## Montogue

Quiz HY101<br>Precipitation

Lucas Montogue

## Problems

## Problem 1

It is estimated that 60\% of annual precipitation in a basin with a drainage area of 20,000 acres is evaporated. If the average annual river flow at the outlet of the basin has been observed to be 3 cfs, determine the long term (annual) precipitation in the basin.


A) $P_{l}=1.37 \mathrm{in} . /$ year
B) $P_{l}=3.26 \mathrm{in} . /$ year
C) $P_{l}=5.15 \mathrm{in}$. year
D) $P_{l}=7.04$ in./year

## Problem 2

During a 2 hour storm, the precipitation over a 40 hectare area is 30 mm . There is a constant infiltration at a rate of $10 \mathrm{~mm} /$ hour. The depression storage is 2.5 hectare-m. Is there any runoff? If yes, how much? Disregard evaporation during the storm.

A) $R=1.2$ hectare -m
B) $R=0.8$ hectare-m
C) $R=0.4$ hectare -m
D) There is no runoff.

## Problem 3

Water from a $250 \mathrm{ft} \times 80 \mathrm{ft}$ lawn converges into a gully. During a 1-hour storm, discharge into the gully is $0.95 \mathrm{ft}^{3} / \mathrm{s}$. The interception by grass is 0.42 acreft . Overall, $60 \%$ of the rain is infiltrated. What is the amount of rainfall?

A) $P=23.69 \mathrm{in}$.
B) $P=32.58 \mathrm{in}$.
C) $P=41.47 \mathrm{in}$.
D) $P=50.36 \mathrm{in}$.

## Problem 4

The following table exists for five stations in a basin for the month of September. Estimate the missing rainfall for September 2018 at station B using the normal ratio method.

| Station | All Years'Average <br> for September $(\mathrm{cm})$ | Measured in <br> September 2018 $(\mathrm{cm})$ |
| :---: | :---: | :---: |
| A | 11.43 | 11.01 |
| B | 7.62 | $?$ |
| C | 5.08 | 4.83 |
| D | 6.35 | 6.86 |
| E | 12.19 | 12.70 |

A) $P_{B}=7.10 \mathrm{~cm}$
B) $P_{B}=7.69 \mathrm{~cm}$
C) $P_{B}=8.05 \mathrm{~cm}$
D) $P_{B}=8.47 \mathrm{~cm}$

## Problem 5 (Mimikou et al., 2016)

A basin consists of a network of 6 rain gauges. The annual rainfall records from these rain gauges are listed in the following table. Calculate the optimum number of rain gauges for the basin, with $10 \%$ error in the calculation of the mean annual rainfall.

| Rain Gauge | 1 | 2 | 3 | 4 | 5 | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Annual Rainfall (cm) | 48 | 75 | 81 | 63 | 104 | 89 |

A) $N=7$
B) $N=8$
C) $N=9$
D) $N=10$

## Problem 6A

The following table shows the mean values of measured annual precipitation in 5 rain gauges located in a certain catchment. Determine the average rainfall by the average method.


| Station | Annual Precipitation (mm) |
| :---: | :---: |
| P1 | 800 |
| P2 | 600 |
| P3 | 900 |
| P4 | 400 |
| P5 | 200 |

A) $P_{\text {Avg }}=500 \mathrm{~mm}$
B) $P_{\text {Avg }}=540 \mathrm{~mm}$
C) $P_{\text {Avg }}=580 \mathrm{~mm}$
D) $P_{\text {Avg }}=620 \mathrm{~mm}$

## Problem 6B

Given that each square corresponds to $1 \mathrm{~km} \times 1 \mathrm{~km}$ surface area, compute the mean annual precipitation for the previous problem by means of the Thiessen method.

## Problem 6C

Solve the previous problem by the isohyetal method.

## Problem 7

Calculate the rainfall at station $A$ in the following figure using the method of inverse distances. The distances of the five adjacent stations from station A, labeled B to F, and the corresponding rainfall depths are provided in the following table.


| Station | $\left\|x_{i}-x_{o}\right\|(\mathrm{km})$ | $\left\|y_{i}-y_{o}\right\|(\mathrm{km})$ | Rainfall (mm) |
| :---: | :---: | :---: | :---: |
| B | 1.2 | 1.1 | 25 |
| C | 1.3 | 1.5 | 35 |
| D | 2.0 | 0.6 | 14 |
| E | 0.8 | 1.0 | 23 |
| F | 0.5 | 1.6 | 19 |

A) $P_{\text {Avg }}=20.4 \mathrm{~mm}$
B) $P_{\text {Avg }}=23.2 \mathrm{~mm}$
C) $P_{\text {Avg }}=26.1 \mathrm{~mm}$
D) $P_{\text {Avg }}=29.0 \mathrm{~mm}$

## Problem 8A

The annual rainfall in millimeters at two stations in the period of 2000 to 2018 is given in the following table. There are gaps in some years due to malfunction in one of the stations. Estimate the missing data for station 2 in the period going from 2006 to 2007, and for the period going from 2013 to 2014, using linear regression.

| Hydrological Year | Station 1 | Station 2 |
| :---: | :---: | :---: |
| $2000-2001$ | 1145.8 | Missing! |
| $2001-2002$ | 1278.5 | 1455.8 |
| $2002-2003$ | 1262.8 | 1422.7 |
| $2003-2004$ | 1186.8 | 1496.1 |
| $2004-2005$ | 975.6 | 1366.7 |
| $2005-2006$ | 863.4 | 1098.9 |
| $2006-2007$ | 1066.8 | Missing! |
| $2007-2008$ | 985.2 | 978.6 |
| $2008-2009$ | 1166.0 | 1318.6 |
| $2009-2010$ | 1306.4 | Missing! |
| $2010-2011$ | 1268.6 | 1502.3 |
| $2011-2012$ | 1297.5 | 1402.5 |
| $2012-2013$ | 1239.6 | 1544.3 |
| $2013-2014$ | 958.3 | Missing! |
| $2014-2015$ | 1322.2 | 1499.8 |
| $2015-2016$ | 1108.1 | 1475.4 |
| $2016-2017$ | 1295.2 | 1399.8 |
| $2017-2018$ | 1100.5 | 1355.9 |

A) $P_{2006-2007}=1144.4 \mathrm{~mm}$ and $P_{2013-2014}=958.3 \mathrm{~mm}$
B) $P_{2006-2007}=1144.4 \mathrm{~mm}$ and $P_{2013-2014}=1205.6 \mathrm{~mm}$
C) $P_{2006-2007}=1295.8 \mathrm{~mm}$ and $P_{2013-2014}=958.3 \mathrm{~mm}$
D) $P_{2006-2007}=1295.8 \mathrm{~mm}$ and $P_{2013-2014}=1205.6 \mathrm{~mm}$

## Problem 8B

Which of the following intervals contains the coefficient of linear regression for the line obtained in the previous part?
A) $r \in(0.2 ; 0.4)$
B) $r \in(0.4 ; 0.6)$
C) $r \in(0.6 ; 0.8)$
D) $r \in(0.8 ; 1.0)$

## Problem 9

The annual precipitation at station $X$ and the mean annual precipitation at 12 surrounding stations are given below. Adjust the data at station $X$. At what year did the breakpoint (change in regime) occur?

|  | Annual Precipitation (mm) |  |  | Annual Precipitation (mm) |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Year | Station X | 12-Station Mean | Year | Station X | 12-Station Mean |
|  |  | 2000 | 447 | 325 |  |
| 1986 | 517 |  | 2001 | 396 | 442 |
| 1987 | 264 | 2002 | 385 | 364 |  |
| 1988 | 462 | 338 | 2003 | 308 | 299 |
| 1989 | 397 | 390 | 2004 | 319 | 429 |
| 1990 | 275 | 338 | 2005 | 352 | 430 |
| 1991 | 385 | 340 | 2008 | 329 | 455 |
| 1992 | 378 | 339 | 2009 | 253 | 338 |
| 1993 | 337 | 364 | 2010 | 407 | 377 |
| 1994 | 407 | 520 | 2011 | 374 | 364 |
| 1995 | 385 | 338 | 2012 | 330 | 442 |
| 1996 | 638 | 312 | 2013 | 308 | 429 |
| 1997 | 358 | 286 | 2014 | 297 | 455 |
| 1998 | 350 |  |  | 438 |  |
| 1999 | 290 |  |  | 325 |  |

A) 1993
B) 1998
C) 2003
D) 2008

## Problem 10 (Guo, 2017)

The following table presents a rainfall event from 16:00 to 17:00. Graph the depth versus duration ( $P-D$ ) curve and the intensity versus duration (I-D) curve for this rainfall event on the same plot.

| Clock Time | Incremental <br> Rainfall Depth (in.) |
| :---: | :---: |
| $16: 00$ | 0.00 |
| $16: 05$ | 0.08 |
| $16: 10$ | 0.11 |
| $16: 15$ | 0.25 |
| $16: 20$ | 0.41 |
| $16: 25$ | 0.55 |
| $16: 30$ | 0.67 |
| $16: 35$ | 1.08 |
| $16: 40$ | 0.59 |
| $16: 45$ | 0.18 |
| $16: 50$ | 0.06 |
| $16: 55$ | 0.06 |
| $17: 00$ | 0.03 |

## Problem 11

Using the data arranged for different durations in the following table, prepare intensity-duration-frequency (IDF) curves for 5-year and 10-year frequencies.

|  | Precipitation (in.) of Duration |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Rank | 5 min. | 10 min. | 15 min. | 20 min. | 30 min. | 60 min. |
| 1 | 0.40 | 0.66 | 0.89 | 1.07 | 1.48 | 2.15 |
| 2 | 0.38 | 0.63 | 0.83 | 0.97 | 1.29 | 1.92 |
| 3 | 0.37 | 0.62 | 0.79 | 0.91 | 1.26 | 1.48 |
| 4 | 0.36 | 0.60 | 0.76 | 0.86 | 0.91 | 1.06 |
| 5 | 0.35 | 0.60 | 0.73 | 0.80 | 0.83 | 0.96 |
| 6 | 0.33 | 0.58 | 0.72 | 0.77 | 0.82 | 0.94 |
| 7 | 0.33 | 0.50 | 0.72 | 0.77 | 0.78 | 0.90 |
| 8 | 0.31 | 0.50 | 0.63 | 0.70 | 0.75 | 0.87 |
| 9 | 0.30 | 0.49 | 0.57 | 0.65 | 0.67 | 0.77 |
| 10 | 0.28 | 0.44 | 0.56 | 0.62 | 0.66 | 0.75 |
| $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ |
| 22 | 0.13 | 0.23 | 0.32 | 0.40 | 0.40 | 0.43 |

## Solutions

## P. 1 ■ Solution

The long term water balance for a basin is

$$
P-E-Q=0
$$

where $P$ is precipitation, $E$ is evapotranspiration, and $Q$ is net flow (outflow minus inflow). We are told that the evaporation is $60 \%$ of the precipitation, or, mathematically, $E=0.6 P$. Substituting this result in the aforementioned equation and solving for $P$, we get

$$
\begin{gathered}
P-\underset{\substack{=0.6 P}}{E}-Q=0 \\
\therefore 0.4 P-Q=0 \\
\therefore P=2.5 Q=2.5 \times 3=7.5 \mathrm{cfs}
\end{gathered}
$$

since $Q=3$ cfs. Some quick unit conversion will allow us to convert this to a longterm - say, annual - quantity,
$P_{\text {depth }}=\frac{P_{\text {vol }}}{A}=\frac{7.5 \frac{\mathrm{ft}^{3}}{\mathrm{~s}}}{20,000 \text { acres }} \times \frac{1}{43,560} \frac{\text { acre }}{\mathrm{ft}^{2}} \times \frac{3600}{1} \frac{\mathrm{~s}}{\mathrm{hr}} \times \frac{24}{1} \frac{\mathrm{~h}}{\text { day }} \times \frac{365}{1} \frac{\text { days }}{\text { year }}=0.271 \frac{\mathrm{ft}}{\text { year }}$
or, in inches,

$$
P_{l}=0.271 \frac{\mathrm{ft}}{\text { year }} \times \frac{12}{1} \frac{\mathrm{in}}{\mathrm{ft}}=3.26 \mathrm{in} . / \text { year }
$$

The correct answer is $\mathbf{B}$.

## P. 2 ■ Solution

The precipitation can be converted to a volume by multiplying it by the area over which the precipitation occurred,

$$
P_{\mathrm{vol}}=P_{\mathrm{depth}} \times A=\left(30 \times 10^{-3}\right) \mathrm{m} \times 40 \text { hectare }=1.2 \text { hectare- } \mathrm{m}
$$

The infiltration rate, in turn, can be converted to a total volume of infiltration by multiplying by the area and the duration of the storm (= 2 hours),

$$
I=10 \frac{\mathrm{~mm}}{\mathrm{hr}} \times 2 \mathrm{hr} \times \frac{10^{-3}}{1} \frac{\mathrm{~m}}{\mathrm{~mm}} \times 40 \text { hectare }=0.8 \text { hectare }-\mathrm{m}
$$

The water balance equation for runoff during a storm is the following,

$$
P-E-I-S_{D}-R=0
$$

in which $P$ is precipitation, $E$ is evapotranspiration, $l$ is infiltration, $S_{D}$ is interception and depression storage, and $R$ is runoff. Ignoring evapotranspiration and solving for $R$, it follows that

$$
\begin{gathered}
P-\not Z-I-S_{D}-R=0 \rightarrow R=P-I-S_{D} \\
\therefore R=1.2-0.8-2.5=-2.1
\end{gathered}
$$

The negative value implies that there is no runoff.

* The correct answer is D.


## P. 3 ■ Solution

The water balance equation for runoff during a storm is

$$
P-E-I-S_{D}-R=0
$$

where $P$ is precipitation, $E$ is evapotranspiration, $l$ is infiltration, $S_{D}$ is interception and depression storage, and $R$ is runoff. Since $60 \%$ of the rain is infiltrated, we can write $I=0.6 E$. The equation then reduces to

$$
\begin{gathered}
P-E-I-S_{D}-R=0 \\
\therefore P-0.6 P-I-S_{D}-R=0 \\
\therefore 0.4 P-\not \subset-S_{D}-R=0 \\
\quad \therefore P=\frac{S_{D}+R}{0.4} \text { (I) }
\end{gathered}
$$

Since precipitation is often expressed as a depth, we should convert $S_{D}$ and $R$ to units of depth. The area of the lawn is $80 \times 250=20,000 \mathrm{ft}^{2}$. The interception by grass, $S_{D}=0.42$ acre-ft, can be converted by dividing it by this area and using the conversion factor 1 acre $=43,560 \mathrm{ft}^{2}$,

$$
S_{D}=\frac{0.42 \text { acre- } \mathrm{ft}}{20,000 \mathrm{ft}^{2}} \times \frac{43,560}{1} \frac{\mathrm{ft}^{2}}{\text { acre }} \times \frac{12}{1} \frac{\mathrm{in} .}{\mathrm{ft}}=10.98 \mathrm{in} .
$$

The runoff, $R=0.95 \mathrm{ft}^{3} / \mathrm{s}$, is converted if we divide it by the lawn area and multiply it by the duration of the storm ( $=1 \mathrm{hr}$ ),

$$
R=\frac{0.95 \mathrm{ft}^{3} / \mathrm{s}}{20,000 \mathrm{ft}^{2}} \times 1 \mathrm{hr} \times \frac{3600}{1} \frac{\mathrm{~s}}{\mathrm{hr}} \times \frac{12}{1} \frac{\mathrm{in} .}{\mathrm{ft}}=2.05 \mathrm{in}
$$

We can then substitute the two previous results into equation (I) and obtain the depth of rainfall,

$$
P=\frac{S_{D}+R}{0.4}=\frac{10.98+2.05}{0.4}=32.58 \mathrm{in} .
$$

$\star$ The correct answer is $\mathbf{B}$.

## P. $4 ■$ Solution

The missing data from station $A$ during the storm can be calculated with the normal-ratio method,

$$
\frac{P_{B}}{N_{B}}=\frac{1}{n}\left(\frac{P_{A}}{N_{A}}+\frac{P_{C}}{N_{C}}+\frac{P_{D}}{N_{D}}+\frac{P_{E}}{N_{E}}\right)
$$

where $P_{X}$ is the average precipitation measured in the month of September at the station labeled $X, N_{X}$ is the all years' average precipitation at the station labeled $X$, and $n$ is the number of stations with complete data. Substituting $n=5$ along with other pertaining data, we obtain

$$
\begin{gathered}
\frac{P_{B}}{N_{B}}=\frac{1}{n}\left(\frac{P_{A}}{N_{A}}+\frac{P_{C}}{N_{C}}+\frac{P_{D}}{N_{D}}+\frac{P_{E}}{N_{E}}\right) \rightarrow \frac{P_{B}}{7.62}=\frac{1}{4}\left(\frac{11.01}{11.43}+\frac{4.83}{5.08}+\frac{6.86}{6.35}+\frac{12.70}{12.19}\right) \\
\therefore P_{B}=7.62 \times \frac{1}{4}\left(\frac{11.01}{11.43}+\frac{4.83}{5.08}+\frac{6.86}{6.35}+\frac{12.70}{12.19}\right)=7.69 \mathrm{~cm}
\end{gathered}
$$

* The correct answer is $\mathbf{B}$.


## P. 5 ■ Solution

The adequacy of a network of rain gauges is determined statistically. The optimum number of rain gauges corresponding to a specified percentage of error is obtained with the formula

$$
N=\left(\frac{C_{u}}{\varepsilon}\right)^{2}
$$

where $N$ is the optimum number of rain gauges, $C_{u}$ is the variation coefficient of the rainfall at the measuring devices, and $\varepsilon$ is the allowed percentage of error in percent. The standard value of $\varepsilon$ is $10 \%$ (otherwise more rain gauges should be deployed). If there are $m$ gauges in a basin, and $P_{1}, P_{2}, \ldots, P_{m}$ are the rainfall depths for a specific time interval, then coefficient $C_{u}$ is $C_{u}=100 S / P$, where $P$ is the mean value of the rainfall recorded in the rain gauges and $S$ is the (sample) standard deviation. The arithmetic mean of the annual rainfall from the six rain gauges is

$$
P=\frac{1}{m} \sum_{i=1}^{m} P_{i}=\frac{(48+75+81+63+104+89)}{6}=76.67 \mathrm{~cm}
$$

The standard deviation is

$$
S=\left[\sum_{i=1}^{m} \frac{\left(x_{i}-\bar{x}\right)^{2}}{m-1}\right]^{0.5}=19.64 \mathrm{~cm}
$$

The $C_{u}$ coefficient is

$$
C_{u}=\frac{100 \times 19.64}{76.67}=25.62 \%
$$

The optimum number of gauges for the basin being investigated follows

$$
N=\left(\frac{25.62}{10}\right)^{2}=6.56 \approx 7
$$

Thus, one more gauge should be installed.

* The correct answer is $\mathbf{A}$.


## P. $6 ■$ Solution

Part A: In the average method, the average areal rainfall is simply the arithmetic mean of the annual precipitation in each station. Accordingly,

$$
P_{\mathrm{Avg}}=\frac{800+600+900+400+200}{5}=580 \mathrm{~mm}
$$

* The correct answer is $\mathbf{C}$.

Part B: The first step in our solution is to outline the Thiessen polygons. We first join the location of one basin with the location of the others and then take the median of each segment. The Thiessen polygons are outlined by the intersections of the medians with each other and with the profile of the basin. Thence, the area of each polygon can be obtained, at least approximatively, by recalling that each square on the grid corresponds to a distance of 1 km .


We then take the product of the area of each polygon and the precipitation of the basin located within it, and finally obtain the average precipitation by dividing the sum of $A \times P$ products by the area of the basin; that is, $P_{\text {Avg }}=\Sigma(A \times P) / A_{\text {Basin }}$. The calculations are summarized in the following table.

| Station | Area $\left(\mathrm{km}^{2}\right)$ | Precipitation $(\mathrm{mm})$ | Prec. $\times$ Area |
| :---: | :---: | :---: | :---: |
| 1 | 11.5 | 800 | 9200 |
| 2 | 16.5 | 600 | 9900 |
| 3 | 13 | 900 | 11,700 |
| 4 | 12.5 | 400 | 5000 |
| 5 | 11.5 | 200 | 2300 |
|  | $A_{\text {Basin }}=65$ |  | $\Sigma(P \times A)=38,100$ |

Thus,

$$
P_{\mathrm{Avg}}=\frac{\Sigma(P \times A)}{A_{\text {Basin }}}=\frac{38,100}{65}=586.2 \mathrm{~mm}
$$

Part C: With the information we were given, we can outline the contours of equal precipitation, or isohyets, and approximate the area encompassed by each one with the aid of the gridlines. The formula to determine the average precipitation is the same as with the Thiessen method, i.e., $P_{\mathrm{Avg}}=\Sigma(A \times P) / A_{\text {Basin }}$ the difference being that $A$ is the area enclosed by a pair of isohyets and $P$ is the mean precipitation of that pair (e.g., the value of $P$ to be used with the area between isohyets of 900 mm and 800 mm is $(900+800) \div 2=850 \mathrm{~mm})$.


| Isohyets $(\mathrm{mm})$ | Area $\left(\mathrm{km}^{2}\right)$ | Precipitation $(\mathrm{mm})$ | Prec. $\times$ Area |
| :---: | :---: | :---: | :---: |
| 900 | 6.0 | 900 | 5400 |
| $900-800$ | 8.0 | 850 | 6800 |
| $800-700$ | 9.0 | 750 | 6750 |
| $700-600$ | 7.0 | 650 | 4550 |
| $600-500$ | 8.5 | 550 | 4675 |
| $500-400$ | 8.5 | 450 | 3825 |
| $400-300$ | 8.0 | 350 | 2800 |
| $300-200$ | 6.0 | 250 | 1500 |
| 200 | 4.0 | 200 | 800 |
|  | $A_{\text {Basin }}=65$ |  | $\Sigma(P \times A)=37,100$ |

Thus,

$$
P_{\mathrm{Avg}}=\frac{\Sigma(P \times A)}{A_{\text {Basin }}}=\frac{37,100}{65}=570.8 \mathrm{~mm}
$$

## P. 7 ■ Solution

According to the method of inverse distances, the rainfall at station $A$ is given by the relation

$$
P_{A}=\sum\left(P_{i} \times w_{i}\right)
$$

where $P_{i}$ is the rainfall depth at each neighboring station and $w_{i}$ is a coefficient that is a function of the distance $d_{i}$ from station A , and can be obtained with the formula

$$
w_{i}=\frac{1 / d_{i}^{2}}{\sum_{j=1}^{k}\left(1 / d_{j}^{2}\right)}
$$

With these expressions in mind, the following table is prepared.

| $n$ | $P_{i}(\mathrm{~mm})$ | $\left\|x_{i}-x_{o}\right\|(\mathrm{km})$ | $\left\|y_{i}-y_{o}\right\|(\mathrm{km})$ | $d_{i}^{2}$ | $d_{i}^{-2}$ | $w_{i}$ | $p_{i} \times w_{i}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| B | 25 | 1.2 | 1.1 | 2.65 | 0.38 | 0.21 | 5.17 |
| C | 35 | 1.3 | 1.5 | 3.94 | 0.25 | 0.14 | 4.86 |
| D | 14 | 2 | 0.6 | 4.36 | 0.23 | 0.13 | 1.76 |
| E | 23 | 0.8 | 1 | 1.64 | 0.61 | 0.33 | 7.68 |
| F | 19 | 0.5 | 1.6 | 2.81 | 0.36 | 0.19 | 3.70 |

The requested rainfall depth is the sum of the $p_{i} \times w_{i}$ products, i.e., the values on the last column. Hence,

$$
P_{\mathrm{Avg}}=\Sigma\left(p_{i} \times w_{i}\right)=23.2 \mathrm{~mm}
$$

The correct answer is $\mathbf{B}$.

## P. 8 ■ Solution

Part A: The missing data can be obtained by linear regression, that is, by correlating the precipitation on one station to the precipitation on the other with an expression of the type $y=a x+b$. Here, we enter the precipitation $x$ on the station for which the value is known and then obtain the corresponding precipitation $y$ on the other station. $a$ and $b$ are coefficients obtained by minimizing the sum of square errors of the estimate. Entering the data in Mathematica (without taking into account the years for which there is no data available for station 2 ) yields the line $y=0.831 x+409.1$. This result can be obtained with the LinearModelFit function, as shown below. Ordinarily, this command creates an object of the FittedModel type, but the functional form of the object can be extracted by using Normal.
$\ln [17]=\operatorname{sts} 12=\{\{1278.5,1455.8\},\{1262.8,1422.7\},\{1186.8,1496.1\},\{975.6,1366.7\}$,
$\{863.4,1098.9\},\{985.2,978.6\},\{1166.0,1318.6\},\{1268.6,1502.3\},\{1297.5,1402.5\}$,
$\{1239.6,1544.3\},\{1322.2,1499.8\},\{1108.1,1475.4\},\{1295.2,1399.8\},\{1100.5,1355.9\}\}$
$O_{t}[17]=((1278.5,1455.8),(1262.8,1422.7),(1186.8,1496.1),(975.6,1366.7),(863.4,1098.9)$, $(985.2,978.6),(1166 ., 1318.6),(1268.6,1502.3),(1297.5,1402.5),(1239.6,1544.3)$, $(1322.2,1499.8),(1108.1,1475.4),(1295.2,1399.8),(1100.5,1355.9))$
$\ln [18]=$ regLine $=$ LinearModelFit[sts12, $\{x, 1\}, x]$
Out[18]= FittedModel
$\operatorname{In}[19]:=$ Normal [regLine]
Out [19] $=409.105+0.831188 x$

Evaluating the function we obtained at $x=1066.8$ yields the missing data point at station 2 in the period 2006-2007, namely, $y=p_{2006-2007}=1295.8 \mathrm{~mm}$; proceeding in the same manner with the period 2013-2014, we get $y=p_{2013-2014}=$ 1205.6 mm.

## * The correct answer is $\mathbf{D}$.

Part B: In Mathematica, the coefficient of determination $r^{2}$ can be computed with the RSquared property of LinearModelFit. The coefficient of
correlation, in turn, is simply the square root of the coefficient of determination. Hence, the following syntax is appropriate.

$$
\begin{aligned}
& \operatorname{In}[20]=\sqrt{\text { regLine ["RSquared"] }} \\
& \text { Out }[20]=0.742421
\end{aligned}
$$

The coefficient of correlation is found to be around 0.74 , which is just above the $r \geq 0.7$ recommendation proposed by Mimikou et al. for the data to be reliable. Coefficient $r$ is contained in the interval ( $0.6 ; 0.8$ ).
$\star$ The correct answer is $\mathbf{C}$.

## P. 9 ■ Solution

We first arrange the years, along with the corresponding data, from latest to earliest. We then perform a running total of annual rainfall at station X , as shown in the blue column below, and another for the 12-station mean, as shown in the red column.

| Year | Station X | 12-Station Mean | Cummulative RF <br> at Station X <br> $\Sigma P_{X}$ | Cumulative RF <br> 12-Station Mean <br> $\Sigma P_{\text {avg }}$ |
| :---: | :---: | :---: | :---: | :---: |
| 1986 | 297 | 325 | 297 | 325 |
| 1987 | 308 | 438 | 605 | 763 |
| 1988 | 330 | 455 | 935 | 1218 |
| 1989 | 374 | 429 | 1309 | 1647 |
| 1990 | 407 | 442 | 1716 | 2089 |
| 1991 | 253 | 364 | 1969 | 2453 |
| 1992 | 330 | 377 | 2299 | 2830 |
| 1993 | 275 | 338 | 2574 | 3168 |
| 1994 | 429 | 455 | 3003 | 3623 |
| 1995 | 352 | 430 | 3355 | 4053 |
| 1996 | 319 | 429 | 3674 | 4482 |
| 1997 | 308 | 299 | 3982 | 4781 |
| 1998 | 385 | 364 | 4367 | 5145 |
| 1999 | 396 | 442 | 4763 | 5587 |
| 2000 | 447 | 325 | 5210 | 5912 |
| 2001 | 290 | 286 | 5500 | 6198 |
| 2002 | 350 | 312 | 5850 | 6510 |
| 2003 | 358 | 338 | 6208 | 6848 |
| 2004 | 638 | 520 | 6846 | 7368 |
| 2005 | 385 | 364 | 7231 | 7732 |
| 2006 | 407 | 339 | 7638 | 8071 |
| 2007 | 337 | 340 | 7975 | 8411 |
| 2008 | 378 | 338 | 8353 | 8749 |
| 2009 | 385 | 390 | 8738 | 9139 |
| 2010 | 275 | 299 | 9013 | 9438 |
| 2011 | 397 | 338 | 9410 | 9776 |
| 2012 | 462 |  | 9872 | 9776 |
| 2013 | 264 |  | 10136 | 9776 |
| 2014 | 517 |  | 10653 | 9776 |
|  |  |  |  |  |
|  |  |  | 7 |  |

Thence, we can plot $\Sigma P_{X}$, the cumulative rainfall at station $X$, against $\Sigma P_{\text {avg, }}$ the 12 -station cumulative rainfall, as shown


The breakpoint occurs at the data point close to the year 2003.

* The correct answer is $\mathbf{C}$.


## P. 10 ■ Solution

The total precipitation depth for this event is 3.29 in. for a period of 60 min , as shown in the third column below. The mass curve for this event is derived in the said column. The highest 5 -min rainfall depth is 0.78 in., observed at 16:35. The highest $10-\mathrm{min}$ rainfall depth is the sum of 0.78 and 0.61 (i.e., the second largest incremental depth), or 1.39 in . Similarly, the sum of the three largest blocks - $0.78,0.61$, and 0.5 for a total of 1.89 in. - represents the 15 -min rainfall depth. Repeating the same procedure generates the $P-D$ curve listed in the blue column below.

| Time | Incremental <br> Rainfall Depth (in.) | Cumulative Depth (in.) | Duration (min) | Highest Depth (in.) | Highest Intensity (in./hr) |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $16: 00$ | 0 | 0.05 | 0.0 |  |  |
| $16: 05$ | 0.04 | 0.08 | 5.0 | 0.78 | 9.36 |
| $16: 10$ | 0.11 | 0.11 | 10.0 | 1.39 | 8.34 |
| $16: 15$ | 0.15 | 0.3 | 15.0 | 1.89 | 7.56 |
| $16: 20$ | 0.32 | 0.62 | 20.0 | 2.37 | 7.11 |
| $16: 25$ | 0.5 | 1.12 | 25.0 | 2.69 | 6.46 |
| $16: 30$ | 0.61 | 1.73 | 30.0 | 2.87 | 5.74 |
| $16: 35$ | 0.78 | 2.51 | 35.0 | 3.02 | 5.18 |
| $16: 40$ | 0.48 | 2.99 | 40.0 | 3.13 | 4.70 |
| $16: 45$ | 0.18 | 3.17 | 45.0 | 3.19 | 4.25 |
| $16: 50$ | 0.06 | 3.23 | 50.0 | 3.23 | 3.88 |
| $16: 55$ | 0.03 | 3.26 | 55.0 | 3.26 | 3.56 |
| $17: 00$ |  | 3.29 | 60.0 | 3.29 | 3.29 |

The P-D curve can be converted to a $I-D$ curve by using the ratio $I=P / T_{d}$, where $I$ is the average intensity in in./h (or $\mathrm{mm} / \mathrm{h}$ ) and $T_{d}$ is the duration in hrs. An $I-D$ curve is a decay curve with respect to duration, starting with the highest 5 -min intensity and ending with the event average intensity. Intensities $I_{5}$ and $I_{10}$, for example, are

$$
\begin{gathered}
I_{5}=\frac{P}{T_{d}}=\frac{0.78}{(5 / 60)}=9.36 \mathrm{in} . / \mathrm{h} \\
I_{10}=\frac{1.39}{(10 / 60)}=8.34 \mathrm{in} . / \mathrm{h}
\end{gathered}
$$

Similar procedures apply to other time duration values. Other values are listed in the red column. The two curves in question are shown below. The blue curve is the rainfall depth-duration ( $P-D$ ) curve, while the red curve is the rainfall intensity-duration (I-D) curve.


It is noticed that the mass curve sharply increases before the peak and then becomes flatter as time increases, while the $P-I$ curve falls continuously with time. The two curves meet at the end of the storm time, on a point that is numerically equal to the largest cumulative depth (= 3.29 in .).

## P. 11 ■ Solution

To develop a IDF curve, we first arrange the precipitation depths in descending order; this was already done in the dataset we were given. The highest value is assigned a rank of 1 and the lowest a rank of 22 . The return periods are obtained with the Weibull formula $T=(n+1) / m$, where $n$ is the total number of years of record $(=22)$ and $m$ is the rank of observed rainfall values in descending order. The precipitation depths can be converted to intensities by means of the formula $I=60 p / t$, where $p$ is the precipitation in linear units (usually in. or mm ) and $t$ is the time period considered in min. For example, a precipitation of 0.5 in . of 30 min . duration has an intensity of $1 \mathrm{in} . / \mathrm{hr}$ (since $60 \times 0.5 / 30=1$ ). These values are plotted as the intensity-duration-curve on arithmetic (ordinary) graph paper or on log-log paper.

|  | Precipitation (in.) of Duration |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Rank | 5 min. | 10 min. | 15 min. | 20 min. | 30 min. | 60 min. | Return period <br> $\mathrm{T}=(\mathrm{n}+1) / \mathrm{m}$ |  |
| 1 | 0.40 | 0.66 | 0.89 | 1.07 | 1.48 | 2.15 | 23.0 |  |
| 2 | 0.38 | 0.63 | 0.83 | 0.97 | 1.29 | 1.92 | 11.5 |  |
| 3 | 0.37 | 0.62 | 0.79 | 0.91 | 1.26 | 1.48 | 7.7 |  |
| 4 | 0.36 | 0.60 | 0.76 | 0.86 | 0.91 | 1.06 | 5.8 |  |
| 5 | 0.35 | 0.60 | 0.73 | 0.80 | 0.83 | 0.96 | 4.6 |  |
| 6 | 0.33 | 0.58 | 0.72 | 0.77 | 0.82 | 0.94 | 3.8 |  |
| 7 | 0.33 | 0.50 | 0.72 | 0.77 | 0.78 | 0.90 | 3.3 |  |
| 8 | 0.31 | 0.50 | 0.63 | 0.70 | 0.75 | 0.87 | 2.9 |  |
| 9 | 0.30 | 0.49 | 0.57 | 0.65 | 0.67 | 0.77 | 2.6 |  |
| 10 | 0.28 | 0.44 | 0.56 | 0.62 | 0.66 | 0.75 | 2.3 |  |
| $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ |  |
| 22 | 0.13 | 0.23 | 0.32 | 0.40 | 0.40 | 0.43 | 10.05 |  |

To obtain the precipitation depths for return periods of 5 or 10 years, we need to interpolate the data between the appropriate time values. For 10 years, we interpolate data for rank 2 , which is associated with a return period of 11.5 years, and rank 3, which pertains to a return period of 7.7 years, as highlighted in blue above. Considering a time of 20 minutes, for example, we interpolate between data points $\{\{7.7,0.91\},\{11.5,0.97\}\}$, which can be easily done with Mathematica's Interpolation function, followed by evaluation of the interpolated polynomial at the desired point of the domain (= 10 years); in this example, one would obtain $p \approx 0.95 \mathrm{in} . / \mathrm{hr}$; the depth thus obtained is then converted to an intensity with the relation $I=60 \times 0.95 / 20=2.85 \mathrm{in}$. We proceed similarly with other values. For the 5-year IDF curve, values should be interpolated between the row for rank 5 , which is associated with a return period $T=4.6 \mathrm{yrs}$, and the row for rank 4, which corresponds to $T=5.8 \mathrm{yrs}$.

|  | Intensity (in./hr) |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Return <br> Period <br> (years) | 5 min. | 10 min. | 15 min. | 20 min. | 30 min. | 60 min. |
| 10 | 4.51 | 3.78 | 3.24 | 2.85 | 2.58 | 1.75 |
| 5 | 4.24 | 3.60 | 2.96 | 2.46 | 1.71 | 0.99 |

Finally, the IDF curves are obtained by plotting intensity, in in./hr, versus duration, in min.


## Answer Summary

| Problem 1 |  | B |
| :---: | :---: | :---: |
| Problem 2 |  | D |
| Problem 3 |  | B |
| Problem 4 |  | B |
| Problem 5 |  | A |
| Problem 6 | 6A | C |
|  | 6B | Open-ended pb. |
|  | 6C | Open-ended pb. |
| Problem 7 |  | B |
| Problem 8 | 8A | D |
|  | 8B | C |
| Problem 9 |  | C |
| Problem 10 |  | Open-ended pb. |
| Problem 11 |  | Open-ended pb. |

## References

- GUO, J. (2017). Urban Flood Mitigation and Stormwater Management. Boca Raton: CRC Press.
- GUPTA, R. (2017). Hydrology and Hydraulic Systems. 4th edition. Long Grove: Waveland Press.
- MIMIKOU, M., BALTAS, E., and TSIHRINTZIS, V. (2016). Hydrology and Water Resources Systems Analysis. Boca Raton: CRC Press

