Quiz AS102
Rocket Propulsion

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Problems

Problem 1

The following data are given for an optimum rocket propulsion system.

→ Chamber pressure: 2.5 MPa
→ Nozzle throat diameter: 120 mm
→ Thrust: 18 kN
→ Propellant mass flow: 7.2 kg/s.
→ Flight velocity: 880 m/s.

The propellant has a calorific value of 25 MJ/kg. True or false?

1.( ) The specific impulse of the rocket engine is greater than 250 sec.
2.( ) The effective velocity of the rocket is greater than 2200 m/s.
3.( ) The specific propellant consumption of the rocket is greater than 0.005 kg/Ns.
4.( ) The thrust power of the rocket is greater than 18 MW.
5.( ) The characteristic velocity of the rocket is greater than 4000 m/s.

Problem 2

The following data are given for an optimum rocket propulsion system.

→ Average molecular mass: 24 kg/kg-mol
→ Chamber pressure: 2.53 MPa
→ External pressure: 0.09 MPa
→ Chamber temperature: 2900 K
→ Throat area: 0.0005 m^2

The specific heat ratio is 1.30. True or false?

1.( ) The throat velocity is greater than 1000 m/s.
2.( ) The specific volume at the throat is greater than 0.7 m^3/kg.
3.( ) The specific impulse of the rocket is greater than 200 sec.
4.( ) The thrust of the rocket is greater than 2000 N.
5.( ) The Mach number at the throat is greater than 1.2.

Problem 3A

Starting from isentropic relations for an ideal exhaust nozzle with discharge coefficient of unity, show that the thrust coefficient can be expressed as

$$C_T = \gamma\sqrt{\frac{2}{\gamma-1}\left(\frac{2}{\gamma+1}\right)^{(\gamma+1)/2(\gamma-1)}}\sqrt{1-\left(\frac{p_e}{p_0}\right)^{(\gamma-1)/\gamma}}$$

Problem 3B

Determine the thrust coefficient for a nozzle with gas having γ = 1.25 and a pressure ratio of 5.0.

A) C_T = 0.811
B) C_T = 1.09
C) C_T = 1.32
D) C_T = 1.51

Problem 4A

Show that the characteristic velocity of a rocket can be expressed as

$$c^* = \frac{\sqrt{\bar{R}T_o/\bar{M}}}{\sqrt{\gamma}\left(\dfrac{2}{\gamma+1}\right)^{(\gamma+1)/2(\gamma-1)}}$$

Problem 4B

Determine the characteristic velocity if γ = 1.25, $\bar{M}$ = 18.0, and T_o = 3200 K. Use R = 8314 whenever needed.

A) c^* = 1250 m/s

B) c^* = 1510 m/s

C) c^* = 1880 m/s

D) c^* = 2070 m/s

Problem 5

Regarding aspects of rocket propulsion and rocket science, true or false?

1.( ) The best efficiency of a rocket nozzle is attained when the expansion ratio is such that the exhaust pressure equals the ambient pressure.

2.( ) The exhaust velocity of a rocket is inversely proportional to the molecular weight of the propellant mixture.

3.( ) All liquid propellant rocket systems require tanks to store propellants prior to injection in the thrust chamber. For space applications, spherical tanks are desirable since the wall surface area of a spherical tank is always less than that of, say, a cylindrical tank.

4.( ) In order to express a rocket's specific impulse in Imperial units, the SI specific impulse must be multiplied by a conversion factor of 0.454.

5.( ) For optimal pumping properties, fuels with high vapor pressures, such as liquid oxygen, are preferred in liquid-propellant rockets.

6.( ) In the second half of the twentieth century, several types of novel propellants were attempted in rocket propulsion. One particularly promising example is liquid fluorine, a powerful oxidizing agent that, in combination with hydrogen, reacts explosively and yields hydrogen fluoride. Fluorine fuels are highly toxic and corrosive.

7.( ) One of the most widely used modern liquid rocket propellants is NTO/MMH, a combination of monomethyl hydrazine (MMH) and nitrogen tetroxide (NTO). This fuel is not hypergolic.

8.( ) Hydrogen peroxide is a well-established monopropellant that decomposes explosively into water and oxygen under appropriate conditions. Since the reaction is quite fast and has a rather low energy of activation, there is no need for a catalyst.

9.( ) The characteristic chamber length L^* for selected liquid propellants are provided in the following table. From these data, we can surmise that the residence time of mixtures containing hydrocarbon molecules is lower than that of lighter fuels such as liquid oxygen-liquid hydrogen.

Propellant combination	Minimum L^* (m)
LOX/RP-1	1.0 – 1.3
LOX/LH_2	0.7 – 1.0
LOX/GH_2	0.5 – 0.7
Hydrazine family/NTO	0.7 – 0.9
H_2O_2/RP-1	1.5 – 1.8

10.( ) Typical solid-propellant fuels include aluminum, magnesium, and beryllium. Aluminum-based fuels are the most energetic of the three.

11.( ) While the products of solid boosters are mostly harmless, the exhaust gases of liquid engines, especially liquid hydrogen-liquid oxygen propellants, include hazardous gases and particulates.

12.( ) Unlike liquid rocket fuels, which are often volatile and prone to accidental explosion, solid rocket fuels tend to be chemically stable. For this reason, the combustion stability of solid fuel motors is superior, allowing for active control of thrust and of the temperature of the burning surface.

13.( ) The rate of consumption of the grain in a solid fuel rocket engine may be expressed by Vielle's law, $\dot{r} = ap_c^n$, where p_c is the combustion chamber pressure and a and n are empirical constants. In order for combustion of a rocket fuel to be stable, exponent n must be less than unity.

Problem **6A**

A rocket has a thrust coefficient of 1.5, an effective jet velocity of 2000 m/s, and a propellant mass flow rate of 6.0 kg/s. The initial mass of the vehicle is 4000 kg. The theoretical thrust for this rocket is:

A) T = 12.2 kN

B) T = 18.0 kN

C) T = 23.1 kN

D) T = 26.4 kN

Problem **6B**

The vehicle specified in the previous problem will have its velocity boosted by 1600 m/s. Determine the mass of propellant required to achieve this velocity increase.

A) m_p = 1000 kg

B) m_p = 1400 kg

C) m_p = 2200 kg

D) m_p = 3000 kg

Problem **7**

A booster rocket engine with nozzle exit diameter of 250 mm is designed to propel a satellite to an altitude of 20 km, at which the ambient pressure is p_a = 5.5 kPa. The chamber pressure at 14 MPa is expanded to an exit pressure of 101 kPa and an exit temperature of 1500 K. The mass flow rate is 16.2 kg/s and the molecular weight of propellant is 24 kg/kmol. Determine the effective jet velocity and the thrust developed by this rocket engine.

A) c_j = 1900 m/s and T = 57.6 kN

B) c_j = 1900 m/s and T = 80.2 kN

C) c_j = 3000 m/s and T = 57.6 kN

D) c_j = 3000 m/s and T = 80.2 kN

Problem **8A**

A research space vehicle in gravity-free and drag-free outer space launches a smaller spacecraft into a meteor shower region. The 2-kg sensitive instrument package of the spacecraft (25 kg total mass) limits the maximum acceleration to no more than 50 m/s^2. It is launched by a solid propellant rocket motor with I_{sp} = 260 s and ζ = 0.88. Determine the minimum allowable burn time, assuming steady propellant mass flow, and the maximum velocity relative to the launch vehicle.

A) $t_{p,\min}$ = 215 s and Δu = 2880 m/s

B) $t_{p,\min}$ = 215 s and Δu = 4210 m/s

C) $t_{p,\min}$ = 360 s and Δu = 2880 m/s

D) $t_{p,\min}$ = 360 s and Δu = 4210 m/s

Problem **8B**

What is the minimum allowable burn time if half of the total impulse is delivered at the previous propellant mass flow rate and the other half at 20% of this mass flow rate?

A) $t'_{p,\min}$ = 420 s

B) $t'_{p,\min}$ = 530 s

C) $t'_{p,\min}$ = 645 s

D) $t'_{p,\min}$ = 730 s

Problem 9

The following table gives the thrust-time characteristics for a rocket engine. The total propellant mass is m_p = 15.0 kg. True or false?

Time (s)	0	0.5	1.0	2.0	2.5	3.0	3.5	4.0	4.25	4.30
Thrust (N)	7200	9000	7800	8400	8400	7500	7600	8800	9200	0

1.( ) The specific impulse of the rocket engine is greater than 250 sec.
2.( ) The average thrust of the rocket is greater than 7.5 kN.
3.( ) The effective jet velocity of the rocket is greater than 2300 m/s.

Problem 10A

A rocket flies at 8800 km/h with an effective exhaust jet velocity of 1400 m/s and propellant flow rate of 5.2 kg/s. If the heat of reaction of the propellants is 6600 kJ/kg of propellant mixture, determine the propulsive power and the propulsion efficiency.

A) P_p = 10.8 MW and η_P = 0.713
B) P_p = 10.8 MW and η_P = 0.864
C) P_p = 17.8 MW and η_P = 0.713
D) P_p = 17.8 MW and η_P = 0.864

Problem 10B

Determine the engine output and the thermal efficiency for the rocket engine introduced in the previous problem.

A) P_T = 20.6 MW and η_P = 0.471
B) P_T = 20.6 MW and η_P = 0.60
C) P_T = 26.2 MW and η_P = 0.471
D) P_T = 26.2 MW and η_P = 0.60

Problem 11

A three-stage rocket is designed to place a small satellite into the low Earth orbit, having a payload weight of 200 kg. Data for the rocket is given in the following table. Determine the increase in its speed.

Stage number	Specific impulse (sec)	Total mass (kg)	Propellant mass (kg)
1	210	15,000	12,500
2	250	4500	3750
3	290	1000	750

A) Δu = 4540 m/s
B) Δu = 5810 m/s
C) Δu = 6120 m/s
D) Δu = 7320 m/s

Problem 12

Tests of solid-propellant grain showed the following results:

Test number	Chamber pressure, p_c (MPa)	Burn rate, $\dot{r}$ (mm/s)
1	18	24.5
2	11	12.5

Find the combustion pressure for a burning rate of 25 mm/s.

A) p_c = 14.2 MPa
B) p_c = 18.3 MPa
C) p_c = 21.3 MPa
D) p_c = 24.2 Mpa

Problem 13A

Compute the thrust produced by an ion thruster producing 2 A of xenon ions at 1500 V. The ion beam has a 20-degree half-angle divergence, the ratio of doubles to singles is 18%, and the thruster mass utilization efficiency is 86%. For xenon, factor $\sqrt{2M/e}$ = 1.65 × 10^{-3} and the atomic mass is M_a = 131.

A) $T' = 115$ mN

B) $T' = 260$ mN

C) $T' = 340$ mN

D) $T' = 450$ mN

Problem 13B

Calculate the specific impulse of the engine introduced in the previous part.

A) $I_{sp} = 1400$ sec

B) $I_{sp} = 2200$ sec

C) $I_{sp} = 3700$ sec

D) $I_{sp} = 5000$ sec

Problem 13C

If the engine requires a 20-A, 25-V discharge to produce the 2-A ion beam, determine the total efficiency of the engine.

A) $\eta_T = 0.312$

B) $\eta_T = 0.423$

C) $\eta_T = 0.594$

D) $\eta_T = 0.725$

Solutions

P.1 ■ Solution

1. True. The specific impulse is determined as

$$I_{\text{sp}} = \frac{F}{\dot{m}_p g} = \frac{18,000}{7.2 \times 9.81} = 255 \text{ s}$$

2. True. The effective velocity is determined as

$$V_{\text{eq}} = I_{\text{sp}} g = 255 \times 9.81 = 2500 \text{ m/s}$$

3. False. The specific propellant consumption is the reciprocal of the specific impulse,

$$SPC = \frac{1}{I_{\text{sp}}} = \frac{1}{255} = 0.00392 \text{ kg/N} \cdot \text{s}$$

4. False. The thrust power is the product of thrust and flight velocity,

$$P_T = TV = 17,000 \times 880 = 15 \text{ MW}$$

5. False. The thrust coefficient is given by

$$C_T = \frac{T}{p_c A_t} = \frac{18,000}{\left(2.5 \times 10^6\right) \times \left(\frac{\pi \times 0.12^2}{4}\right)} = 0.637$$

The characteristic velocity is the ratio of effective velocity to thrust coefficient,

$$c* = \frac{V_{\text{eq}}}{C_T} = \frac{2500}{0.637} = 3920 \text{ m/s}$$

P.2 ■ Solution

1. True. The throat velocity is estimated as

$$V_t = \sqrt{\frac{2\gamma R}{(\gamma+1)M} T_c} = \sqrt{\frac{2 \times 1.30 \times 8314}{(1.30+1) \times 24} \times 2900} = 1070 \text{ m/s}$$

2. False. The specific volume at the throat follows as

$$v_t = \frac{RT_c}{Mp_c}\left(\frac{\gamma+1}{2}\right)^{1/(\gamma-1)} = \frac{8314 \times 2900}{24 \times \left(2.53 \times 10^6\right)} \times \left(\frac{1.30+1}{2}\right)^{1/(1.3-1)} = 0.633 \text{ m}^3/\text{kg}$$

3. True. The specific impulse is given by $I_{sp} = V_{eq} \times g$, where effective velocity V_{eq} is given by

$$V_{eq} = \sqrt{\frac{2\gamma}{\gamma-1}\frac{RT_c}{M}\left[1-\left(\frac{p_2}{p_1}\right)^{(\gamma-1)/\gamma}\right]} = \sqrt{\frac{2\times 1.30}{1.30-1}\times\frac{8314\times 2900}{24}\times\left[1-\left(\frac{0.09}{2.53}\right)^{0.3/1.3}\right]} = 2160 \text{ m/s}$$

so that

$$I_{sp} = \frac{2160}{9.81} = 220 \text{ sec}$$

4. False. The mass flow rate is given by

$$\dot{m} = \frac{A_t V_t}{v_t} = \frac{\left(5\times 10^{-4}\right)\times 1070}{0.633} = 0.845 \text{ kg/s}$$

and the thrust follows as

$$T = \dot{m} V_{eq} = 0.845 \times 2160 = 1830 \text{ N}$$

5. False. The Mach number at the throat is 1.0 because the throat is operating supersonically.

P.3 ■ Solution

Part A: Gas velocity at the nozzle exit for isentropic flow is given by

$$c_e = \sqrt{\frac{2\gamma}{\gamma-1} RT_o\left[1-\left(\frac{p_e}{p_o}\right)^{(\gamma-1)/\gamma}\right]}$$

Assuming the flow to have reached critical conditions at the throat corresponding to a nozzle pressure ratio p^*/p_o lesser than

$$\frac{p^*}{p_o} = \left(\frac{2}{\gamma+1}\right)^{\gamma/(\gamma-1)}$$

The propellant flow rate through the nozzle is

$$\dot{m}_p = \rho^* c^* A^* = \frac{p^*}{RT^*} A^* \sqrt{\gamma RT^*} = p^* A^* \sqrt{\frac{\gamma}{RT^*}}$$

Since $T = \dot{m}_p \times c_e$, we can write

$$C_T = p^* A^* \sqrt{\frac{\gamma}{RT^*}} \sqrt{\frac{2\gamma}{\gamma-1} RT_o\left[1-\left(\frac{p_e}{p_o}\right)^{(\gamma-1)/\gamma}\right]}$$

$$\therefore C_T = \frac{T}{p_o A^*} = \gamma\left(\frac{p^*}{p_o}\right)\left(\frac{T_o}{T^*}\right)^{1/2} \sqrt{\frac{2}{\gamma-1}\left[1-\left(\frac{p_e}{p_o}\right)^{(\gamma-1)/\gamma}\right]}$$

However, $p^*/p_o = [2/(\gamma+1)]^{\gamma/(\gamma-1)}$ and

$$\frac{T_o}{T^*} = \left(\frac{2}{\gamma+1}\right)^{(\gamma+1)/(\gamma-1)}$$

Accordingly, C_T becomes

$$C_T = \gamma\left(\frac{2}{\gamma-1}\right)^{1/2}\left(\frac{2}{\gamma+1}\right)^{(\gamma+1)/2(\gamma-1)} \sqrt{1-\left(\frac{p_e}{p_o}\right)^{(\gamma-1)/\gamma}}$$

Part B: Substituting $\gamma = 1.25$ and $p_e/p_o = 1/5.0$, the thrust coefficient becomes

$$C_T = 1.25\times\left(\frac{2}{1.25-1}\right)^{1/2}\left(\frac{2}{1.25+1}\right)^{(1.25+1)/2(1.25-1)} \sqrt{1-(1/5.0)^{(1.25-1)/1.25}} = \boxed{1.09}$$

★ The correct answer is **B**.

P.4 ■ Solution

Part A: By definition, the characteristic velocity is the ratio of effective jet velocity and thrust coefficient,

$$c^* = \frac{c_j}{C_T}$$

Variable c_j is given by

$$c_j = \sqrt{\frac{2\gamma}{\gamma-1} RT_o \left[1 - \left(\frac{p_e}{p_o}\right)^{(\gamma-1)/\gamma}\right]}$$

while, as shown in Problem 3, C_T is given by

$$C_T = \gamma \left(\frac{2}{\gamma-1}\right)^{1/2} \left(\frac{2}{\gamma+1}\right)^{(\gamma+1)/2(\gamma-1)} \left[1 - \left(\frac{p_e}{p_o}\right)^{(\gamma-1)/\gamma}\right]^{1/2}$$

Accordingly, c^* becomes

$$c^* = \frac{\sqrt{\dfrac{2}{\gamma-1}} \times \sqrt{\gamma RT_o}}{\gamma \left(\dfrac{2}{\gamma-1}\right)^{1/2} \left(\dfrac{2}{\gamma+1}\right)^{(\gamma+1)/2(\gamma-1)}} = \frac{\sqrt{\gamma RT_o}}{\gamma \left(\dfrac{2}{\gamma+1}\right)^{(\gamma+1)/2(\gamma-1)}}$$

However, $a_0 = \sqrt{\gamma RT_o} = \sqrt{\gamma \bar{R} T_o / M}$, with the result that

$$c^* = \frac{\sqrt{\bar{R} T_o / M}}{\sqrt{\gamma} \left(\dfrac{2}{\gamma+1}\right)^{(\gamma+1)/2(\gamma-1)}}$$

Part B: Substituting $\gamma = 1.25$, $\bar{M} = 18.0$ and $T_o = 3200$ K gives

$$c^* = \frac{\sqrt{8314 \times 3300/18.0}}{\sqrt{1.25} \left(\dfrac{2}{1.25+1}\right)^{(1.25+1)/2(1.25-1)}} = \boxed{1880 \text{ m/s}}$$

★ The correct answer is **C**.

P.5 ■ Solution

1. True. The velocity achieved is governed by the nozzle area ratio, which in turn is determined by the design ambient pressure – the atmosphere into which the nozzle discharges. Low ambient pressure, encountered at high altitudes, leads to a high nozzle exit area, higher gas exit velocity, and ultimately more thrust.

2. True. One way to express exhaust velocity is

$$V_e = C_T \times \left[\gamma \left(\frac{2}{\gamma+1}\right)^{(\gamma+1)/(\gamma-1)} \frac{MW}{RT_e}\right]^{-1/2}$$

Accordingly, the exhaust velocity is inversely dependent on molecular weight to the ½ power. Lower molecular weight produces a higher velocity. The typical exhaust velocity of some propellant mixtures are provided in the following table.

Oxidant	Fuel	V_e (m/s)
O_2	H_2	4550
O_2	RP1	3580
F_2	H_2	4790
N_2O_4	MMH	3420
N_2O_4	N_2H_4 + UDMH	3420

Observe that the hydrogen-fluorine and hydrogen-oxygen propellants, which are the lightest-MW fuel-oxidizer combinations in the table, also happen to constitute the mixtures with highest V_e.

3. True. For a given propellant tank, it is easy to show that the wall surface area of a spherical tank is always less than that of a cylindrical tank, namely,

$$\frac{S_{\text{sp}}}{S_{\text{cyl}}} = \frac{1.5^{2/3}}{\frac{1}{2(L/D)^{2/3}} + (L/D)^{1/3}}$$

where L/D is the length-to-diameter ratio of the cylindrical tank. This relation shows that the surface area ratio is always less than unity when the tank length/diameter ratio is greater than unity, so that a spherical tank is always the most efficient way to house propellant (i.e., tank weight is proportional to surface area).

4. False. One of the inherent advantages of specific impulse as a rocket performance parameter is the fact that its value is the same regardless of the system of units being used. In general, specific impulse varies from about 175 to 250 sec for solid propellants and from 230 to 440 sec for liquid propellants.

5. False. On the contrary, high vapor pressures are associated with greater problems in handling and greater susceptibility to pump cavitation. Propellants with high vapor pressures – including LOX – require special design provisions, unusual handling techniques, and special low-temperature materials.

6. True. Fluorine is a powerful oxidizing agent, but both fluorine and its its combustion byproducts, including hydrogen fluoride, are poisonous to humans. In addition, fluorine is relatively expensive. These factors have led to the abandonment of fluorine-based fuels in modern rocketry. Fluorine-derived compounds such as oxygen difluoride (OF_2) and chlorine trifluoride (ClF_3) have also been employed to no avail.

7. False. In fact, NTO/MMH is the most widely used hypergolic liquid propellant. Some of the properties of monomethyl hydrazine and nitrogen tetroxide are listed below.

Component	Formula	Density (g/cm³)	Vapor Pressure (torr, at 20°C)	Boiling Point (°C)
MMH	CH_3HNNH_2	0.86	36	87
NTO	N_2O_4	1.44	720	21.1

8. False. The non-catalyzed reaction of hydrogen peroxide is slow and unsuitable for thruster applications. Consequently, a catalyst is needed to increase the reaction speed. The most widely used catalyst used with rocket grade hydrogen peroxide (RGHP) is silver, typically arranged in screens (either pure or coated on stainless steel or another alloy). Other feasible catalysts include platinum, iron oxide, and manganese oxide.

9. False. The residence time of a fuel in the combustion chamber can be written as

$$t_r = \frac{c^*}{RT_c}(A_c l_c / A_t)$$

The fraction on the right-hand side of this relation is a function of the propellant combination alone and is constant for a given propellant. The term $A_c l_c / A_t$, in turn, represents the ratio of the chamber volume to the throat area of the nozzle and is therefore a unit of length; it is known as the characteristic length, L^*. The higher the characteristic length of a nozzle, the greater the residence time in the combustion chamber. Accordingly, the fact that hydrocarbon fuels such as LOX/RP-1 have greater characteristic lengths than lighter fuels indicates that they tend to burn more slowly.

10. False. Beryllium-based fuels are the most energetic fuels for solid-rocket propellants. However, the toxic exhaust gases (BeO_x) created by these materials have led to their abandonment in most rocket applications. The most common solid fuel in modern practice is aluminum, which is included in solid fuels in concentrations ranging anywhere from 2 to 21% Al. Magnesium has been implemented in “clean” formulations with reduced exhaust emissions.

11. False. The exhaust gases produced by liquid engines tend to be much less pernicious than those of solid fuels. For one, the main product of liquid hydrogen-liquid oxygen combustion is water, and usually includes no particulates at all. The chlorine in the oxidants of solid boosters, on the other hand, produces substantial amounts of hydrogen chloride. The HCl cloud from a large solid motor can combine with water vapor to form acid rain in the area near the launch pad. Further, solid fuels also include a chemically diverse series of additives, such as inorganic salts, which may give rise to dangerous particulate emissions.

12. False. The fact that solid fuels are supposedly more stable says nothing about the stability of solid-fuel motors. In fact, the performance of a solid propellant rocket motor, if anything, is erratic. For one, in a liquid rocket the thrust is actively controlled by the rate of supply of propellants, and in the majority of cases it is stabilized at a constant value. For a solid rocket, the thrust depends on the rate of supply of combustible propellant, which in turn depends upon the pressure and temperature at the burning surface, which cannot be actively controlled. What's more, a liquid-fueled engine can be shut down by closing valves, whereas the solid motor continues to burn until all the propellant is exhausted.

13. True. We know that the mass flow rate out of the chamber depends linearly on the pressure. Accordingly, if $n > 1.0$, the supply of gas from the burning grain increases faster than with pressure than the rate of exhaust, and an uncontrolled rise in burning rate and pressure could result from a small initial increase. Conversely, a small initial decrease in pressure could result in a catastrophic drop in burning rate. If, on the other hand, $n < 1.0$, then the rate of change of burning rate is always less than the (linear) rate of change of mass flow through the exhaust, and the pressure in the chamber will stabilize after any positive or negative change in burning rate. Typical ranges of n range from 0.4 to 0.7.

P.6 ■ Solution

Part A: The thrust developed by the rocket is given by

$$T = \dot{m}c_j C_T = 6.0\times 2000\times 1.5 = \boxed{18.0\text{ kN}}$$

★ The correct answer is **B**.

Part B: The mass ratio is calculated as

$$MR = \exp\left(-\frac{\Delta u}{c_j}\right) = \exp\left(-\frac{1600}{2000}\right) = 0.449$$

The corresponding propellant mass follows as

$$m_p = m_0\left(1-MR\right) = 4000\times\left(1-0.449\right) = \boxed{2200\text{ kg}}$$

★ The correct answer is **C**.

P.7 ■ Solution

The exhaust velocity can be determined by solving the mass flow rate equation for V_e,

$$\dot{m} = \rho_e A_e V_e \rightarrow V_e = \frac{\dot{m}}{\rho_e A_e}$$

However, $\rho_e = MW \times p_e/R_e T_e$ and $A_e = \pi D_e^2/4$, so that

$$V_e = \frac{\dot{m}}{\rho_e A_e} \rightarrow V_e = \frac{4R_e T_e \dot{m}}{MW p_e \pi D_e^2}$$

$$\therefore V_e = \frac{4\times 8.314\times 1500\times 16.2}{24\times 101\times 3.14\times 0.25^2} = 1700\text{ m/s}$$

The effective jet velocity follows as

$$c_j = V_e + \frac{\left(p_e - p_a\right)}{\dot{m}}A_e = 1700 + \frac{\left[\left(101-5.5\right)\times 10^3\right]}{24}\times\left(\frac{\pi\times 0.25^2}{4}\right) = \boxed{1900\text{ m/s}}$$

Lastly, the total thrust is the product of effective velocity and propellant mass flow,

$$T = \dot{m}c_j = 24 \times 2400 = \boxed{57.6\ \text{kN}}$$

★ The correct answer is **A**.

P.8 ■ Solution

Part A: The mass of propellant is determined first,

$$m_0 = m_{pl} + m_{\text{motor}} \rightarrow 25 = 2 + \frac{m_p}{0.88}$$

$$\therefore m_p = 20.2\ \text{kg}$$

Given m_p/m_0 = 20.2/25 = 0.808, the mass ratio is calculated as

$$MR = 1 - \frac{20.2}{25} = 0.192$$

The minimum allowable burning time is then

$$t_{p,\min} = \frac{I_{\text{sp}}g\left(m_p/m_0\right)}{\alpha_{\max}MR} = \frac{260 \times 9.81 \times 0.808}{50 \times 0.192} = \boxed{215\ \text{s}}$$

The maximum velocity relative to the launch vehicle is calculated to be

$$\Delta u = I_{\text{sp}}g\ln\left(\frac{1}{MR}\right) = 215 \times 9.81 \times \ln\left(\frac{1}{0.192}\right) = \boxed{4210\ \text{m/s}}$$

★ The correct answer is **B**.

Part B: The initial mass flow is $\dot{m}_0$ = 20.2/215 = 0.0940 kg/s, and the total impulse is $I = cm_p$ = 260 × 9.81 × 20.2 = 51,500 N·s. Half of the impulse is delivered at the previous propellant flow rate, which corresponds to a time of $t_{p,1}$ = 215/2 = 108 sec. The other half of the impulse is delivered at a mass flow rate such that $\dot{m}_2$ = 0.20 × 0.0940 = 0.0188 kg/s, and corresponds to a time interval

$$t_{p,2} = \frac{0.5 \times I}{\dot{m}_2 I_{\text{sp}} g} = \frac{0.5 \times 51,500}{0.0188 \times 260 \times 9.81} = 537\ \text{s}$$

The minimum allowable burn time is then

$$t_p' = t_{p,1} + t_{p,2} = 108 + 537 = \boxed{645\ \text{s}}$$

★ The correct answer is **C**.

P.9 ■ Solution

1. False. The total impulse is the area under the curve of thrust versus time. Since we were given a discrete series of thrust-time data points, the impulse is determined by numerical integration. The area of each trapezoid is

$$\text{Area} = \text{Base} \times \frac{(\text{Height 1} + \text{Height 2})}{2}$$

Accordingly,

$$I = 0.5 \times \begin{bmatrix} 0.5 \times (7200 + 9000) + 0.5 \times (9000 + 7800) + 1.0 \times (7800 + 8400) \\ +0.5 \times (8400 + 8400) + 0.5 \times (8400 + 7500) + 0.5 \times (7500 + 7600) \\ +0.5 \times (7600 + 8800) + 0.25 \times (8800 + 9200) + 0.05 \times (9200 + 0) \end{bmatrix} = 34,900\ \text{N}\cdot\text{s}$$

The specific impulse is obtained by dividing I by the product of propellant mass m_p = 15.0 kg and g = 9.81 m/s^2; that is,

$$I_{\text{sp}} = \frac{I}{m_p g} = \frac{34,900}{15.0 \times 9.81} = 237\ \text{s}$$

2. True. The average thrust is the ratio of impulse and the time encompassed by the thrust-time data; that is,

$$T = \frac{I}{t} = \frac{34,900}{4.30} = 8.12 \text{ kN}$$

3. True. The effective jet velocity follows as

$$I_{sp} = \frac{c_j}{g} \rightarrow c_j = I_{sp} \times g$$

$$\therefore c_j = 237 \times 9.81 = 2330 \text{ m/s}$$

P.10 ■ Solution

Part A: The propulsive power is given by

$$P_p = \dot{m}_p c_j u$$

where $\dot{m}_p$ = 5.2 kg/s is the propellant flow rate, c_j = 1400 m/s is the effective jet velocity, and u = 8800/3.6 = 2440 m/s is the flight speed, so that

$$P_p = \dot{m}_p c_j u = 5.2 \times 1400 \times 2440 = \boxed{17.8 \text{ MW}}$$

The propulsion efficiency is given by

$$\eta_p = \frac{\text{Propulsive power}}{\text{Engine output power}}$$

where the propulsive power = $\dot{m}_p c_j u$ as proposed above and the engine output power $\dot{m}_p (c_j^2 + u^2)/2$, with the result that

$$\eta_p = \frac{\dot{m}_p \times c_j \times u}{\dfrac{\dot{m}_p \left(c_j^2 + u^2\right)}{2}} = \frac{2u/c_j}{1 + u^2/c_j^2} = \frac{2\sigma}{1+\sigma^2}$$

Here, we have introduced the velocity ratio $\sigma = u/c_j$, which in the present case becomes

$$\sigma = \frac{u}{c_j} = \frac{2440}{1400} = 1.74$$

Accordingly,

$$\eta_p = \frac{2 \times 1.74}{1 + 1.74^2} = \boxed{0.864}$$

★ The correct answer is **D**.

Part B: The engine output can be determined by dividing the propulsive power by the propulsive efficiency,

$$\eta_p = \frac{P_p}{P_T} \rightarrow P_T = \frac{P_p}{\eta_p}$$

$$\therefore P_T = \frac{17.8}{0.864} = \boxed{20.6 \text{ MW}}$$

Another way to arrive at the same result is to apply the aforementioned relation

$$P_T = \frac{1}{2} \dot{m}_p \left(c_j^2 + u^2\right) = 0.5 \times 5.2 \times \left(1400^2 + 2440^2\right) = 20.6 \text{ MW}$$

The thermal efficiency is given by the ratio of engine output power to the power input through fuel,

$$\eta_T = \frac{\text{Engine output power}}{\text{Power input through fuel}} = \frac{P_T}{\dot{m}_p Q_R}$$

where Q_R = 6600 kJ/kg is the heat of reaction of the propellants. Hence,

$$\eta_T = \frac{20.6}{5.2 \times 6.6} = \boxed{0.60}$$

Lastly, we may also determine the overall efficiency, which is stated as the product of the two foregoing efficiency terms; that is,

$$\eta_o = \eta_p \eta_T = 0.864 \times 0.60 = 0.518$$

The overall efficiency of the rocket is close to 52 percent.

★ The correct answer is **B**.

P.11 ■ Solution

The increase in speed is given by

$$\Delta u = g \sum_{i=1}^{N} \left(I_{\text{sp}}\right)_i \times \ln\left(\frac{m_0}{m_0 - m_p}\right)_i$$

Consider the first stage. Since the payload is carried atop the third stage into orbit, the total mass of the first stage is

$$m_{0,1} = m_{t,1} + m_{t,2} + m_{t,3} + m_L$$

$$\therefore m_{0,1} = 15,000 + 4500 + 1000 + 200 = 20,700 \text{ kg}$$

and, given the propellant mass $\dot{m}_{p,1}$ = 12,500 kg, it follows that

$$\left(I_{\text{sp}}\right)_1 \times \ln\left(\frac{m_{0,1}}{m_{0,1} - m_{p,1}}\right) = 210 \times \ln\left(\frac{20,700}{20,700 - 12,500}\right) = 194 \text{ s}$$

Next, consider the second stage. As the propellant of the first stage is burned off during powered ascent, the near-empty tank and structure of the first stage are dropped off to reduce the weight of the vehicle to achieve orbital velocity. Smaller amount of propellant is retained in the second - and third -stage tanks. Accordingly, mass $m_{0,2}$ is determined as

$$m_{0,2} = m_{t,2} + m_{t,3} + m_L = 4500 + 1000 + 200 = 5700 \text{ kg}$$

and, given the propellant mass $\dot{m}_{p,2}$ = 3750 kg, it follows that

$$\left(I_{\text{sp}}\right)_2 \times \ln\left(\frac{m_{0,2}}{m_{0,2} - m_{p,2}}\right) = 250 \times \ln\left(\frac{5700}{5700 - 3750}\right) = 268 \text{ s}$$

Next, consider the third stage. Since the second stage is dropped off as well, mass $m_{0,3}$ is determined as

$$m_{0,3} = m_{t,3} + m_L = 1000 + 200 = 1200 \text{ kg}$$

and, given the propellant mass $m_{p,3}$ = 750 kg, it follows that

$$\left(I_{\text{sp}}\right)_3 \times \ln\left(\frac{m_{0,3}}{m_{0,3} - m_{p,3}}\right) = 290 \times \ln\left(\frac{1200}{1200 - 750}\right) = 284 \text{ s}$$

Gleaning our results, the increase in speed is calculated to be

$$\Delta u = 9.81 \times (194 + 268 + 284) = \boxed{7320 \text{ m/s}}$$

★ The correct answer is **D**.

P.12 ■ Solution

The burning rate is expressed by Vielle's law,

$$\dot{r} = a p_c^n$$

where a is an empirical constant influenced by the ambient temperature of the propellant prior to ignition, n is the burning rate pressure exponent, and p_c is the chamber pressure. For test 1, we write

$$\dot{r}_1 = a p_{c,1}^n$$

Likewise, for test 2,

$$\dot{r}_2 = a p_{c,2}^n$$

Since the energetic material is the same in both tests, coefficients a and n do not change. Dividing the two preceding equations gives

$$\frac{\dot{r}_1}{\dot{r}_2}=\left(\frac{p_{c,1}}{p_{c,2}}\right)^n \rightarrow \ln\left(\frac{\dot{r}_1}{\dot{r}_2}\right)=n\times\ln\left(\frac{p_{c,1}}{p_{c,2}}\right)$$

$$\therefore n=\frac{\ln\left(\dot{r}_1/\dot{r}_2\right)}{\ln\left(p_{c,1}/p_{c,2}\right)}=\frac{\ln\left(24.5/12.5\right)}{\ln\left(18/11\right)}=1.37$$

Returning to Vielle's law with this result, we have

$$\dot{r}=ap_c^n \rightarrow a=\frac{\dot{r}}{p_c^n}$$

$$\therefore a=\frac{24.5}{18^{1.37}}=0.467$$

Accordingly, the combustion pressure for a burning rate of 25 mm/s is calculated as

$$p_c=\left(\frac{\dot{r}}{a}\right)^{\frac{1}{n}}=\left(\frac{25}{0.467}\right)^{\frac{1}{1.37}}=\boxed{18.3\text{ MPa}}$$

★ The correct answer is **B**.

P.13 ■ Solution

Part A: The thrust for an ion thruster with a singly charged propellant, in Newtons, is given by

$$T=\sqrt{\frac{2M}{e}}I_b\sqrt{V_b}$$

where M is the ion mass, e is the elementary charge, I_b is the ion beam current, and V_b is the net voltage through which the ion was accelerated. If the propellant is xenon, the first term $(2M/e)^{0.5}$ = 1.65 × 10^{-3} and the equation for T, expressed in mN, becomes

$$T=1.65I_b\sqrt{V_b}\quad[\text{mN}]$$

The foregoing expression is the basic thrust equation for a unidirectional, singly ionized, monoenergetic beam of ions. The equation must be modified to account for (1) the divergence of the ion beam and (2) the presence of multiply charged ions. The first correction is represented by a factor F_t such that

$$F_t=\cos\theta$$

where θ is the average half-angle divergence of the beam. The second correction is expressed by a factor α given by

$$\alpha=\frac{1+0.707\dfrac{I^{++}}{I^{+}}}{1+\dfrac{I^{++}}{I^{+}}}$$

where I^{++}/I^{+} is the fraction of double ion current in the beam. Both correction factors can be stated as a single factor γ such that

$$\gamma=\alpha F_t$$

so that the corrected thrust becomes

$$T'=\gamma\sqrt{\frac{2M}{e}}I_b\sqrt{V_b}$$

If xenon is the propellant, we have

$$T'=1.65\gamma I_b\sqrt{V_b}\quad[\text{mN}]$$

Consider the data at hand. For a 15-degree half-angle beam divergence, we have

$$F_t=\cos 20^\circ=0.940$$

For a 18% doubles-to-singles ratio, in turn, we find that

$$\alpha = \frac{1+0.707\times0.18}{1+0.18} = 0.955$$

with the result that

$$\gamma = 0.940\times0.955 = 0.898$$

For a thruster producing 2 A of xenon ions at 1500 V, the thrust produced follows as

$$T' = 1.65\times0.898\times2\times\sqrt{1500} = \boxed{115\ \text{mN}}$$

This calculation illustrates the fact that the thrust of an ion engine is really small. Ion engines are best used for very high-velocity increment missions in which the time penalty associated with small thrust is not important. They are generally suitable for interplanetary missions and for station keeping, where the low thrust is not a disadvantage.

★ The correct answer is **A**.

Part B: The specific impulse of an ion thruster may be approximated by the relation

$$I_{\text{sp}} = 1417\gamma\eta_m\sqrt{\frac{V_b}{M_a}}$$

where, in addition to the variables introduced before, we have the mass utilization efficiency η_m = 0.86 and the atomic mass M_a = 131. Accordingly,

$$I_{\text{sp}} = 1417\times0.898\times0.86\sqrt{\frac{1500}{131}} = \boxed{3700\ \text{sec}}$$

★ The correct answer is **C**.

Part C: The total efficiency of the engine is given by

$$\eta_T = \gamma^2\eta_e\eta_m$$

Here, in addition to aforementioned variables, we have the electrical efficiency η_e of the thruster, which is given by

$$\eta_e = \frac{I_bV_b}{I_bV_b+P_o}$$

where P_o is the power input required to create the thrust beam. In the present case, we require a 20-A, 25-V discharge to produce a 2-A ion beam, hence P_o = 20 × 25/2 = 250 eV/ion. Accordingly, the electrical efficiency follows as

$$\eta_e = \frac{I_bV_b}{I_bV_b+P_o} = \frac{2\times1500}{2\times1500+250\times2} = 0.857$$

Substituting in the first equation, the total efficiency is determined to be

$$\eta_T = \gamma^2\eta_e\eta_m = 0.898^2\times0.857\times0.86 = \boxed{0.594}$$

★ The correct answer is **C**.

Answer Summary

Problem 1		T/F
Problem 2		T/F
Problem 3	3A	Open-ended pb.
	3B	B
Problem 4	4A	Open-ended pb.
	4B	C
Problem 5		T/F
Problem 6	6A	B
	6B	C
Problem 7		A
Problem 8	8A	B
	8B	C
Problem 9		T/F
Problem 10	10A	D
	10B	B
Problem 11		D
Problem 12		B
Problem 13	13A	A
	13B	C
	13C	C

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