Montogue

Quiz EL107
Power System Stability

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PROBLEMS

Problem 1 (Saadat, 1999, w/ permission)

A four-pole, 60-Hz synchronous generator has a rating of 200 MVA, 0.8 power factor lagging. The moment of inertia of the rotor is 45,100 kg·m^2. Determine the inertia constants *M* and *H*.

Problem 2 (Nasar, 1990, w/ permission)

Problem 2.1: A 60-Hz, four-pole turbogenerator rated 500 MVA, 22 kV has an inertia constant of *H* = 7.5 MJ/MVA. Find the kinetic energy stored in the rotor at synchronous speed and the angular acceleration if the electrical power developed is 400 MW when the input less the rotational losses is 740,000 hp.

Problem 2.2: The generator of Problem 2.1 is delivering rated megavolt-amperes at 0.8 power factor lag when a fault reduces the electric power output by 40%. Determine the accelerating torque in newton-meters at the time the fault occurs. Neglect losses and assume constant power input to the shaft.

Problem 3 (Saadat, 1999, w/ permission)

Problem 3.1: The swing equations of two interconnected synchronous machines are written as

$$\frac{H_1}{\pi f_0}\frac{d^2\delta_1}{dt^2} = P_{m,1} - P_{e,1}$$

and

$$\frac{H_2}{\pi f_0}\frac{d^2\delta_2}{dt^2} = P_{m,2} - P_{e,2}$$

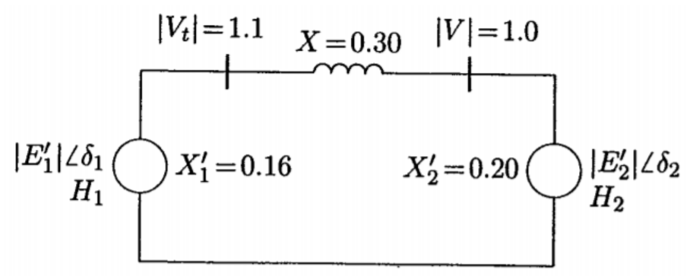
Denote the relative power angle between the two machines by $\delta = \delta_1 - \delta_2$. Obtain a swing equation equivalent to that of a single machine in terms of δ, and show that

$$\frac{H}{\pi f_0}\frac{d^2\delta}{dt^2} = P_m - P_e$$

where

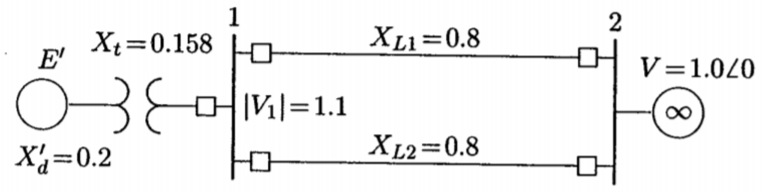
$$H = \frac{H_1 H_2}{H_1 + H_2} \; ; \; P_m = \frac{H_2 P_{m,1} - H_1 P_{m,2}}{H_1 + H_2} \; ; \; P_e = \frac{H_2 P_{e,1} - H_1 P_{e,2}}{H_1 + H_2}$$

‘ **Problem 3.2**: Two synchronous generators represented by a constant voltage behind transient reactance are connected by a pure reactance X = 0.3 per unit, as shown in the following figure. The generator inertia constants are H_1 = 4.0 MJ/MVA and H_2 = 6.0 MJ/MVA, and the transient reactances are X_1' = 0.16 and X_2' = 0.20 per unit. The system is operating in the steady state with E_1' = 1.2, $P_{m,1}$ = 1.5 and E_2' = 1.1, $P_{m,2}$ = 1.0 per unit. Denote the relative power angle between the two machines by $\delta = \delta_1 - \delta_2$. Referring to Problem 3.1, reduce the two-machine system to an equivalent one-machine against an infinite bus. Find the inertia constant of the equivalent machine, the mechanical input power, and the amplitude of its power angle curve, and obtain the equivalent swing equation in terms of δ.



Problem 4 (Saadat, 1999, w/ permission)

Problem 4.1: A 60-Hz synchronous generator has a transient reactance of 0.2 per unit and an inertia constant of 5.66 MJ/MVA. The generator is connected to an infinite bus through a transformer and a double circuit transmission line, as shown below. Resistances are neglected and reactances are expressed on a common MVA base and are marked on the diagram. The generator is delivering a real power of 0.77 per unit to bus bar 1. Voltage magnitude at bus 1 is 1.1. The infinite bus voltage V = 1.0∠0° per unit. Determine the generator excitation voltage and obtain the swing equation.



Problem 4.2: The machine in the power system of Problem 4.1 has a per unit damping coefficient of D = 0.15. The generator excitation voltage is E' = 1.25 per unit and the generator is delivering a real power of 0.77 per unit to the infinite bus at a voltage of V = 1.0 per unit. Write the linearized swing equation for this power system. Use equations 2 and 3 in the Additional Information section to find the expressions describing the motion of the rotor angle and the generator frequency for a small disturbance of $\Delta\delta$ = 15°. Use MATLAB to obtain plots of rotor angle and frequency.

Problem 4.3: The generator of Problem 4.1 is operating in the steady state at δ_0 when the input power is increased by a small amount ΔP = 0.15 per unit. The generator excitation and the infinite bus voltage are the same as before. Use equations 4 and 5 to establish the motion of the rotor angle and the generator frequency for a small disturbance of ΔP = 0.15 per unit. Use MATLAB to obtain the plots of rotor angle and frequency.

Problem 4.4: The machine of Problem 4.1 is delivering a real power input of 0.77 per unit to the infinite bus at a voltage of 1.0 per unit. The generator excitation voltage is E' = 1.25 per unit. Use *eacpower*(P_m, *E*, *V*, *X*) (download the *.m* file in our website) to find the maximum power input that can be added without loss of synchronism. Repeat the calculation with zero initial power input, assuming the generator internal voltage remains constant at the value computed in the first situation.

Problem 5 (Kothari and Nagrath, 2003, w/ permission)

A synchronous motor is drawing 30% of the maximum steady state power from an infinite bus bar. If the load on motor is suddenly increased by 100 percent, would the synchronism be lost? If not, what is the maximum excursion of torque angle about the new steady state rotor position? Use the equal area criterion.

Problem 6 (Grainger and Stevenson Jr., 1994)

A generator having H = 6.0 MJ/MVA is delivering power of 1.0 per unit to an infinite bus through a purely reactive network when the occurrence of a fault reduces the generator output power to zero. The maximum power that could be delivered is 2.5 per unit. When the fault is cleared the original network conditions again exist. Using equations 6 and 7 in the Additional Information section, determine the critical clearing angle and critical clearing time.

Problem 7 (Kothari and Nagrath, 2003, w/ permission)

A synchronous generator is feeding 250 MW to a large 50 Hz network over a double circuit transmission line. The maximum steady-state power that can be transmitted over the line with both circuits in operation is 500 MW or 350 MW with any one of the circuits. A solid three-phase fault occurring at the network-end of one of the lines causes it to trip. Estimate the critical clearing angle in which the circuit breakers must trip. Estimate the critical clearing angle in which the circuit breakers must trip so that synchronism is not lost. What further information is needed to estimate the critical clearing time?

Problem 8 (Kothari and Nagrath, 2003, w/ permission)

The transfer reactances between a generator and an infinite bus bar operating at 200 kV under various conditions on the interconnector are:

Pre-fault: 150 Ω per phase
During fault: 400 Ω per phase
Post-fault: 200 Ω per phase

If the fault is cleared when the rotor has advanced 60 degrees electrical from its prefault position, determine the maximum load that could be transferred without loss of stability. Try solving the problem *without* equation 8 in the Additional Information section.

Problem 9 (Kothari and Nagrath, 2003, w/ permission)

A 60-Hz generator is supplying 60% of P_{max} to an infinite bus through a reactive network. A fault occurs which increases the reactance of the network between the generator internal voltage and the infinite bus by 400%. When the fault is cleared the maximum power that can be delivered is 80% of the original maximum value. Determine the critical clearing angle for the condition described. Use equation 8 in the Additional Information section.

Problem 10 (Nasar, 1990, w/ permission)

Problem 10.1: The kinetic energy stored in the rotor of a 50-MVA, six-pole, 60-Hz synchronous machine is 200 MJ. The input to the machine is 25 MW at a developed power of 22.5 MW. Calculate the accelerating power and the acceleration.

Problem 10.2: If the acceleration of the machine of Problem 10.1 remains constant for ten cycles, what is the power angle at the end of the ten cycles?

Problem 10.3: The generator of Problem 10.1 has an internal voltage of 1.2 pu and is connected to an infinite bus operating at a voltage of 1.0 pu through a 0.3-pu reactance. A three-phase short circuit flows on the line. Subsequently, circuit breakers operate and the reactance between the generator and the bus becomes 0.4 pu. Calculate the critical clearing angle.

Problem 10.4: Plot the swing curve for the machine considered in Problems 10.1 to 10.3.

Problem 10.5: From the results of Problems 10.3 and 10.4, find the critical clearing time in cycles for an appropriately set circuit breaker.

ADDITIONAL INFORMATION

Equations

1 → *Linearized swing equation*

$$\frac{d^2\Delta\delta}{dt^2}+2\zeta\omega_n\frac{d\Delta\delta}{dt}+\omega_n^2\Delta\delta=0$$

where $\Delta\delta$ is a small disturbance in power angle, t is time, ζ is damping ratio, and ω_n is the angular frequency of oscillation.

2 → *Motion of a rotor relative to a synchronously revolving field – small disturbances in power angle*

$$\delta=\delta_0+\frac{\Delta\delta_0}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin\left(\omega_d t+\theta\right)$$

where δ_0 is initial power angle, $\Delta\delta_0$ is the (small) deviation in power angle, ζ is the damping ratio, ω_n is the angular frequency of oscillation, ω_d is the damped frequency of oscillation, t is time, and $\theta = \cos^{-1}\zeta$.

3 → *Angular frequency of a rotor relative to a synchronously revolving field – small disturbances in power angle*

$$\omega=\omega_0-\frac{\omega_n\Delta\delta_0}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin\omega_d t$$

where ω_0 is the initial angular frequency of oscillation and other variables are as defined in equation 2.

4 → *Motion of a rotor relative to a synchronously revolving field – small disturbances in power input*

$$\delta = \delta_0 + \frac{\pi f_0 \Delta P}{H\omega_n^2}\left[1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t}\sin\left(\omega_d t + \theta\right)\right]$$

where ΔP is the disturbance in power input, f_0 is the initial linear frequency, H is the per unit inertia constant, and other variables are as defined for equation 2.

5 → *Angular frequency of a rotor relative to a synchronously revolving field – small disturbances in power angle*

$$\omega = \omega_0 + \frac{\pi f_0 \Delta P}{H\omega_n\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t}\sin\omega_d t$$

where variables are as defined for equations 3 and 4.

6 → *Critical clearing angle (radians)*

$$\delta_{\text{cr}} = \cos^{-1}\left[\left(\pi - 2\delta_0\right) - \cos\delta_0\right]$$

where δ_0 is the initial power angle.

7 → *Critical clearing time*

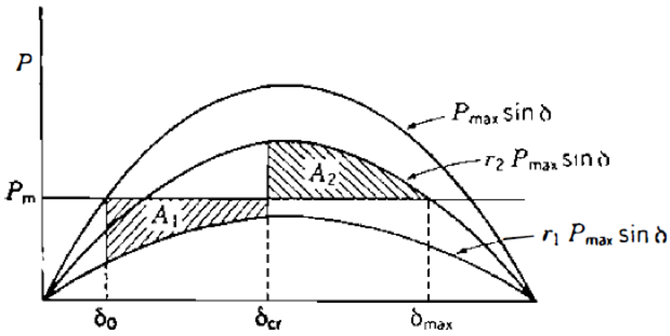
$$t_{\text{cr}} = \sqrt{\frac{4H\left(\delta_{\text{cr}} - \delta_0\right)}{\omega_s P_m}}$$

where H is the per-unit inertia constant, δ_{cr} is the critical clearing angle determined from equation 6, δ_0 is the initial power angle, ω_s is the angular synchronous speed, and P_m is mechanical input power.

8 → *Critical clearing angle for fault clearing*

$$\cos\delta_{\text{cr}} = \frac{\left(P_m/P_{\max}\right)\left(\delta_{\max} - \delta_0\right) + r_2\cos\delta_{\max} - r_1\cos\delta_0}{r_2 - r_1}$$

where P_m is mechanical input power, P_{max} is maximum power, $\delta_{\max}$ is the angle indicated below, and δ_0 is the initial clearing angle. Further, r_1 is a coefficient in $r_1 P_{max} \sin\delta$, the power that can be transmitted during fault, and r_2 is a coefficient in $r_2 P_{max} \sin\delta$, the power that can be transmitted after the fault is cleared by switching at the instant when $\delta = \delta_{\text{cr}}$.



Appendix – Step-by-step solution of the swing equation

The swing equation may be solved iteratively with the step-by-step procedure illustrated in the next figure. In the solution, it is assumed that the accelerating power P_a and the relative rotor angular frequency ω_r are constant within each of a succession of intervals (top and middle graphs); their values are used to find the change in δ during each interval.

To begin the iterations, we need $P_a(0+)$, which we evaluate as

$$P_a\left(0+\right) = P_i - P_e\left(0+\right)$$

Then, the swing equation may be written

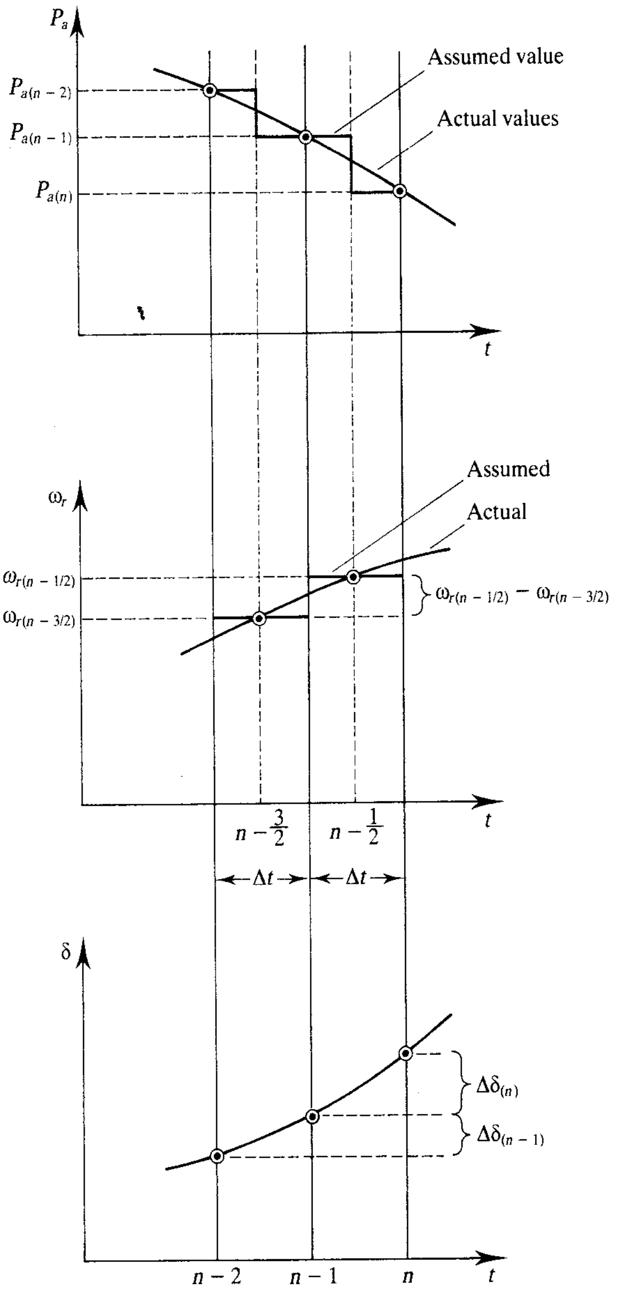
$$\frac{d^2\delta}{dt^2} = \alpha\left(0+\right) = \frac{P_a\left(0+\right)}{M}$$

and the change in ω_r is given by

$$\Delta\omega_r = \alpha\left(0+\right)\Delta t$$

Then,

$$\omega_r = \omega_0 + \Delta\omega_r = \omega_0 + \alpha\left(0+\right)\Delta t$$



Similarly, the change in power angle for the first interval is

$$\Delta\delta_1 = \Delta\omega_r \Delta t$$

and so

$$\delta_1 = \delta_0 + \Delta\delta_1 = \delta_0 + \alpha(0+)(\Delta t)^2 \quad \text{(I)}$$

Evaluation of P_a. If there is no discontinuity in the swing curve during an iteration interval, then $P_a(0+)$ is equal to half of P_a immediately after the fault. If there is a discontinuity at the beginning of the i-th interval, then

$$P_{a(i-1)} = \frac{1}{2}\left(P_{a(i-1)-} + P_{a(i-1)+}\right)$$

where $P_{a(i-1)-}$ and $P_{a(i-1)+}$ are, respectively, the accelerating power immediately before and immediately after the fault is cleared.

If the discontinuity occurs at the middle of an interval, then for that interval

$$P_a = P_i - \text{output during the fault}$$

For this case, at the beginning of the interval immediately following the clearing of the fault, P_a is given by

$$P_a = P_i - \text{output after the fault is cleared}$$

Finally, if the discontinuity occurs neither at the beginning nor at the middle of an interval, P_a may still be evaluated from the three preceding equations.

Algorithm for the iterations. Returning now to (I), we see that δ_1 gives one point on the swing curve. The algorithm for the iterative process is as follows:

$$P_{a(n-1)} = P_i - P_{e(n-1)}$$

$$P_{e(n-1)} = \frac{|E||V|}{X} \sin \delta_{(n-1)}$$

$$\alpha_{(n-1)} = \frac{P_{a(n-1)}}{M}$$

$$\Delta\omega_{r(n)} = \alpha_{(n-1)}\Delta t$$

$$\omega_{r(n)} = \omega_{r(n-1)} + \alpha_{(n-1)}\Delta t$$

$$\Delta\delta_{(n)} = \Delta\delta_{(n-1)} + \frac{P_{a(n-1)}}{M}(\Delta t)^2$$

$$\delta_{(n)} = \delta_{(n-1)} + \Delta\delta_{(n)}$$

SOLUTIONS

P.1 ➡ Solution

Given f_0 = 60 Hz and the number of poles P = 4, we first convert the generator operating frequency as

$$n_s = \frac{120 f_0}{P} = \frac{120\times 60}{4} = 1800\,\text{rpm}$$

The corresponding angular velocity is

$$\omega_{sm} = \frac{2\pi n_s}{60} = \frac{2\pi\times 1800}{60} = 188\text{ rad/s}$$

Given the mass moment of inertia J = 45,100 kg·m^2, the kinetic energy of the rotor is found as

$$W_k = \frac{1}{2}J\omega_{sm}^2 = \frac{1}{2}\times 45{,}100\times 188^2 = 797\text{ MJ}$$

The inertia constant M is obtained by dividing $2W_k$ by the rotor angular velocity; in mathematical terms,

$$M = \frac{2W_k}{\omega_{sm}} = \frac{2\times 797}{188} = 8.48 \approx \boxed{8.5\,\text{MJ}\cdot\text{sec}}$$

The H constant, in turn, is obtained by dividing kinetic energy by the machine rating in MVA,

$$H = \frac{W_k}{S_B} = \frac{797}{200} = 3.99 \approx \boxed{4.0\text{ MJ/MVA}}$$

P.2 ➡ Solution

Problem 2.1: The kinetic energy is given by the product of generator rating and inertia constant H,

$$W_k = 500\times 7.5 = \boxed{3750\,\text{MJ}}$$

Converting the input power to MW,

$$\text{Input power} = 740{,}000\times\left(746\times 10^{-6}\right) = 552\,\text{MW}$$

Appealing to the swing equation,

$$\frac{H}{\pi f_0}\frac{d^2\delta}{dt^2} = \text{Input power} - \text{Rotational loss}$$

$$\therefore \frac{7.5}{\pi\times 60}\frac{d^2\delta}{dt^2} = \frac{552-400}{500}$$

$$\therefore \frac{d^2\delta}{dt^2} = 438\,\text{mech. degrees/s}^2$$

For a four-pole machine,

$$\frac{d^2\delta}{dt^2} = \frac{438}{2} = 219\,\text{mech. degres/s}^2$$

Converting to rpm/s^2,

$$\frac{d^2\delta}{dt^2} = 219\times\frac{60}{360} = \boxed{36.5\,\text{rpm/s}^2}$$

Problem 2.2: The power P_a developed by the generator at the time of fault is calculated as

$$P_a = \omega_m T_a = (1.0-0.6)\times 0.8\times 500 = 160\,\text{MW}$$

where

$$\omega_m = \frac{2\pi f}{2} = \frac{2\pi\times 60}{2} = 189\,\text{mech. radians/s}$$

so that

$$P_a = \omega_m T_a \rightarrow T_a = \frac{P_a}{\omega_m}$$

$$\therefore T_a = \frac{160\times 10^6}{189} = \boxed{847{,}000\,\text{N-m}}$$

P.3 ➡ Solution

Problem 3.1: We restate the two swing equations provided as follows,

$$\frac{H_!}{\pi f_0}\frac{d^2\delta_1}{dt^2} = P_{m,1} - P_{e,1} \rightarrow \frac{1}{\pi f_0}\frac{d^2\delta_1}{dt^2} = \frac{P_{m,1}}{H_!} - \frac{P_{e,1}}{H_1}$$

$$\frac{H_2}{\pi f_0}\frac{d^2\delta_2}{dt^2} = P_{m,2} - P_{e,2} \rightarrow \frac{1}{\pi f_0}\frac{d^2\delta_2}{dt^2} = \frac{P_{m,2}}{H_2} - \frac{P_{e,2}}{H_2}$$

Then, we subtract the second equation from the first,

$$\frac{1}{\pi f_0}\frac{d^2\delta_1}{dt^2} - \frac{1}{\pi f_0}\frac{d^2\delta_2}{dt^2} = \left(\frac{P_{m,1}}{H_1} - \frac{P_{e,1}}{H_1}\right) - \left(\frac{P_{m,2}}{H_2} - \frac{P_{e,2}}{H_2}\right)$$

$$\therefore \frac{1}{\pi f_0}\frac{d^2(\delta_1-\delta_2)}{dt^2} = \left(\frac{P_{m,1}}{H_1} - \frac{P_{m,2}}{H_2}\right) - \left(\frac{P_{e,1}}{H_1} - \frac{P_{e,2}}{H_2}\right)$$

$$\therefore \frac{1}{\pi f_0}\frac{d^2(\delta_1-\delta_2)}{dt^2} = \left(\frac{H_2P_{m,1} - H_1P_{m,2}}{H_1H_2}\right) - \left(\frac{H_2P_{e,1} - H_1P_{e,2}}{H_1H_2}\right)$$

Multiplying both sides by $H_1H_2/(H_1+H_2)$ and using the definitions provided in the problem statement,

$$\frac{1}{\pi f_0}\underbrace{\frac{H_1H_2}{H_1+H_2}}_{=H}\frac{d^2\left(\underbrace{\delta_1-\delta_2}_{=\delta}\right)}{dt^2} = \underbrace{\left(\frac{H_2P_{m,1}-H_1P_{m,2}}{H_1+H_2}\right)}_{=P_m} - \underbrace{\left(\frac{H_2P_{e,1}-H_1P_{e,2}}{H_1+H_2}\right)}_{=P_e}$$

$$\therefore \frac{H}{\pi f_0}\frac{d^2\delta}{dt^2} = P_m - P_e$$

as we intended to show.

Problem 3.2: Using the formulas developed in Problem 3.1, the equivalent parameters are calculated to be

$$H = \frac{H_1H_2}{H_1+H_2} = \frac{4.0\times 6.0}{4.0+6.0} = 2.4\,\text{MJ/MVA}$$

$$P_m = \frac{H_2P_{m,1}-H_1P_{m,2}}{H_1+H_2} = \frac{6.0\times 1.5 - 4.0\times 1.0}{4.0+6.0} = 0.5\,\text{pu}$$

$$P_{e,1} = \frac{|E_1||E_2|}{X}\sin(\delta_1-\delta_2) = \frac{|1.2|\times|1.1|}{(0.16+0.30+0.20)}\times\sin\delta = 2.0\sin\delta$$

Since $P_{e,2} = -P_{e,1}$, we may write

$$P_e = \frac{H_2 P_{e,1} - H_1 P_{e,2}}{H_1 + H_2} = \frac{6.0\times(2.0\sin\delta) - 4.0\times(-2.0\sin\delta)}{4.0+6.0} = \frac{12.0\sin\delta + 8.0\sin\delta}{10.0} = 2\sin\delta$$

Therefore, the equivalent swing equation is

$$\frac{H}{\pi f_0}\frac{d^2\delta}{dt^2} = P_m - P_e \rightarrow \frac{2.4}{180^{\text{o}}\times 60}\frac{d^2\delta}{dt^2} = 0.5 - 2\sin\delta$$

$$\therefore 0.000222\frac{d^2\delta}{dt^2} = 0.5 - 2\sin\delta$$

$$\therefore \frac{d^2\delta}{dt^2} = 4500(0.5 - 2\sin\delta),\ \text{where } \delta \text{ is in degrees}$$

P.4 ➡ *Solution*

Problem 4.1: We have the voltage magnitude at bus 1, but the phase angle is missing. Noting that the two 0.8-pu reactances can be reduced to a single 0.8/2 = 0.4 pu line, we may write

$$P = \frac{|V_1||V_2|}{X_L}\sin\delta_1 \rightarrow 0.77 = \frac{|1.1|\times|1.0|}{0.4}\sin\delta_1$$

$$\therefore \sin\delta_1 = 0.280$$

$$\therefore \delta_1 = 16.3^{\text{o}}$$

so that

$$I = \frac{V_1 - V_2}{jX_L} = \frac{1.1\angle 16.3^{\text{o}} - 1.0\angle 0^{\text{o}}}{j0.4} = 0.772 - j0.140 = 0.784\angle -10.2^{\text{o}}\,\text{pu}$$

The total reactance is X = 0.2 + 0.158 + 0.4 = 0.758, and the generator excitation voltage is

$$E' = 1.0 + j0.758\times(0.784\angle -10.2^{\text{o}}) = 1.25\angle 27.9^{\text{o}}\,\text{pu}$$

Finally, the swing equation with δ in radians is

$$\frac{H}{\pi f_0}\frac{d^2\delta}{dt^2} = P_m - P_{\max}\sin\delta \rightarrow \frac{5.66}{\pi\times 60}\frac{d^2\delta}{dt^2} = 0.77 - \frac{1.25\times 1.0}{0.758}\sin\delta$$

$$\therefore 0.030\frac{d^2\delta}{dt^2} = 0.77 - 1.65\sin\delta$$

Problem 4.2: The linearized swing equation is given by

$$\frac{d^2\Delta\delta}{dt^2} + 2\zeta\omega_n\frac{d\Delta\delta}{dt} + \omega_n^2\Delta\delta = 0$$

where ω_n is the angular frequency of oscillation and ζ is the damping ratio. The former is given by

$$\omega_n = \sqrt{\frac{\pi f_0}{H}P_s}$$

where f_0 is the linear frequency, H is inertia constant, and P_s is the synchronizing power coefficient. The damping ratio, in turn, is given by

$$\zeta = \frac{D}{2}\sqrt{\frac{\pi f_0}{HP_s}}$$

where D is the per-unit damping coefficient. Using the given data, we first determine the initial power angle

$$0.77 = \frac{1.25\times 1.0}{0.758}\sin\delta_0 \rightarrow \sin\delta_0 = 0.467$$

$$\therefore \delta_0 = 27.8^{\text{o}}$$

Then, the synchronizing power coefficient is

$$P_s = P_{\max}\cos\delta_0 = \frac{1.25\times 1.0}{0.758}\cos 27.8^\circ = 1.46$$

We proceed to determine ω_n and ζ,

$$\omega_n = \sqrt{\frac{\pi f_0}{H}P_s} = \sqrt{\frac{\pi\times 60}{5.66}\times 1.46} = 6.97\,\text{rad/s}$$

$$\zeta = \frac{D}{2}\sqrt{\frac{\pi f_0}{HP_s}} = \frac{0.15}{2}\times\sqrt{\frac{\pi\times 60}{5.66\times 1.46}} = 0.358$$

It follows that the linearized force-free equation that determines the mode of oscillation is, with δ in radians,

$$\frac{d^2\Delta\delta}{dt^2} + 2\zeta\omega_n\frac{d\Delta\delta}{dt} + \omega_n^2\Delta\delta = 0$$

$$\therefore \frac{d^2\Delta\delta}{dt^2} + 2\times 0.358\times 6.97\frac{d\Delta\delta}{dt} + 6.97^2\Delta\delta = 0$$

$$\therefore \frac{d^2\Delta\delta}{dt^2} + 4.99\frac{d\Delta\delta}{dt} + 48.6\Delta\delta = 0$$

The damped circular frequency of oscillation is

$$\omega_d = \omega_n\sqrt{1-\zeta^2} = 6.97\times\sqrt{1-0.358^2} = 6.51\,\text{rad/s}$$

and corresponds to a linear frequency such that

$$f_d = \frac{\omega_d}{2\pi} = 1.04\,\text{Hz}$$

Also, $\theta = cos^{-1}\,\zeta = cos^{-1}$ 0.358 = 69.0°. Now, the motion of the rotor relative to the synchronously revolving field is described by the equation

$$\delta = \delta_0 + \frac{\Delta\delta_0}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin\left(\omega_d t + \theta\right)\ \ \text{(I)}$$

Also, the rotor angular frequency is given by

$$\omega = \omega_0 - \frac{\omega_n\Delta\delta_0}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin\omega_d t$$

which can be normalized to yield

$$f = f_0 - \frac{f_n\Delta\delta_0}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin\omega_d t\ \ \text{(II)}$$

Here, in addition to familiar variables, we were given $\Delta\delta_0$ = 15°. Substituting in equations (I) and (II) brings to

$$\delta = 27.8^\circ + \frac{15^\circ}{\sqrt{1-0.358^2}}e^{-0.358\times 6.97\times t}\sin\left(6.51t + 69.0^\circ\right)$$

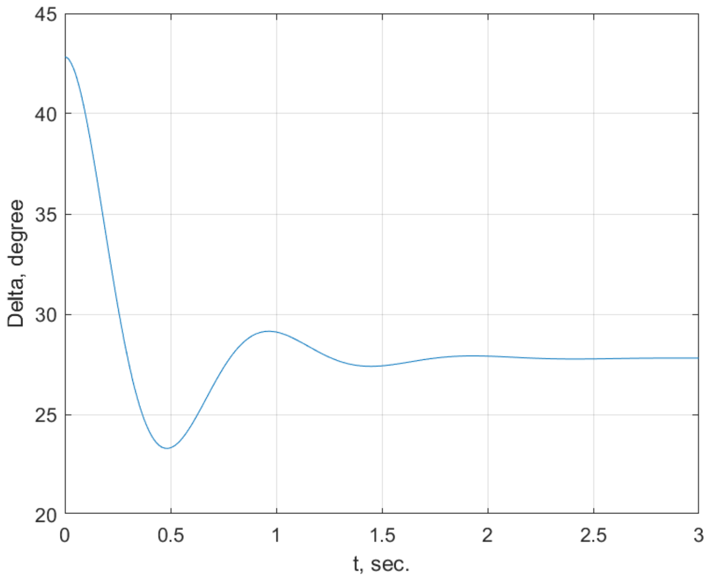
$$\therefore \delta = 27.8^\circ + 16.1e^{-2.50t}\sin\left(6.51t + 69.0^\circ\right)$$

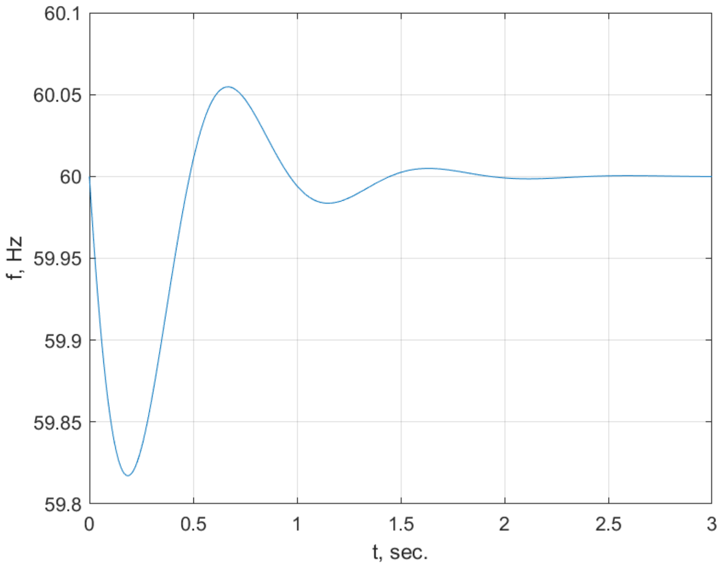
$$f = 60 - 0.311e^{-2.50t}\sin\left(6.51t\right)$$

The remaining step is to plot rotor angle and frequency. For $t \in (0, 3)$ s, using MATLAB,

```
t = 0:.01:3;
d = 27.8 + 16.1*exp(-2.50*t).*sin(6.51*t + 69.0*pi/180);
f = 60 - 0.311*exp(-2.50*t).*sin(6.51*t);
figure(1), plot(t,d), grid
xlabel('t, sec.'), ylabel('Delta, degree')
figure(2), plot(t,f), grid
xlabel('t, sec.'), ylabel('f, Hz')
```

The resulting graphs are shown below.





Problem 4.3: The equation that describes the motion of the rotor is, in the case at hand,

$$\delta = \delta_0 + \frac{\pi f_0 \Delta P}{H\omega_n^2}\left[1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t}\sin\left(\omega_d t + \theta\right)\right]$$

while the rotor angular frequency is given by

$$\omega = \omega_0 + \frac{\pi f_0 \Delta P}{H\omega_n\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t}\sin\omega_d t$$

or, equivalently,

$$f = f_0 + \frac{f_0 \Delta P}{2H\omega_n\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t}\sin\omega_d t$$

Substituting our data in the equation for δ, we get

$$\delta = 27.8^{\text{o}} + \frac{180\times60\times0.15}{5.66\times6.97^2}\times\left[1 - \frac{1}{\sqrt{1-0.358^2}}\times e^{-0.358\times6.97\times t}\sin\left(6.51t+69.0^{\text{o}}\right)\right]$$

$$\therefore \delta = 27.8^{\text{o}} + 5.89\left[1 - 1.07e^{-2.50t}\sin\left(6.97t+69.0^{\text{o}}\right)\right]$$

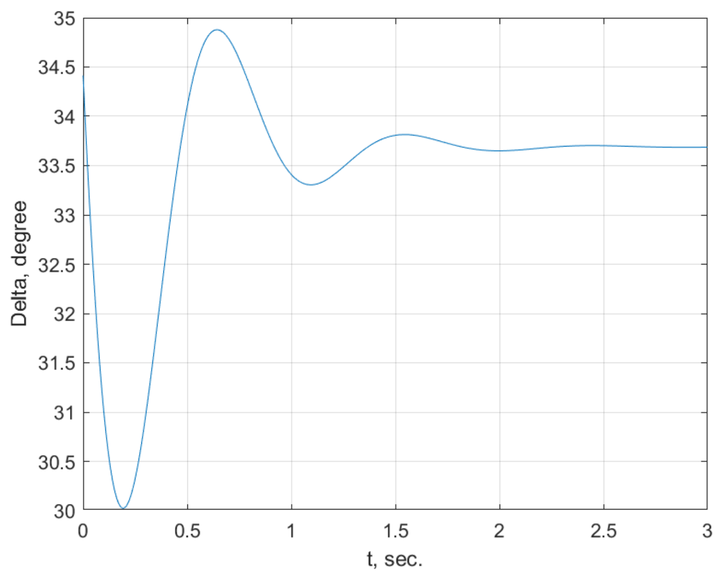
Likewise for f,

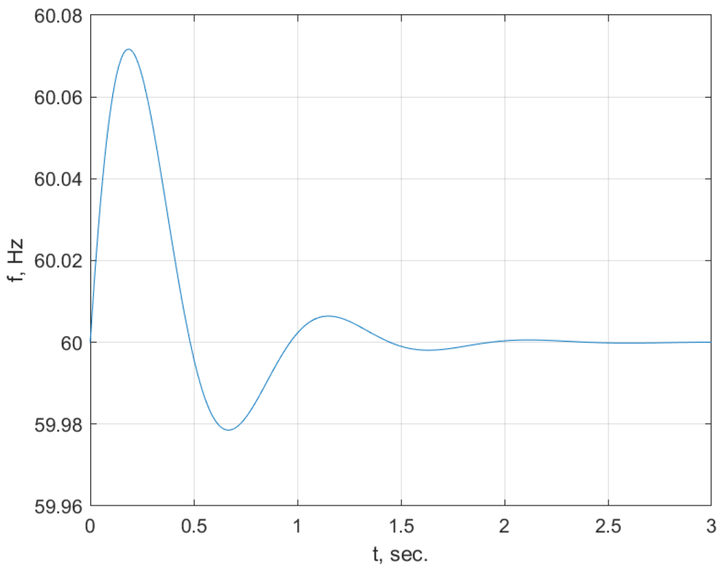
$$f = 60 + 0.122e^{-2.50t}\sin 6.51t$$

To plot these expressions, write the MATLAB code

```
t = 0:.01:3;
d = 27.8 + 5.89*(1-1.07*exp(-2.5*t).*sin(6.97*t+69));
f = 60 + 0.122*exp(-2.5*t).*sin(6.51*t);
figure(1), plot(t,d), grid
xlabel('t, sec.'), ylabel('Delta, degree')
figure(2), plot(t,f), grid
xlabel('t, sec.'), ylabel('f, Hz')
```

The outputs are shown below.





Problem 4.4: The real power input is P_0 = 0.77 pu; the generator excitation voltage is E' = 1.25 pu; the voltage of the machine is V = 1.0 pu; the transfer reactance is X = 0.758 pu. Appealing to *eacpower*, we apply the code

```
disp('(a) Initial real power P0 = 0.77')
P0 = 0.77; E = 1.25; V = 1.0; X = 0.758;
h=figure;
eacpower(P0,E,V,X)
h=figure;
disp('(b) Zero initial power')
P0 = 0;
eacpower(P0,E,V,X)
```

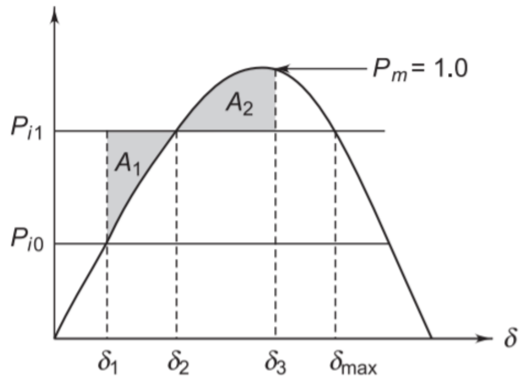
Running this piece of code yields

```
(a) Initial real power PO = 0.77
Initial power                        =  0.770 p.u.
Initial power angle                  = 27.835 degrees
Sudden additional power              =  0.649 p.u.
Total power for critical stability  =  1.419 p.u.
Maximum angle swing                  =120.617 degrees
New operating angle                  = 59.383 degrees

Current plot held
(b) Zero initial power
Initial power                        =  0.000 p.u.
Initial power angle                  =  0.000 degrees
Sudden additional power              =  1.195 p.u.
Total power for critical stability  =  1.195 p.u.
Maximum angle swing                  =133.563 degrees
New operating angle                  = 46.437 degrees
```

P.5 ➡ *Solution*

Refer to the following graph.



Initially,

$$P_{i0} = 0.3 = \sin\delta_1$$
$$\therefore \delta_1 = 17.5^{\circ}$$

After the load is doubled,

$$P_{i1} = 0.6 = \sin\delta_2$$
$$\therefore \delta_2 = 36.9^{\circ}$$

Area A_1 is given by

$$A_1 = 0.6(\delta_2 - \delta_1) - \int_{\delta_1}^{\delta_2} \sin\delta d\delta = 0.049$$

Likewise, area A_2 is

$$A_2 = \int_{\delta_2}^{\delta_3} \sin\delta d\delta - 0.6(\delta_3 - \delta_2)$$

Subtracting A_2 from A_1,

$$A_2 - A_1 = \int_{\delta_1}^{\delta_3} \sin\delta d\delta - 0.6(\delta_3 - \delta_1) = 0$$
$$\therefore \cos\delta_3 + 0.6\delta_3 = 1.14$$

This transcendental equation can be easily solved with Mathematica's *FindRoot* command,

```
In[21]:= FindRoot[Cos[δ3] + 0.6 δ3 - 1.14, {δ3, 1}]
Out[21]= {δ3 → 1.00125}
```

That is, δ_3 = 1.00 rad = 57.4°. Synchronism will not be lost. Angle δ_{max} is obtained by symmetry,

$$\delta_{max} = 180^{\circ} - \delta_2 = 180^{\circ} - 36.9^{\circ} = 143^{\circ}$$

Area $A_{2,max}$ is to be compared to A_1 to ascertain system stability,

$$A_{2,max} = \int_{\delta_2}^{\delta_{max}} \sin\delta d\delta - 0.6(\delta_{max} - \delta_2) = -\cos\delta\Big|_{\delta_2}^{\delta_{max}} - 0.6(\delta_{max} - \delta_2)$$

$$\therefore A_{2,max} = -\cos 143^{\circ} + \cos 36.9^{\circ} - 0.6 \times (143^{\circ} - 36.9^{\circ}) = 0.487$$

Since $A_{2,max} > A_1$, the system is stable. The maximum excursion angle Δ about the new rotor position that can be attained while still retaining synchronism is

$$\Delta = \delta_3 - \delta_2 = 57.4^{\circ} - 36.9^{\circ} = \boxed{20.5^{\circ}}$$

P.6 ➡ Solution

We first determine power angle δ_0,

$$2.5\sin\delta_0 = 1.0 \rightarrow \delta_0 = \arcsin\left(\frac{1.0}{2.5}\right)$$
$$\therefore \delta_0 = 23.6^{\circ} = 0.412\,\text{rad}$$

The critical angle can be computed with equation 6,

$$\delta_{cr} = \cos^{-1}\left[(\pi - 2\delta_0)\sin\delta_0 - \cos\delta_0\right] = \cos^{-1}\left[(\pi - 2\times 0.412)\times\sin 23.6^{\circ} - \cos 23.6^{\circ}\right]$$

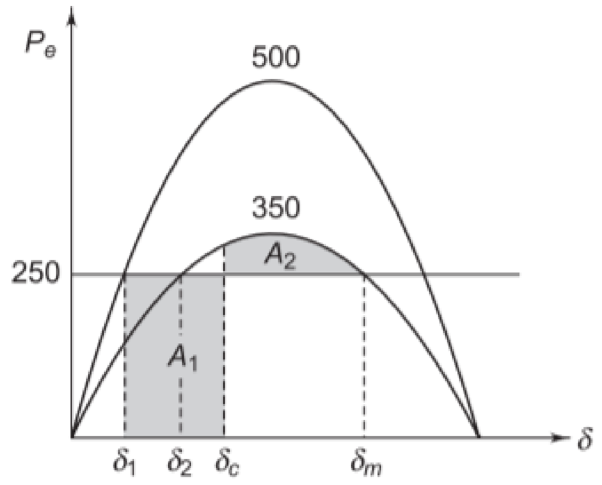
$$\therefore \delta_{cr} = 1.56\,\text{rad} = \boxed{89.4^\circ}$$

The critical clearing time is given by equation 7,

$$t_{cr} = \sqrt{\frac{4H(\delta_{cr} - \delta_0)}{\omega_s P_m}} = \sqrt{\frac{4 \times 6 \times (1.56 - 0.412)}{(2\pi \times 60) \times 1.0}} = \boxed{0.270\,\text{s}}$$

P.7 ➡ Solution

Refer to the following graph.



$$\delta_1 = \sin^{-1}\frac{250}{500} = 30^\circ$$

$$\delta_2 = \sin^{-1}\frac{350}{500} = 45.6^\circ$$

From symmetry of the lower curve, $\delta_m = 180^\circ - \delta_2 = 134^\circ$. Let δ_c denote the critical clearing angle. Rectangular area A_1 is given by

$$A_1 = \frac{\pi}{180} \times (\delta_c - 30^\circ) \times 250 = 4.36\delta_c - 131$$

Area A_2 is, in turn,

$$A_2 = \int_{\delta_c}^{\delta_m} (350\sin\delta - 250)\,d\delta = 350\cos\delta_c + 250\delta_c - 342$$

Expressing the term in the middle in degrees for homogeneity,

$$A_2 = 350\cos\delta_c + 250\delta_c \times \frac{\pi}{180} - 342 = 350\cos\delta_c + 4.36\delta_c - 342$$

Equating A_1 and A_2 and solving for δ_c,

$$\cancel{4.36\delta_c} - 131 = 350\cos\delta_c + \cancel{4.36\delta_c} - 342$$

$$\therefore -131 = 350\cos\delta_c - 342$$

$$\therefore \cos\delta_c = 0.603$$

$$\therefore \boxed{\delta_c = 53.0^\circ}$$

Writing the swing equation up to the critical angle and integrating twice,

$$\frac{d^2\delta}{dt^2} = \frac{250}{M} \to \frac{d\delta}{dt} = \frac{250}{M}t \to \delta(t) = \frac{125}{M}t^2 + \delta_1$$

where we have used zero initial velocity and initial power angle = δ_1 as boundary conditions. Evaluating the resulting function at the critical clearing time,

$$\delta(t_c) = \delta_c = \frac{125}{M}t_c^2 + \delta_1$$

$$\therefore \delta_c - \delta_1 = \frac{125}{M}t_c^2$$

$$\therefore (53.0^\circ - 30^\circ) \times \frac{\pi}{180} = \frac{125}{M}t_c^2$$

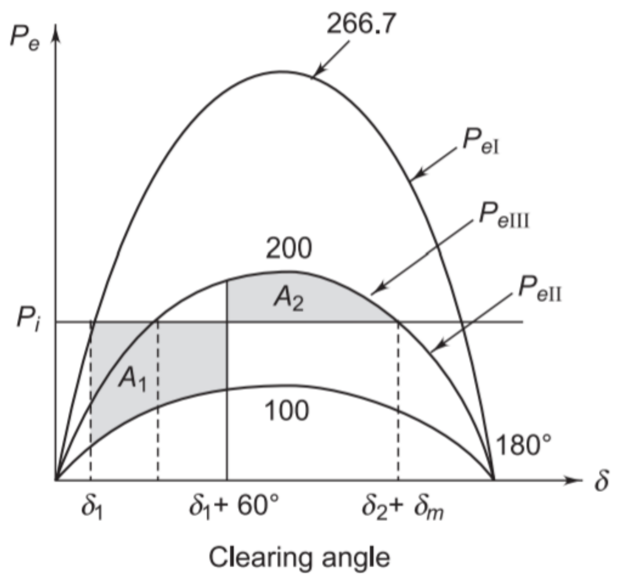
$$\therefore 0.401 \text{ rad} = \frac{125}{M} t_c^2$$

$$\therefore t_c = \sqrt{0.00321M} = 0.0567M^{1/2} \text{ s}$$

To determine the critical clearing time, we need the inertia constant M.

P.8 ➡ *Solution*

Three pertaining power versus clearing angle plots are shown below.



Using the three reactance values in turn, we have

$$P_{e,I}\left(\text{pre-fault}\right) = \frac{200^2}{150}\sin\delta = 267\sin\delta$$

$$P_{e,II}\left(\text{during fault}\right) = \frac{200^2}{400}\sin\delta = 100\sin\delta$$

$$P_{e,III}\left(\text{post-fault}\right) = \frac{200^2}{200}\sin\delta = 200\sin\delta$$

Maximum load transfer corresponds to $A_1 = A_2$, so that

$$A_1 = \int_{\delta_1}^{\delta_1+60^\circ}\left(P_1 - 100\sin\delta\right)d\delta = P_i \times \frac{\pi}{180}\times 60^\circ + 100\times\left[\cos\left(\delta_1+60^\circ\right) - \cos\delta_1\right]$$

With $P_I = 267 sin\ \delta_1$,

$$A_1 = 267\times\frac{\pi}{3}\times\sin\delta_1 + 100\times\cos\left(\delta_1+60^\circ\right) - 100\times\cos\delta_1 \quad \text{(I)}$$

Now, with

$$\delta_2 = 180^\circ - \sin^{-1}\left(\frac{P_i}{200}\right) = 180^\circ - \sin^{-1}\left(\frac{267}{200}\sin\delta_1\right) \quad \text{(i)}$$

we have, for A_2,

$$A_2 = \int_{\delta_1+60^\circ}^{\delta_2}\left(200\sin\delta - P_i\right)d\delta = -200\cos\delta\Big|_{\delta_1+180^\circ}^{\delta_2} - P_i\left(\delta_2 - \delta_1 - 60^\circ\right)\times\pi/180$$

$$\therefore A_2 = -200\cos\delta\Big|_{\delta_1+60^\circ}^{\delta_2} - P_i\left(\delta_2 + \delta_1 - 60^\circ\right)\times\pi/180$$

$$\therefore A_2 = -200\cos\delta_2 + 200\cos\left(\delta_1+60^\circ\right) - 4.65\left(\delta_2 - \delta_1 - 60^\circ\right)\sin\delta_1 \quad \text{(II)}$$

Equating (I) and (II) brings to

$$280\sin\delta_1 + 100\cos\left(\delta_1+60^\circ\right) - 100\cos\delta_1 =$$
$$-200\cos\delta_2 + 200\cos\left(\delta_1+60^\circ\right) - 4.65\left(\delta_2 - \delta_1 - 60^\circ\right)\sin\delta_1$$

with δ_2 given by (i). This trigonometric equation can be easily solved with Mathematica's *FindRoot* command, yielding $\delta_1 = 28.5°$. The maximum load follows as

$$P_{i(\max)} = 267\sin 28.5^\circ = \boxed{127\,\text{MW}}$$

P.9 ➡ *Solution*

We begin by determining δ_0, the phase angle before fault,

$$P_{\max}\sin\delta_0 = 0.6P_{\max} \rightarrow \delta_0 = \arcsin 0.6 = 36.9° = 0.644\,\text{rad}$$

Since the fault causes the reactance between the generator internal voltage and the infinite bus to increase by 400%, $r_1 = 1/4.0 = 0.25$. Further, because the maximum power that can be delivered is 80% of the original maximum, $r_2 = 0.8$. This extreme state of operation corresponds to a phase angle $\delta_{\max}$ such that

$$r_2P_{\max}\sin\left(180°-\delta_{\max}\right) = P_m \rightarrow \sin\left(180°-\delta_{\max}\right) = \frac{P_m}{P_{\max}}\times\frac{1}{r_2}$$

$$\therefore \sin\left(180°-\delta_{\max}\right) = 0.6\times\frac{1}{0.8} = 0.75$$

$$\therefore 180°-\delta_{\max} = \sin^{-1}0.75 = 48.6°$$

$$\therefore \delta_{\max} = 180°-48.6° = 131° = 2.29\,\text{rad}$$

Using equation 8 yields the critical angle

$$\cos\delta_{cr} = \frac{\left(P_m/P_{\max}\right)\left(\delta_{\max}-\delta_0\right)+r_2\cos\delta_{\max}-r_1\cos\delta_0}{r_2-r_1}$$

$$\therefore \cos\delta_{cr} = \frac{0.6\times\left(2.29-0.644\right)+0.8\times\cos 2.29-0.25\times\cos 0.644}{0.8-0.25} = 0.474$$

$$\delta_{cr} = \cos^{-1}0.474 = \boxed{61.7°}$$

P.10 ➡ *Solution*

Problem 10.1: The accelerating power is

$$P_a = P_i - P_e = 25-22.5 = \boxed{2.5\,\text{MW}}$$

Inertia constant H is determined as

$$H = \frac{W_k}{S_B} = \frac{200}{50} = 4.0$$

and can be used to determine M,

$$M = \frac{S_BH}{\pi f_0} = \frac{50\times 4.0}{180\times 60} = 0.0185\,\text{MJ}\cdot\text{s/degree} = 1.06\,\text{MJ}\cdot\text{s/rad}$$

Lastly, dividing the accelerating power by inertia constant M gives the acceleration

$$M\frac{d^2\delta}{dt^2} = P_a \rightarrow \frac{d^2\delta}{dt^2} = \frac{P_a}{M}$$

$$\therefore \frac{d^2\delta}{dt^2} = \frac{2.5}{1.06} = \boxed{2.36\,\text{rad/s}^2}$$

Problem 10.2: Integrating $\ddot{\delta} = 2.36$ with respect to t brings to

$$\frac{d^2\delta}{dt^2} = 2.36 \rightarrow \frac{d\delta}{dt} = 2.36t + C_1$$

Since $\dot{\delta} = 0$ at $t = 0$, constant $C_1 = 0$. Integrating a second time yields

$$\frac{d\delta}{dt} = 2.36t \rightarrow \delta = 1.18t^2 + C_2$$

At $t = 0$, let $\delta = \delta_0$ (the initial power angle). It follows that

$$\delta(t) = 1.18t^2 + \delta_0$$

At 60 Hz, the time required for ten cycles is $t = 10/60 = 0.167$ s. For this value of t,

$$\delta(0.167) = 1.18 \times 0.167^2 + \delta_0 = \boxed{0.0329 + \delta_0 \text{ rad}}$$

Problem 10.3: The critical clearing angle can be determined from equation 8, which we restate as

$$\cos\delta_{cr} = \frac{(\delta_{max} - \delta_0)\sin\delta_0 + r_2\cos\delta_{max} - r_1\cos\delta_0}{r_2 - r_1}$$

The maximum power before the fault is

$$P_{max} = \frac{1.2 \times 1.0}{0.3} = 4.0\,\text{pu}$$

and can be used to determine the initial power angle δ_0, namely

$$4.0\sin\delta_0 = 1.0 \rightarrow \sin\delta_0 = 0.25$$
$$\therefore \delta_0 = \sin^{-1} 0.25 = 14.5^\circ = 0.253\,\text{rad}$$

During the fault, $P_{max,2}$ = 0 and r_2 = 0. After the fault is cleared, the reactance between the generator and the bus becomes 0.4 pu and $P_{max,3}$ is computed as

$$P_{max,3} = \frac{1.2 \times 1.0}{0.4} = 3.0\,\text{pu}$$

Thus, r_2 = 3.0/4.0 = 0.75. The last quantity we need is δ_{max},

$$\sin(180^\circ - \delta_{max}) = \frac{1}{3.0} \rightarrow 180^\circ - \delta_{max} = \sin^{-1}\left(\frac{1}{3.0}\right) = 19.5^\circ$$
$$\therefore \delta_{max} = 180^\circ - 19.5^\circ = 161^\circ = 2.81\,\text{rad}$$

Finally, the critical clearing angle is calculated to be

$$\cos\delta_{cr} = \frac{(2.81 - 0.253)\sin 0.253 + 0.75 \times \cos 2.81 - 0 \times \cos 0.253}{0.75 - 0} = -0.0921$$
$$\therefore \delta_{cr} = \cos^{-1} -0.0921 = \boxed{95.3^\circ}$$

Problem 10.4: The critical clearing angle can be determined from equation 8, which we restate as

$$\cos\delta_{cr} = \frac{(\delta_{max} - \delta_0)\sin\delta_0 + r_2\cos\delta_{max} - r_1\cos\delta_0}{r_2 - r_1}$$

The maximum power before the fault is

$$P_{max} = \frac{1.2 \times 1.0}{0.3} = 4.0\,\text{pu}$$

and can be used to determine the initial power angle δ_0, namely

$$4.0\sin\delta_0 = 1.0 \rightarrow \sin\delta_0 = 0.25$$

$$\therefore \delta_0 = \sin^{-1} 0.25 = 14.5^\circ = 0.253\,\text{rad}$$

During the fault, $P_{max,2}$ = 0 and r_2 = 0. After the fault is cleared, the reactance between the generator and the bus becomes 0.4 pu and $P_{max,3}$ is computed as

$$P_{max,3} = \frac{1.2 \times 1.0}{0.4} = 3.0\,\text{pu}$$

Thus, r_2 = 3.0/4.0 = 0.75. The last quantity we need is δ_{max},

$$\sin(180^\circ - \delta_{max}) = \frac{1}{3.0} \rightarrow 180^\circ - \delta_{max} = \sin^{-1}\left(\frac{1}{3.0}\right) = 19.5^\circ$$

$$\therefore \delta_{max} = 180^\circ - 19.5^\circ = 161^\circ = 2.81\,\text{rad}$$

Finally, the critical clearing angle is calculated to be

$$\cos\delta_{cr} = \frac{(2.81-0.253)\sin 0.253 + 0.75\times\cos 2.81 - 0\times\cos 0.253}{0.75-0} = -0.0921$$

$$\therefore \delta_{cr} = \cos^{-1} - 0.0921 = \boxed{95.3^{\circ}}$$

The per-unit value of the angular momentum, based on the machine rating, is

$$M = \frac{1.0\times 4}{180\times 60} = 3.70\times 10^{-4}\text{ pu}$$

Following the algorithm outlined in the Appendix, we initially have, using Δt = 0.05 s,

$$P_a(0+) = \frac{1.0-0.0}{2} = 0.5$$

$$\alpha(0+) = \frac{P_a(0+)}{M} = \frac{0.5}{3.7\times 10^{-4}} = 1351^{\circ}/\text{s}$$

$$\Delta\omega_{r(1)} = \alpha(0+)\Delta t = 1351\times 0.05 = 67.55^{\circ}/\text{s}$$

$$\omega_{r(1)} = \omega_{r(0)} + \Delta\omega_{r(1)} = 0 + 67.57 = 67.55^{\circ}/\text{s}$$

$$\Delta\delta_{(1)} = \omega_{r(1)}\Delta t = 67.55\times 0.05 = 3.379^{\circ}$$

Finally, with δ_0 = 14.5° determined in Problem 10.3,

$$\delta_{(1)} = \delta_{(0)} + \Delta\delta_{(1)} = 14.5^{\circ} + 3.379^{\circ} = 17.88^{\circ}$$

Proceeding similarly with the second interval,

$$P_{a(1)} = 1.0 - 0.0 = 1.0$$

$$\alpha_{(1)} = \frac{1.0}{3.7\times 10^{-4}} = 2703^{\circ}/\text{s}$$

$$\Delta\omega_{r(2)} = \alpha_{(1)}\Delta t = 2703\times 0.05 = 135.2^{\circ}/\text{s}$$

$$\omega_{r(2)} = \omega_{r(1)} + \Delta\omega_{r(2)} = 67.55 + 135.2 = 202.8^{\circ}/\text{s}$$

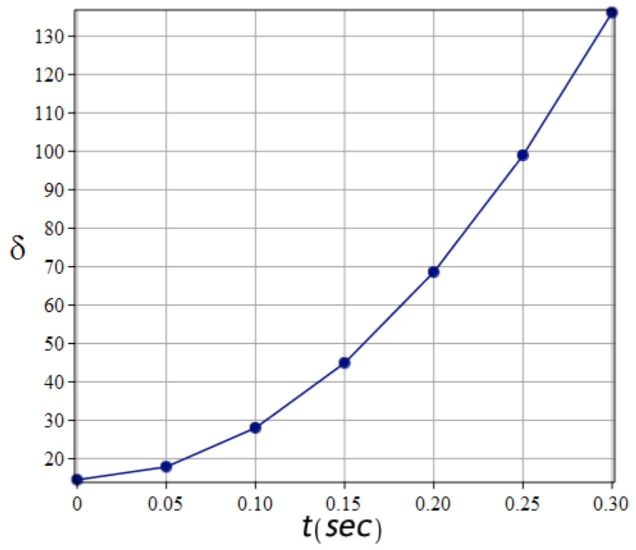
$$\Delta\delta_{(2)} = \omega_{r(2)}\Delta t = 202.8\times 0.05 = 10.14^{\circ}$$

$$\delta_{(2)} = \delta_{(1)} + \Delta\delta_{(2)} = 17.88^{\circ} + 10.14^{\circ} = 28.02^{\circ}$$

Calculations for other steps are tabulated below.

t	P_a	α	$\Delta\omega_r$	ω_r	$\Delta\delta$	δ
0	0.5	1351				14.50
0.05	1.0	2703	67.55	67.55	3.378	17.88
0.1	1.0	2703	135.15	202.70	10.14	28.01
0.15	1.0	2703	135.15	337.85	16.89	44.91
0.2	1.0	2703	135.15	473.00	23.65	68.56
0.25	1.0	2703	135.15	608.15	30.41	98.96
0.3	1.0	2703	135.15	743.30	37.17	136.13

The swing curve is obtained by plotting power angle (red column) versus time (blue column), as shown on the next page.



Problem 10.5: In Problem 10.3 we established the critical clearing angle to be 95.3°. Entering this ordinate into the preceding graph, we read a time of 0.245 s. Hence the fault must be cleared within $60 \times 0.245 = 14.7$ cycles.

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