Montogue

QUIZ GT203
Pile Foundations

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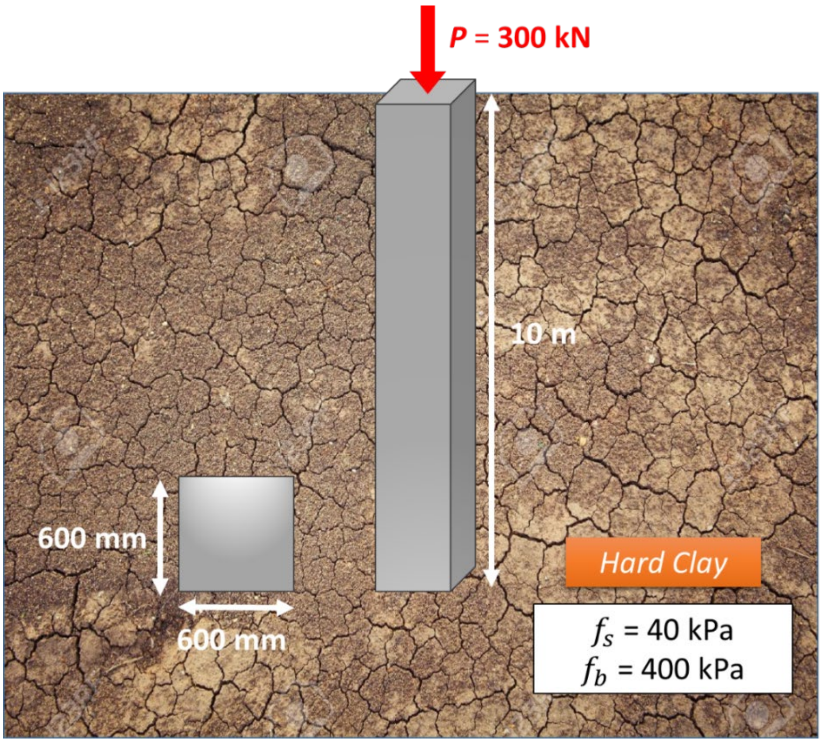
▸ PROBLEMS

PROBLEM 1

A 300 kN compressive load is to be imposed on a 10-m long reinforced concrete pile with square cross-section having a side of 600-mm width, driven through a homogeneous layer of hard clay as illustrated below. The unit skin friction is f_s = 40 kPa and the unit end-bearing stress is f_b = 400 kPa. Compute the downward load capacity using a factor of safety equal to 3 and determine whether the design is acceptable.



A) Q_{all} = 136 kN < P → The design is not acceptable.

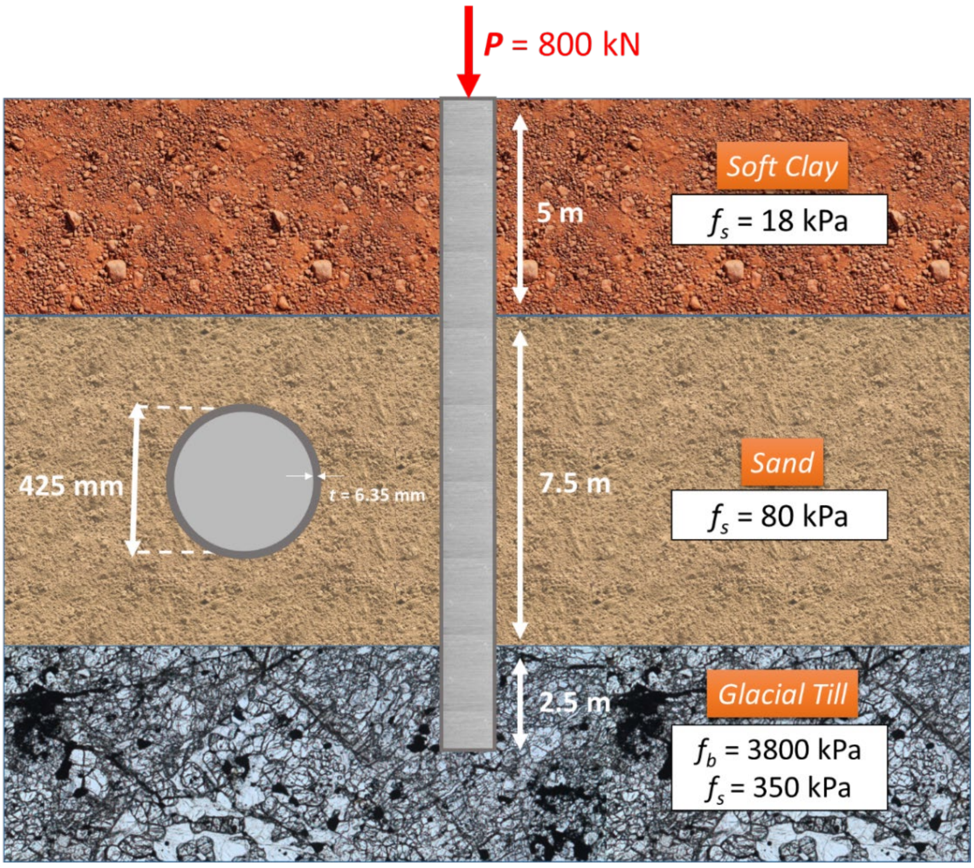
B) Q_{all} = 217 kN < P → The design is not acceptable.

C) Q_{all} = 368 kN > P → The design is acceptable.

D) Q_{all} = 429 kN > P → The design is acceptable.

PROBLEM 2

A 800 kN compressive load is to be imposed on a 425-mm diameter, 15-m long steel pipe pile driven into the soil profile shown in the figure below. The net end-bearing and unit skin friction resistances are as shown. Compute the downward load capacity using a factor of safety of 3 and determine whether the design is acceptable.



A) Q_{all} = 654 kN < P → The design is not acceptable.

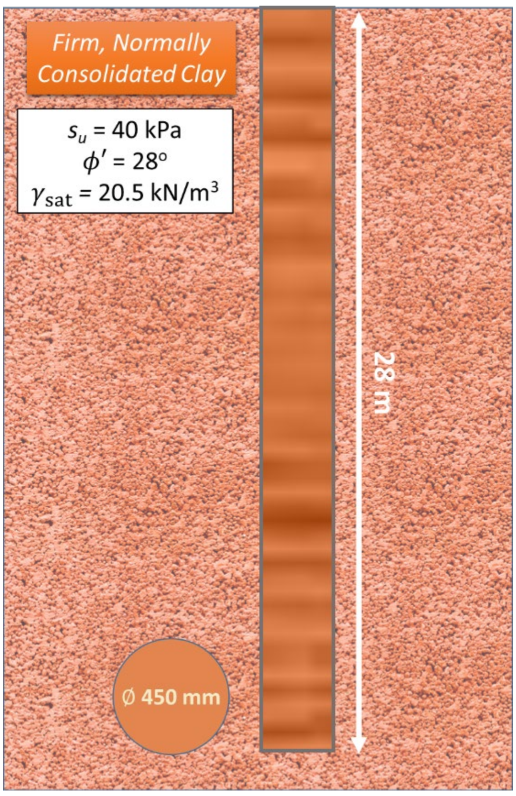
B) Q_{all} = 765 kN < P → The design is not acceptable.

C) Q_{all} = 876 kN > P → The design is acceptable.

D) Q_{all} = 987 kN > P → The design is acceptable.

PROBLEM 3

A cylindrical timber pile of diameter 450 mm is driven to a depth of 28 m into firm, homogeneous normally consolidated clay. The soil parameters are s_u = 40 kPa, ϕ' = 28°, and γ_{sat} = 20.5 kN/m^3. Groundwater level is at the surface. Estimate the allowable load, Q_{all}, for a factor of safety of 1.5. Which of the following intervals contains Q_{all}? <u>Use a total stress analysis</u>.



A) Q_{all} ∈ (650 ; 850) kN

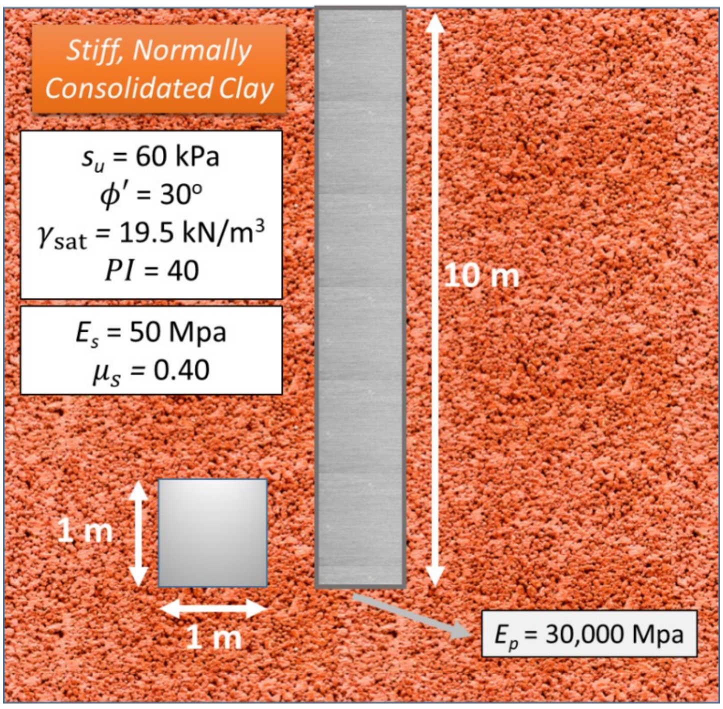
B) Q_{all} ∈ (850 ; 1050) kN

C) Q_{all} ∈ (1050 ; 1250) kN

D) Q_{all} ∈ (1250 ; 1450) kN

PROBLEM 4A

A reinforced concrete pile of square cross-section with a 1-m side is driven to a depth of 10 m into stiff, homogeneous normally consolidated clay. The soil parameters are s_u = 60 kPa, ϕ' = 30°, and γ_{sat} = 19.5 kN/m^3. More information on the soil and pile materials is provided in the illustration. Groundwater level is at the surface. Estimate the allowable load, Q_{all}, for a factor of safety of 1.5. Which of the following intervals contains Q_{all}? Use an effective stress analysis.



A) $Q_{all} \in$ (300 ; 600) kN
B) $Q_{all} \in$ (600 ; 900) kN
C) $Q_{all} \in$ (900 ; 1200) kN
D) $Q_{all} \in$ (1200 ; 1500) kN

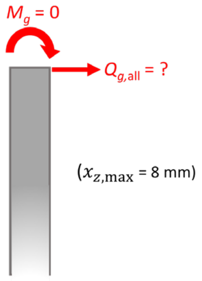
PROBLEM 4B

Consider, now, that the allowable lateral displacement of the top of the pile is 8 mm. Compute the allowable lateral load, Q_g, by the *limiting displacement* and the *moment capacity* conditions. Note that M_g = 0, that is, there is no moment at the ground surface (z = 0). In your calculations, use coefficient R, given by

$$R = \left(\frac{E_p I_p}{k_s}\right)^{\frac{1}{4}}$$

where E_p is the modulus of elasticity of the pile, I_p is the moment of inertia of the pile cross-section, and k_s is the modulus of subgrade reaction of the soil, which, in the case of cohesive soils, can be approximated by Vesic's expression

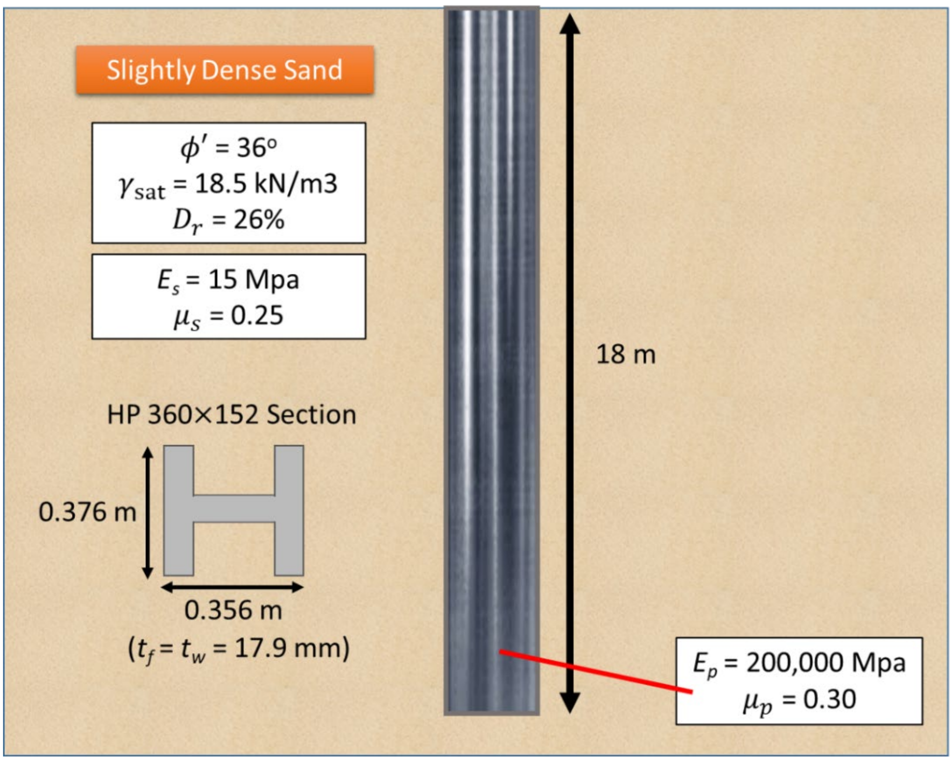
$$k_s = 0.65\left(\frac{E_s D^4}{E_p I_p}\right)^{\frac{1}{12}}\left(\frac{E_s}{1-\mu_s^2}\right)$$



A) $Q_{g,all}$ = 211 kN
B) $Q_{g,all}$ = 471 kN
C) $Q_{g,all}$ = 2341 kN
D) $Q_{g,all}$ = 8027 kN

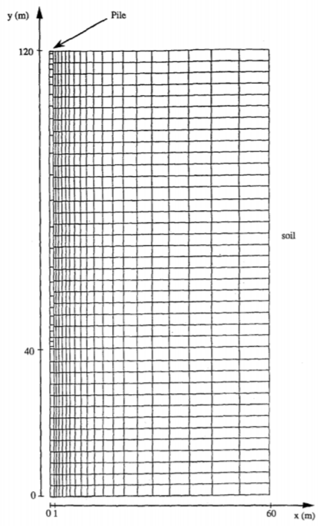
PROBLEM 5A

A steel H-pile of square cross-section of type HP 360×152 is driven 18 m into a deposit of homogeneous, slightly dense sand with parameters $\phi' = 36°$, $\gamma_{sat} = 18.5$ kN/m^3, and $D_r = 26\%$. More information on the soil and pile materials is provided on the illustration. Groundwater level is at the surface. True or false?



1.( ) The skin friction on the pile by the β-method, with a β factor obtained by means of the Burland formula ($\beta = (1 - \sin\phi')\tan\delta$), will be no less than 400 kN.
2.( ) The skin friction on the pile by the β-method, using a β factor determined with the Bhushan formula ($\beta = 0.18 + 0.65D_r$, with D_r given as a decimal), is less than 60% of the skin friction obtained with a β factor taken from the trend line drawn by Fellenius (Figure 3).
3.( ) The end-bearing loads obtained by Berezantsev's and Meyerhof's curves and methodologies are within 400 kN of each other.
4.( ) The end-bearing load obtained with the Coyle & Castello curve (Figure 5) is greater than 925 kN.

PROBLEM 5B



The uplift resistance and the compressive resistance are approximately the same for fine-grained soils, but not exactly so for piles in sands. In such cases, Nicola & Randolph executed a numerical study, their 2D mesh shown above, and proposed the following expression for the uplift skin frictional stress ($f_{s,up}$), expressed as a fraction of the unit skin friction,

$$\frac{f_{s,up}}{f_s} = \left[1 - 0.2\log_{10}\frac{100}{\left(L/D\right)}\right]\left(1 - 8\eta + 25\eta^2\right)$$

where η is a compressibility factor given by

$$\eta = \mu_p \tan\delta\left(\frac{L}{D}\right)\left(\frac{\bar{G}_s}{E_p}\right)$$

in which μ_p is Poisson's ratio for the pile material, δ is the soil-pile interfacial friction angle, *L/D* is the embedment ratio, $\bar{G}_s$ is the averaged shear modulus of the soil profile in which the pile is inserted, and E_p is Young's modulus for the pile material. Consider $\delta = (2/3)\phi'$ and soil to be an isotropic material. Compute unit skin friction using Bhushan's formula ($\beta = 0.18 + 0.65D_r$). Then, obtain a reduction factor, *R*, that can be applied to the formula

$$\left(Q_f\right)_{\text{up}} = R \times Q_f$$

to produce a skin friction load equivalent to the downward skin friction Q_f as obtained with the β-method and Bhushan's formula, where $(Q_f)_{\text{up}}$ is the uplift resistance as obtained with the expressions outlined above.

A) R = 0.606

B) R = 0.717

C) R = 0.828

D) R = 0.939

PROBLEM 5C

Estimate the settlement of the pile introduced above. To do so, consider the three settlement components of the pile using the formulas outlined below. Use the end-bearing resistance computed with Meyerhof's method, and the skin friction resistance computed with the β-method, with a β calculated with Bhushan's formula. As a simple approximation, the total settlement *S* is to be given as the algebraic sum of the three settlement components. Use a factor of safety *FS* = 1.5.

Components of the Elastic Settlement of a Single Pile		
Settlement from Elastic Theory	Settlement Caused by Load Carried by Pile Shaft	Settlement Caused by Load Carried by Pile End
$s_{e,1} = \frac{(Q_{p,\text{all}} + 0.6Q_{f,\text{all}})L}{A_b E_p}$ Legend: $Q_{p,\text{all}}$ → End-bearing resistance under working load condition $Q_{f,\text{all}}$ → Skin friction resistance under working load condition L → Pile length A_b → Pile cross-section E_p → Pile material elastic modulus	$s_{e,2} = \left(\frac{Q_{f,\text{all}}}{pL}\right)\frac{D}{E_s}(1-\mu_s^2)I_{ws}$ Legend: $Q_{f,\text{all}}$ → Skin friction resistance under working load condition p → Pile perimeter L → Pile length D → Pile width E_s → Elastic modulus of soil surrounding pile shaft μ_s → Poisson's ratio of soil surrounding pile shaft I_{ws} → Influence factor, given by Vesic as $I_{ws} = 2 + 0.35\sqrt{\frac{L}{D}}$	$s_{e,3} = \frac{Q_{p,\text{all}}C_p}{Dq_{p,\text{ult}}}$ Legend: $Q_{p,\text{all}}$ → End-bearing resistance under working load condition D → Pile width $q_{p,\text{ult}}$ → Ultimate unit end-bearing resistance of the pile C_p → Empirical coefficient, with values given below. Choose 0.025.

Type of soil	Driven pile	Bored pile
Sand (dense to loose)	0.02 – 0.04	0.09 – 0.18
Clay (stiff to soft)	0.02 – 0.03	0.03 – 0.06
Silt (dense to loose)	0.03 – 0.05	0.09 – 0.12

A) S = 5.54 mm

B) S = 8.65 mm

C) S = 11.76 mm

D) S = 14.87 mm

PROBLEM 6

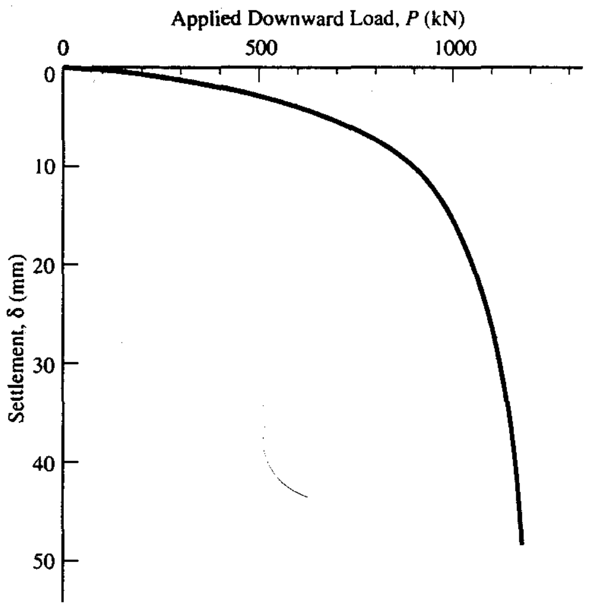
A 15-m long tubular steel pile (E_p = 200,000 MPa) is driven into a normally consolidated clay and has a computed ultimate side friction capacity ($\Sigma f_s A_s$) of 580 kN and an ultimate end-bearing capacity ($f_b A_b$) of 300 kN. Develop a load-settlement curve using the equations below, then determine the adjusted settlement when the foundation is subjected to the allowable load, given a factor of safety of 2. Use a = 0.40 and b = 0.60.

<table>
<tr><th colspan="2">Load-Settlement Response of Deep Foundations</th></tr>
<tr><th>Skin Friction Component</th><th>End-bearing Component</th></tr>
<tr><td>$$\frac{(f_s)_m}{f_s} = \left(\frac{\delta}{\delta_u}\right)^a \leq 1$$
$(f_s)_m$ → Mobilized unit skin friction resistance
f_s → Unit skin friction resistance
δ → Settlement
δ_u → Settlement required to mobilize ultimate resistance = 10 mm for skin friction
a → Exponent ∈ (0.02 ; 0.5)</td><td>$$\frac{(f_b)_m}{f_b} = \left(\frac{\delta}{\delta_u}\right)^b$$
$(f_b)_m$ → Mobilized end-bearing resistance
f_b → Unit end-bearing resistance
δ → Settlement
δ_u → Settlement required to mobilize ultimate resistance = B/10 for end-bearing
b → Exponent ∈ (0.5 (clay) ; 1.0 (sand))</td></tr>
<tr><td colspan="2">Deep foundations also experience elastic compression, which is another source of apparent “settlement.” This component of settlement can be computed with the expression
$$\delta_e = \frac{P z_c}{A_{\sec} E_p}$$
P → Total downward load
z_c → Depth to centroid of soil resistance ≈ 0.75L (where L = depth of embedment)
$A_{\sec}$ → Cross-sectional area of pile excluding soil plugs, if any
E_p → Young’s modulus of pile material
The adjusted settlement δ_{adj} is obtained by summing assumed values of δ used with the two foregoing formulas with the elastic component δ_e obtained with the aforementioned expression.</td></tr>
</table>

A) δ_{adj} = 3.80 mm
B) δ_{adj} = 5.41 mm
C) δ_{adj} = 7.92 mm
D) δ_{adj} = 10.73 mm

PROBLEM 7

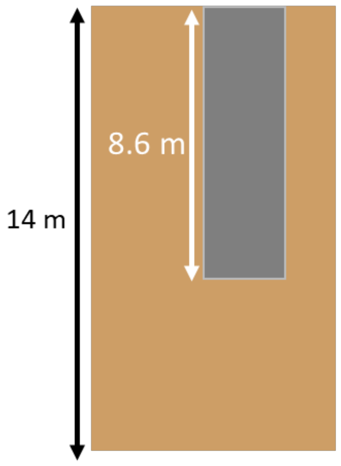
A 250-mm square, 15-m long prestressed concrete pile (f_c' = 40 MPa) was driven at a site in Amsterdam as described by Heijnen & Janse (1985). A conventional load test conducted 31 days later produced the load-settlement curve shown below. Using Davisson’s method, compute the ultimate load capacity of the pile.



A) P_{ult} = 710 kN
B) P_{ult} = 830 kN
C) P_{ult} = 970 kN
D) P_{ult} = 1100 kN

PROBLEM 8

The following is the variation of N_{60} with depth in a granular soil deposit. A concrete pile 8.6 m long, with 0.410 m × 0.410 m cross-section, is driven into the sand and fully embedded into the sand. Estimate the allowable load-carrying capacity of the pile using Meyerhof's equations (reproduced below) for a factor of safety *FS* = 3.5.



Depth (m)	N_{60}
1.5	4
3.0	8
4.5	6
6.0	5
7.5	16
9.0	18
10.5	21
11.0	24
12.5	20
14.0	19

Meyerhof's formulas for bearing capacity based on SPT results	
End-bearing capacity	Skin Friction
$Q_p = 0.4p_a(N_{60})_{\text{sp}}\left(\frac{L}{D}\right)A_b \leq 4p_a(N_{60})_{\text{sp}}A_b$ p_a → Atmospheric pressure (≈ 100,000 Pa) (L/D) → Embedment ratio (length/width) A_b → Cross-sectional area $(N_{60})_{\text{sp}}$ → N_{60} averaged about 10 widths above and 4 widths below pile end	$Q_f = 0.01p_a\bar{N}_{60}pL$ p_a → ≈ 100 kPa $\bar{N}_{60}$ → Average SPT number <u>throughout length of pile</u> p → Pile perimeter L → Pile length

A) Q_{all} = 295 kN
B) Q_{all} = 388 kN
C) Q_{all} = 491 kN
D) Q_{all} = 589 kN

PROBLEM 9

A reinforced concrete pile weighing 45 kN (including helmet and dolly) is driven by a drop hammer weighing 30 kN with an effective fall of 0.8 m. The average penetration per blow is 16 mm. The total temporary elastic compression of the pile, pile cap and soil may be taken as 19 mm. The coefficient of restitution is 0.32. What is the allowable load of this pile? Use Hiley's formula with a factor of safety of 1.5.

A) Q_{all} = 190 kN
B) Q_{all} = 231 kN
C) Q_{all} = 289 kN
D) Q_{all} = 360 kN

▸ ADDITIONAL INFORMATION

Table 1 Adhesion factors α based on undrained shear strength (s_u), soil consistency and pile type - NAVFAC data

Pile type	Soil consistency	Undrained shear strength s_u [kPa]	α
Timber and concrete piles	Very soft	0–12	1.00
	Soft	12–24	1.00–0.96
	Medium stiff	24–48	0.96–0.75
	Stiff	48–96	0.75–0.48
	Very stiff	96–192	0.48–0.33
Steel piles	Very soft	0–12	1.00
	Soft	12–24	1.00–0.92
	Medium stiff	24–48	0.92–0.70
	Stiff	48–96	0.70–0.36
	Very stiff	96–192	0.36–0.19

Table 2 Young's modulus (E_s) estimates for soil (MPa)

Description	Very Soft	Soft	Medium	Hard	Sandy
Clay	2 – 15	5 – 25	15 – 50	50 – 100	25 – 250
Description	Silty	Loose	Dense	Sand + Gravel – Loose	Sand + Gravel - Dense
Sand	5 – 20	10 – 25	50 – 81	50 – 150	100 – 200

Table 3 Poisson's ratio for soil and rock

Soil/Rock	Poisson's Ratio
Clay, saturated	0.4 – 0.5
Clay, unsaturated	0.1 – 0.3
Silt	0.3 – 0.35
Dense sand, drained conditions	0.30 – 0.40
Loose sand, drained conditions	0.10 – 0.30
Sandstone	0.25 – 0.30
Granite	0.23 – 0.27

Table 4 Rigidity index (I_r) versus undrained cohesion (s_u, normalized with atmospheric pressure) for saturated conditions

s_u/p_a	I_r
0.24	50
0.48	150
$\geq$ 0.96	250 – 300

Table 5 Lateral earth pressure coefficient (K) for pile design with β-method

Steel piles	K (piles under compression)	K (piles under tension)
Driven H-piles	0.5 – 1.0	0.3 – 0.5
Driven displacement piles	1.0 – 1.5	0.6 – 1.0
Driven displacement tapered piles	1.5 – 2.0	1.0 – 1.3
Driven jetted piles	0.4 – 0.9	0.3 – 0.6
Bored piles (less than 60 cm in diameter)	0.7	0.4

Table 6 Pile-soil interface friction angle δ

Steel piles	$\delta = \frac{2}{3}\phi'$ to $0.8\phi'$
Concrete piles	$\delta = 0.9\phi'$ to $1.0\phi'$
Timber piles	$\delta = 0.8\phi'$ to $1.0\phi'$

Table 7 Approximate ranges of β-coefficients (Fellenius, 2009)

Soil Type	β range
Clay	0.15 – 0.35
Silt	0.25 – 0.50
Sand	0.30 – 0.90
Gravel	0.35 – 0.80

Table 8 Approximate ranges of N_q coefficients

ϕ'	N_q
25° – 30°	3 – 30
28° – 34°	20 – 40
32° – 40°	30 – 150
35° – 45°	60 – 300

Table 9 N_q^* coefficients based on Meyerhof's theory for end-bearing capacity on sand

Soil friction angle, ϕ (deg)	N_q^*
20	12.4
21	13.8
22	15.5
23	17.9
24	21.4
25	26.0
26	29.5
27	34.0
28	39.7
29	46.5
30	56.7
31	68.2
32	81.0
33	96.0
34	115.0
35	143.0
36	168.0
37	194.0
38	231.0
39	276.0
40	346.0
41	420.0
42	525.0
43	650.0
44	780.0
45	930.0

Table 10 Variation of α_u factor with s_u/σ_z' ratio

s_u/σ_z'	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6
α_u	0.95	0.70	0.62	0.56	0.53	0.50	0.48	0.42
s_u/σ_z'	1.8	2.0	2.2	2.4	2.6	2.8	3.0	3.2
α_u	0.41	0.41	0.40	0.40	0.40	0.39	0.39	0.39

Figure 1 β-coefficient for piles in clay versus plasticity index (Fellenius, 2006).

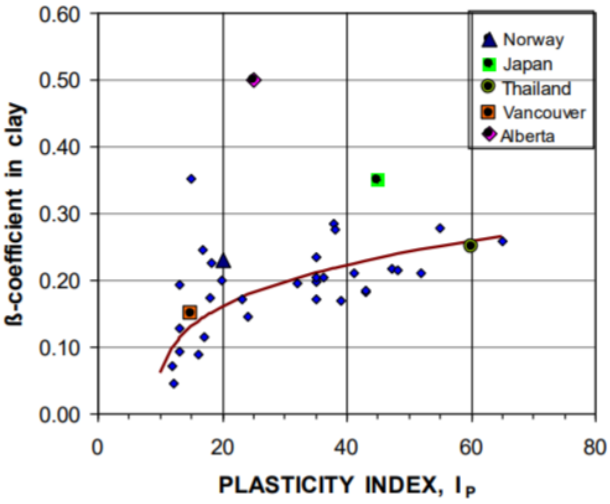


Figure 2 Variation of A'_x and A'_m (Figure a), B'_x, and B'_m (Figure b) with dimensionless depth Z.

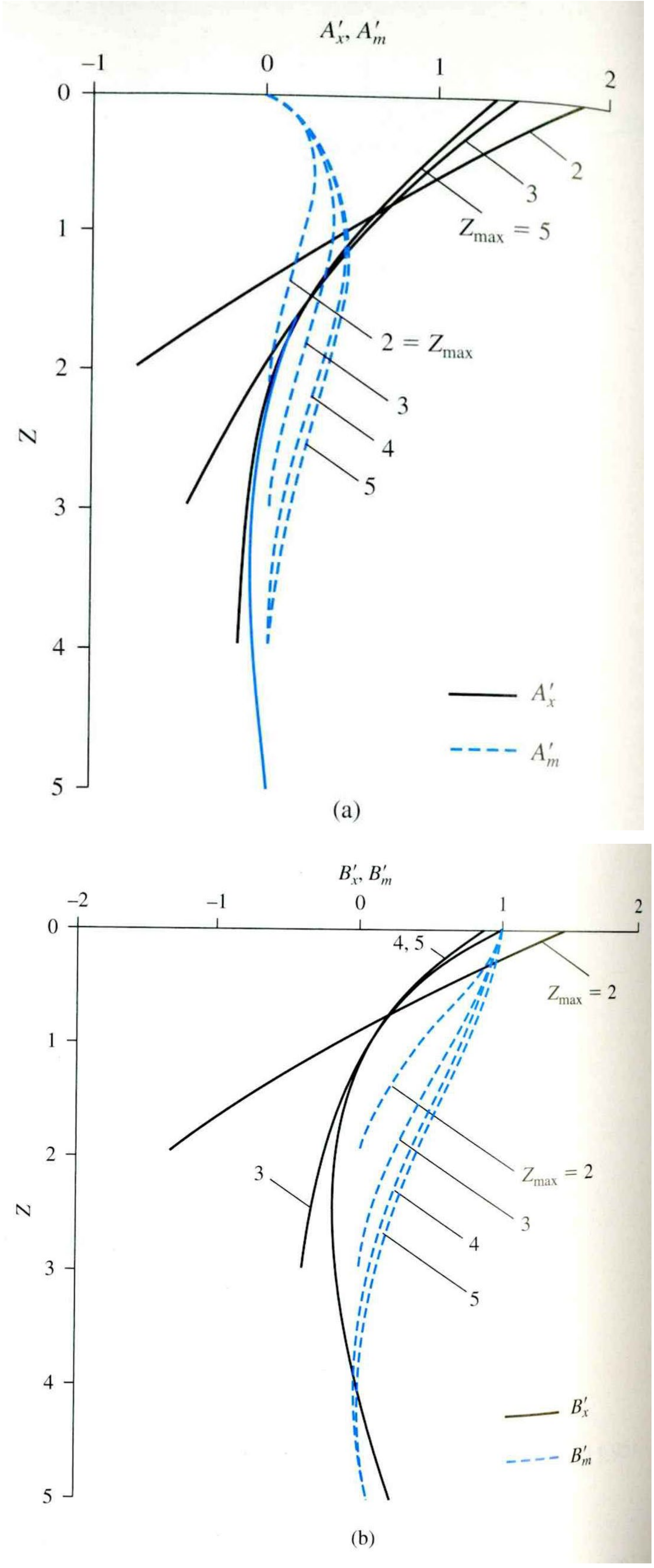


Figure 3 β-coefficient for piles in sand versus embedment length (Fellenius, 2006).

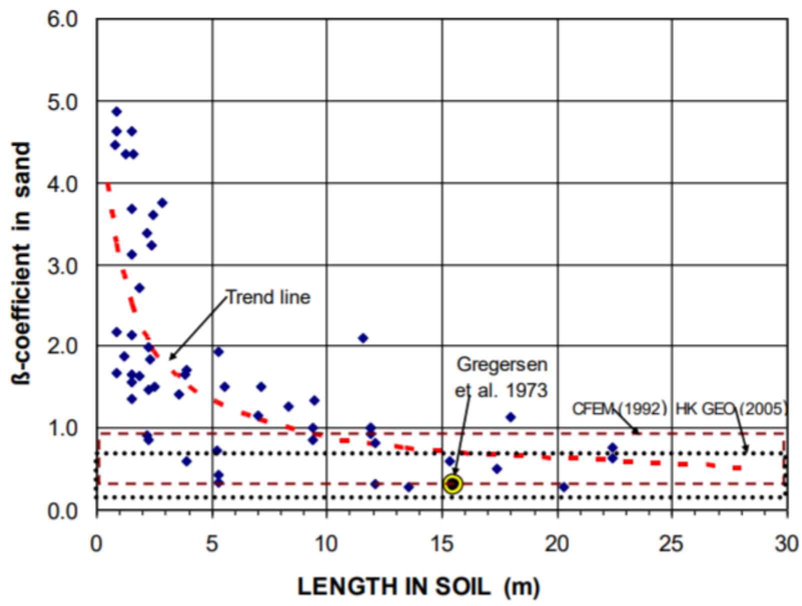


Figure 4 Variation of bearing capacity factor N_q in sand following Berezantsev's 1961 theory.

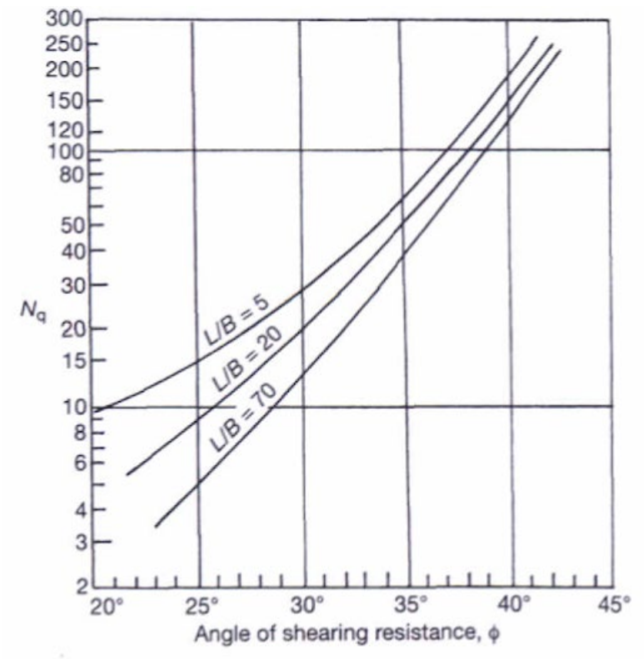


Figure 5 Variation of N_q^* in sand with *L/D* and friction angle ϕ' following Coyle & Castello's theory.

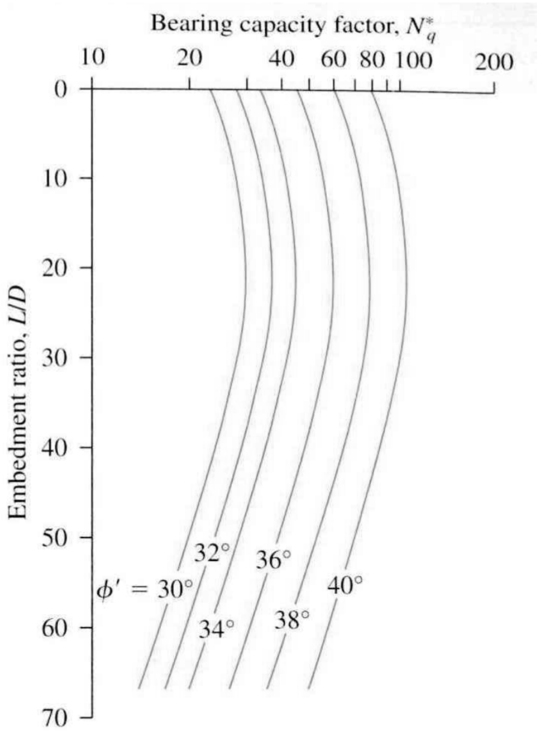
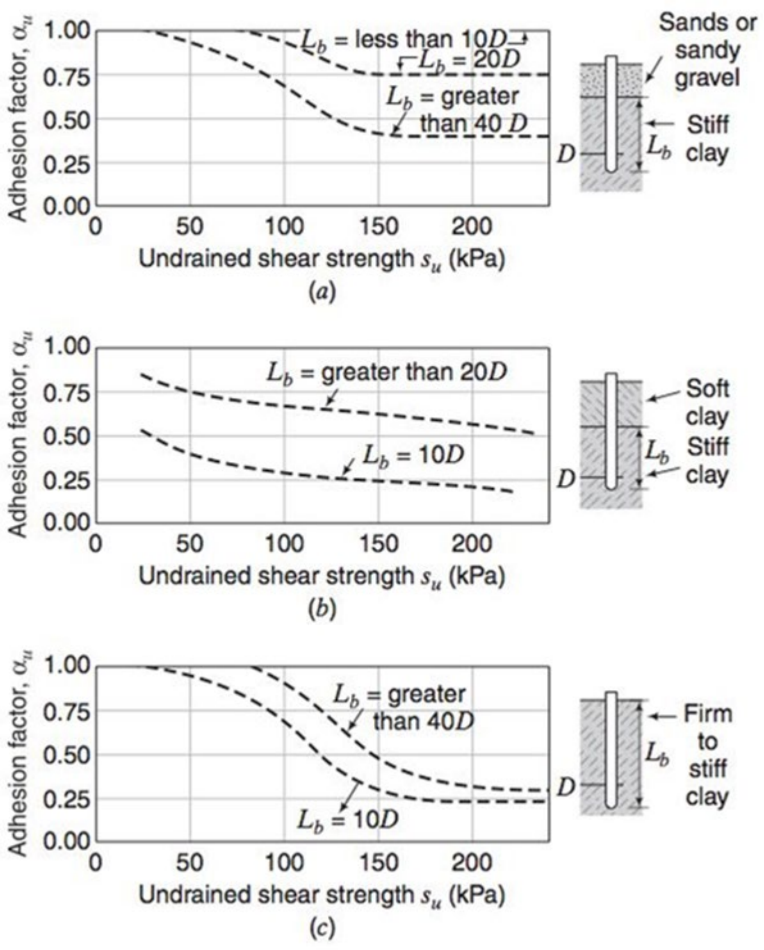


Figure 6 Variation of α_u factor for different fine-grained soil profiles.



▶ SOLUTIONS

P.1 ■ Solution

The component of resistance associated with skin friction, Q_f, is given by the product of unit friction stress f_s = 40 kPa and the surface area $A_s = p \times L$, where p = 4 × 0.6 = 2.4 m is the perimeter of the cross-section of the pile and L = 10 m is its length, so that A_s = 2.4×10 = 24 m^2. The skin friction load follows as

$$Q_f = f_s A_s = 40 \times 24 = 960 \text{ kN}$$

The contribution to resistance due to point bearing, Q_p, is the product of the unit end bearing resistance f_b = 400 kPa and the cross-sectional area of the pile A_b = 0.6^2 = 0.36 m^2; that is,

$$Q_p = f_b A_b = 400 \times 0.36 = 144 \text{ kN}$$

The ultimate load capacity is then Q_{ult} = 960 + 144 = 1104 kN, and the allowable load Q_{all}, given a factor of safety of 3, is calculated to be

$$Q_{\text{all}} = \frac{Q_{\text{ult}}}{FS} = \frac{1104}{3} = \boxed{368 \text{ kN}}$$

Since $Q_{\text{all}} > P$, the design is acceptable.

▶ The correct answer is **C**.

P.2 ■ Solution

Consider, first, side friction for the soft clay. Let $A_{s,1}$ be the surface area of the pile exposed to this soil, and $f_{s,1}$ = 18 kPa be the unit side friction resistance of this layer of soil. The side friction load $Q_{f,\text{ clay}}$ that this layer contributes to the load capacity of the pile is

$$Q_{f,\text{clay}} = f_{s,1} A_{s,1} = f_{s,1} \times \left(\pi \times \underbrace{D}_{=\text{Pile diameter}} \times \underbrace{L_1}_{\text{Soil layer depth}} \right)$$

$$\therefore Q_{f,\text{clay}} = 18 \times (\pi \times 0.425 \times 5) = 120 \text{ kN}$$

Similarly, suppose $A_{s,2}$ is the surface area of the pile that penetrates the sandy deposit underlying the clay, and $f_{s,2}$ = 80 kPa is its corresponding unit side friction resistance. The side friction load of this layer of sand is determined as

$$Q_{f,\text{sand}} = f_{s,2}A_{s,2} = 80\times(\pi\times0.425\times7.5) = 801\text{ kN}$$

The glacial till contributes to the friction load by an amount $Q_{f,\text{till}}$, which is calculated as

$$Q_{f,\text{till}} = f_{s,3}A_{s,3} = 350\times(\pi\times0.425\times2.5) = 1168\text{ kN}$$

It remains to compute the end-bearing resistance of the pile. This is done by multiplying the unit end-bearing resistance f_b = 3800 kPa of the soil at the toe of the pile, which is the glacial till, by the contact area A_b, so that $Q_p = f_bA_b$. Accordingly,

$$Q_p = f_bA_b = 3800\times\left(\frac{\pi\times D^2}{4}\right) = 3800\times\left(\frac{\pi\times0.425^2}{4}\right) = 539\text{ kN}$$

The ultimate load capacity of the pile is then

$$Q_{\text{ult}} = Q_{f,\text{clay}} + Q_{f,\text{sand}} + Q_{f,\text{till}} + Q_p$$
$$\therefore Q_{\text{ult}} = 120+801+1168+539 = 2628\text{ kN}$$

For a factor of safety of 3, the allowable load Q_{all} becomes

$$Q_{\text{all}} = \frac{Q_{\text{ult}}}{FS} = \frac{2628}{3} = \boxed{876\text{ kN}}$$

Since $Q_{\text{all}} > P$, the design is acceptable.

► The correct answer is **C**.

P.3 ■ Solution

A total stress analysis begs use of the α-method. In this approach, the undrained shear strength s_u is related to the skin frictional stress f_s by the adhesion coefficient, α_u, so that $f_s = \alpha_u s_u$. One quick way to obtain this coefficient is to resort to Table 1, which is provided in the NAVFAC guidelines. For the present soil, which is in the "medium stiff" range, and the current pile, which is made of timber, α_u ranges from 0.96 to 0.75, and linear interpolation yields α_u = 0.820. The skin friction Q_f is given by the product of f_s and the surface area of the shaft, i.e. perimeter × length. Accordingly,

$$Q_{f,1} = f_s\times A_s\times L = \alpha_u s_u\times\pi D\times L = (0.820\times40)\times(\pi\times0.45)\times28 = 1298\text{ kN}$$

Another approach would be to employ the rule prescribed by the American Petroleum Institute (1987), which suggests values of α_u as a function of s_u such that

$$\alpha_u = \begin{cases} 1-\left(\dfrac{s_u-25}{90}\right),\ 25 < s_u < 70\text{ kPa} \\ 1.0,\ s_u \le 25\text{ kPa} \\ 0.5,\ s_u \ge 70\text{ kPa} \end{cases}$$

which in the present case becomes

$$\alpha_u = 1-\left(\frac{40-25}{90}\right) = 0.833$$

This quantity is only marginally above the coefficient extracted from Table 1, and would produce a skin friction such that

$$Q_{f,2} = \alpha_u s_u\pi DL = 0.833\times40\times\pi\times0.45\times28 = 1319\text{ kN}$$

The previous method gave a slightly more conservative result. Another way to establish the α_u coefficient, as posed by Sladen (1992), is to employ the formula

$$\alpha_u = C\left(\frac{\sigma'_{vo}}{s_u}\right)^{0.45}$$

where σ'_{vo} is the vertical effective stress at mid-depth and $C \geq 0.5$ for driven piles, but we take C = 0.5 for a more careful design. Also,

$$\sigma'_{vo} = \frac{L}{2}\times\gamma' = \frac{28}{2}\times(20.5-9.81) = 149.7\,\text{kPa}$$

and hence

$$\alpha_u = 0.5\times\left(\frac{149.7}{40}\right)^{0.45} = 0.906$$

This is the largest α_u we've obtained, and should yield the most daring skin friction result,

$$Q_{f,3} = \alpha_u s_u \pi DL = 0.906\times 40\times\pi\times 0.45\times 28 = 1435\text{ kN}$$

The α-method thusly provides friction loads in the range of 1298 – 1435 kN. A second step in pile design is to compute the end-bearing load Q_p, which is given by

$$Q_{p,1} = N_c\,(s_u)_b\,A_b$$

where N_c is a bearing capacity coefficient that can be taken as 9 for saturated clays under undrained conditions (assuming they have undrained strength greater than 25 kPa), $(s_u)_b$ is the undrained cohesion of the soil surrounding the end of the pile, and A_b is the cross-sectional area. Substituting 9 for N_c, 40 kPa for $(s_u)_b$, and $\pi\times 0.45^2/4$ = 0.159 m^2 for A_b gives

$$Q_{p,1} = 9\times 40\times 0.159 = 57\text{ kN}$$

A second, more complex approach would be to apply Vesic's expansion of cavity theory (1977), according to which the end-bearing load for a saturated clay (ϕ = 0 condition) is

$$Q_{p,2} = N_c^*\,(s_u)_b\,A_b$$

where N_c^* is a coefficient given by

$$N_c^* = 1.33(\ln I_{rr} + 1) + 2.57$$

in which I_{rr} is the reduced rigidity index of the soil. At first, I_{rr} is to be computed with the formula

$$I_{rr} = \frac{I_r}{1+I_r\varepsilon_v}$$

in which I_r is the (non-reduced) rigidity index, and ε_v is the volumetric strain of the soil. A simplified, short-term approach would have ε_v = 0 and, consequently, $I_{rr} = I_r$. in undrained loading of saturated soil there is no volume change, but if construction stops, the excess pore pressures dissipates, consolidation occurs, and the volume changes. Volumetric strain can be estimated with the relationship

$$\varepsilon_v = 0.005\left(1-\frac{\phi'-25}{20}\right)\frac{(\sigma'_{vo})_b}{p_a}$$

where $(\sigma'_{vo})_b$ is the vertical effective stress at the level of the pile end, and p_a = 100 kPa is the atmospheric pressure. Computation of I_r, in turn, also involves knowledge of Young's modulus and Poisson's ratio for the soil, some values of which are provided in Tables 2 and 3 for future reference. Nonetheless, for a total stress analysis, we should have ε_v = 0, and the expression for I_{rr} simplifies to

$$I_{rr} = \frac{I_r}{1 + I_r \underbrace{\varepsilon_v}_{=0}} = \frac{I_r}{1} \rightarrow I_{rr} = I_r$$

Conveniently, a simple relationship for the rigidity index under saturated (ϕ = 0) conditions as a function of the normalized undrained cohesion is given in Table 4. In the present case, we have s_u/p_a = 40/100 = 0.40, so we can obtain, by means of linear interpolation, I_r = 109.5. Noting once again that $I_r = I_{rr}$ when there is no volume change, we can easily obtain the bearing capacity coefficient N_c^*,

$$N_c^* = 1.33(\ln I_{rr} + 1) + 2.57 = 1.33(\ln 109.5 + 1) + 2.57 = 10.1$$

We can then determine the end-bearing load $Q_{p,2}$,

$$Q_{p,2} = N_c^* (s_u)_b A_b = 10.1 \times 40 \times 0.159 = 64 \text{ kN}$$

. In summary, the ultimate load imparted on the pile is in the range

$$Q_{\text{ult}} = Q_f + Q_p = (1298 ; 1435) + (53 ; 64) = (1351 ; 1499) \text{kN}$$

and the allowable load Q_{all} will be in the range

$$Q_{\text{all}} = \frac{Q_{\text{ult}}}{FS} = \frac{(1351 ; 1499)}{1.5} = \boxed{(901 ; 999) \text{ kN}}$$

The interval that encompasses this set of values is (850 ; 1050) kN.

► The correct answer is **B**.

P.4 ■ Solution

Part A: An effective stress analysis can be done by dint of the β-method. In this approach, the skin friction of the pile is established by means of the equation

$$f_s = \mu_x \sigma_h' = \underbrace{\mu_x}_{=\tan\delta} K\sigma_{vo}' = \underbrace{K \tan\delta}_{=\beta} \times \sigma_{vo}'$$
$$\therefore f_s = \beta\sigma_{vo}'$$

That is, the unit skin resistance f_s is obtained by applying a friction factor, μ_x, to the horizontal effective stress, σ_h', which in turn is related to its vertical counterpart by the coefficient of lateral earth pressure, K. The friction factor is usually given as the tangent of the pile-soil skin friction angle δ. Finally, the product $K \tan\delta$ is often condensed in a single constant of proportionality β, hence the name of the method in question. One of the simplest means to compute this factor is to resort to tabulated values of K and δ, such as those given in Tables 5 and 6, which are based on data by NAVFAC. For a driven displacement pile, we could take a cautious K = 1.0; also, for an interface between concrete and soil, we could apply a conservative δ = 0.9ϕ' = 0.9×30° = 27°. Combining the two quantities produces a factor $\beta = K\tan\delta = 1.0 \times \tan 27^\circ = 0.509$. The ensuing unit friction resistance is, accordingly,

$$f_s = \beta\sigma_{vo}' = \beta \times \left(\frac{L}{2}\gamma'\right) = 0.509 \times \left[\frac{10}{2} \times (19.5 - 9.81)\right] = 24.7 \text{ kPa}$$

Note that the effective stress is conventionally taken at mid-depth of the pile – in this case, 10/2 = 5 m. The skin friction load $Q_{f,1}$ is then

$$Q_{f,1} = f_s \times p \times L = 24.7 \times (4 \times 1) \times 10 = 988 \text{ kN}$$

One of the earlier expressions for this the β factor, valid for fine-grained soils, is due to Burland (1973) and makes use of the at-rest lateral earth pressure coefficient,

$$\beta = K \tan\delta = K_o^{oc} \tan\delta = (1 - \sin\phi')(OCR)^{0.5} \tan\delta$$

Substituting ϕ' = 30°, OCR = 1.0 for a normally consolidated clay, and taking a conservative $\delta = 0.9\phi' = 0.9 \times 30 = 27°$ as the interface angle between soil and concrete, we get

$$\beta = \left(1 - \sin 30^{\circ}\right) \times 1.0^{0.5} \times \tan 27^{\circ} = 0.255$$

We observe that this factor is only about 50% of the β value used for the previous skin friction computation, suggesting that either the previous method is exceedingly heedless or that the present one is too conservative. One investigator (Fellenius, in his *Basics of Foundation Design*) has proposed a reasonable interval of β values ranging from 0.15 to 0.35 for clay (Table 7), which would preclude the first value we obtained from being used. Nonetheless, the same author acknowledges that β values can deviate substantially from his range. The unit skin friction resistance with this β coefficient is

$$f_s = \beta\sigma'_{vo} = 0.255 \times \left(\frac{L}{2}\gamma'\right) = 0.255 \times \left[\frac{10}{2} \times (19.5 - 9.81)\right] = 12.4 \text{ kPa}$$

The skin friction $Q_{f,2}$ is then

$$Q_{f,2} = f_s \times p \times L = 12.4 \times (4 \times 1) \times 10 = 496 \text{ kN}$$

A third approach to establish unit friction in the β-method, valid for clays, is to resort to the trend line given in Figure 1. For a plasticity index PI = 40, there corresponds a β coefficient approximately equal to 0.225. The unit skin friction is found as

$$f_s = \beta\sigma'_{vo} = 0.225 \times \left(\frac{L}{2}\gamma'\right) = 0.225 \times \left[\frac{10}{2} \times (19.5 - 9.81)\right] = 10.9 \text{ kPa}$$

The corresponding skin friction $Q_{f,3}$ is

$$Q_{f,3} = f_s \times p \times L = 10.9 \times (4 \times 1) \times 10 = 436 \text{ kN}$$

A reasonable approach to determine a final value of β would be to reject the larger value, i.e., the one obtained from sample NAVFAC data, and instead pick the average of Q_f values obtained from the other two methods, yielding

$$Q_f = \frac{\sum_i Q_{f,i} - Q_{f,1}}{2} = \frac{436 + 496}{2} = 466 \text{ kN}$$

Nevertheless, we shall include $Q_{f,1}$ in our final computations to give an upper limit for the ultimate load. The second step is to assess the end-bearing capacity, Q_p. As before, for piles in saturated clays under undrained conditions, the end-bearing load can be approximated as

$$Q_p = 9(s_u)_b A_b$$

where $(s_u)_b$ is the undrained cohesion of the soil at the pile end, and A_b is the cross-sectional area of the pile. Substituting $(s_u)_b$ = 60 kPa and A_b = 1×1 = 1 m^2 gives

$$Q_{p,1} = 9 \times 60 \times 1 = 540 \text{ kN}$$

In an effective stress analysis, however, the contribution of the pile end to loading is best determined with a relationship of the form

$$Q_p = f_b A_b = N_q (\sigma'_{vo})_b A_b$$

where $f_b = N_q(\sigma'_{vo})_b$ is the base resistance stress, given by the product of the effective vertical stress at the end of the pile, $(\sigma'_{vo})_b$, and a bearing capacity coefficient, N_q, which can be obtained by a number of different approaches. An approximate range of N_q coefficients is provided in Table 8. In compiling correlations and trend lines in a N_q versus ϕ' graph, one author found that some of the lowest values were reported by Janbu's 1976 expression, namely,

$$N_q = \left(\tan\phi' + \sqrt{1 + \tan^2\phi'}\right)^2 e^{2\psi \tan\phi'}$$

where ψ is the angle of plastification associated with pile penetration, which should not exceed $\pi/3$ for soft fine-grained soils (after all, in such cases the pile tip is capable of piercing the soil without causing significant plastic zones). Taking ψ = $\pi/3$ and ϕ' = 30° gives

$$N_q = \left(\tan 30^\circ + \sqrt{1+\tan^2 30^\circ}\right)^2 e^{2\times\frac{\pi}{3}\times\tan 30^\circ} = 10.05$$

It should be regarded that N_q is applicable only up to a certain critical depth of penetration into the end bearing layer, that is, a *critical depth*, L_c, that is dependent on friction angle, soil compressibility, and method of installation. One expression that approximates L_c in the range of friction angles 20° ≤ ϕ' ≤ 40° is

$$L_e \le 0.556D\exp\left(0.085\phi'\right)$$

which in the present case becomes

$$L_e \le 0.556\times 1\times\exp\left(0.085\times 30\right) = 7.12\text{ m}$$

Hence, we will not use the full length of 10 m of the pile in computing the end-bearing resistance. This affects the effective stress $(\sigma'_{vo})_b$, which will be such that

$$\left(\sigma'_{vo}\right)_{b,\text{mod}} = L_c\gamma' = 7.12\times\left(19.5-9.81\right) = 69\text{ kPa}$$

The end-bearing resistance with Janbu's factor is then

$$Q_{p,2} = 10.05\times 69\times 1 = 693\text{ kN}$$

A third option would be to resort to Vesic's expansion of cavities-based approach, which has already been introduced in the previous example. In this method, the end-bearing capacity of the pile is given by

$$Q_p = N_c^*\left(\sigma'_{vo}\right)_{b,\text{mod}} A_b$$

where the modified effective stress $(\sigma'_{vo})_{b,\text{mod}}$ = 69 kPa replaces the undrained shear strength $(s_u)_b$ at the end of the pile. Coefficient N_c^*, as before, is given by

$$N_c^* = 1.33\left(\ln I_{rr}+1\right)+2.57$$

in which the reduced rigidity index is

$$I_{rr} = \frac{I_r}{1+I_r\varepsilon_v}$$

wherein I_r is the (non-reduced) rigidity index and ε_v is the volumetric strain. In undrained loading of saturated soil there is no volume change, but if construction stops the excess pore pressures dissipate, consolidation occurs, and the volume changes. Thus, a long-term, effective stress-based analysis would include volumetric strain, i.e., $\varepsilon_v \neq 0$. As stated before, ε_v can be approximated with the expression

$$\varepsilon_v = 0.005\left(1-\frac{\phi'-25}{20}\right)\frac{\left(\sigma'_{vo}\right)_b}{p_a}$$

where $(\sigma'_{vo})_b$ is the vertical effective stress at the level of the pile tip, and p_{atm} = 100,000 kPa is the atmospheric pressure. We have

$$\left(\sigma'_{vo}\right)_b = L\gamma' = 10\times\left(19.5-9.81\right) = 96.9\text{ kPa}$$

Substituting the available data in the expression for ε_v gives

$$\varepsilon_v = 0.005\times\left[1-\frac{(30)-25}{20}\right]\times\frac{96,900}{100,000} = 0.00363$$

We then proceed to compute I_r, which is given by

$$I_r = \frac{E_s}{2(1+\mu_s)(\sigma'_{vo})_b \tan\phi'}$$

in which E_s is the modulus of elasticity of the soil, and μ_s is Poisson's ratio for the soil. We were given E_s = 50 MPa, which is a reasonable value for a somewhat hard clay, and μ_s = 0.40. Substituting the available data in the foregoing relation yields

$$I_r = \frac{E_s}{2(1+\mu_s)(\sigma'_{vo})_b \tan\phi'} = \frac{50,000}{2\times(1+0.4)\times 96.9\times\tan 30^\circ} = 319.2$$

We can then obtain I_{rr}

$$I_{rr} = \frac{I_r}{1+I_r\varepsilon_v} = \frac{319.2}{1+319.2\times 0.00363} = 147.9$$

Then, we can return to the expression for the bearing capacity coefficient N_c^*, i.e.,

$$N_c^* = 1.33(\ln 147.9+1)+2.57 = 10.55$$

Accordingly, the end-bearing load as computed with Vesic's formula is

$$Q_{p,3} = N_c^*(\sigma'_{vo})_{b,\text{mod}} A_b = 10.55\times 69\times(1^2) = 728\text{ kN}$$

There is significant disparity between the three approaches for end-bearing capacity. The initial short-term approach gave us a predicted end-bearing load $Q_{p,1}$ = 540 kN; the calculation based on Janbu's expression for the bearing capacity coefficient produced a result $Q_{p,2}$ = 693 kN, which is 28% higher than the first estimate; finally, a computation based on Vesic's expansion of cavities approach gave $Q_{p,3}$ = 728 kN, which in turn is almost 35% more than the first estimate. In summary, the ultimate load imparted on the pile is in the range

$$Q_{\text{ult}} = Q_f + Q_p = (436\ ;\ 496)+(540\ ;\ 728) = (976\ ;\ 1224)\text{ kN}$$

The corresponding range for the allowable load capacity, given a factor of safety FS = 1.5, is

$$Q_{\text{all}} = \frac{Q_{\text{ult}}}{FS} = \frac{(976\ ;1224)}{1.5} = \boxed{(651\ ;816)\text{ kN}}$$

The interval that encompasses this set of values is (600 ; 900) kN.

► The correct answer is **B**.

Part B: In contrast with what is usually thought with granular soils, the coefficient of subgrade reaction for cohesive soils can be estimated to be constant with depth, and could be approximated with the following expression, proposed by Vesic in 1961,

$$k_s = 0.65\times\left(\frac{E_sD^4}{E_pI_p}\right)^{\frac{1}{12}}\times\left(\frac{E_s}{1-\mu_s^2}\right)$$

where E_s and E_p are Young's modulus for the soil and the pile material, respectively, μ_s is Poisson's ratio for the soil, D is the width of the pile and I_p is the moment of inertia for the pile cross-section. Substituting E_s = 50 MPa, E_p = 30,000 MPa, μ_s = 0.40, D = 1 m, and $I_p = Bd^3/12 = 1\times 1^3/12$ = 0.0833 m^4 gives

$$k_s = 0.65\times\left(\frac{E_sD^4}{E_pI_p}\right)^{\frac{1}{12}}\times\left(\frac{E_s}{1-\mu_s^2}\right) = 0.65\times\left[\frac{50\times 1^4}{30,000\times\left(\frac{1\times 1^3}{12}\right)}\right]^{1/12}\times\left(\frac{50,000}{1-0.40^2}\right) = 27,930\text{ kN/m}^3$$

We can now return to the expression for R and substitute E_p = 30,000 MPa, I_p = 0.0833 m^4, and the just obtained k_s = 27,930 kN/m^3 = 27.93 MN/m^3, giving

$$R=\left(\frac{30,000\times 0.0833}{27.93}\right)^{\frac{1}{4}}=3.076$$

The lateral displacement of the pile as a function of depth can be described with the equation

$$x_z(z)=A'_x\frac{Q_gR^3}{E_pI_p}+B'_x\frac{M_gR^2}{E_pI_p}$$

where Q_g and M_g are the lateral force and the moment applied at the ground surface (z = 0), whereas A'_x and B'_x are coefficients whose variations with the dimensionless depth $Z=z/R$, with $Z_{\max}=L/R$ as a parameter, are provided in Figure 2. Similarly, the variation of moment $M_z(z)$ with depth is given by the expression

$$M_z(z)=A'_mQ_gR+B'_mM_g$$

in which coefficients A'_m and B'_m are coefficients provided in the aforementioned figure. Referring to the deflection equation, we have zero moment at the top of the pile, so $x_z(z)$ reduces to

$$x_z(z)=A'_x\frac{Q_gR^3}{E_pI_p}$$

which can be solved for the lateral load Q_g,

$$x_z(z)=A'_x\frac{Q_gR^3}{E_pI_p}\rightarrow Q_g=\frac{x_z(z)E_pI_p}{A'_xR^3}$$

To obtain Q_g, we require coefficient A'_x, which in turn is a function of $Z_{\max}=L/R$ = 10/3.076 = 3.25. The results stemming from the theory used here would be more precise if the pile were such that $L\geq 5T$, at which point it is considered a *long pile;* (T is a coefficient analogous to R, the difference being that it applies for the calculation of deflections of piles in *granular* soils). But we will apply the theory outlined here nonetheless. Charting Z = 0 in Figure 2, we read coefficient $A'_x\approx$ 1.458. Substituting this coefficient, along with the allowable deflection $x_{z,\max}$ = 8 mm, E_p = 30 × 10^6 kPa, I_p = 0.0833 m^4, and R = 3.076, the allowable lateral load is determined as

$$Q_{g,\text{all}}=\frac{0.008\times 30,000,000\times 0.0833}{1.458\times 3.076^3}=471\text{ kN}$$

The allowable lateral load on the pile is just short of 470 kilonewtons. However, this magnitude of Q_g is based on the *limiting displacement condition* only, and a full design should also consider the allowable load by the *moment capacity* of the pile. In this case, we resort to the function for $M_z(z)$, which, with M_g = 0, simplifies to

$$M_z(z)=A'_mQ_gR+\cancel{B'_mM_g}$$

$$\therefore M_z(z)=A'_mQ_gR$$

$$\therefore Q_g=\frac{M_z(z)}{A'_mR}$$

The maximum allowable moment the pile can carry is given by the product of yield stress and section modulus,

$$M_{z,\max}=\sigma_YS=\sigma_Y\frac{I_p}{\bar{y}}$$

where $\bar{y}$ is the distance from the neutral axis of the section to its upper or lower extremity, which in this case equals 1.0/2 = 0.5 m, I_p = 0.0833 m^4, and the yield stress $\sigma_Y=\epsilon E$, where the strain at yield conditions for concrete is about 0.2%, so it follows that

$$M_{z,\max} = \left(\frac{0.2}{100}\times 30\times 10^{9}\right)\times\frac{0.0833}{0.5} = 10^{7}\ \mathrm{N\cdot m} = 10{,}000\ \mathrm{kN\cdot m}$$

From Figure 2, using the curve for $Z_{\max} = 3$ as a reference, the largest A'_m we can obtain is around 0.416. Substituting this coefficient, the accompanying moment, and the value of R in the previous formula, we obtain the allowable load as

$$Q_{g,\mathrm{all}} = \frac{M_{z,\max}}{A'_m R} = \frac{10{,}000}{0.416\times 3.076} = 7815\ \mathrm{kN}$$

This is much greater than the allowable load predicted by the limit displacement theory. This is comprehensible, since the bulky cross-section of this pile, with one meter of width, results in high tolerable moments and, consequently, high tolerable loads. Nevertheless, $Q_{g,\,\mathrm{all}}$ = 471 kN controls.

► The correct answer is **B**.

P.5 ■ Solution

Part A: An effective stress analysis of skin friction load on a pile should involve the β-method. Recall that the β-factor, in the formula proposed by Burland, is given by

$$\beta = (1-\sin\phi')\tan\delta$$

where δ is the soil-pile interfacial angle, which, following Table 6, ranges from (2/3)ϕ' to 0.8ϕ' for steel piles; let us take a conservative δ = (2/3) × 36° = 24°. Then,

$$\beta = (1-\sin\phi')\tan\delta = (1-\sin 36^{\circ})\tan 24^{\circ} = 0.184$$

Comparing this to the range of values in Table 7, this may be an underestimate. However, the variation of the β factor in granular soils is broader than with other types of soil, so there is no need to outright reject our computed value. The unit friction resistance is, in this case,

$$f_s = \beta\sigma'_{vo} = 0.184\times\left(\frac{L}{2}\gamma'\right) = 0.184\times\left[\frac{18}{2}\times(18.5-9.81)\right] = 14.4\ \mathrm{kPa}$$

and the corresponding load follows as

$$Q_{f,1} = f_s\times p\times L = 14.4\times\left[2\times(0.356+0.376)\right]\times 18 = 380\ \mathrm{kN}$$

This is not greater than 400 kN, and hence denies statement 1. Another possibility to assess the friction resistance by the β-method in sandy soils is to resort to the formula recommended by Bhushan (1982),

$$\beta = 0.18+0.65D_r$$

where D_r is the relative density of the sand expressed in decimal form. We were given D_r = 0.26, which corresponds to a loose sand, albeit one close to the interval for medium-dense granular soils. Thus, β is given by

$$\beta = 0.18+0.65D_r = 0.18+0.65\times 0.26 = 0.349$$

This is almost 90% above the estimate obtained from the Burland formula, and serves to illustrate the inexactitude of frictional resistance calculations for piles in sand. Nevertheless, proceeding with this quantity, the unit friction resistance would be

$$f_s = \beta\sigma'_{vo} = 0.349\times\left(\frac{L}{2}\gamma'\right) = 0.349\times\left[\frac{18}{2}\times(18.5-9.81)\right] = 27.3\ \mathrm{kPa}$$

and the corresponding load is

$$Q_{f,2} = f_s\times p\times L = 27.3\times\left[2\times(0.356+0.376)\right]\times 18 = 719\ \mathrm{kN}$$

A third option to determine the β coefficient is to employ the trend line traced in the β-coefficient versus pile length plane of Figure 3, which is Fellenius's condensation of a number of experimental results and design recommendations established over the years. Entering L = 18 m in the red trend line given in this graph, we read β = 0.680. This is almost twice the value of β obtained with the Bhushan formula. The corresponding unit skin friction is

$$f_s = \beta\sigma'_{vo} = 0.680\times\left(\frac{L}{2}\gamma'\right) = 0.680\times\left[\frac{18}{2}\times(18.5-9.81)\right] = 53.2\text{ kPa}$$

The skin friction load follows as

$$Q_{f,3} = f_s\times p\times L = 53.2\times\left[2\times(0.356+0.376)\right]\times 18 = 1402\text{ kN}$$

Notice that the result obtained with the Bhushan formula is only about (719/1402) × 100 = 51.3% of this result, thus confirming statement 2. Observe, by the way, that the skin friction on the H-pile could be close to 400 kN as predicted by the Burland $K\tan\delta$ formula; or it could be around 700 kN as anticipated by the Bhushan formula; or, yet, it may very well be as high as 1400 kN, if we are to follow the trend line outlined by Fellenius.

We now turn to the end-bearing resistance that acts on the pile. One simple approach to assess the end-bearing resistance in a sand deposit is due to Berezantsev, who proposed, in 1961, that Q_p be given by

$$Q_p = f_b A_b = N_q\left(\sigma'_{vo}\right)_b A_b$$

where N_q is a bearing capacity factor that varies with the friction angle ϕ' as given in Figure 4. $(\sigma'_{vo})_b$ is the vertical effective stress at the pile end, and A_b is the cross-sectional area of the pile (which, in the case of an H-pile, is in fact the area of the rectangle that circumscribes the cross-section, thus including the soil plugs). Since we have L/B = 18/0.356 ≈ 50, we could trace an intermediary curve between those that represent embedment ratios L/B = 20 and L/B = 70 and see which N_q corresponds to ϕ' = 36°. This procedure yields approximately N_q = 50. We can then compute the end-bearing resistance,

$$Q_{p,1} = N_q\left(\sigma'_{vo}\right)_b A_b = 50\times\left[18\times(18.5-9.81)\right]\times(0.356\times0.376) = 1047\text{ kN}$$

Another simple approach to assess the end-bearing resistance in a sand deposit is due to Meyerhof, who proposed, in 1976, that Q_p be given by

$$Q_p = f_b A_b = N_q^*\left(\sigma'_{vo}\right)_b A_b \le q_l A_b$$

where N_q^* once again is a bearing capacity factor that varies with the friction angle ϕ', this time following the interpolated values given in Table 9, while $(\sigma'_{vo})_b$ and A_b are the same as before. For ϕ' = 36°, we have N_q^* = 168. Accordingly,

$$Q_p = N_q^*\left(\sigma'_{vo}\right)_b A_b = 168\times\left[18\times(18.5-9.81)\right]\times(0.356\times0.376) = 3517\text{ kN}$$

As noted in the foregoing equation, however, Meyerhof's theory stipulates that the end-bearing stress should not be greater than a limiting value $q_l A_b$, where q_l is a unit point bearing resistance determined by the formula

$$q_l = 0.5 p_a N_q^* \tan\phi'$$

In which p_a = 100 kPa is the atmospheric pressure. Accordingly, the maximum end-bearing load is

$$q_l A_b = \left(0.5 p_a N_q^*\tan\phi'\right)\times A_b = \left(0.5\times100\times168\times\tan 36^\circ\right)\times(0.356\times0.376) = 817\text{ kN}$$

This is sensibly less than the initial value of Q_p that we computed; indeed, the point resistance obtained by Meyerhof's initial formula may be as much as seven times higher than the maximum $q_l A_b$ established by the same theory. Therefore, we take $Q_{p,2}$ = 817 kN. Notice that the end-bearing resistances computed by means of Berezantsev's and Meyerhof's theories are within 400 kN of each other (that is, $Q_{p,1} - Q_{p,2} = 1047 - 817 = 230$ kN). A third method to determine the contribution of the pile's end to resistance is that of Coyle & Castello, who have analyzed 24 large-scale field load tests of driven piles in sand and suggested that the end-bearing resistance be computed with the expression

$$Q_p = N_q^* \left(\sigma'_{vo}\right)_b A_b$$

which would be identical to the Meyerhof formula were it not for the bearing capacity factor N_q^*, which, in this case, varies with the embedment ratio L/D and the friction angle ϕ' in accordance with Figure 5. In the present case, we have L/D = 18/0.356 ≈ 50 and ϕ' = 36°. Entering these quantities in the aforementioned figure, we read N_q^* = 40. Substituting this factor and other pertaining variables in the expression for Q_p gives

$$Q_{p,3} = 40\times\left[18\times\left(18.5-9.81\right)\right]\times\left(0.356\times0.376\right) = 838 \text{ kN}$$

which is less than 925 kN and thus implies that statement 4 is false. The end-bearing resistances we've obtained range from 817 kN to 1047 kN; inasmuch as the results from the Meyerhof (= 817 kN) and the Coyle & Castello (= 838 kN) approaches are quite close and the result stemming from Berezantsev's chart (= 1047 kN) deviates substantially from them, a prudent designer could simply reject the latter and take the average of the former two; after all, the Berezantsev data is the oldest of the three methods. This approach would lead to an ultimate end-bearing capacity $Q_{p,\text{ult}}$ = (817 + 838)/2 = 828 kN.

► Statements **2** and **3** are true, whereas statements **1** and **4** are false.

Part B: Since the soil deposit is homogeneous, it can be modeled by a single shear modulus, which in the case of an isotropic material is given by

$$\overline{G}_s = G_s = \frac{E_s}{2\left(1+\mu_s\right)}$$

We were given E_s = 15 MPa and μ_s = 0.25. Accordingly,

$$G_s = \frac{E_s}{2\left(1+\mu_s\right)} = \frac{15}{2\times\left(1+0.25\right)} = 6 \text{ MPa}$$

Also, we have μ_s = 0.30, the interfacial angle $\delta = (2/3)\phi' = (2/3) \times 36^\circ$ = 24°, the embedment ratio L/D = 18/0.356 = 50.6, and E_p = 200,000 MPa. With these data, factor η becomes

$$\eta = 0.30\times\tan 24^\circ\times\left(50.6\right)\times\left(\frac{6}{200{,}000}\right) = 2.03\times10^{-4}$$

If we were to compute skin friction with a β-factor using Bhushan's formula, we would obtain a unit skin friction f_s = 27.3 kPa, as demonstrated in the previous part. Thus, if we substitute f_s, the embedment ratio L/D = 50.6, and η = 2.03×10^{-4} in the formula we were given, the uplift skin frictional stress becomes

$$f_{s,\text{up}} = f_s\left[1-\log_{10}\frac{100}{\left(L/D\right)}\right]\left[1-8\eta+25\eta^2\right]$$

$$\therefore f_{s,\text{up}} = 27.3\times\left[1-0.2\log_{10}\frac{100}{\left(50.6\right)}\right]\left[1-8\times\left(2.03\times10^{-4}\right)+25\times\left(2.03\times10^{-4}\right)^2\right] = 25.6 \text{ kPa}$$

Consequently, the ultimate uplift skin friction load is such that

$$\left(Q_f\right)_{\text{up}} = f_{s,\text{up}}\times p\times L = 25.6\times\left[2\times\left(0.356+0.376\right)\right]\times18 = 675 \text{ kN}$$

The reduction factor R that, when applied to the formula for the downward skin friction Q_f, would produce the same force as that obtained with the β-method and Bhushan's formula in the previous part, which was about 719 kN, is

$$\left(Q_f\right)_{\text{up}} = R \times Q_{f,2} \rightarrow R = \frac{\left(Q_f\right)_{\text{up}}}{Q_{f,2}} = \frac{675}{719} = \boxed{0.939}$$

That is to say, the uplift skin friction has about 94% of the intensity of the downward frictional resistance.

► The correct answer is **D**.

Part C: The skin friction resistance computed using the β-method conjugated with Bhushan's formula was Q_f = 719 kN, whereas the end-bearing resistance obtained with Meyerhof's technique for granular soils was Q_p = 817 kN. With *FS* = 1.5, the working loads become $Q_{f,\text{all}}$ = 479.3 kN and $Q_{p,\text{all}}$ = 544.7 kN. The length of the pile is L = 18 m, A_b = 0.356 × 0.376 = 0.134 m^2, and E_p = 200,000 MPa. Substituting these in the formula for settlement from elastic theory, $s_{e,1}$, we obtain

$$s_{e,1} = \frac{\left(544.7 + 0.6 \times 479.3\right) \times 18}{0.134 \times \left(200 \times 10^6\right)} = 0.000559 = 0.56 \text{ mm}$$

Next, let us consider the contribution to settlement due to the load carried by the pile shaft. Here, we also need D = 0.356 m, p = 2(0.356 + 0.376) = 1.464 m, E_s = 15 MPa, and μ_s = 0.25. We also require influence factor I_{ws}, which is determined as

$$I_{ws} = 2 + 0.35\sqrt{\frac{L}{D}} = 2 + 0.35\sqrt{\frac{18}{0.356}} = 4.49$$

Substituting in the expression for $s_{e,2}$ gives

$$s_{e,2} = \left(\frac{479.3}{1.464 \times 18}\right) \times \left(\frac{0.356}{15{,}000}\right) \times \left(1 - 0.25^2\right) \times 4.49 = 0.00182 = 1.82 \text{ mm}$$

It remains to calculate the contribution to settlement caused by load carried by the pile end. Here, we require the ultimate unit end-bearing resistance $q_{p,\text{ult}} = Q_{p,\text{ult}}/A_b$ = 817/0.134 = 6097 kPa and the empirical coefficient C_p = 0.03. Substituting the pertaining variables brings to

$$s_{e,3} = \frac{544.7 \times 0.025}{0.356 \times 6097} = 0.00627 = 6.27 \text{ mm}$$

In a simplified analysis, the total settlement S of the pile would be simply the sum of the contribution of each component s_e, or

$$S = s_{e,1} + s_{e,2} + s_{e,3} = 0.56 + 1.82 + 6.27 = \boxed{8.65 \text{ mm}}$$

The ASCE's 1997 *Standard Guidelines for the Design and Installation of Pile Foundations* stipulates that one way to conceive a failure load is to define it as the load that causes a pile tip movement of 3.8 mm plus one percent of the tip diameter. In the present case, this would correspond to a tip movement of 3.8 + 0.01 × 356 = 7.36 mm, which is 15% less than the settlement that we have computed. Thus, if we were to follow such regulations, a larger factor of safety and smaller working loads would have to be employed.

► The correct answer is **B**.

P.6 ■ Solution

The allowable load, given a factor of safety of 2, is

$$Q_{\text{all}} = \frac{Q_{\text{ult}}}{FS} = \frac{Q_f + Q_b}{FS} = \frac{300 + 580}{2} = 440 \text{ kN}$$

We have δ_u = 10 mm for side friction and δ_u = 127/10 = 12.7 mm for end-bearing resistance. Also, we make use of the exponents a = 0.40 and b = 0.60 in the formulas just provided. The settlement due to elastic compression is determined as

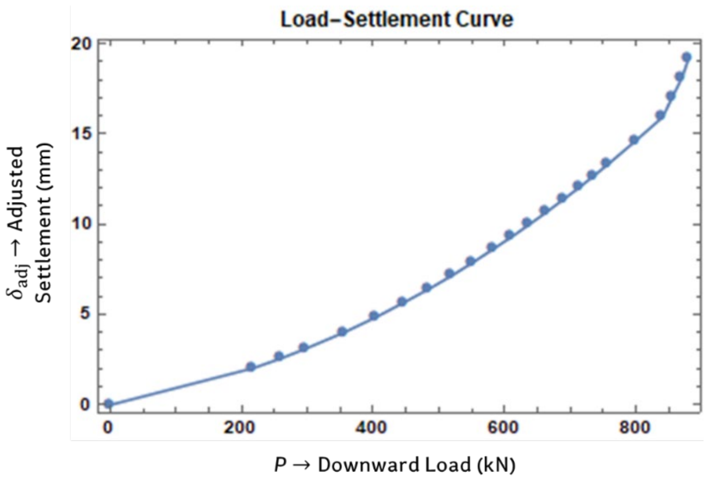
$$\delta_e = \frac{Pz_c}{A_{\text{sec}}E_p} = \frac{P\times 0.75L}{A_{\text{sec}}E_p} = \frac{0.75L}{A_{\text{sec}}E_p}P$$

$$\therefore \delta_e = \frac{0.75\times 15}{0.00798\times 200,000}P = 0.007049P\ [\text{mm}]$$

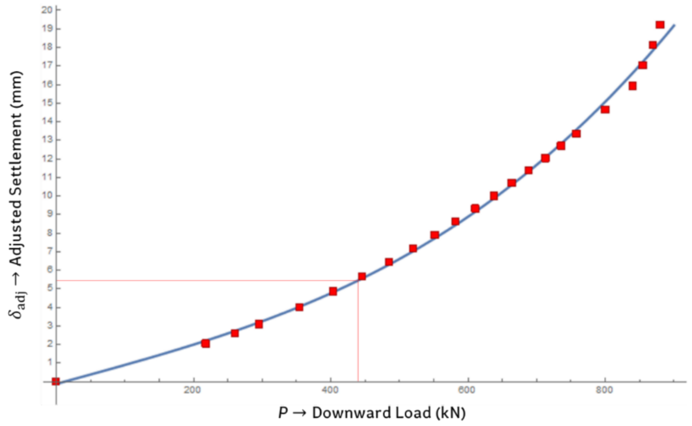
The following table summarizes the calculations.

	Side Friction			End-Bearing					
δ (mm)	δ/δu	(fs)m/fs	(fs*As)m (kN)	δ/δu	(fb)m/fb	(fb*Ab)m (kN)	*P* (kN)	δe (mm)	δ_Adj (mm)
Settlement	Normalized Settlement (SF comp.)	Ratio of mobilized unit SF to unit SF	Mobilized skin friction load	Normalized Settlement (EB comp.)	Ratio of mobilized unit EBR to unit EBR	Mobilized end-bearing load	Mobilized load = SF + EBR	Settlement due to Elastic Compression	Adjusted Settlement
0.00	0.00	0.00	0	0.00	0.00	0	0	0.00	0.00
0.50	0.05	0.30	175	0.04	0.14	43	218	1.54	2.04
0.75	0.08	0.35	206	0.06	0.18	55	261	1.84	2.59
1.00	0.10	0.40	231	0.08	0.22	65	296	2.09	3.09
1.50	0.15	0.47	272	0.12	0.28	83	355	2.50	4.00
2.00	0.20	0.53	305	0.16	0.33	99	404	2.85	4.85
2.50	0.25	0.57	333	0.20	0.38	113	446	3.15	5.65
3.00	0.30	0.62	358	0.24	0.42	126	485	3.42	6.42
3.50	0.35	0.66	381	0.28	0.46	138	520	3.66	7.16
4.00	0.40	0.69	402	0.31	0.50	150	552	3.89	7.89
4.50	0.45	0.73	421	0.35	0.54	161	582	4.11	8.61
5.00	0.50	0.76	440	0.39	0.57	171	611	4.31	9.31
5.50	0.55	0.79	457	0.43	0.61	182	638	4.50	10.00
6.00	0.60	0.82	473	0.47	0.64	191	664	4.68	10.68
6.50	0.65	0.84	488	0.51	0.67	201	689	4.86	11.36
7.00	0.70	0.87	503	0.55	0.70	210	713	5.02	12.02
7.50	0.75	0.89	517	0.59	0.73	219	736	5.19	12.69
8.00	0.80	0.91	530	0.63	0.76	227	758	5.34	13.34
9.00	0.90	0.96	556	0.71	0.81	244	800	5.64	14.64
10.00	1.00	1.00	580	0.79	0.87	260	840	5.92	15.92
11.00	1.00	1.00	580	0.87	0.92	275	855	6.03	17.03
12.00	1.00	1.00	580	0.94	0.97	290	870	6.13	18.13
13.00	1.00	1.00	580	1.00	1.00	300	880	6.20	19.20

The adjusted settlement versus mobilized load data points, respectively obtained from the red and blue columns above, are graphed below.



A curve fit, along with the plot markers, is provided in the following. The point corresponding to the allowable load Q_{all} = 440 kN is mapped, yielding δ_{adj} = 5.41 mm.



► The correct answer is **B**.

P.7 ■ Solution

Davisson's method requires plotting a line on the P–δ (load–settlement) plane following the relation

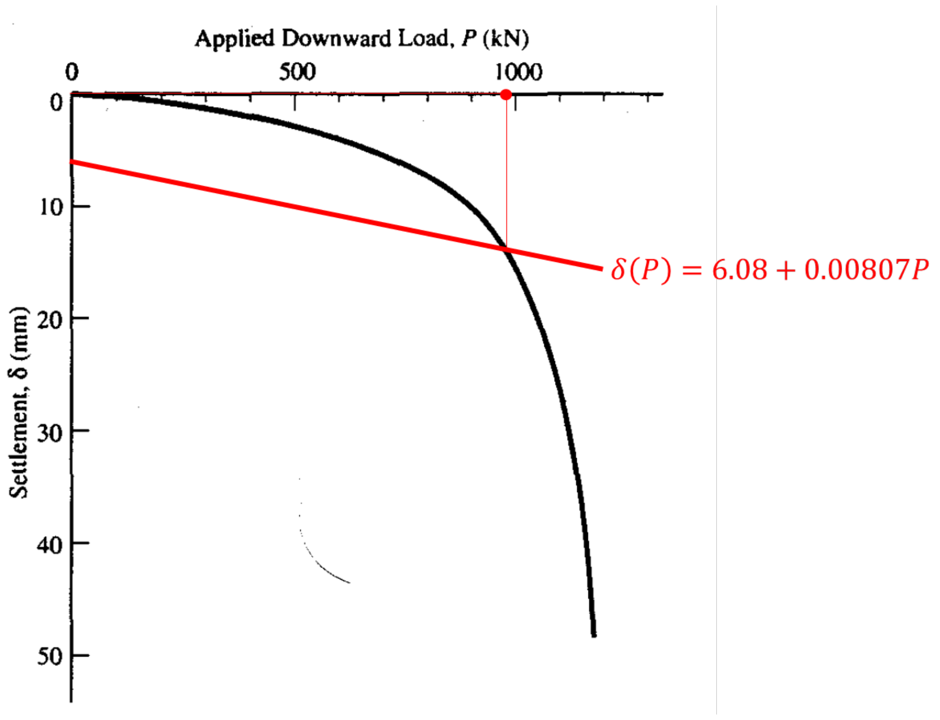
$$\delta(P) = 4[\text{mm}] + \frac{B}{120} + \frac{PL}{AE}$$

where P is the axial load applied on the foundation, B is the foundation diameter or width, L is the foundation depth, A is the cross-sectional area, and E is the modulus of elasticity of the foundation material (in this case, concrete). E for the concrete can be computed with the formula $E = 4700\sqrt{f_c'} = 4700\sqrt{40\text{ [N/mm}^2]} =$ 29.73 N/mm^2 = 29,725,000 kPa. Substituting this quantity, along with B = 250 mm, L = 15,000 mm, and A = 0.25^2 = 0.0625 m^2, we obtain

$$\delta(P) = 4 + \frac{B}{120} + \frac{PL}{AE} = 4[\text{mm}] + \frac{250[\text{mm}]}{120} + \frac{P \times (15{,}000\,[\text{mm}])}{(0.0625[\text{m}^2]) \times \left(29{,}725{,}000\left[\frac{\text{kN}}{\text{m}^2}\right]\right)}$$

$$\therefore \delta(P) = 6.08 + 0.00807P$$

Plotting this line in the P–δ plane and looking for the intercept with the horizontal axis, we conclude that the ultimate load capacity P_{ult} is around 970 kN, as shown.



► The correct answer is **C**.

P.8 ■ Solution

The influence length of the pile is 10 diameters above the tip and 4 diameters below it. Hence, the upper limit of influence is $8.6 - 10 \times 0.410 = 4.5$ m, whereas the lower limit is $8.6 + 4 \times 0.410 = 10.24 \approx 10.5$ m. Given these approximations, we shall take the average blow count in the zone between a depth of 4.5 m and 10.5 m; that is,

$$(N_{60})_{\text{sp}} = \frac{6+5+16+18+21}{5} = 13.2$$

We choose not to round the number up or down because our aim is to use it in calculations, not report it in a soils investigation communiqué. We then write Meyerhof's expression to compute the end-bearing resistance of a pile using SPT results,

$$Q_p = 0.4p_a(N_{60})_{\text{sp}}\left(\frac{L}{D}\right)A_b \le 4p_a(N_{60})_{\text{sp}}A_b$$

where p_a is the atmospheric pressure, $(N_{60})_{\text{sp}}$ is the special averaged standard penetration number, L/D is the embedment ratio, and A_b is the cross-sectional area of the pile. Substituting p_a = 100 kPa, $(N_{60})_{sp}$ = 13.2, L/D = 8.6/0.41 = 21, and A_b = 0.41^2 = 0.17 m^2 on the left-hand side of the inequality gives

$$Q_p = 0.4 \times 100 \times 13.2 \times (21) \times 0.17 = 1885 \text{ kN}$$

We compare this result with the right-hand side of the inequality,

$$Q_p = 4 \times 100 \times 13.2 \times 0.17 = 898 \text{ kN}$$

The latter result is lower, so we take Q_p = 898 kN. Next, to compute the shaft resistance from SPT results, we resort to the formula

$$Q_f = 0.01 p_a \bar{N}_{60} p L$$

where $\bar{N}_{60}$ is the average number of SPT blow counts throughout the length of the pile, and p is the perimeter of the pile section. $\bar{N}_{60}$ is calculated as

$$\bar{N}_{60} = \frac{4+8+6+5+16+18}{6} = 9.5$$

Accordingly,

$$Q_f = 0.01 \times 100 \times 9.5 \times (4 \times 0.410) \times 8.6 = 134 \text{ kN}$$

The ultimate load imparted on the pile is then

$$Q_{\text{ult}} = Q_p + Q_f = 898 + 134 = 1032 \text{ kN}$$

and, given a factor of safety FS = 3.5, the allowable load is

$$Q_{\text{all}} = \frac{Q_{\text{ult}}}{FS} = \frac{Q_p + Q_f}{FS} = \frac{1032}{3.5} = \boxed{295 \text{ kN}}$$

► The correct answer is **A**.

P.9 ■ Solution

As per the Hiley formula, the ultimate load is

$$Q_{\text{ult}} = \eta_b \frac{W \times H}{S + \frac{C}{2}}$$

where W = 30 kN is the weight of the hammer, H = 0.9 m is the effective fall, S = 0.015 m is the average penetration per blow, C = 0.018 m is the temporary compression of the pile, pile cap and soil. Lastly, η_b is the efficiency factor, which is given by

$$\eta_b = \frac{W + e^2 P}{W + P}$$

in which W = 30 kN, P = 45 kN, and e = 0.32 is the coefficient of restitution, so that

$$\eta_b = \frac{30 + 0.32^2 \times 45}{30 + 45} = 0.461$$

Substituting the pertaining variables in the equation for Q_{ult}, we obtain

$$Q_{\text{ult}} = 0.461 \times \frac{30 \times 0.8}{0.016 + 0.019/2} = 434 \text{ kN}$$

Applying the factor of safety, the allowable load becomes

$$Q_{\text{allow}} = \frac{Q_{\text{ult}}}{FS} = \frac{434}{1.5} = \boxed{289 \text{ kN}}$$

► The correct answer is **C**.

▸ ANSWER SUMMARY

Problem 1		C
Problem 2		C
Problem 3		B
Problem 4	**4A**	B
	4B	B
Problem 5	**5A**	T/F
	5B	D
	5C	B
Problem 6		B
Problem 7		C
Problem 8		A
Problem 9		C

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