Montogue

Quiz EL103

Power Flow Studies

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PROBLEMS

Problem 1

Problem 1.1: For a two-bus power system, a $0.7 + j0.4$ unit per load at bus 2 is supplied by a generator at bus 1 through a transmission line with series impedance of $0.05 + j0.1$ per unit. With bus 1 as the slack bus with a fixed per-unit voltage of $1.0\angle 0$, use the Gauss-Seidel method to calculate the voltage at bus 2 after three iterations.

Problem 1.2: Repeat the previous problem with the slack bus voltage changed to $1.0\angle 30^\circ$ per unit.

Problem 2

For the three-bus system whose bus admittance matrix Y_{bus} is given, calculate the second iteration value of V_3 using the Gauss-Seidel method. Assume bus 1 as the slack (with $V_1 = 1.0\angle 0^\circ$) and buses 2 and 3 are load buses with a per-unit load of $S_2 = 1 + j0.5$ and $S_3 = 1.5 + j0.75$. Use voltage guesses of $1.0\angle 0^\circ$ at both buses 2 and 3. The bus admittance matrix for the three-bus system is

$$\mathbf{Y}_{\text{bus}} = \begin{bmatrix} -j10 & j5 & j5 \\ j5 & -j10 & j5 \\ j5 & j5 & -j10 \end{bmatrix}$$

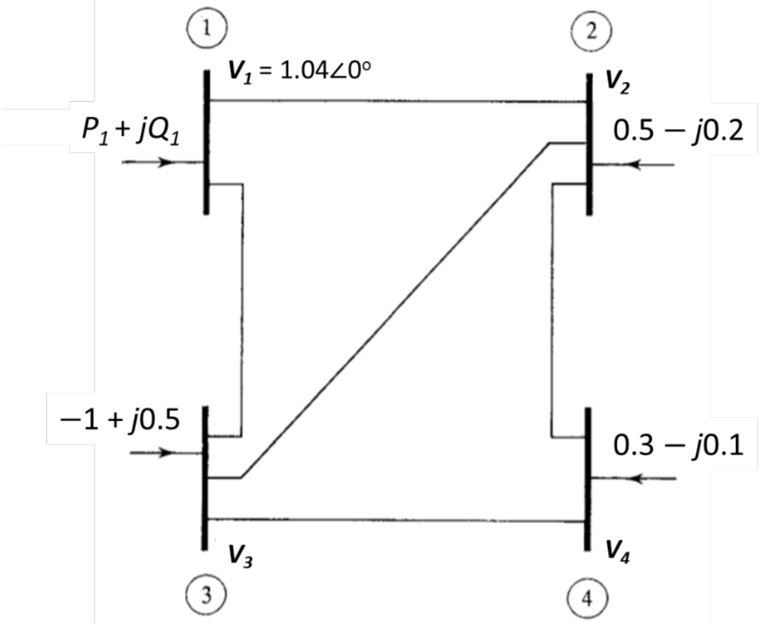
Problem 3

Problem 3.1: For the system shown below, the bus admittance matrix is

$$\mathbf{Y}_{\text{bus}} = \begin{bmatrix} 3-j9 & -2+j6 & -1+j3 & 0 \\ -2+j6 & 3.666-j11 & -0.666+j2 & -1+j3 \\ -1+j3 & -0.666+j2 & 3.666-j11 & -2+j6 \\ 0 & -1+j3 & -2+j6 & 3-j9 \end{bmatrix} \text{pu}$$

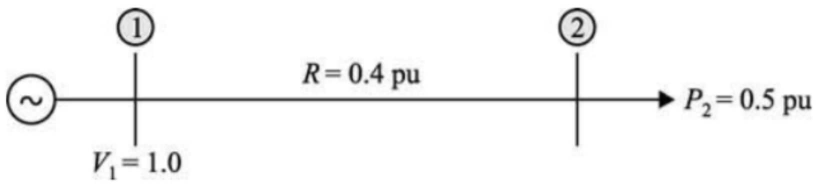
With the complex power on buses 2, 3, and 4 as shown in the figure, determine the value of V_2 that is produced by the first iteration of the Gauss-Seidel procedure. Use $1.0\angle 0^\circ$ pu as the initial guess for all voltages.

Problem 3.2: Determine the value of V_2 as produced by the second iteration of the Gauss-Seidel procedure.



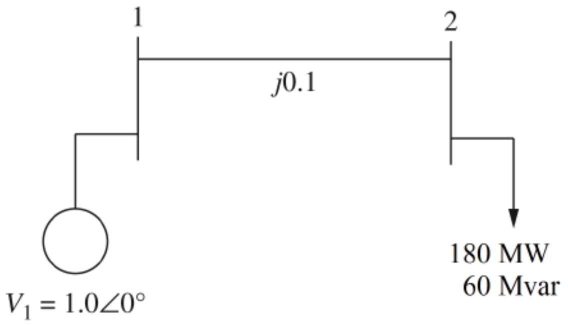
Problem 4

The resistive network shown below is supplying a load of 0.5 pu over a line with resistance 0.4 pu. If bus 1 is assumed to be the slack bus having a voltage of 1.0 pu, use the Newton-Raphson iterative method to determine the voltage at load bus 2, the current in line 1-2, the slack bus power, and the power loss in the line.



Problem 5

A generator bus (with a 1.0 per unit voltage) supplies a 180 MW, 60 Mvar load through a lossless transmission line with per unit (100 MVA base) impedance of $j0.1$ and no line charging. Starting with an initial voltage guess of $1.0\angle 0°$, iterate until converged using the Newton-Raphson power flow method. For convergence criteria, use a maximum power flow mismatch of 0.1 MVA.



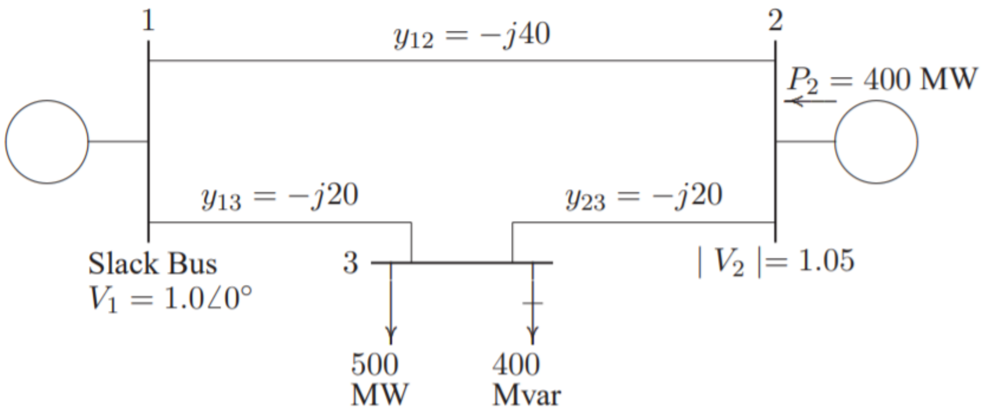
Problem 6

Problem 6.1: For a three-bus power system, assume bus 1 is the slack with a per unit voltage of $1.0\angle 0°$, bus 2 is a PQ bus with a per unit load of $2.0 + j0.5$, and bus 3 is a PV bus with 1.0 per unit generation and a 1.0 voltage setpoint. The per unit line impedances are $j0.1$ between buses 1 and 2, $j0.4$ between buses 1 and 3, and $j0.2$ between buses 2 and 3. Using a flat start, use the Newton-Raphson approach to determine the first iteration phasor voltages at buses 2 and 3.

Problem 6.2: Repeat the previous problem except with the bus 2 real power load changed to 1.0 per unit.

Problem 7

The following figure shows the one-line diagram of a simple three-bus power system with generation at buses 1 and 2. The voltage at bus 1 is $V = 1.0\angle 0°$ per unit. Voltage magnitude at bus 2 is fixed at 1.05 pu with a real power generation of 400 MW. A load consisting of 500 MW and 400 Mvar is taken from bus 3. Line admittances are marked in per unit on a 100 MVA base. Using the Newton-Raphson method, start with the initial estimates of $V_2(i = 0) = 1.05\angle 0°$ and $V_3(i = 0) = 1.0\angle 0°$, and keeping $|V_2| = 1.05$ pu, determine the phasor values of V_2 and V_3. Perform two iterations.



Problem 8

For the situation specified in the previous problem, obtain the power flow solution using the fast decoupled algorithm. Perform two iterations.

SOLUTIONS

P.1 ➡ Solution

Problem 1.1: The bus admittance is given by

$$Y=\frac{1}{Z_L}=\frac{1}{0.05+j0.1}\times\left(\frac{0.05-j0.1}{0.05-j0.1}\right)=\frac{1}{0.0125}\times(0.05-j0.1)$$

$$\therefore Y=4-j8=8.94\angle-63.4^{\circ}\,\text{pu}$$

The bus admittance matrix is

$$\mathbf{Y}_{\text{bus}}=\begin{bmatrix}Y_{11} & Y_{12}\\ Y_{21} & Y_{22}\end{bmatrix}=\begin{bmatrix}Y & -Y\\ -Y & Y\end{bmatrix}=\begin{bmatrix}4-j8 & -4+j8\\ -4+j8 & 4-j8\end{bmatrix}$$

In the Gauss-Seidel method, voltage in a bus k is given by

$$V_k(i+1)=\frac{1}{Y_{kk}}\left[\frac{P_k-jQ_k}{V_k^*(i)}-\sum_{n=1}^{k-1}Y_{kn}V_n(i+1)-\sum_{n=k+1}^{N}Y_{kn}V_n(i)\right]$$

where i is the iteration number and N is the total number of buses. In the first iteration, applying the Gauss-Seidel method to bus 2 brings to

$$V_2(i=1)=\frac{1}{4-j8}\left[\frac{-0.7-j(-0.4)}{1\angle0^{\circ}}-(-4+j8)\times1\right]$$

$$\therefore V_2(i=1)=0.925-j0.05=0.9264\angle-3.0941^{\circ}\,\text{pu}$$

Proceeding similarly in iteration 2,

$$V_2(2)=\frac{1}{Y_{22}}\left[\frac{P_2-jQ_2}{V_2^*(1)}-Y_{21}V_1(2)\right]=\frac{1}{4-j8}\left[\frac{-0.7-j(-0.4)}{(0.9264\angle3.0941^{\circ})}-(-4+j8)(1)\right]$$

$$\therefore V_2(2)=0.9176\angle-3.0953^{\circ}$$

In iteration 3,

$$V_2(3)=\frac{1}{4-j8}\left[\frac{-0.7-j(-0.4)}{(0.9176\angle3.0953^{\circ})}-(-4-j8)(1)\right]$$

$$\therefore V_2(3)=0.9168\angle-3.1249^{\circ}$$

Thus, after 3 iterations,

$$\boxed{V_2=0.9168\angle-3.1249^{\circ}}$$

Problem 1.2: In a first iteration, applying the Gauss-Seidel method to bus 2 brings to

$$V_2(i=1)=\frac{1}{4-j8}\left[\frac{-0.7-j(-0.4)}{1\angle-30^{\circ}}-(-4+j8)\times1\right]$$

$$\therefore V_2(1)=0.9101\angle-4.8109^{\circ}$$

In the second iteration,

$$V_2(2)=\frac{1}{4-j8}\left[\frac{-0.7-j(-0.4)}{0.9101\angle4.8109^{\circ}}-(-4+j8)\times1\right]=0.9145\angle-2.9982^{\circ}$$

In the third iteration,

$$V_2(3)=\frac{1}{4-j8}\left[\frac{-0.7-j(-0.4)}{0.9145\angle2.9982^{\circ}}-(-4+j8)\times1\right]=0.9166\angle-3.1464^{\circ}$$

Thus, after 3 iterations,

$$\boxed{V_2=0.9166\angle-3.1464^{\circ}}$$

P.2 ➡ Solution

We have $V_1=1.0\angle0$. For V_2, at iteration number $i+1$,

$$V_2(i+1)=\frac{1}{Y_{22}}\left[\frac{P_2-jQ_2}{V_2^*(i)}-Y_{21}V_1-Y_{23}V_3(i)\right]$$

$$\therefore V_2(i+1)=\frac{1}{-j10}\left[\frac{-1+j0.5}{V_2^*(i)}-j5V_1-j5V_3(i)\right]$$

As for V_3,

$$V_3(i+1)=\frac{1}{Y_{33}}\left[\frac{P_3-jQ_3}{V_3^*(i)}-Y_{31}V_1-Y_{32}V_2(i+1)\right]$$

$$\therefore V_3(i+1)=\frac{1}{-j10}\left[\frac{-1.5+j0.75}{V_3^*(i)}-j5V_1-j5V_2(i+1)\right]$$

In the first iteration,

$$V_2(0+1)=V_2(1)=\frac{1}{-j10}\left[\frac{-1+j0.5}{1\angle 0^{\circ}}-j5\times 1\angle 0^{\circ}-j5\times 1\angle 0^{\circ}\right]=0.9553\angle-6.0090^{\circ}$$

$$V_3(0+1)=V_3(1)=\frac{1}{-j10}\left[\frac{-1.5+j0.75}{1\angle 0^{\circ}}-j5\times 1\angle 0^{\circ}-j5\times 0.9553\angle-6.0090^{\circ}\right]$$

$$\therefore V_3(1)=0.9220\angle-12.5286^{\circ}$$

In the second iteration,

$$V_2(2)=\frac{1}{-j10}\left[\frac{-1+j0.5}{(0.9553\angle 6.0090^{\circ})}-j5\times(1\angle 0^{\circ})-j5\times(0.9220\angle-12.5286^{\circ})\right]$$

$$\therefore V_2(2)=0.9090\angle-12.6220^{\circ}$$

$$V_3(2)=\frac{1}{-j10}\left[\frac{-1.5+j0.75}{(0.9220\angle 12.5286^{\circ})}-j5\times(1\angle 0^{\circ})-j5\times(0.9090\angle-12.6220^{\circ})\right]$$

$$\therefore V_3(2)=0.8630\angle-16.1804^{\circ}$$

Thus, after two iterations,

$$\boxed{V_2=0.9090\angle-12.6220^{\circ}\ ;\ V_3=0.8630\angle-16.1804^{\circ}}$$

P.3 ➡ *Solution*

Problem 3.1: As stated, we have $V_2(0)=V_3(0)=V_4(0)=1.0\angle 0^{\circ}$ pu. Then, $V_2(1)$ is calculated to be

$$V_2(1)=\frac{1}{Y_{22}}\left[\frac{P_2-jQ_2}{V_2^*(i)}-Y_{21}V_1-Y_{23}V_3(0)-Y_{24}V_4(0)\right]$$

$$\therefore V_2(1)=\frac{1}{3.666-j11}\left[\frac{0.5+j0.2}{1\angle 0^{\circ}}-(-2+j6)\times(1.04\angle 0^{\circ})-(-0.666+j2)\times(1\angle 0^{\circ})-(-1+j3)\times(1\angle 0^{\circ})\right]$$

$$\therefore V_2(1)=1.0191+j0.0464=\boxed{1.0202\angle 2.6050^{\circ}\,\text{pu}}$$

Problem 3.2: We are looking for $V_2(2)$, which is given by

$$V_2(2)=\frac{1}{Y_{22}}\left[\frac{P_2-jQ_2}{V_2^*(1)}-Y_{21}V_1-Y_{23}V_3(1)-Y_{24}V_4(1)\right]$$

Before proceeding, we need to determine V_3 and V_4 for the first iteration, namely

$$V_3(1)=\frac{1}{Y_{33}}\left[\frac{P_3-jQ_3}{V_3^*(0)}-Y_{31}V_1-Y_{32}V_2(1)-Y_{34}V_4(0)\right]$$

$$\therefore V_3(1)=1.028-j0.087\text{ pu}$$

$$V_4(1)=\frac{1}{Y_{44}}\left[\frac{P_4-jQ_4}{V_4^*(0)}-Y_{41}V_1-Y_{42}V_2(1)-Y_{43}V_3(1)\right]$$

$$\therefore V_4(1) = 1.025 - j0.0093 \text{ pu}$$

so that

$$V_2(2) = \frac{1}{3.666 - j11.0}\left[\begin{array}{r} \dfrac{0.5 + j0.2}{1.0191 + j0.0464} - 1.04 \times (-2 + j6) \\ -(-0.666 + j2.0) \times (1.028 - j0.087) - (-1 + j3) \times (1.025 - j0.0093) \end{array}\right]$$

$$\therefore V_2(2) = 1.033 + j0.0272 = \boxed{1.0334\angle 1.5083^\circ \text{pu}}$$

P.4 ➡ Solution

The power at bus 2 in terms of the bus voltages V_1 = 1.0 and V_2 = ? and the line resistance R = 0.38 pu can be written as

$$V_2 \frac{(V_1 - V_2)}{R} = \frac{V_1 V_2 - V_2^2}{0.4} = 0.5$$

$$\therefore -V_2^2 + V_1 V_2 = 0.2$$

$$\therefore V_2^2 - V_1 V_2 = -0.2$$

$$\therefore V_2^2 - V_1 V_2 + 0.2 = 0$$

This is a quadratic equation in V_2 and every student knows how to solve it effortlessly. The point of the problem, however, is to illustrate use of the Newton-Raphson method in power flow analysis. We begin by writing the equation above in function form,

$$f(V_2) = V_2^2 - V_1 V_2 + 0.2$$

Next, we derive $f(V_2)$ with respect to V_2,

$$f'(V_2) = 2V_2 - V_1$$

Now, in the NR method an equation of the form $y = f(x)$ is solved so that, in the i-th iteration,

$$\Delta y^i = y - f(x^i)$$

$$\Delta x^i = \frac{\Delta y^i}{f'(x^i)} = \frac{y - f(x^i)}{f'(x^i)}$$

$$x^{i+1} = x^i + \Delta x^i = x^i + \frac{y - f(x^i)}{f'(x^i)}$$

With the notation at hand,

$$\Delta V^i = -\frac{f(V^i)}{\dfrac{df(V^i)}{dV}} = -\frac{f(V)}{f'(V)}$$

and

$$V^{i+1} = V^i + \Delta V^i$$

where we have dropped the subscript 2 for clarity. The calculations are tabulated below.

i	V	$f(V)$	$\dfrac{df(V)}{dV}$	ΔV^i	$\Delta V^{i+1} = V^i + \Delta V^i$
0	1.00000	0.2	1.00000	-0.20000	0.80000
1	0.80000	0.04	0.60000	-0.06667	0.73333
2	0.73333	0.004444	0.46667	-0.00952	0.72381
3	0.72381	9.07E-05	0.44762	-0.00020	0.72361
4	0.72361	4.11E-08	0.44721	0.00000	0.72361
5	0.72361	8.38E-15	0.44721	0.00000	0.72361
6	0.72361	0.000000	0.44721	0.00000	0.72361

By the fifth iteration, it becomes clear that the root is V_2 = 0.72361 pu. We can proceed to determine the line current,

$$\text{Line current} = \frac{V_1 - V_2}{R} = \frac{1.0 - 0.7236}{0.4} = \boxed{0.691\,\text{pu}}$$

the slack bus power,

$$\text{Slack bus power} = V_1 \times \text{line current} = 1.0 \times 0.691 = \boxed{0.691\,\text{pu}}$$

and the power loss in the line,

$$\text{Power loss in line} = 0.691 - 0.50 = \boxed{0.191\,\text{pu}}$$

P.5 ➡ Solution

We first convert all values to per-unit,

$$P_{\text{pu}} = \frac{P}{S_{\text{base}}} = -\frac{180\text{ MW}}{100\,\text{MVA}} = -1.8\,\text{pu}$$

$$Q_{\text{pu}} = -\frac{60\,\text{MW}}{100\,\text{MVA}} = -0.6\,\text{pu}$$

$$\Delta y_{\text{max,pu}} = \frac{0.1\,\text{MVA}}{100\,\text{MVA}} = 10^{-4}\text{ pu}$$

Since there are n = 2 buses, we need to solve 2(n − 1) = 2 equations. The impedance matrix is

$$\mathbf{Y}_{\text{bus}} = \begin{bmatrix} -j10 & j10 \\ j10 & -j10 \end{bmatrix} = \begin{bmatrix} 10\angle -90^\circ & 10\angle 90^\circ \\ 10\angle 90^\circ & 10\angle -90^\circ \end{bmatrix}$$

The base equations are

$$P_2(\delta_2, V_2) = V_2\left[Y_{21}V_1\cos(\delta_2 - \delta_1 - \theta_{21}) + Y_{22}V_2\cos(-\theta_{22})\right]$$

$$Q_2(\delta_2, V_2) = V_2\left[Y_{21}V_1\sin(\delta_2 - \delta_1 - \theta_{21}) + Y_{22}V_2\sin(-\theta_{22})\right]$$

and the Jacobian matrix elements are, for the first iteration,

$$J1_{22} = -V_2Y_{21}V_1\sin(\delta_2 - \delta_1 - \theta_{21}) = -1\times 10\times 1\times\sin(0-0-90^\circ) = 10$$

$$J2_{22} = V_2Y_{22}\cos(\theta_{22}) + Y_{21}V_1\cos(\delta_2 - \delta_1 - \theta_{21}) + Y_{22}V_2\cos(-\theta_{22})$$

$$\therefore J2_{22} = 1\times 10\times\cos(-90^\circ) + 10\times 1\times\cos(0-0-90^\circ) + 10\times 1\times\cos\left[-(-90^\circ)\right] = 0$$

$$J3_{22} = V_2Y_{21}V_1\cos(\delta_2 - \delta_1 - \theta_{21}) = 1\times 10\times 1\times\cos(0-0-90^\circ) = 0$$

$$J4_{22} = -V_2Y_{22}\sin\theta_{22} + Y_{21}V_1\sin(\delta_2 - \delta_1 - \theta_{21}) + Y_{22}V_2\sin(-\theta_{22})$$

$$\therefore J4_{22} = -1\times 10\times\sin(-90^\circ) + 10\times 1\times\sin(0-0-90^\circ) + 10\times 1\times\sin(90^\circ) = 10$$

Accordingly, we assemble the matrix equation

$$\begin{bmatrix} \Delta P_2 \\ \Delta Q_2 \end{bmatrix} = \begin{bmatrix} J1_{22} & J2_{22} \\ J3_{22} & J4_{22} \end{bmatrix}\begin{bmatrix} \Delta\delta_2(1) \\ \Delta V_2(1) \end{bmatrix} = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}\begin{bmatrix} \Delta\delta_2(1) \\ \Delta V_2(1) \end{bmatrix}$$

or, equivalently,

$$\begin{bmatrix} \Delta\delta_2(1) \\ \Delta V_2(1) \end{bmatrix} = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}^{-1}\begin{bmatrix} -1.8 \\ -0.6 \end{bmatrix} = \begin{bmatrix} -0.18 \\ -0.06 \end{bmatrix}$$

Thus,

$$\delta_2(1) = \delta_2(0) + \Delta\delta_2(1) = 0 - 0.18 = -0.18\,\text{rad} = -10.313^\circ$$
$$V_2(1) = V_2(0) + \Delta V_2(1) = 1 - 0.06 = 0.940\,\text{pu}$$

The updated V_2 is $0.940\angle -10.313^{\circ}$. Reformulating the Jacobian entries, we have

$$J1_{22} = -V_2Y_{21}V_1\sin(\delta_2-\delta_1-\theta_{21}) = -0.940\times10\times1\times\sin(-10.313^{\circ}-0-90^{\circ}) = 9.2481$$

$$J2_{22} = V_2Y_{22}\cos(\theta_{22}) + Y_{21}V_1\cos(\delta_2-\delta_1-\theta_{21}) + Y_{22}V_2\cos(-\theta_{22})$$

$$\therefore J2_{22} = 0.940\times10\times\cos(-90^{\circ}) + 10\times1\times\cos(-10.313^{\circ}-0-90^{\circ}) + 10\times0.940\times\cos\left[-(-90^{\circ})\right] = -1.7903$$

$$J3_{22} = V_2Y_{21}V_1\cos(\delta_2-\delta_1-\theta_{21}) = 0.940\times10\times1\times\cos(-10.313^{\circ}-0-90^{\circ}) = -1.6828$$

$$J4_{22} = -V_2Y_{22}\sin\theta_{22} + Y_{21}V_1\sin(\delta_2-\delta_1-\theta_{21}) + Y_{22}V_2\sin(-\theta_{22})$$

$$\therefore J4_{22} = -0.940\times10\times\sin(-90^{\circ}) + 10\times1\times\sin(-10.313^{\circ}-0-90^{\circ}) + 10\times0.940\times\sin(90^{\circ}) = 8.9616$$

so that

$$\begin{bmatrix}\Delta P_2\\ \Delta Q_2\end{bmatrix} = \begin{bmatrix}J1_{22} & J2_{22}\\ J3_{22} & J4_{22}\end{bmatrix}\begin{bmatrix}\Delta\delta_2(2)\\ \Delta V_2(2)\end{bmatrix} = \begin{bmatrix}9.2481 & -1.7903\\ -1.6828 & 8.9616\end{bmatrix}\begin{bmatrix}\Delta\delta_2(2)\\ \Delta V_2(2)\end{bmatrix}$$

or

$$\begin{bmatrix}\Delta\delta_2(2)\\ \Delta V_2(2)\end{bmatrix} = \begin{bmatrix}9.2481 & -1.7903\\ -1.6828 & 8.9616\end{bmatrix}^{-1}\begin{bmatrix}\Delta P_2\\ \Delta Q_2\end{bmatrix}$$

Real power P_2 is

$$P_2(-10.313^{\circ}, 0.940) = 0.940\times\left\{10\times1\times\cos(-10.313^{\circ}-0-90^{\circ}) + 10\times0.940\times\cos\left[-(-90^{\circ})\right]\right\}$$

$$\therefore P_2 = -1.6828$$

so the real power mismatch is calculated as

$$\Delta P_2(1) = P_2 - P_2(1) = -1.8 - (-1.6828) = -0.1172$$

Likewise, reactive power Q_2 is

$$Q_2(-10.313^{\circ}, 0.940) = 0.940\times\left\{10\times1\times\sin(-10.313^{\circ}-0^{\circ}-90^{\circ}) + 10\times0.940\times\sin\left[-(-90^{\circ})\right]\right\}$$

$$\therefore Q_2 = -0.4121$$

so the reactive power mismatch is computed as

$$\Delta Q_2(1) = Q_2 - Q_2(1) = -0.6 - (-0.4121) = -0.1879$$

Returning to the matrix equation,

$$\begin{bmatrix}\Delta\delta_2(2)\\ \Delta V_2(2)\end{bmatrix} = \begin{bmatrix}9.2481 & -1.7903\\ -1.6828 & 8.9616\end{bmatrix}^{-1}\begin{bmatrix}-0.1172\\ -0.1879\end{bmatrix} = \begin{bmatrix}-0.0174\\ -0.0242\end{bmatrix}$$

Thus,

$$\delta_2(2) = \delta_2(1) + \Delta\delta_2(2) = -0.18 - 0.01736 = -0.1974\text{ rad} = -11.308^{\circ}$$

$$V_2(2) = V_2(1) + \Delta V_2(2) = 0.940 - 0.0242 = 0.9158\,\text{pu}$$

The updated V_2 is $0.9158\angle -10.313^{\circ}$. Moving on to the third iteration, we update the Jacobian entries

$$J1_{22} = -V_2Y_{21}V_1\sin(\delta_2-\delta_1-\theta_{21}) = -0.9158\times10\times1\times\sin(-11.308^{\circ}-0-90^{\circ}) = 8.9802$$

$$J2_{22} = V_2Y_{22}\cos(\theta_{22}) + Y_{21}V_1\cos(\delta_2-\delta_1-\theta_{21}) + Y_{22}V_2\cos(-\theta_{22})$$

$$\therefore J2_{22} = 0.9158\times10\times\cos(-90^{\circ}) + 10\times1\times\cos(-11.308^{\circ}-0-90^{\circ}) + 10\times0.9158\times\cos\left[-(-90^{\circ})\right] = -1.9608$$

$$J3_{22} = V_2Y_{21}V_1\cos(\delta_2-\delta_1-\theta_{21}) = 0.9158\times10\times1\times\cos(-11.308^{\circ}-0-90^{\circ}) = -1.7957$$

$$J4_{22} = -V_2Y_{22}\sin\theta_{22} + Y_{21}V_1\sin(\delta_2-\delta_1-\theta_{21}) + Y_{22}V_2\sin(-\theta_{22})$$

$$\therefore J4_{22} = -0.9158\times10\times\sin(-90^o)+10\times1\times\sin(-11.308^o-0-90^o)+10\times0.9158\times\sin(90^o)=8.5101$$

so that

$$\begin{bmatrix}\Delta P_2\\ \Delta Q_2\end{bmatrix}=\begin{bmatrix}J1_{22} & J2_{22}\\ J3_{22} & J4_{22}\end{bmatrix}\begin{bmatrix}\Delta\delta_2(3)\\ \Delta V_2(3)\end{bmatrix}=\begin{bmatrix}8.9802 & -1.9608\\ -1.7957 & 8.5101\end{bmatrix}\begin{bmatrix}\Delta\delta_2(3)\\ \Delta V_2(3)\end{bmatrix}$$

or

$$\begin{bmatrix}\Delta\delta_2(3)\\ \Delta V_2(3)\end{bmatrix}=\begin{bmatrix}8.9802 & -1.9608\\ -1.7957 & 8.5101\end{bmatrix}^{-1}\begin{bmatrix}\Delta P_2\\ \Delta Q_2\end{bmatrix}$$

Real power P_2 is

$$P_2(-11.308^o,0.9158)=0.9158\times\{10\times1\times\cos(-11.308^o-0-90^o)+10\times0.9158\times\cos[-(-90^o)]\}$$

$$\therefore P_2=-1.7957$$

so the real power mismatch is calculated as

$$\Delta P_2(2)=P_2-P_2(2)=-1.8-(-1.7957)=-0.0043$$

Similarly, reactive power Q_2 is

$$Q_2(-11.308^o,0.9158)=0.9158\times\{10\times1\times\sin(-11.308^o-0^o-90^o)+10\times0.9158\times\sin[-(-90^o)]\}$$

$$\therefore Q_2=-0.5933$$

so the reactive power mismatch is computed as

$$\Delta Q_2(2)=Q_2-Q_2(2)=-0.6-(-0.5933)=-0.0067$$

Returning to the matrix equation,

$$\begin{bmatrix}\Delta\delta_2(2)\\ \Delta V_2(2)\end{bmatrix}=\begin{bmatrix}8.9802 & -1.9608\\ -1.7957 & 8.5101\end{bmatrix}^{-1}\begin{bmatrix}-0.0043\\ -0.0067\end{bmatrix}=\begin{bmatrix}-0.0007\\ -0.0009\end{bmatrix}$$

Therefore,

$$\delta_2(3)=\delta_2(2)+\Delta\delta_2(3)=-0.1974-0.0007=-0.1981\text{ rad}=-11.350^o$$

$$V_2(3)=V_2(2)+\Delta V_2(3)=0.9158-0.0009=0.9149\,\text{pu}$$

Thus, after 3 iterations, $V_2 = 0.9149\angle -11.350^o$.

P.6 ➡ Solution

Problem 6.1: We first write the impedance matrix

$$\mathbf{Y}_{\text{bus}}=\begin{bmatrix}-j12.5 & j10.0 & j2.5\\ j10.0 & -j15.0 & j5.0\\ j2.5 & j5.0 & -j7.5\end{bmatrix}$$

Initially, $\delta_2(0) = 0^o$, $\delta_3(0) = 0^o$, and $V_2(0) = 1.0$. We proceed to determine the Jacobian entries,

$$J1_{22}=\frac{\partial P_2}{\partial\delta_2}=-V_2\left[Y_{21}V_1\sin(\delta_2-\delta_1-\theta_{21})+Y_{23}V_3\sin(\delta_2-\delta_3-\theta_{23})\right]$$

$$\therefore J1_{22}=-1\times\left[10\times1\times\sin(0-0-90^o)+5\times1\times\sin(0-0-90^o)\right]=15$$

$$J1_{23}=\frac{\partial P_2}{\partial\delta_3}=V_2Y_{23}V_3\sin(\delta_2-\delta_3-\theta_{23})=1\times5\times\sin(-90^o)=-5$$

$$J1_{32}=\frac{\partial P_3}{\partial\delta_2}=V_3Y_{32}V_2\sin(\delta_3-\delta_2-\theta_{32})=1\times5\times1\times\sin(0-0-90^o)=-5$$

$$J1_{33}=\frac{\partial P_3}{\partial\delta_3}=-V_3\left[Y_{31}V_1\sin(\delta_3-\delta_1-\theta_{31})+Y_{32}V_2\sin(\delta_3-\delta_2-\theta_{32})\right]$$

$$\therefore J1_{33}=-1.0\times\left[2.5\times1\times\sin(0-0-90^o)+5\times1\times\sin(0-0-90^o)\right]=7.5$$

$$J2_{22} = \frac{\partial P_2}{\partial V_2} = V_2 Y_{22} \cos(\theta_{22}) + Y_{21} V_1 \cos(\delta_2 - \delta_1 - \theta_{21}) + Y_{22} V_2 \cos(-\theta_{22}) + Y_{23} V_3 \cos(\delta_2 - \delta_3 - \theta_{23})$$

$$\therefore J2_{22} = 1 \times (-15) \times \cos(-90^o) + 10 \times 1 \times \cos(0 - 0 - 90^o)$$
$$+(-15) \times 1 \times \cos[-(-90^o)] + 5 \times 1 \times \cos(0 - 0 - 90^o) = 0$$

$$J2_{32} = \frac{\partial P_3}{\partial V_2} = V_3 Y_{32} \cos(\delta_3 - \delta_2 - \theta_{32}) = 1 \times 5 \times \cos(0 - 0 - 90^o) = 0$$

$$J3_{22} = \frac{\partial Q_2}{\partial \delta_2} = V_2 [Y_{21} V_1 \cos(\delta_2 - \delta_1 - \theta_{21}) + Y_{23} V_3 \cos(\delta_2 - \delta_3 - \theta_{23})]$$

$$\therefore J3_{22} = 1.0 \times [10 \times 1 \times \cos(0 - 0 - 90^o) + 5 \times 1 \times \cos(0 - 0 - 90^o)] = 0$$

$$J4_{22} = \frac{\partial Q_2}{\partial V_2} = -V_2 Y_{22} \sin(\theta_{22}) + Y_{21} V_1 \sin(\delta_2 - \delta_1 - \theta_{21}) + Y_{22} V_2 \sin(-\theta_{22}) + Y_{23} V_3 \sin(\delta_2 - \delta_3 - \theta_{23})$$

$$\therefore J4_{22} = -1 \times 15 \times \sin(-90^o) + 10 \times 1 \times \sin(0 - 0 - 90^o)$$
$$+15 \times 1 \times \sin[-(-90^o)] + 5 \times 1 \times \sin(0 - 0 - 90^o) = 15$$

We can proceed to write the Jacobian,

$$\underline{J}(0) = \left[\begin{array}{c|c} \underline{J1} & \underline{J2} \\ \hline \underline{J3} & \underline{J4} \end{array}\right] = \left[\begin{array}{cc|c} 15 & -5 & 0 \\ -5 & 7.5 & 0 \\ \hline 0 & 0 & 15 \end{array}\right] \text{pu}$$

The next step is to solve

$$\left[\begin{array}{c|c} \mathbf{J1}(i) & \mathbf{J2}(i) \\ \hline \mathbf{J3}(i) & \mathbf{J4}(i) \end{array}\right] \begin{bmatrix} \Delta\boldsymbol{\delta}(i) \\ \Delta\mathbf{V}(i) \end{bmatrix} = \begin{bmatrix} \Delta\mathbf{P}(i) \\ \Delta\mathbf{Q}(i) \end{bmatrix}$$

where, in the present case, vector $\Delta y(0)$ has the form

$$\Delta y(0) = \begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \Delta Q_2 \end{bmatrix} = \begin{bmatrix} P_2 - P_2(X) \\ P_3 - P_3(X) \\ Q_2 - Q_2(X) \end{bmatrix}$$

with

$$P_2(X) = 1.0[10\cos(-90^o) + 5\cos(-90^o)] = 0$$
$$P_3(X) = 2.5\cos(-90^o) + 5\cos(-90^o) = 0$$
$$Q_2(X) = 1.0[10\sin(-90^o) + 15 + 5\sin(-90^o)] = 1.0(-10 + 15 - 5) = 0$$

so that

$$\Delta y(0) = \begin{bmatrix} -2.0 - 0 \\ 1.0 - 0 \\ -0.5 - 0 \end{bmatrix} = \begin{bmatrix} -2.0 \\ 1.0 \\ -0.5 \end{bmatrix}$$

and

$$\begin{bmatrix} 15 & -5 & 0 \\ -5 & 7.5 & 0 \\ 0 & 0 & 15 \end{bmatrix} \begin{bmatrix} \Delta\delta_2 \\ \Delta\delta_3 \\ \Delta V_2 \end{bmatrix} = \begin{bmatrix} -2.0 \\ 1.0 \\ -0.5 \end{bmatrix}$$

This system is almost triangulated as it stands. We multiply the first row by 5/15 = 0.3333 and add to the second row, giving

$$\begin{bmatrix} 15 & -5 & 0 \\ -5 + 0.3333(15) & 7.5 + 0.3333(-5) & 0 + 0.3333(0) \\ 0 & 0 & 15 \end{bmatrix} \begin{bmatrix} \Delta\delta_2 \\ \Delta\delta_3 \\ \Delta V_2 \end{bmatrix} = \begin{bmatrix} -2.0 \\ 1.0 + 0.3333(-2.0) \\ -0.5 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 15 & -5 & 0 \\ 0 & 5.8335 & 0 \\ 0 & 0 & 15 \end{bmatrix} \begin{bmatrix} \Delta\delta_2 \\ \Delta\delta_3 \\ \Delta V_2 \end{bmatrix} = \begin{bmatrix} -2.0 \\ 0.3334 \\ -0.5 \end{bmatrix}$$

Next, multiply the second row by 5/5.8335 = 0.8571 and add to the first row,

$$\begin{bmatrix} 15+0.8571(0) & -5+0.8571(5.8335) & 0+0.8571(0) \\ 0 & 5.8335 & 0 \\ 0 & 0 & 15 \end{bmatrix} \begin{bmatrix} \Delta\delta_2 \\ \Delta\delta_3 \\ \Delta V_2 \end{bmatrix} = \begin{bmatrix} -2.0+0.8571(0.3334) \\ 0.3334 \\ -0.5 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 15 & 0 & 0 \\ 0 & 5.8335 & 0 \\ 0 & 0 & 15 \end{bmatrix} \begin{bmatrix} \Delta\delta_2 \\ \Delta\delta_3 \\ \Delta V_2 \end{bmatrix} = \begin{bmatrix} -1.7142 \\ 0.3334 \\ -0.5 \end{bmatrix}$$

Thus,

$$\Delta V_2 = -\frac{0.5}{15} = -0.03333$$

$$\Delta\delta_3 = \frac{0.3334}{5.8335} = 0.05715$$

$$\Delta\delta_2 = \frac{-1.7142}{15} = -0.11428$$

Then, we assemble the vector

$$\Delta x = \begin{bmatrix} \Delta\delta_2 \\ \Delta\delta_3 \\ \Delta V_2 \end{bmatrix} = \begin{bmatrix} -0.11428 \\ 0.05715 \\ -0.03333 \end{bmatrix}$$

and update the unknowns

$$x(1) = \begin{bmatrix} \delta_2(1) \\ \delta_3(1) \\ V_2(1) \end{bmatrix} = x(0) + \Delta x = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} -0.11428 \\ 0.05715 \\ -0.03333 \end{bmatrix}$$

$$\therefore x(1) = \begin{bmatrix} -0.11428\,\text{rad} \\ 0.05715\,\text{rad} \\ 0.96667\text{ pu} \end{bmatrix}$$

Finally, we must ascertain whether 3 remains a voltage-controlled bus. Noting that δ_2 = –0.11428 rad = –6.54678° and δ_3 = 0.05715 rad = 3.27445°, we proceed to determine reactive power Q_3,

$$Q_3 = V_3\left[Y_{31}V_1\sin\left(\delta_3-\delta_1-\theta_{31}\right)+Y_{32}V_2\sin\left(\delta_3-\delta_2-\theta_{32}\right)+Y_{33}V_3\sin\left(-\theta_{33}\right)\right]$$

$$\therefore Q_3 = 1\times\left\{\begin{array}{c} 2.5\times1\times\sin\left(3.27445^\circ-0-90^\circ\right)+5\times0.96667\times\sin\left[3.27445^\circ-\left(-6.54678^\circ\right)-90^\circ\right] \\ +7.5\times1\times\sin\left[-\left(-90^\circ\right)\right] \end{array}\right\}$$

$$\therefore Q_3 = 0.24157\,\text{pu}$$

$$\therefore Q_{G3} = Q_3 + G_{L3} = 0.24157 + 0 = 0.2416\,\text{pu}$$

Since Q_{G3} = 0.2416 is within the limits (–5.0, +5.0), bus 3 remains a voltage-controlled bus. This completes the first Newton-Raphson iteration.

Problem 6.2: We are to repeat the procedure used in the previous problem, only this time P_{L2} = 1 pu. $\boldsymbol{Y}_{\text{bus}}$ stays the same. The Jacobian

$$J(0) = \begin{bmatrix} 15 & -5 & 0 \\ -5 & 7.5 & 0 \\ 0 & 0 & 15 \end{bmatrix}$$

also continues to be valid. There's a small tweak in vector Δy,

$$\Delta y(0) = \begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \Delta Q_2 \end{bmatrix} = \begin{bmatrix} P_2 - \Delta P_2(X) \\ P_3 - \Delta P_3(X) \\ Q_2 - \Delta Q_2(X) \end{bmatrix} = \begin{bmatrix} -1.0 - 0 \\ 1.0 - 0 \\ -0.5 - 0 \end{bmatrix} = \begin{bmatrix} -1.0 \\ 1.0 \\ -0.5 \end{bmatrix}$$

so that

$$\begin{bmatrix} 15 & -5 & 0 \\ -5 & 7.5 & 0 \\ 0 & 0 & 15 \end{bmatrix} \begin{bmatrix} \Delta\delta_2 \\ \Delta\delta_3 \\ \Delta V_2 \end{bmatrix} = \begin{bmatrix} -1.0 \\ 1.0 \\ -0.5 \end{bmatrix}$$

Multiply the second row by 0.333 and add to the first row,

$$\begin{bmatrix} 15 & -5 & 0 \\ -5 + 0.333(15) & 7.5 + 0.333(-5) & 0 + 0.333(0) \\ 0 & 0 & 15 \end{bmatrix} \begin{bmatrix} \Delta\delta_2 \\ \Delta\delta_3 \\ \Delta V_2 \end{bmatrix} = \begin{bmatrix} -1.0 \\ 1.0 + 0.333(-1.0) \\ -0.5 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 15 & -5 & 0 \\ 0 & 5.8335 & 0 \\ 0 & 0 & 15 \end{bmatrix} \begin{bmatrix} \Delta\delta_2 \\ \Delta\delta_3 \\ \Delta V_2 \end{bmatrix} = \begin{bmatrix} -1.0 \\ 0.66667 \\ -0.5 \end{bmatrix}$$

Next, multiply the second row by 5/5.8335 = 0.8571 and add to the first row,

$$\begin{bmatrix} 15 + 0.8571(0) & -5 + 0.8571(5.8335) & 0 + 0.8571(0) \\ 0 & 5.8335 & 0 \\ 0 & 0 & 15 \end{bmatrix} \begin{bmatrix} \Delta\delta_2 \\ \Delta\delta_3 \\ \Delta V_2 \end{bmatrix} = \begin{bmatrix} -1.0 + 0.8571(0.66667) \\ 0.3334 \\ -0.5 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 15 & 0 & 0 \\ 0 & 5.8335 & 0 \\ 0 & 0 & 15 \end{bmatrix} \begin{bmatrix} \Delta\delta_2 \\ \Delta\delta_3 \\ \Delta V_2 \end{bmatrix} = \begin{bmatrix} -0.7142 \\ 0.66667 \\ -0.5 \end{bmatrix}$$

Thus,

$$\Delta V_2 = -\frac{0.5}{15} = -0.03333$$

$$\Delta\delta_3 = \frac{0.6667}{5.8335} = 0.11428$$

$$\Delta\delta_2 = \frac{-0.7142}{15} = -0.02857$$

Then, we write the vector

$$\Delta x = \begin{bmatrix} \Delta\delta_2 \\ \Delta\delta_3 \\ \Delta V_2 \end{bmatrix} = \begin{bmatrix} -0.02857 \\ 0.11428 \\ -0.03333 \end{bmatrix}$$

and update the unknowns

$$x(1) = \begin{bmatrix} \delta_2(1) \\ \delta_3(1) \\ V_2(1) \end{bmatrix} = x(0) + \Delta x = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} -0.02857 \\ 0.11428 \\ -0.03333 \end{bmatrix}$$

$$\therefore x(1) = \begin{bmatrix} -0.02857 \text{ rad} \\ 0.11428 \text{ rad} \\ 0.96667 \text{ pu} \end{bmatrix}$$

Noting that $\delta_2 = -0.02857$ rad $= -1.63694°$ and $\delta_3 = 0.11428$ rad $= 6.54678°$, we proceed to check reactive power Q_3,

$$Q_3 = V_3\left[Y_{31}V_1 \sin(\delta_3 - \delta_1 - \theta_{31}) + Y_{32}V_2 \sin(\delta_3 - \delta_2 - \theta_{32}) + Y_{33}V_3 \sin(-\theta_{33})\right]$$

$$\therefore Q_3 = 1 \times \left\{ \begin{array}{c} 2.5 \times 1 \times \sin\left(6.54678^{\circ} - 0 - 90^{\circ}\right) + 5 \times 0.96667 \times \sin\left[6.54678^{\circ} - \left(-1.63694^{\circ}\right) - 90^{\circ}\right] \\ +7.5 \times 1 \times \sin\left[-\left(-90^{\circ}\right)\right] \end{array} \right\}$$

$$\therefore Q_3 = 0.2322\,\text{pu}$$

Since $Q_{G3} = 0.2322$ is within the limits (−5.0, +5.0), bus 3 remains a voltage-controlled bus. This completes the first Newton-Raphson iteration.

P.7 ➡ *Solution*

By inspection, the bus admittance matrix in polar form is

$$\mathbf{Y}_{\text{bus}} = \begin{bmatrix} -60j & 40j & 20j \\ 40j & -60j & 20j \\ 20j & 20j & -40j \end{bmatrix}$$

The load and generation expressed in per units are

$$P_2 = \frac{400}{100} = 4.0\,\text{pu}$$

$$S_3 = -\frac{(500 + j400)}{100} = -5.0 - j4.0\,\text{pu}$$

The slack bus voltage is $V_1 = 1.0\angle 0^{\circ}$ and the bus 2 voltage magnitude is $V_2 = 1.05$ pu. Starting with initial estimates $V_3 = 1.0$, $\delta_2 = 0$, and $\delta_3 = 0$, we note that

$$P_i = \sum_{j=1}^{n} |V_i||V_j||Y_{ij}|\cos\left(\theta_{ij} - \delta_1 + \delta_j\right) \quad \text{(I)}$$

$$Q_i = -\sum_{j=1}^{n} |V_i||V_j||Y_{ij}|\sin\left(\theta_{ij} - \delta_i + \delta_j\right) \quad \text{(II)}$$

and proceed to compute the Jacobian entries

$$\frac{\partial P_2}{\partial \delta_2} = -V_2\left[Y_{21}V_1 \sin\left(\delta_2 - \delta_1 - \theta_{21}\right) + Y_{23}V_3 \sin\left(\delta_2 - \delta_3 - \theta_{23}\right)\right]$$

$$\therefore \frac{\partial P_2}{\partial \delta_2} = -1.05 \times \left[40 \times 1 \times \sin\left(0 - 0 - 90^{\circ}\right) + 20 \times 1 \times \sin\left(0 - 0 - 90^{\circ}\right)\right] = 63$$

$$\frac{\partial P_2}{\partial \delta_3} = V_2 Y_{32} V_3 \sin\left(\delta_2 - \delta_3 - \theta_{23}\right) = 1.05 \times 20 \times 1 \times \sin\left(0 - 0 - 90^{\circ}\right) = -21$$

$$\frac{\partial P_3}{\partial \delta_2} = V_3 Y_{32} V_2 \sin\left(\delta_3 - \delta_2 - \theta_{32}\right) = 1 \times 20 \times 1.05 \times \sin\left(0 - 0 - 90^{\circ}\right) = -21$$

$$\frac{\partial P_3}{\partial \delta_3} = -V_3\left[Y_{31}V_1 \sin\left(\delta_3 - \delta_1 - \theta_{31}\right) + Y_{32}V_2 \sin\left(\delta_3 - \delta_2 - \theta_{32}\right)\right]$$

$$\therefore \frac{\partial P_3}{\partial \delta_3} = -1 \times \left[20 \times 1 \times \sin\left(0 - 0 - 90^{\circ}\right) + 20 \times 1.05 \times \sin\left(0 - 0 - 90^{\circ}\right)\right] = 41$$

$$\frac{\partial P_2}{\partial V_3} = 20V_2 \cos\left(90^{\circ} - \delta_2 + \delta_3\right) = 20 \times 1.05 \times \cos\left(90^{\circ} - 0 + 0\right) = 0$$

$$\frac{\partial P_3}{\partial V_3} = 20V_1 \cos\left(90^{\circ} - \delta_3 + \delta_1\right) + 20V_2 \cos\left(90^{\circ} - \delta_3 + \delta_2\right)$$

$$\therefore \frac{\partial P_3}{\partial V_3} = 20 \times 1 \times \cos\left(90^{\circ} - 0 + 0\right) + 20 \times 1.05 \times \cos\left(90^{\circ} - 0 + 0\right) = 0$$

$$\frac{\partial Q_3}{\partial \delta_2} = -20V_3V_2 \cos\left(90^{\circ} - \delta_3 + \delta_2\right) = -20 \times 1 \times 1.05 \times \cos\left(90^{\circ} - 0 + 0\right) = 0$$

$$\frac{\partial Q_3}{\partial \delta_3} = 20V_3V_1 \cos\left(90^{\circ} - \delta_3 + \delta_1\right) + 20V_3V_2 \cos\left(90^{\circ} - \delta_3 + \delta_2\right)$$

$$\therefore \frac{\partial Q_3}{\partial \delta_3} = 20 \times 1 \times 1 \cos\left(90^{\circ} - 0 + 0\right) + 20 \times 1 \times 1 \cos\left(90^{\circ} - 0 + 0\right) = 0$$

$$\frac{\partial Q_3}{\partial V_3} = -20V_1 \sin\left(90^\circ - \delta_3 + \delta_1\right) - 20V_2 \sin\left(90^\circ - \delta_3 + \delta_2\right) + 80V_3$$

$$\therefore \frac{\partial Q_3}{\partial V_3} = -20\times 1\times \sin\left(90^\circ - 0 + 0\right) - 20\times 1.05\times \sin\left(90^\circ - 0 + 0\right) + 80\times 1 = 39$$

We then determine the residuals

$$\Delta P_2 = P_2 - P_2(0) = 4.0 - 0 = 4.0$$
$$\Delta P_3 = P_3 - P_3(0) = -5.0 - 0 = -5.0$$
$$\Delta Q_3 = Q_3 - Q_3(0) = -4.0 - (-1.0) = -3.0$$

and assemble the system

$$\begin{bmatrix} 4.0 \\ -5.0 \\ -3.0 \end{bmatrix} = \begin{bmatrix} 63 & -21 & 0 \\ -21 & 41 & 0 \\ 0 & 0 & 39 \end{bmatrix} \begin{bmatrix} \Delta\delta_2(0) \\ \Delta\delta_3(0) \\ \Delta V_3(0) \end{bmatrix}$$

which can be solved to yield

$$\begin{bmatrix} \Delta\delta_2(0) \\ \Delta\delta_3(0) \\ \Delta V_3(0) \end{bmatrix} = \begin{bmatrix} 63 & -21 & 0 \\ -21 & 41 & 0 \\ 0 & 0 & 39 \end{bmatrix}^{-1} \begin{bmatrix} 4.0 \\ -5.0 \\ -3.0 \end{bmatrix} = \begin{bmatrix} 0.02754 \\ -0.1078 \\ -0.0769 \end{bmatrix}$$

Accordingly,

$$\delta_2(i=1) = 0 + 0.02754 = 0.02754\,\text{rad} = 1.5779^\circ$$
$$\delta_3(i=1) = 0 - 0.1078 = -0.1078\,\text{rad} = -6.1765^\circ$$
$$V_3(i=1) = 1 - 0.0769 = 0.9231\,\text{pu}$$

We are now ready to proceed to the second iteration. The updated Jacobian entries are

$$\frac{\partial P_2}{\partial \delta_2} = -V_2\left[Y_{21}V_1 \sin\left(\delta_2 - \delta_1 - \theta_{21}\right) + Y_{23}V_3 \sin\left(\delta_2 - \delta_3 - \theta_{23}\right)\right]$$

$$\therefore \frac{\partial P_2}{\partial \delta_2} = -1.05\times\left\{40\times 1\times \sin\left(1.5779^\circ - 0 - 90^\circ\right) + 20\times 0.9231\times \sin\left[1.5779^\circ - \left(-6.1795^\circ\right) - 90^\circ\right]\right\} = 61.1939$$

$$\frac{\partial P_2}{\partial \delta_3} = V_2 Y_{32} V_3 \sin\left(\delta_2 - \delta_3 - \theta_{23}\right) = 1.05\times 20\times 0.9231\times \sin\left[1.5779^\circ - \left(-6.1765^\circ\right) - 90^\circ\right] = -19.2078$$

$$\frac{\partial P_3}{\partial \delta_2} = V_3 Y_{32} V_2 \sin\left(\delta_3 - \delta_2 - \theta_{32}\right) = 0.9231\times 20\times 1.05\times \sin\left(-6.1765^\circ - 1.5779^\circ - 90^\circ\right) = -19.2078$$

$$\frac{\partial P_3}{\partial \delta_3} = -V_3\left[Y_{31}V_1 \sin\left(\delta_3 - \delta_1 - \theta_{31}\right) + Y_{32}V_2 \sin\left(\delta_3 - \delta_2 - \theta_{32}\right)\right]$$

$$\frac{\partial P_3}{\partial \delta_3} = -0.9231\times\left[20\times 1\times \sin\left(-6.1765^\circ - 0 - 90^\circ\right) + 20\times 1.05\times \sin\left(-6.1765^\circ - 1.5779^\circ - 90^\circ\right)\right] = 37.5627$$

$$\frac{\partial P_2}{\partial V_3} = 20V_2 \cos\left(90^\circ - \delta_2 + \delta_3\right) = 20\times 1.05\times \cos\left(90^\circ - 1.5779^\circ - 6.1765^\circ\right) = 2.8346$$

$$\frac{\partial P_3}{\partial V_3} = 20V_1 \cos\left(90^\circ - \delta_3 + \delta_1\right) + 20V_2 \cos\left(90^\circ - \delta_3 + \delta_2\right)$$

$$\therefore \frac{\partial P_3}{\partial V_3} = 20\times 1\times \cos\left[90^\circ - \left(-6.1795^\circ\right) + 0\right] + 20\times 1.05\times \cos\left[90^\circ - \left(-6.1795^\circ\right) + 1.5779^\circ\right] = -4.9874$$

$$\frac{\partial Q_3}{\partial \delta_2} = -20V_3V_2 \cos\left(90^\circ - \delta_3 + \delta_2\right) = -20\times 0.9231\times 1.05\times \cos\left[90^\circ - \left(-6.1795^\circ\right) + 1.5779^\circ\right] = 2.6166$$

$$\frac{\partial Q_3}{\partial \delta_3} = 20V_3V_1 \cos\left(90^\circ - \delta_3 + \delta_1\right) + 20V_3V_2 \cos\left(90^\circ - \delta_3 + \delta_2\right)$$

$$\therefore \frac{\partial Q_3}{\partial \delta_3} = 20\times 0.9231\times 1\times \cos\left[90^\circ - \left(-6.1765^\circ\right) + 0\right] + 20\times 0.9231\times 1.05\times \cos\left[90^\circ - \left(-6.1765^\circ\right) + 1.5779^\circ\right] = -4.6019$$

$$\frac{\partial Q_3}{\partial V_3} = -20V_1 \sin\left(90^\circ - \delta_3 + \delta_1\right) - 20V_2 \sin\left(90^\circ - \delta_3 + \delta_2\right) + 80V_3$$

$$\therefore \frac{\partial Q_3}{\partial V_3} = -20\times 1\times \sin\left[90^\circ - \left(-6.1765^\circ\right) + 0\right] - 20\times 1.05\times \sin\left[90^\circ - \left(-6.1765^\circ\right) + 1.5779^\circ\right] + 80\times 0.9231 = 33.1563$$

Equations (I) and (II) can be used to give

$$P_2 = V_2V_1Y_{21}\cos\left(\theta_{21} - \delta_2 + \delta_1\right) + V_2^2Y_{22}\cos\left(\theta_{22}\right) + V_2V_3Y_{23}\cos\left(\theta_{23} - \delta_2 + \delta_3\right)$$

$$\therefore P_2(1) = 1.05\times 1.0\times 40\cos\left(90^\circ - 1.5779^\circ + 0\right) + 1.05^2\times\left(-60\right)\times\cos\left(-90^\circ\right)$$
$$+1.05\times 0.925\times 20\cos\left(90^\circ - 1.5779^\circ - 6.1765^\circ\right) = 3.7721$$

$$P_3 = V_3V_1Y_{31}\cos\left(\theta_{31} - \delta_3 + \delta_1\right) + V_3V_2Y_{32}\cos\left(\theta_{32} - \delta_3 + \delta_2\right) + V_3^2Y_{33}\cos\left(\theta_{33}\right)$$

$$\therefore P_3(1) = 0.925\times 1.0\times 20\cos\left[90^\circ - \left(-6.1765^\circ\right) + 0\right]$$
$$+0.925\times 1.05\times 20\cos\left[90^\circ - \left(-6.1765^\circ\right) + 1.5779^\circ\right] + 0.925^2\times\left(-40\right)\times\cos\left(-90^\circ\right) = -4.6019$$

$$Q_3 = -V_3V_1Y_{31}\sin\left(\theta_{31} - \delta_3 + \delta_1\right) - V_3V_2Y_{32}\sin\left(\theta_{32} - \delta_3 + \delta_2\right) - V_3^2Y_{33}\sin\left(\theta_{33}\right)$$

$$\therefore Q_3(1) = -0.925\times 1.0\times 20\times\sin\left[90^\circ - \left(-6.1765^\circ\right) + 0\right]$$
$$-0.925\times 1.05\times 20\times\sin\left[90^\circ - \left(-6.1765^\circ\right) + 1.5779^\circ\right] - 0.925^2\times\left|-40\right|\times\sin\left(-90^\circ\right) = -3.4781$$

We then determine the residuals

$$\Delta P_2 = P_2 - P_2(1) = 4.0 - 3.7721 = 0.2279$$
$$\Delta P_3 = P_3 - P_3(1) = -5.0 - \left(-4.6019\right) = -0.3981$$
$$\Delta Q_3 = Q_3 - Q_3(1) = -4.0 - \left(-3.4781\right) = -0.5219$$

and draw up the system

$$\begin{bmatrix} 0.2279 \\ -0.3981 \\ -0.5219 \end{bmatrix} = \begin{bmatrix} 61.1939 & -19.2078 & 2.8346 \\ -19.2078 & 37.5627 & -4.9874 \\ 2.6166 & -4.6019 & 33.1563 \end{bmatrix}\begin{bmatrix} \Delta\delta_2(1) \\ \Delta\delta_3(1) \\ \Delta V_3(1) \end{bmatrix}$$

which can be solved to yield

$$\begin{bmatrix} \Delta\delta_2(1) \\ \Delta\delta_3(1) \\ \Delta V_3(1) \end{bmatrix} = \begin{bmatrix} 61.1939 & -19.2078 & 2.8346 \\ -19.2078 & 37.5627 & -4.9874 \\ 2.6166 & -4.6019 & 33.1563 \end{bmatrix}^{-1}\begin{bmatrix} 0.2279 \\ -0.3981 \\ -0.5219 \end{bmatrix} = \begin{bmatrix} 0.0006 \\ -0.0126 \\ -0.0175 \end{bmatrix}$$

Lastly,

$$\delta_2(i=2) = 0.02754 + 0.0006 = 0.02814\,\text{rad} = 1.6123^\circ$$
$$\delta_3(i=2) = -0.1078 - 0.0126 = -0.1204\,\text{rad} = -6.8984^\circ$$
$$V_3(i=2) = 0.9231 - 0.0175 = 0.9056\,\text{pu}$$

This completes the second iteration.

P.8 ➡ *Solution*

In the fast decoupled algorithm, ΔP and $\Delta\delta$ are related by the matrix equation

$$\frac{\Delta P}{\left|V_i\right|} = -B'\Delta\delta \quad \text{(I)}$$

In the system in question, bus 1 is the slack bus, therefore the corresponding bus susceptance matrix for evaluation of phase angles $\Delta\delta_2$ and $\Delta\delta_3$ is

$$\mathbf{Y}_{\text{bus}} = \begin{bmatrix} \cancel{-60j} & \cancel{40j} & \cancel{20j} \\ \cancel{40j} & -60j & 20j \\ \cancel{20j} & 20j & -40j \end{bmatrix} \Rightarrow B' = \begin{bmatrix} -60 & 20 \\ 20 & -40 \end{bmatrix}$$

The inverse of this matrix is

$$B'^{-1} = \begin{bmatrix} -0.02 & -0.01 \\ -0.01 & -0.03 \end{bmatrix}$$

We proceed to determine the residuals

$$\Delta P_2 = P_2 - P_2(0) = 4.0 - 0 = 4.0$$
$$\Delta P_3 = P_3 - P_3(0) = -5.0 - 0 = -5.0$$
$$\Delta Q_3 = Q_3 - Q_3(0) = -4.0 - (-1.0) = -3.0$$

Next, noting that V_2 = 1.05 and V_3 = 1.0, equation (I) is restated as

$$\frac{\Delta P}{|V_i|} = -B'\Delta\delta \rightarrow \Delta\delta = -B'^{-1}\frac{\Delta P}{|V_i|}$$

so that

$$\begin{bmatrix} \Delta\delta_2 \\ \Delta\delta_3 \end{bmatrix} = -\begin{bmatrix} -0.02 & -0.01 \\ -0.01 & -0.03 \end{bmatrix}\begin{bmatrix} \frac{4.0}{1.05} \\ \frac{-5.0}{1.0} \end{bmatrix} = \begin{bmatrix} 0.0262 \\ -0.1119 \end{bmatrix}$$

In continuation, ΔQ is related to ΔV by the matrix equation

$$\frac{\Delta Q}{|V_i|} = -B''\Delta V \quad \text{(II)}$$

Since bus 2 is a regulated bus, the corresponding row and column of B' are eliminated and we get

$$B' = \begin{bmatrix} \cancel{-60} & \cancel{20} \\ \cancel{20} & -40 \end{bmatrix} \Rightarrow B'' = [-40]$$

which obviously has an inverse

$$B''^{-1} = \left[\frac{1}{-40}\right] = [-0.025]$$

Now, equation (II) can be adjusted to give

$$\frac{\Delta Q}{|V_i|} = -B''\Delta V \rightarrow \Delta V = -B''^{-1}\frac{\Delta Q}{|V_i|}$$

so that

$$[\Delta V_3] = -[-0.025]\left[\frac{-3.0}{1.0}\right] = [-0.075],$$

Updating the unknowns, we find

$$\delta_2(i=1) = 0 + 0.0262 = 0.0262\,\text{rad} = 1.5012^\circ$$
$$\delta_3(i=1) = 0 - 0.1119 = -0.1119\,\text{rad} = -6.4114^\circ$$
$$V_3(i=1) = 1 + (-0.075) = 0.925\,\text{pu}$$

$$P_2 = V_2V_1Y_{21}\cos(\theta_{21} - \delta_2 + \delta_1) + V_2^2Y_{22}\cos(\theta_{22}) + V_2V_3Y_{23}\cos(\theta_{23} - \delta_2 + \delta_3)$$
$$\therefore P_2(1) = 1.05\times1.0\times40\cos(90^\circ - 1.5012^\circ + 0) + 1.05^2\times(-60)\times\cos(-90^\circ)$$
$$+1.05\times0.925\times20\cos(90^\circ - 1.5012^\circ - 6.4114^\circ) = 3.7744$$

$$P_3 = V_3V_1Y_{31}\cos(\theta_{31} - \delta_3 + \delta_1) + V_3V_2Y_{32}\cos(\theta_{32} - \delta_3 + \delta_2) + V_3^2Y_{33}\cos(\theta_{33})$$
$$\therefore P_3(1) = 0.925\times1.0\times20\cos[90^\circ - (-6.4114^\circ) + 0]$$
$$+0.925\times1.05\times20\cos[90^\circ - (-6.4114^\circ) + 1.5012^\circ] + 0.925^2\times(-40)\times\cos(-90^\circ) = -4.7399$$

$$Q_3 = -V_3V_1Y_{31}\sin(\theta_{31} - \delta_3 + \delta_1) - V_3V_2Y_{32}\sin(\theta_{32} - \delta_3 + \delta_2) - V_3^2Y_{33}\sin(\theta_{33})$$
$$\therefore Q_3(1) = -0.925\times1.0\times20\times\sin[90^\circ - (-6.4114^\circ) + 0]$$
$$-0.925\times1.05\times20\times\sin[90^\circ - (-6.4114^\circ) + 1.5012^\circ] - 0.925^2\times|-40|\times\sin(-90^\circ) = -3.3994$$

For the second iteration, the power residuals are

$$\Delta P_2 = P_2 - P_2(1) = 4 - 3.7744 = 0.2256$$
$$\Delta P_3 = P_3 - P_3(1) = -5.0 - (-4.7399) = -0.2601$$
$$\Delta Q_3 = Q_3 - Q_3(1) = -4.0 - (-3.3994) = -0.6006$$

Proceeding with the second iteration, we have, for the phase angles,

$$\begin{bmatrix} \Delta\delta_2 \\ \Delta\delta_3 \end{bmatrix} = -\begin{bmatrix} -0.02 & -0.01 \\ -0.01 & -0.03 \end{bmatrix} \begin{bmatrix} \dfrac{0.2256}{1.05} \\ \dfrac{-0.2601}{0.9250} \end{bmatrix} = \begin{bmatrix} 0.0015 \\ -0.0063 \end{bmatrix}$$

and for voltage V_3,

$$[\Delta V_3] = -[-0.025]\left[\frac{-0.6006}{0.9250}\right] = [-0.0162]$$

Updating the unknowns, we obtain

$$\delta_2(i=2) = 0.0262 + 0.0015 = 0.0277\,\text{rad} = 1.5871^\circ$$
$$\delta_3(i=2) = -0.1119 - 0.0063 = -0.1182\,\text{rad} = -6.7724^\circ$$
$$V_3(i=2) = 0.925 - 0.0162 = 0.9088\,\text{pu}$$

This completes the second iteration.

REFERENCES

- GLOVER, J.D., OVERBYE, T.J., and SARMA, M.S. (2017). *Power System Analysis and Design*. 6th edition. Boston: Cengage Learning.
- NAGSARKAR, T.K. and SUKHIJA, M.S. (2014). *Power System Analysis*. 2nd edition. Oxford: Oxford University Press.
- NASAR, S.A. (1990). *Schaum's Outline of Electric Power Systems*. New York: McGraw-Hill.
- SAADAT, H. (1999). *Power System Analysis*. 2nd edition. New York: McGraw-Hill.