 300) = A + \frac{B}{300^2} = 1.601 \quad \text{(II)}$$

Subtracting (I) from (II),

$$\left(A + \frac{B}{300^2} \right) - \left(A + \frac{B}{600^2} \right) = 1.601 - 1.508$$

$$\therefore \frac{B}{300^2} - \frac{B}{600^2} = 1.601 - 1.508$$

$$\therefore 8.333 \times 10^{-6} B = 0.093$$

$$\therefore B = \frac{0.093}{8.333 \times 10^{-6}} = 11{,}160.4$$

Substituting in (I),

$$A + \frac{11{,}160}{600^2} = 1.508$$

$$\therefore A = 1.477$$

The equation that describes the variation in refractive index is

$$n(\lambda_0) = 1.477 + \frac{11{,}160.4}{\lambda_0^2}$$

Substituting λ_0 = 425 nm gives

$$n(\lambda_0) = 1.477 + \frac{11{,}160.4}{425^2} = \boxed{1.5388}$$

10 ➡ 2.67

Evoking the thin lens equation, we may write

$$P = \frac{1}{f} = +6.0 = (n - 1.0) \times \left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \text{(I)}$$

Writing a second equation for lens power in water, we have

$$P_w = \frac{n_w}{f_w} = (n - n_w) \times \left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \text{(II)}$$

Dividing (II) by (I),

$$\frac{P_w}{P} = \frac{(n - n_w)}{(n - 1.0)} \times \frac{\cancel{\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}}{\cancel{\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}}$$

$$\therefore P_w = \frac{(n - n_w)}{(n - 1.0)} \times P = \frac{1.6 - 1.333}{1.6 - 1.0} \times 6.0 = \boxed{+2.67 \text{ D}}$$

11 ➡ 0.450 – 0.500

The entire cable has a loss of 3 × 4 dB + 1 dB + 1 dB = 14 dB in 3 kilometers, hence the absorption coefficient is α_{dB} = 14 dB/3 km. Then, appealing to the definition of absorption coefficient and solving for output power, we obtain

$$\alpha_{\text{dB}} = (10 \text{ dB/km}) \log_{10}(P_1/P_2) \rightarrow P_2 = P_1\left(10^{-\alpha_{dB}L/10(\text{dB/km})}\right)$$

$$\therefore P_2 = 12 \times \left(10^{-(14/3)\times 3/10}\right) = \boxed{0.478 \text{ mW}}$$

12 ➡ 2.00 – 2.20

This is a straightforward application of the Stefan-Boltzmann law:

$$\Pi = \varepsilon\sigma T^4 = 0.36 \times \left(5.67 \times 10^{-8}\right) \times 3173^4 = 2.07 \times 10^6 \text{ W/m}^2$$

$$\boxed{\Pi = 2.07 \text{ MW/m}^2}$$

13 ➡ B

The energy contained in one 320-nm photon is $E = hc/\lambda$ = $(6.63\times10^{-34}) \times (3\times10^{8})/(320\times10^{-9}) = 6.22\times10^{-19}$ J. If the substance was irradiated at 80 W continuously for 30 min, an amount of energy equal to 30 × 80 × 60 = 144,000 J was involved in the process. The number n of photons involved in the procedure is then

$$n = \frac{144{,}000 \text{ J}}{6.22 \times 10^{-19} \text{ J/photon}} = 2.32 \times 10^{23} \text{ photons}$$

Converting to einsteins,

$$Q = \frac{2.32 \times 10^{23} \text{ photons}}{6.02 \times 10^{23} \text{ photon/mol}} = 0.385 \text{ mol}$$

If the intensity of the transmitted light is 0.257 times the intensity of the incident light, the number of photons absorbed should equal (1 – 0.257) × 0.385 = 0.286 mol. It remains to compute the quantum yield ϕ,

$$\phi = \frac{\text{No. of moles decomposed}}{\text{No. of einsteins absorbed}} = \frac{0.34 \text{ mol}}{0.286 \text{ mol}} = \boxed{1.19}$$

14 ➡ B

Obtaining the resolving power in the third order is straightforward:

$$\Re = mN = 3\times\left(6000\,\frac{1}{\text{cm}}\right)\times(6.4\text{ cm}) = \boxed{115{,}200}$$

In the second order, $\Re = 2 \times 6000 \times 6.4 = 76{,}800$ and the minimum allowable wavelength $\Delta\lambda$ becomes

$$\Delta\lambda = \frac{\lambda}{\Re} = \frac{550}{76{,}800} = 0.00716\text{ nm} = \boxed{0.0716\text{ Å}}$$

15 ➡ A

The radius of curvature R of the lens can be expressed as

$$R = \frac{r_{10}^2 - t_{10}^2}{2t_{10}}\ \text{(I)}$$

where $r_{10} = 7.89/2 = 3.945$ mm and thickness t_{10} can be determined as

$$t_m = \left(m - \frac{1}{2}\right)\frac{\lambda}{2} = (10-0.5)\times\frac{546.1\times10^{-6}}{2} = 2.59\times10^{-3}\text{ mm}$$

Substituting in (I) brings to

$$R = \frac{3.945^2 - \left(2.59\times10^{-3}\right)}{2\times\left(2.59\times10^{-3}\right)} = 3004\text{ mm} = \boxed{3.0\text{ m}}$$

16 ➡ D

We first compute the coefficient of finesse F,

$$F = \frac{4R}{(1-R)^2} = \frac{4\times0.825}{(1-0.825)^2} = 107.8$$

The resolving power follows as

$$\rho = \frac{\pi t\sqrt{F}}{\lambda} = \frac{\pi\times1\times\sqrt{107.8}}{4900\times10^{-7}} = \boxed{51{,}920}$$

17 ➡ D

The radius R_1 of the first Fresnel half-zone is

$$R_1 = \sqrt{n\lambda r_0} = \sqrt{1\times\left(600\times10^{-7}\right)\times30} = 0.0424\text{ cm} = \boxed{0.424\text{ mm}}$$

The number N of half-period zones corresponding to the specified aperture is

$$N = \frac{R_1^2}{\lambda r_0} = \frac{1^2}{\left(6\times10^{-5}\right)\times30} = \boxed{556}$$

18 ➡ C

The first step is to compute the general variable

$$v_0 = -\sqrt{\frac{2}{\lambda d}}\,y = -\sqrt{\frac{2}{\left(5\times10^{-5}\right)\times100}}\,y = -20y$$

with y given in centimeters. For y = 1 mm = 0.1 cm, we have

$$v_0 = -20y = -20\times0.1 = -2$$

Referring to the table given in the problem statement, $C(-2) = -0.4882$ and $S(-2) = -0.3434$; also, $C(\infty) = S(\infty) = 0.5$. The relative irradiance follows as

$$\frac{I}{I_0} = \frac{1}{2}\left\{\left[0.5 - C(v_0)\right]^2 + \left[0.5 - S(v_0)\right]^2\right\}$$

$$\therefore \frac{I}{I_0} = \frac{1}{2}\left\{\left[0.5-(-0.4882)\right]^2 + \left[0.5-(-0.3434)\right]^2\right\} = \boxed{0.844}$$

19 ➡ D

Note that for the given equation, component $|E_x| \neq |E_y|$; thus, the wave is elliptically polarized. Note further that, because **E** plots as a counterclockwise-rotating vector, the wave meets the criterion for a left-handed elliptically polarized wave.

20 ➡ A

In general, Malus' law gives the irradiance I_1 afforded by a polarizer relatively to the pre-polarization irradiance I_0:

$$I_1 = \frac{1}{2} I_0 \cos^2\theta$$

With $\theta = 90^{\text{o}} - 0^{\text{o}} = 90^{\text{o}}$, we obtain

$$I_1 = \frac{1}{2} I_0 \cos^2\left(90^{\text{o}} - 0^{\text{o}}\right) = \frac{1}{2} I_0 \underbrace{\cos^2 90^{\text{o}}}_{=0} = \boxed{0}$$

21 ➡ B

The angle between successive polarizers is always 15°, therefore,

$$I_1 = \frac{1}{2} I_0 \left[\cos^2\left(15^{\text{o}}\right)\right]^6 = 0.3298 I_0 \approx \boxed{0.33 I_0}$$

22 ➡ C

Note first that $\theta' = NA/n = 0.2/1.5 = 0.133$. Let the number N of reflections in a distance L be $N = L\theta'/D$. If the reflectance is R, we may write

$$R^N = 10^{-7} \quad \rightarrow \quad N \log_{10} R = -7$$

$$\therefore L = \frac{-7D}{\theta' \log_{10} R}$$

$$\therefore L = \frac{-7 \times \left(300 \times 10^{-6}\right)}{0.133 \times (-1)} = \boxed{15.8 \text{ mm}}$$

23 ➡ C

The power reflectivity on each facet is

$$R = \left(\frac{n-1}{n+1}\right)^2 = \left(\frac{3.6-1}{3.6+1}\right)^2 = 0.3195$$

The mirror loss is expressed as

$$\alpha_m = -\frac{\ln\left(R_1 R_2\right)}{4L}$$

Setting $R_1 = R_2 = 0.309$ and solving for cavity length L, we obtain

$$\alpha_m = -\frac{\ln\left(R_1 R_2\right)}{4L} \quad \rightarrow \quad L = -\frac{\ln\left(R^2\right)}{4\alpha_m}$$

$$\therefore L = -\frac{2\ln(R)}{4\alpha_m}$$

$$\therefore L = -\frac{\ln(R)}{2\alpha_m}$$

$$\therefore L = -\frac{\ln(0.3195)}{2 \times 20} = 0.0285 \text{ cm} = \boxed{285\ \mu\text{m}}$$

In order for the mirror loss to equal the absorption loss in the laser cavity, the cavity length should be close to 285 micrometers.

24 ➡ A

The waist spot size w_0 for a small divergence angle θ_d is determined to be

$$w_0 = \frac{\lambda}{\pi \theta_d} = \frac{632.8 \times 10^{-9}}{\pi \times 10^{-3}} = 2.01 \times 10^{-4}\ \text{m} = \boxed{0.201 \text{ mm}}$$

Now, noting that the power of a Gaussian beam is related to its peak intensity by

$$P = \left(\frac{\pi w_0^2}{2}\right) I_0$$

we can solve for I_0 to obtain

$$I_0 = \frac{2P}{\pi w_0^2} = \frac{2\times 8}{\pi \times 0.201^2} = \boxed{126 \text{ mW/mm}^2}$$

25 ➡ B

The total image blur is the sum of the contributions from the finite size of the source and its spectral bandwidth; mathematically,

$$\Delta x_L = z_O\left(\frac{\Delta x_P}{z_P}\right) + z_O\left(\frac{x_P}{z_P}\right)\left(\frac{\Delta\lambda_2}{\lambda_2}\right)$$

Here, Δx_L = 0.5 mm, Δx_P = 6 mm, z_P = 1 m, (x_P/z_P) = *tan* 35° = 0.700, $\Delta\lambda_2$ = 6 nm, and λ_2 = 546 nm, giving

$$0.5\times 10^{-3} = z_O \times \left(\frac{6}{1000}\right) + z_O \times \tan 35^{\circ} \times \left(\frac{6}{546}\right)$$

$$\therefore 0.5\times 10^{-3} = 0.006 z_O + 0.0077 z_O$$

$$\therefore z_O = 0.0365 \text{ m} = \boxed{36.5 \text{ mm}}$$

Thus, the distance of any point in the image from the hologram should not exceed approximately 37 millimeters.

26 ➡ A

Per Rayleigh theory, the scattering cross-section σ_s is given by

$$\sigma_x = \frac{8\pi a^2 x^4}{3}\left(\frac{n_{\text{rel}}^2 - 1}{n_{\text{rel}}^2 + 2}\right)^2 \quad \text{(I)}$$

Here, a = 9 nm is the radius of the spherical particle; n_{rel} is the ratio of the sphere's refractive index, n_s = 1.58, to the refractive index of the surrounding medium, n_b = 1.32; parameter x is the product of angular wavenumber k and sphere radius a. We proceed to compute n_{rel},

$$n_{\text{rel}} = \frac{n_s}{n_b} = \frac{1.58}{1.33} = 1.19$$

and the angular wavenumber k,

$$k = \frac{2\pi n_b}{\lambda} = \frac{2\pi\times 1.32}{425\times 10^{-9}} = 1.95\times 10^7 \text{ m}^{-1}$$

so that

$$x = ka = \left(1.95\times 10^7\right)\times\left(9\times 10^{-9}\right) = 0.176$$

Substituting in (I),

$$\sigma_x = \frac{8\pi\times\left(9\times 10^{-9}\right)^2 \times 0.176^4}{3} \times \left(\frac{1.19^2 - 1}{1.19^2 + 2}\right)^2 = \boxed{9.66\times 10^{-21} \text{ m}^2}$$

27 ➡ B

From the Compton formula, we may write

$$\frac{v - v'}{vv'} = \frac{h}{m_0 c^2}\left(1 - \cos\theta\right)$$

or, alternatively,

$$v' = \frac{v}{1 + vA\left(1 - \cos\theta\right)}$$

where $A = h/m_0c^2$. For θ = 90$^{\text{O}}$, the equation above further simplifies to

$$\nu' = \frac{\nu}{1+\nu A\left(1-\underbrace{\cos 90^\circ}_{=0}\right)} = \frac{\nu}{1+\nu A}$$

so that

$$A = \frac{h}{m_0 c^2} = \frac{6.63\times10^{-34}}{\left(9.11\times10^{-31}\right)\times\left(3.0\times10^{8}\right)^2} = 8.09\times10^{-21}$$

$$\nu' = \frac{\nu}{1+\nu A} = \frac{3.2\times10^{19}}{1+\left(3.2\times10^{19}\right)\times\left(8.09\times10^{-21}\right)} = 2.54\times10^{19}\text{ Hz}$$

The energy loss by the photon then becomes

$$\Delta W_{\text{ph}} = h\left(\nu-\nu'\right) = \left(4.14\times10^{-15}\right)\times\left[\left(3.2-2.54\right)\times10^{19}\right] = \boxed{27.3\text{ keV}}$$

28 ➡ 0.250 – 0.350

Given the internal efficiency η_q = 0.75, the external efficiency η_{ext} = 0.02, the wavelength λ = 1550 nm, and the injection current I = 25 mA, we may write

$$P_{\text{opt}} = \eta_q \eta_{\text{ext}} \frac{hc}{\lambda q} I = 0.75\times0.02\times\frac{\left(6.63\times10^{-34}\right)\times\left(3\times10^{8}\right)}{\left(1550\times10^{-9}\right)\times\left(1.6\times10^{-19}\right)}\times\left(25\times10^{-3}\right)$$

$$\therefore \boxed{P_{\text{opt}} = 0.301\text{ mW}}$$

29 ➡ 1.5

In general, a resonator constructed with two mirrors will remain stable if

$$R_1 + R_2 < L < R_2 \text{ if } |R_1| < R_2$$

or

$$0 < L < R_2 \text{ if } |R_1| > R_2$$

In the situation at hand, $|R_1|$ = 2 > $|R_2|$ = 1.5 m, hence the latter condition applies; the resonator will remain stable so long as 0 < L < 1.5 m. The maximum spacing is 1.5 m.

30 ➡ 1.9 – 2.1

The energy associated with one 327-nm photon is hc/λ = (4.14×10^{-15}) × (3×10^{8})/ (327×10^{-9}) = 3.80 eV. The maximum kinetic energy KE of electrons as they are irradiated with these photons is then

$$KE = \frac{hc}{\lambda} - W = 3.80 - 1.8 = \boxed{2.0\text{ eV}}$$

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