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## Quiz SM207

## PRESSURE VESSELS

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## () PROBLEMS

## Problem $\mathbf{1}$ (Gere \& Goodno, 2009, w/ permission)

A rubber ball is inflated to a pressure of 60 kPa . At that pressure the diameter of the ball is 230 mm and the wall thickness is 1.2 mm . The rubber has modulus of elasticity $E=3.5 \mathrm{MPa}$ and Poisson's ratio $v=0.45$. Determine the maximum stress and strain in the ball. Determine the maximum stress and strain in the ball.

A) $\sigma_{\text {max }}=1.44 \mathrm{MPa}$ and $\varepsilon_{x}=0.226$
B) $\sigma_{\text {max }}=1.44 \mathrm{MPa}$ and $\varepsilon_{x}=0.452$
C) $\sigma_{\text {max }}=2.88 \mathrm{MPa}$ and $\varepsilon_{x}=0.226$
D) $\sigma_{\text {max }}=2.88 \mathrm{MPa}$ and $\varepsilon_{x}=0.452$

## Problem 2 (Gere \& Goodno, 2009, w/ permission)

A large spherical tank contains gas at a pressure of 450 psi . The tank is 42 ft in diameter and is constructed of high-strength steel having a yield stress in tension of 80 ksi . Determine the required thickness of the wall of the tank if a factor of safety of 3.5 with respect to yielding is required.

A) $t=1.61 \mathrm{in}$
B) $t=2.05 \mathrm{in}$.
C) $t=2.48 \mathrm{in}$.
D) $t=3.02 \mathrm{in}$.

## Problem 3 (Hibbeler, 2014, w/ permission)

The tank of the air compressor is subjected to an internal pressure of 90 psi . If the internal diameter of the tank is 22 in . and the wall thickness is 0.25 in., determine the hoop stress and the longitudinal stress components acting at point $A$.

A) $\sigma_{\text {hoop }}=3.96 \mathrm{ksi}$ and $\sigma_{\text {long }}=0.97 \mathrm{ksi}$
B) $\sigma_{\text {hoop }}=3.96 \mathrm{ksi}$ and $\sigma_{\text {long }}=1.98 \mathrm{ksi}$
C) $\sigma_{\text {hoop }}=5.25 \mathrm{ksi}$ and $\sigma_{\text {long }}=0.97 \mathrm{ksi}$
D) $\sigma_{\text {hoop }}=5.25 \mathrm{ksi}$ and $\sigma_{\text {long }}=1.98 \mathrm{ksi}$

## Problem 4A (Philpot, 2013, w/ permission)

A tall open-topped standpipe has an inside diameter of 2750 mm and a wall thickness of 6 mm . The standpipe contains water of mass density of $1000 \mathrm{~kg} / \mathrm{m}^{3}$. What height of water will produce a circumferential stress of 16 MPa in the wall of the standpipe?

A) $h=3.24 \mathrm{~m}$
B) $h=5.21 \mathrm{~m}$
C) $h=7.12 \mathrm{~m}$
D) $h=9.04 \mathrm{~m}$

## Problem 4B

What is the longitudinal stress in the wall of the standpipe due to the water pressure?
A) $\sigma_{\text {long }}=0$
B) $\sigma_{\text {long }}=4 \mathrm{MPa}$
C) $\sigma_{\text {long }}=8 \mathrm{MPa}$
D) $\sigma_{\text {long }}=12 \mathrm{MPa}$

## Problem 5 (Philpot, 2013, w/ permission)

The pressure tank shown in the next figure is fabricated from spirally wrapped metal plates that are welded at the seams in the orientation shown, where $\beta=40^{\circ}$. The tank has an inside diameter of 480 mm and a wall thickness of 8 mm . Determine the largest gage pressure that can be used inside the tank if the allowable normal stress perpendicular to the weld is 100 MPa and the allowable shear stress parallel to the weld is 25 MPa .

A) $p_{\text {allow }}=2.03 \mathrm{MPa}$
B) $p_{\text {allow }}=3.38 \mathrm{MPa}$
C) $p_{\text {allow }}=4.20 \mathrm{MPa}$
D) $p_{\text {allow }}=5.54 \mathrm{MPa}$

## Problem 6 (Hibbeler, 2014, w/ permission)

The ring, having the dimensions shown, is placed over a flexible membrane which is pumped up with a pressure $p$. Determine the change in the internal radius of the ring after this pressure is applied. The modulus of elasticity of the ring is $E$. What is the change in internal radius if $p=80 \mathrm{kPa}, r_{o}=20 \mathrm{~mm}, r_{i}=19 \mathrm{~mm}$, and $E=200 \mathrm{GPa}$ ?

A) $\delta r_{i}=1.44 \mu \mathrm{~m}$
B) $\delta r_{i}=4.29 \mu \mathrm{~m}$
C) $\delta r_{i}=7.05 \mu \mathrm{~m}$
D) $\delta r_{i}=9.71 \mu \mathrm{~m}$

## Problem 7 (Hibbeler, 2014, w/ permission)

The inner ring $A$ has an inner radius $r_{1}$ and outer radius $r_{2}$. Before heating, the outer ring $B$ has an inner radius $r_{3}$ and an outer radius $r_{4}$, and $r_{2}>r_{3}$. If the outer ring is heated and then fitted over the inner ring, determine the pressure between the two rings when ring $B$ reaches the temperature of the inner ring. The material has a modulus of elasticity of $E$ and a coefficient of thermal expansion of $\alpha$.


## Problem 8A (Hibbeler, 2014, w/ permission)

A closed-ended pressure vessel is fabricated by cross-winding glass filaments over a mandrel, so that the wall thickness $t$ of the vessel is composed entirely of filament and an epoxy binder as shown in the figure. Consider a segment of the vessel of width $w$ and wrapped at an angle $\theta$. If the vessel is subjected to an internal pressure $p$, show that the force in the segment is $F_{\theta}=\sigma_{0} w t$, where $\sigma_{0}$ is the stress in the filaments. Also, show that the stresses in the hoop and longitudinal directions are $\sigma_{h}=\sigma_{0} \sin ^{2} \theta$ and $\sigma_{l}=\sigma_{0} \cos ^{2} \theta$, respectively.


## Problem 8B

At what angle $\theta$ (optimum winding angle) would the filaments have to be so that the hoop and longitudinal stresses are equivalent?
A) $\theta=25.4^{\circ}$
B) $\theta=40.2^{\circ}$
C) $\theta=54.7^{\circ}$
D) $\theta=69.3^{\circ}$

## Problem 9 (Gere \& Goodno, 2009, w/ permission)

A cylindrical tank subjected to internal pressure $p$ is simultaneously compressed by an axial force $F=72 \mathrm{kN}$ (see figure). The cylinder has diameter $d=100$ mm and wall thickness $t=4 \mathrm{~mm}$. Calculate the maximum allowable internal pressure based upon an allowable shear stress in the wall of the tank of 60 MPa .

A) $p_{\text {allow }}=9.6 \mathrm{MPa}$
B) $p_{\text {allow }}=14.1 \mathrm{MPa}$
C) $p_{\text {allow }}=19.7 \mathrm{MPa}$
D) $p_{\text {allow }}=28.4 \mathrm{MPa}$

## Problem 10 (Gere \& Goodno, 2009, w/ permission)

A cylindrical tank having diameter $d=2.5 \mathrm{in}$. is subjected to an internal gas pressure $p=600 \mathrm{psi}$ and an external tensile load $T=1000 \mathrm{lb}$ (see figure). Determine the minimum thickness of the wall of the tank based upon an allowable shear stress of 3000 psi.

A) $t_{\text {min }}=0.0413 \mathrm{in}$.
B) $t_{\text {min }}=0.0584 \mathrm{in}$.
C) $t_{\text {min }}=0.0837 \mathrm{in}$.
D) $t_{\text {min }}=0.125 \mathrm{in}$.

## Problem 11A (Gere \& Goodno, 2009, w/ permission)

A pressurized cylindrical tank with flat ends is loaded by torques $T$ and tensile forces $P$ (see figure). The tank has a radius of $r=125 \mathrm{~mm}$ and wall thickness $t=6.5 \mathrm{~mm}$. The internal pressure $p=7.25 \mathrm{MPa}$ and the torque $T=850 \mathrm{~N} \cdot \mathrm{~m}$. What is the maximum permissible value of the force $P$ if the allowable tensile stress in the wall of the cylinder is 80 MPa ?

A) $P_{\text {max }}=15.4 \mathrm{kN}$
B) $P_{\text {max }}=27.1 \mathrm{kN}$
C) $P_{\text {max }}=39.8 \mathrm{kN}$
D) $P_{\text {max }}=52.5 \mathrm{kN}$

## Problem 11B

If forces $P=114 \mathrm{kN}$, what is the maximum acceptable internal pressure in the tank?
A) $p_{\text {max }}=2.35 \mathrm{MPa}$
B) $p_{\text {max }}=6 \mathrm{MPa}$
C) $p_{\text {max }}=10 \mathrm{MPa}$
D) $p_{\text {max }}=14.5 \mathrm{MPa}$

## () SOLUTIONS

## P. $1 \rightarrow$ Solution

The tensile stress in the wall is given by

$$
\sigma_{\max }=\frac{p r}{2 t}=\frac{\left(60 \times 10^{3}\right) \times(230 / 2)}{2 \times 1.2}=2.88 \mathrm{MPa}
$$

The maximum strain in the ball is determined as

$$
\begin{gathered}
\varepsilon_{x}=\frac{\sigma}{E}(1-v) \rightarrow \varepsilon_{x}=\frac{p r}{2 t E}(1-v) \\
\therefore \varepsilon_{x}=\frac{\left(60 \times 10^{3}\right) \times(230 / 2)}{2 \times 1.2 \times\left(3.5 \times 10^{6}\right)} \times(1-0.45)=0.452
\end{gathered}
$$

C The correct answer is $\mathbf{D}$.

## P. $2 \rightarrow$ Solution

The radius of the tank is $r=(42 \times 12) / 2=252 \mathrm{in}$. The tensile stress in the wall of the vessel is given by

$$
\sigma_{\max }=\frac{p r}{2 t}
$$

From the definition of factor of safety, we have

$$
F S=\frac{\sigma_{Y}}{\sigma_{\max }} \rightarrow \sigma_{\max }=\frac{\sigma_{Y}}{F S}
$$

Substituting in the first equation gives

$$
\begin{aligned}
& \sigma_{\max }=\frac{\sigma_{Y}}{F S}=\frac{p r}{2 t} \rightarrow t=\frac{p r \times F S}{2 \sigma_{Y}} \\
& \therefore t=\frac{450 \times 252 \times 3.5}{2 \times\left(80 \times 10^{3}\right)}=2.48 \mathrm{in} .
\end{aligned}
$$

C The correct answer is $\mathbf{C}$.

## P. $3 \rightarrow$ Solution

The hoop stress follows from the formula

$$
\sigma_{\text {hoop }}=\frac{p r}{t}=\frac{90 \times(22 / 2)}{0.25}=3.96 \mathrm{ksi}
$$

The longitudinal stress, in turn, is determined as

$$
\sigma_{\text {long }}=\frac{p r}{2 t}=1.98 \mathrm{ksi}
$$

C The correct answer is $\mathbf{B}$.

## P. $4 \Rightarrow$ Solution

Part A: The hoop stress in the wall is given by

$$
\sigma_{h}=\frac{p r}{t}
$$

The hydrostatic pressure $p=\rho g h$. Substituting and solving for the height $h$, we obtain

$$
\begin{gathered}
\sigma_{h}=\frac{p r}{t}=\frac{\rho g h r}{t} \rightarrow h=\frac{\sigma_{h} t}{\rho g r} \\
\therefore h=\frac{\left(16 \times 10^{6}\right) \times\left(6 \times 10^{-3}\right)}{1000 \times 9.81 \times(2.75 / 2)}=7.12 \mathrm{~m}
\end{gathered}
$$

O The correct answer is $\mathbf{C}$.
Part B: Since the standpipe is open to the atmosphere at its upper end, the fluid pressure will not create stress in the standpipe's longitudinal direction.
Therefore, $\sigma_{\text {long }}=0$.
$\bigcirc$ The correct answer is $\mathbf{A}$.

## P. $5 \rightarrow$ Solution

The longitudinal axis of the cylinder is defined as the $x$-axis and the circumferential direction is defined as the $y$-axis. Accordingly, the normal and shear stresses on longitudinal and circumferential faces of a stress element are

$$
\begin{gathered}
\sigma_{x}=\frac{p r}{2 t}=\frac{p \times 0.24}{2 \times 0.008}=15 p \\
\sigma_{y}=\frac{p r}{t}=30 p \\
\tau_{x y}=0
\end{gathered}
$$

The weld is oriented at $40^{\circ}$ as shown, but the angle $\theta$ required for the stress transformation equations is the angle normal to the weld. The angle in question is $\theta=$ $40^{\circ}+90^{\circ}=130^{\circ}$. Now, the normal stress perpendicular to the weld can be determined with the transformation equation

$$
\sigma=\left|\sigma_{x} \cos ^{2} \theta+\sigma_{y} \sin ^{2} \theta+2 \tau_{x y} \sin \theta \cos \theta\right| \leq 100 \mathrm{MPa}
$$

Substituting the pertaining variables yields

$$
\begin{gathered}
\left|15 p \times \cos ^{2} 130^{\circ}+30 p \times \sin ^{2} 130^{\circ}+2 \times 0 \times \sin 130^{\circ} \cos 130^{\circ}\right| \leq 100 \mathrm{MPa} \\
\therefore|6.2 p+17.6 p+0| \leq 100 \mathrm{MPa} \\
\therefore p_{\max }=4.20 \mathrm{MPa}
\end{gathered}
$$

The shear stress parallel to the weld can be determined with the transformation equation

$$
\tau=-\left(\sigma_{x}-\sigma_{y}\right) \sin \theta \cos \theta+\tau_{x y}\left(\cos ^{2} \theta-\sin ^{2} \theta\right) \leq 25 \mathrm{MPa}
$$

Substituting the pertaining variables yields

$$
\begin{gathered}
\tau=\left|-(30 p-15 p) \sin 130^{\circ} \cos 130^{\circ}+0 \times\left(\cos ^{2} 130^{\circ}-\sin ^{2} 130^{\circ}\right)\right| \leq 25 \mathrm{MPa} \\
\therefore 15 p \sin 130^{\circ} \cos 130^{\circ} \leq 25 \mathrm{MPa} \\
\therefore p_{\max }=3.38 \mathrm{MPa}
\end{gathered}
$$

The lower result governs the analysis, and hence $p_{\text {allow }}=3.38 \mathrm{MPa}$.
C The correct answer is $\mathbf{B}$.

## P. $6 \Rightarrow$ Solution

Using a free-body diagram, the force $P$ exerted on the ring is determined as

$$
2 P-2 p r_{i} w=0 \rightarrow P=p r_{i} w
$$

The corresponding stress is

$$
\sigma_{1}=\frac{P}{A}=\frac{p r_{i} w}{\left(r_{o}-r_{i}\right) w}=\frac{p r_{i}}{\left(r_{o}-r_{i}\right)}
$$

Using the stress-strain relation, we have

$$
\begin{gathered}
\sigma_{1}=E \varepsilon_{1} \rightarrow \varepsilon_{1}=\frac{\sigma_{1}}{E} \\
\therefore \varepsilon_{1}=\frac{p r_{i}}{E\left(r_{o}-r_{i}\right)}
\end{gathered}
$$

Since strain $\varepsilon_{1}=\delta r_{i} / r_{i}$, we can write

$$
\varepsilon_{1}=\frac{\delta r_{i}}{r_{i}}=\frac{p r_{i}}{E\left(r_{o}-r_{i}\right)} \rightarrow \delta r_{i}=\frac{p r_{i}^{2}}{E\left(r_{o}-r_{i}\right)}
$$

Inserting the data we received, it follows that

$$
\delta r_{i}=\frac{\left(80 \times 10^{3}\right) \times 0.019^{2}}{\left(200 \times 10^{9}\right) \times(0.02-0.019)}=1.44 \mu \mathrm{~m}
$$

C The correct answer is $\mathbf{A}$.

## P. $7 \rightarrow$ Solution

Using a free-body diagram, the force $P$ exerted on the ring is determined as

$$
2 P-2 p r_{i} w=0 \rightarrow P=p r_{i} w
$$

where $r_{i}$ denotes the inner radius of either ring and $w$ is the width of the ring. The corresponding stress is

$$
\sigma_{1}=\frac{P}{A}=\frac{p r_{i} w}{\left(r_{o}-r_{i}\right) w}=\frac{p r_{i}}{\left(r_{o}-r_{i}\right)}
$$

where $r_{o}$ denotes the outer radius of either ring. Using the stress-strain relation, we have

$$
\begin{gathered}
\sigma_{1}=E \varepsilon_{1} \rightarrow \varepsilon_{1}=\frac{\sigma_{1}}{E} \\
\therefore \varepsilon_{1}=\frac{p r_{i}}{E\left(r_{o}-r_{i}\right)}
\end{gathered}
$$

Since strain $\varepsilon_{1}=\delta r_{i} / r_{i}$, we can write

$$
\varepsilon_{1}=\frac{\delta r_{i}}{r_{i}}=\frac{p r_{i}}{E\left(r_{o}-r_{i}\right)} \rightarrow \delta r_{i}=\frac{p r_{i}^{2}}{E\left(r_{o}-r_{i}\right)}(\mathrm{I})
$$

Compatibility between the two rings requires that

$$
\delta r_{2}+\delta r_{3}=r_{2}-r_{3}
$$

In view of equation (I), we have for rings $A$ and $B$ respectively,

$$
\delta r_{2}=\frac{p r_{2}^{2}}{E\left(r_{2}-r_{1}\right)} ; \delta r_{3}=\frac{p r_{3}^{2}}{E\left(r_{4}-r_{3}\right)}
$$

Substituting and solving for pressure, we find that

$$
\begin{aligned}
& \frac{p r_{2}^{2}}{E\left(r_{2}-r_{1}\right)}+\frac{p r_{3}^{2}}{E\left(r_{4}-r_{3}\right)}=r_{2}-r_{3} \\
& \therefore \frac{p}{E}\left[\frac{r_{2}^{2}}{\left(r_{2}-r_{1}\right)}+\frac{r_{3}^{2}}{\left(r_{4}-r_{3}\right)}\right]=r_{2}-r_{3} \\
& \therefore p=\frac{E\left(r_{2}-r_{3}\right)}{\frac{r_{2}^{2}}{\left(r_{2}-r_{1}\right)}+\frac{r_{3}^{2}}{\left(r_{4}-r_{3}\right)}}
\end{aligned}
$$

## P. $8 \rightarrow$ Solution

Part A: Considering a portion of the cylinder, it is easy to see that, given the area $A=w t$ of the filament wire, the force in the $\theta$-direction is given by

$$
\sigma_{\theta}=\frac{F_{\theta}}{A} \rightarrow F_{\theta}=\sigma_{\theta} w t
$$

The force in the circumferential direction is $F_{h}=F_{\theta} \sin \theta=\sigma_{0} w t \sin \theta$, and the area is $A=w t / \sin \theta$. The corresponding hoop stress is determined as

$$
\sigma_{\text {hoop }}=\frac{F_{h}}{A}=\frac{\sigma_{0} w t \sin \theta}{w t / \sin \theta}=\sigma_{0} \sin ^{2} \theta
$$

In a similar manner, the force in the longitudinal direction is $F_{l}=F_{\theta} \cos \theta=$ $\sigma_{0} w t \cos \theta$, and the corresponding longitudinal stress follows as

$$
\sigma_{\text {long }}=\frac{F_{l}}{A}=\frac{\sigma_{0} w t \cos \theta}{w t / \cos \theta}=\sigma_{0} \cos ^{2} \theta
$$

Part B: We know that the hoop stress can be calculated with the expression $\sigma_{\text {hoop }}=p r / t$, while the longitudinal stress can be computed with $\sigma_{\text {long }}=p r / 2 t$. Combining these expressions with the results of the previous part, we have, at the optimum wrap angle,

$$
\begin{aligned}
& \frac{\sigma_{\text {hoop }}}{\sigma_{\text {long }}}=\frac{p r / t}{p r / 2 t}=\frac{\sigma_{0} \sin ^{2} \theta}{\sigma_{0} \cos ^{2} \theta} \\
& \therefore \tan ^{2} \theta=2 \\
& \therefore \tan \theta=1.41
\end{aligned}
$$

Solving this trigonometric equation, we obtain the optimum angle $\theta=54.7^{\circ}$. With some additional calculations, it is possible to derive an interesting conclusion about this result. To begin, note that the hoop force on the filament could have been written as

$$
F_{\text {hoop }}=\frac{p d}{2 t} \times \frac{w t}{\sin \theta}=\frac{p d w}{2 \sin \theta}
$$

while the longitudinal force could have been cast as

$$
F_{\text {long }}=\frac{p d}{4 t} \times \frac{w t}{\cos \theta}=\frac{p d w}{4 \cos \theta}
$$

The force in the $\theta$-direction is the resultant of the two preceding forces; that is,

$$
\begin{gathered}
F_{\theta}=\sqrt{F_{\text {hoop }}^{2}+F_{\text {long }}^{2}}=\sqrt{\left(\frac{p d w}{2 \sin \theta}\right)^{2}+\left(\frac{p d w}{4 \cos \theta}\right)^{2}} \\
\therefore F_{\theta}=\frac{p d w}{2} \sqrt{\frac{4}{\sin ^{2} \theta}+\frac{1}{\cos ^{2} \theta}}
\end{gathered}
$$

Plotting the trigonometric component of the equation above, we obtain the following graph.


There is a point at which this trigonometric term, and therefore the force $F_{\theta}$, attains a minimum value. Inspecting the graph, we see that the abscissa of this point happens to be $54.7^{\circ}$. That is, the angle at which the stresses in the hoop and longitudinal stresses become equivalent is also the angle for which the resultant force in the cylinder reaches a minimum. To verify this mathematically, we can differentiate the term in question with respect to $\theta$ and, after some simplification, obtain

$$
\frac{d F_{\theta}}{d \theta}=\frac{\left(1-4 \cot ^{4} \theta\right) \sec ^{2} \theta \tan \theta}{\sqrt{4 \csc ^{2} \theta+\sec ^{2} \theta}}=0
$$

Setting the term in parentheses to zero, we get

$$
\begin{gathered}
1-4 \cot ^{4} \theta=0 \rightarrow \cot ^{4} \theta=\frac{1}{4} \\
\therefore \tan ^{4} \theta=4 \\
\therefore \tan \theta=1.41
\end{gathered}
$$

with the result that $\theta=54.7^{\circ}$, as expected.
C The correct answer is $\mathbf{C}$.

## P. $9 \Rightarrow$ Solution

The circumferential stress in the wall of the tank is

$$
\sigma_{\text {hoop }}=\frac{p r}{t}=\frac{p \times 50}{4}=12.5 p
$$

The longitudinal stress, in sequence, consists of contributions from the pressure of the fluid and the axial load F. In mathematical terms,

$$
\begin{gathered}
\sigma_{\text {long }}=\frac{p r}{2 t}-\frac{F}{2 \pi r t} \\
\therefore \sigma_{\text {long }}=6.25 p-\frac{72,000}{2 \pi \times 50 \times 4}=6.25 p-57.3
\end{gathered}
$$

where the units are in MPa. The cylinder of course is subjected to biaxial stress. Using the relation for maximum in-plane shear stress, which we label case 1 , the maximum allowable internal pressure is determined as

$$
\begin{gathered}
\tau_{\max }=\frac{\sigma_{\text {hoop }}-\sigma_{\text {long }}}{2}=\frac{12.5 p-(6.25 p-57.3)}{2}=60 \\
\therefore p_{\max }=10 \mathrm{MPa}
\end{gathered}
$$

Considering out-of-plane shear we have, for one of the stresses (tag it as case 2),

$$
\begin{gathered}
\tau_{\max }=\frac{\sigma_{\text {hoop }}}{2} \rightarrow 60=\frac{12.5 p}{2} \\
\therefore p_{\max }=9.6 \mathrm{MPa}
\end{gathered}
$$

Finally, considering the remaining out-of-plane shear, we find that (case 3)

$$
\begin{gathered}
\tau_{\max }=\frac{\sigma_{\text {long }}}{2} \rightarrow 60=\frac{6.25 p-57.3}{2} \\
\therefore p_{\max }=28.4 \mathrm{MPa}
\end{gathered}
$$

Case 2 controls, and we conclude that the pressure in the cylinder should be no larger than $p_{\text {allow }}=9.6 \mathrm{MPa}$.

C The correct answer is $\mathbf{A}$.

## P. $10 \rightarrow$ Solution

The circumferential stress in the wall of the tank is

$$
\sigma_{\mathrm{hoop}}=\frac{p r}{t}=\frac{600 \times 1.25}{t}=\frac{750}{t}
$$

The longitudinal stress, in sequence, consists of contributions from the pressure of the fluid and the axial load T. Mathematically,

$$
\begin{gathered}
\sigma_{\text {long }}=\frac{p r}{2 t}+\frac{T}{2 \pi r t} \\
\therefore \sigma_{\text {long }}=\frac{375}{t}+\frac{1000}{2 \pi \times 1.25 \times t}=\frac{502}{t}
\end{gathered}
$$

The cylinder of course is subjected to biaxial stress. From the relation for maximum in-plane shear stress, which we label case 1 , the required thickness is calculated as

$$
\begin{aligned}
\tau_{\max }=\frac{\sigma_{\text {hoop }}-\sigma_{\text {long }}}{2} & =\frac{\frac{750}{t}-\frac{502}{t}}{2}=\frac{124}{t}=3000 \\
\therefore t_{\min } & =0.0413 \mathrm{in}
\end{aligned}
$$

Considering out-of-plane shear, we have, for one of the stresses (call it case 2),

$$
\begin{gathered}
\tau_{\max }=\frac{\sigma_{\text {hoop }}}{2} \rightarrow 3000=\frac{1}{2} \times \frac{750}{t} \\
\therefore t_{\min }=\frac{375}{3000}=0.125 \mathrm{in} .
\end{gathered}
$$

Lastly, considering the other out-of-plane shear, we find that (case 3)

$$
\begin{gathered}
\tau_{\max }=\frac{\sigma_{\text {long }}}{2} \rightarrow 3000=\frac{1}{2} \times \frac{502}{t} \\
\therefore t_{\min }=\frac{251}{3000}=0.0837 \mathrm{in} .
\end{gathered}
$$

Case 2 controls, and we take $t_{\min }=0.125$ in. as the minimum thickness of the tank wall.

C The correct answer is $\mathbf{D}$.

## P. $11 \Rightarrow$ Solution

Part A: The circumferential stress in the cylinder is given by the usual equation

$$
\sigma_{\text {hoop }}=\frac{p r}{t}=\frac{\left(7.25 \times 10^{6}\right) \times 0.125}{0.0065}=139 \mathrm{MPa}
$$

The longitudinal stress, in turn, is made up of contributions from the pressure of the fluid and the axial load P. Mathematically,

$$
\begin{gathered}
\sigma_{\text {long }}=\frac{p r}{2 t}+\frac{P}{2 \pi r t}=\frac{\left(7.25 \times 10^{6}\right) \times 0.125}{2 \times 0.0065}+\frac{P}{2 \pi \times 0.125 \times 0.0065} \\
\therefore \sigma_{\text {long }}=69.7+1.96 \times 10^{-4} P[\mathrm{MPa}]
\end{gathered}
$$

We should also consider the shear stress created by torque $T$, which follows from the torsion formula,

$$
\tau=\frac{\operatorname{Tr}}{J}=\frac{850 \times 0.125}{\left(2 \pi \times 0.125^{3} \times 0.0065\right)}=1.33 \mathrm{MPa}
$$

The maximum tensile stress in the wall is obtained by dint of the stress transformation equation

$$
\sigma_{\max }=\frac{\sigma_{\text {hoop }}+\sigma_{\text {long }}}{2}+\sqrt{\left(\frac{\sigma_{\text {hoop }}-\sigma_{\text {long }}}{2}\right)^{2}+\tau^{2}} \leq 80 \mathrm{MPa}
$$

Substituting the pertaining variables, it follows that

$$
\begin{aligned}
& \frac{139+\left(69.7+1.96 \times 10^{-4} P\right)}{2}+\sqrt{\left[\frac{139-\left(69.7+1.96 \times 10^{-4} P\right)}{2}\right]^{2}+1.33^{2}}=80 \mathrm{MPa} \\
& \\
& \therefore P_{\max }=52.5 \mathrm{kN} \\
& \text { OThe correct answer is } \mathbf{D} .
\end{aligned}
$$

Part B: The circumferential stress is now

$$
\sigma_{\text {hoop }}=\frac{p r}{t}=\frac{p \times 125}{6.5}=19.2 p
$$

while the longitudinal stress becomes

$$
\begin{gathered}
\sigma_{\text {long }}=\frac{p r}{2 t}+\frac{P}{2 \pi r t}=\frac{p \times 0.125}{2 \times 0.0065}+\frac{114 \times 10^{3}}{2 \pi \times 0.125 \times 0.0065} \\
\therefore \sigma_{\text {long }}=9.62 p+2.23 \times 10^{7} \\
\therefore \sigma_{\text {long }}=9.62 p+22.3[\mathrm{MPa}]
\end{gathered}
$$

The shear stress due to torque $T$ remains $\tau=1.33 \mathrm{MPa}$. Considering the equation for maximum tensile stress, we have

$$
\begin{aligned}
\frac{19.2+(9.62 p+22.3)}{2}+ & {\left[\frac{19.2-(9.62 p+22.3)}{2}\right]^{2}+1.33^{2} }
\end{aligned}=80 \mathrm{MPa}
$$

C The correct answer is $\mathbf{B}$.

## () ANSWER SUMMARY

| Problem 1 |  | D |
| :---: | :---: | :---: |
| Problem 2 |  | C |
| Problem 3 |  | B |
| Problem 4 | 4A | C |
|  | 4B | A |
| Problem 5 |  | B |
| Problem 6 |  | A |
| Problem 7 |  | Open-ended pb. |
| Problem 8 | 8A |  |
|  | 8B | Open-ended pb. |
| Problem 9 |  | C |
| Problem 10 |  | D |
| Problem 11 | 11A | D |
|  | 11B | B |

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