Quiz HD301

Sediment Transport

Lucas Montogue

Note:

- *Use ρ = 1000 kg/m^3, γ = 9800 N/m^3 and ν = 1.006 × 10^{-6} m^2/s as the density, unit weight, and kinematic viscosity of water, respectively. Use G_s = 2.65 as the specific gravity whenever needed.*

Problems

Problem 1

A reservoir with a capacity of 50 Mm^3 is proposed at a location in a river with an annual sediment inflow of 0.15 Mm^3. Estimate the time required for the loss of 30% of initial capacity of the reservoir due to sedimentation. Assume 3 equal steps of capacity loss.

A) t_{30} = 41 years

B) t_{30} = 65 years

C) t_{30} = 84 years

D) t_{30} = 102 years

Problem 2

A reservoir has an initial capacity of 90,000 ha-m and the annual sediment load in the stream is estimated as 600 ha-m. If the average annual inflow into the reservoir is 400,000 ha-m, estimate the time in years for the reservoir to lose 50% of its initial capacity. Use 5 time steps of equal capacity loss. In the relevant range, suppose the trap efficiency can be estimated with the usual relation

$$\eta_f = 6.064\ln\left(\frac{C}{I}\right) + 101.48$$

A) t_{50} = 44 years

B) t_{50} = 60 years

C) t_{50} = 83 years

D) t_{50} = 101 years

Problem 3

A proposed reservoir has a catchment of 2660 km^2. It has a capacity of 360 Mm^3 and the annual yield of the catchment is estimated as 40 cm. Assuming the average composition of the sediment as 20% sand, 35% silt and 45% clay, estimate the probable life of the reservoir to a point where 40% of the reservoir capacity is lost by sedimentation. The sediment yield is estimated independently as 360 tonnes/km^2/year. Assume the reservoir to have a moderate drawdown.

A) t_{40} = 144 years

B) t_{40} = 195 years

C) t_{40} = 238 years

D) t_{40} = 277 years

Problem 4

For a trapezoidal channel flowing at normal depth with q = 5 m^3/s, bottom width b = 8 m, 2H : 1V side slopes, channel slope S_0 = 0.003, and Manning's n = 0.030, determine the average sediment particle size that will be on the verge of motion within the channel. Assume conditions at 10°C, for which $\mu = 1.307\times10^{-3}$ Pa·s. Use the following approximation for the Shields parameter,

$$\theta = 0.118\,\mathrm{Re}_*^{-0.979} + 0.056\exp\left(-5.31\,\mathrm{Re}_*^{-0.679}\right)$$

where Re_* is the particle Reynolds number.

A) d = 5.17 mm

B) d = 10.1 mm

C) d = 15.3 mm

D) d = 20.6 mm

Problem 5

Calculate the total bed load transport rate associated with the channel and flow conditions specified in the previous problem. The bed material of the channel is composed as indicated in the following table. Use the Einstein-Brown approach.

Sieve Opening (mm)	% Passing
16	100
13.2	80
11.2	70
8	54
6.3	41
4.75	33
4	25
2	18
1	12
< 1	8

A) g_s = 82.4 N/s

B) g_s = 112 N/s

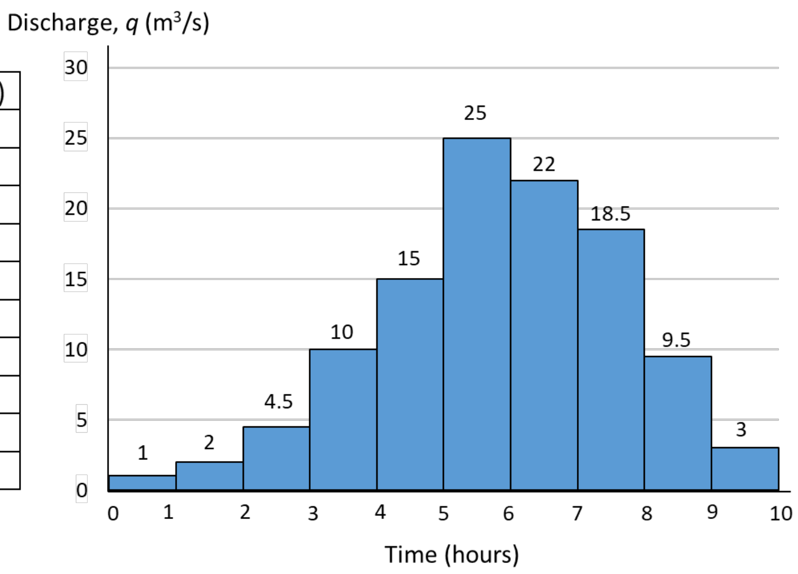
C) g_s = 140 N/s

D) g_s = 177 N/s

Problem 6A

The cross-section of a natural stream is reasonably approximated by a trapezoidal section with bottom width b = 7.5 m and 3H:1V side slopes. The bankfull depth is 2.1 m, Manning's n = 0.064, and the longitudinal slope S_0 = 0.0025. It is found that the total sediment transport can be estimated by $g_s = 0.0018q^{1.2}(\tau_0 - 13.5)$, with g_s in N/s, q in m^3/s, and τ_0 in N/m^2. An hourly time series of discharge is presented in the graph below and corresponds to a single flood event. Determine the discharge at which sediment transport begins.

Hour	q (m^3/s)
0 → 1	1
1 → 2	2
2 → 3	4.5
3 → 4	10
4 → 5	15
5 → 6	25
6 → 7	22
7 → 8	18.5
8 → 9	9.5
9 → 10	3



A) $q_{\min}$ = 0.885 m^3/s

B) $q_{\min}$ = 3.42 m^3/s

C) $q_{\min}$ = 6.08 m^3/s

D) $q_{\min}$ = 10.2 m^3/s

Problem 6B

Use the rating curve to estimate the total sediment load transported by the flood event.

A) L = 18.5 kN

B) L = 23.7 kN

C) L = 30.2 kN

D) L = 37.4 kN

Problem 7A

The flow depth in a river is 2.8 m, the bottom width is 45 m, the flow velocity is 1.3 m/s, and the energy slope is 0.001. The flow is uniform within the measuring reach. The bed sediment has a median size d_{50} = 1.5 mm, the static angle of repose is 32°, and the dynamic angle of repose is 20°. Compute the bed-load transport rate with the van Rijn equation.

A) q_s = 0.0781 m^3/s

B) q_s = 0.226 m^3/s

C) q_s = 0.604 m^3/s

D) q_s = 0.933 m^3/s

Problem 7B

Compute the bed-load transport rate with the Yalin equation.

A) q_s = 0.0849 m^3/s

B) q_s = 0.156 m^3/s

C) q_s = 0.665 m^3/s

D) q_s = 0.951 m^3/s

Problem 7C

Compute the bed-load transport rate with the Engelund and Fredsøe equation.

A) q_s = 0.272 m^3/s

B) q_s = 0.677 m^3/s

C) q_s = 0.904 m^3/s

D) q_s = 1.22 m^3/s

Problem 7D

True or false?

1.( ) The saltation length of the river as calculated with van Rijn's formula is greater than 0.5 m.

2.( ) The saltation height of the river as calculated with van Rijn's formula is greater than 0.05 m.

3.( ) The mean particle velocity of the river as calculated with van Rijn's formula is greater than 1.2 m/s.

Problem 8A

A natural stream has a flow depth of 1.5 m and a bed slope of 0.0026. The bed consists of sand with a median size of 0.45 mm. Suppose the bed concentration at an elevation of 0.05 m from the bed is 0.85. Noting that β = 1, calculate the Rouse number,

$$\zeta = \frac{w_s}{\beta \kappa u_*}$$

A) ζ = 0.312

B) ζ = 0.521

C) ζ = 0.720

D) ζ = 0.914

Problem 8B

Plot the sediment concentration distribution between 0.05 m and the free surface.

Problem 9A

Water flows through a channel of flow depth 2.8 m and width of 45 m. The flow rate per unit width is 12 m^2/s, the channel has a streamwise bed slope of 0.001, and Manning's n is 0.150. The bed sediment has a median size d_{50} = 0.5 mm, the specific gravity is 2.65, and the terminal fall velocity of the sediment particles is 0.075 m/s. Compute the suspended-load transport rate with the Lane and Kalinske method. Use a = 0.3 m as the reference level and C_a = 10^{-3} as the reference concentration.

A) g_s = 185 N/s

B) g_s = 1200 N/s

C) g_s = 4900 N/s

D) g_s = 12,200 N/s

Problem 9B

Compute the suspended-load transport rate with the Einstein method. Use a = 10^{-3} m as the reference level, C_a = 0.4 as the reference concentration, and Δ_k = 10^{-4} m as the apparent roughness.

A) g_s = 438 N/s

B) g_s = 6660 N/s

C) g_s = 25,300 N/s

D) g_s = 61,200 N/s

Problem 9C

Compute the suspended-load transport rate with the Brooks method. Use a = 4×10^{-4} m as the reference level and C_a = 1.0 as the reference concentration.

A) g_s = 855 N/s

B) g_s = 2320 N/s

C) g_s = 14,500 N/s

D) g_s = 52,000 N/s

Problem 9D

Compute the suspended-load transport rate with the Chang et al. method. Use a = 5.6 $\times10^{-4}$ m as the reference level and C_a = 0.715 as the reference concentration.

A) g_s = 475 N/s

B) g_s = 3130 N/s

C) g_s = 15,100 N/s

D) g_s = 42,660 N/s

Problem 10A

Data points of suspended load rating curve and flow duration curve of a river at a gauging site are given below. Plot the respective curves and use them to estimate the total sediment yield at the gauging station. Assume the bed load to be 10% of the suspended load.

Flow Duration Curve		Suspended Sediment Rating Curve	
Percent Times Flow Equaled or Exceeded	Average Daily Discharge (m^3/s)	Water Discharge (m^3/s)	Suspended Load (tonnes/day)
0.5 - 1.0	2550	2550	355000
1.5 - 5.0	1275	1250	200000
5.0 - 15.0	735	750	62500
15.0 - 35.0	450	450	22500
35.0 - 55.0	200	350	17500
55.0 - 75.0	110	225	10000
75.0 - 95.0	50	125	4000
95.0 - 99.0	20	85	2000
		50	500
		25	50

A) m_s = 3.14 × 10^6 tonnes/year
B) m_s = 5.25 × 10^6 tonnes/year
C) m_s = 7.09 × 10^6 tonnes/year
D) m_s = 9.75 × 10^6 tonnes/year

Problem 10B

Calculate the concentration of suspended load in parts per million.

A) C_s = 225 ppm
B) C_s = 435 ppm
C) C_s = 603 ppm
D) C_s = 883 ppm

Problem 11A

Consider a wide river with flow depth of 3.4 m, depth-averaged flow velocity of 1.8 m/s, and energy slope of 0.0002. The bed sediment is fine sand with d_{50} = 0.2 mm and d_{65} = 0.3 mm. Use the Simons & Richardson approach to predict the type of bed form in this river.

A) Plane bed.
B) Ripples.
C) Dunes.
D) Antidunes.

Problem 11B

Classify the bed form according to the Chabert and Chauvin chart.

A) Plane bed.
B) Dunes.
C) Ripples.
D) There should be no bed motion.

Problem 11C

Classify the bed form according to the van Rijn method.

A) Sand waves (symmetrical).
B) Washed-out dunes and sand waves (asymmetrical).
C) Dunes.
D) Plane bed and/or antidunes.

Problem 12

A wide river has a flow depth of 2.0 m, a flow velocity of 1.0 m/s, and a streamwise bed slope of 0.00035. The flow is fairly uniform within the measuring reach. The characteristics of bed sediment are a median size d_{50} = 0.6 mm and d_{65} = 0.75 mm. True or false?

1.( ) According to Julien, dunes can be formed if the particle parameter $D_* \in [3, 70]$, the particle Reynolds number $Re_* \in [11.6, 70]$, and the bed shear stress due to particle roughness τ_0' satisfies the inequality

$$\tau_0' < \frac{1}{D_* \kappa} \ln\left(\frac{y}{20 d_{50}}\right)$$

For the river considered herein, at least two of these requirements are fulfilled. Use $k_s \approx d_{65}$ when calculating the Reynolds number, and approximate τ_0' as the bed shear τ_0.

2.( ) The dune height calculated with the Julien and Klaassen approximation is greater than 0.5 m.

3.( ) The dune height calculated with the van Rijn formula is greater than 75 mm.

4.( ) The dune height calculated with the Watanabe formula is greater than 0.4 m.

5.( ) The dune length calculated with the van Rijn formula is greater than 12 m.

Additional Information

Equations

1 → *Shields parameter*:

$$\theta = \frac{\tau_0}{\gamma (G_s - 1) d_{50}}$$

where τ_0 is shear stress, γ is the unit weight of water, G_s is the specific gravity, and d_{50} is the median size of sediment grains.

2 → *Particle parameter*:

$$D_* = \left[\frac{(G_s - 1) g}{\nu^2}\right]^{1/3} d_{50}$$

where G_s is the specific gravity, g = 9.81 m/s^2, ν is the kinematic viscosity of water, and d_{50} is the median size of sediment grains.

3 → *Van Rijn's relationships between critical Shields parameter and D_*:*

$$\theta_{cr} = 0.24 D_*^{-1} \; ; \; 1 < D_* \le 4$$
$$\theta_{cr} = 0.14 D_*^{-0.64} \; ; \; 4 < D_* \le 10$$
$$\theta_{cr} = 0.04 D_*^{-0.1} \; ; \; 10 < D_* \le 20$$
$$\theta_{cr} = 0.013 D_*^{0.29} \; ; \; 20 < D_* \le 150$$
$$\theta_{cr} = 0.055 \; ; \; D_* > 150$$

where D_* is the particle parameter.

4 → *Dimensionless bed-load transport intensity:*

$$\Phi_b = \frac{q_b}{\sqrt{g (G_s - 1) d_{50}^3}}$$

where q_b is the bed-load transport rate, g = 9.81 m/s^2, G_s is specific gravity, and d_{50} is the median size of sediment grains.

5 → *Einstein-Brown formulas for bed-load transport intensity:*

$$\Phi_g = 0 \ ; \ \theta \le 0.056$$

$$\Phi_g = 0.425\sqrt{\theta} - 0.1 \ ; \ 0.056 \le \theta \le 0.08$$

$$\Phi_g = 40\theta^3 \ ; \ \theta > 0.08$$

where θ is the Shields parameter. The dimensionless transport rate used with these equations should not be confused with Φ_b as given in eq. 4 and is given by

$$\Phi_g = \frac{g_b}{\gamma G_s d_{50} w_s}$$

in which g_b is the unit weight transport rate per unit time, γ is the unit weight of water, G_s is specific gravity, d_{50} is the median size of sediment grains, and w_s is the terminal fall velocity.

6 → *Van Rijn (1984) equation for bed-load transport intensity:*

$$\Phi_b = \frac{0.053}{D_*^{0.3}}\left(\frac{\theta}{\theta_{\mathrm{cr}}} - 1\right)^{2.1}$$

where D_* is the particle parameter (eq. 2), θ is the Shields parameter, and θ_{cr} is the critical Shields parameter (eq. 3).

7 → *Yalin (1977) equation for bed-load transport intensity:*

$$\Phi_b = 0.635\theta^{0.5}\left(\frac{\theta}{\theta_{\mathrm{cr}}} - 1\right)\left\{1 - \frac{\theta_{\mathrm{cr}}}{a_1\left(\theta - \theta_{\mathrm{cr}}\right)}\ln\left[1 + a_1\left(\frac{\theta}{\theta_{\mathrm{cr}}} - 1\right)\right]\right\}$$

where θ is the Shields parameter, θ_{cr} is the critical Shields parameter (eq. 3), and a_1 is a factor given by

$$a_1 = \frac{2.45\theta_{\mathrm{cr}}^{0.5}}{G_s^{0.4}}$$

in which G_s is the specific gravity.

8 → *Engelund and Fredsøe (1976) equation for bed-load transport intensity:*

$$\Phi_b = \frac{9.3}{\mu_d}\left(\theta - \theta_{\mathrm{cr}}\right)\left(\theta^{0.5} - 0.7\theta_{\mathrm{cr}}^{0.5}\right)$$

where θ is the Shields parameter, θ_{cr} is the critical Shields parameter (eq. 3), and μ_d is the dynamic coefficient of friction of the bed particles.

9 → *Van Rijn equations for saltation length and saltation height:*

$$\frac{\lambda_b}{d_{50}} = 3D_*^{0.6}\left(\frac{\theta}{\theta_{\mathrm{cr}}} - 1\right)^{0.9}$$

$$\frac{h_s}{d_{50}} = 0.3D_*^{0.7}\left(\frac{\theta}{\theta_{\mathrm{cr}}} - 1\right)^{0.5}$$

where d_{50} is the median size of soil grains, D_* is the particle parameter (eq. 2), θ is the Shields parameter, and θ_{cr} is the critical Shields parameter (eq. 3).

10 → *Van Rijn equation for mean particle velocity:*

$$\frac{\overline{u}_b}{u_*} = 9 + 2.6\log D_* - 8\left(\frac{\theta}{\theta_{\mathrm{cr}}}\right)^{0.5}$$

where u_* is the shear velocity, D_* is the particle parameter (eq. 2), θ is the Shields parameter, and θ_{cr} is the critical Shields parameter (eq. 3).

11 → *Rouse number:*

$$\zeta = \frac{w_s}{\beta\kappa u_*}$$

where w_s is the terminal fall velocity of sediment, β is a coefficient related to eddy diffusivity, κ = 0.41 is the Von Kármán constant, and u_* is the shear velocity. Although β is usually taken as unity, analysis by van Rijn has verified that, depending on the ratio w_s/u_*, a better approximation might be

$$\beta\left(0.1<\frac{w_s}{u_*}<1\right)=1+2\left(\frac{w_s}{u_*}\right)^2$$

12 → *Lane and Kalinske (1941) equation for suspended-load transport rate:*

$$q_b = qC_aP_C\exp\left(\frac{15w_s a}{u_* y}\right)$$

where q is the unit water discharge, C_a is the reference concentration, w_s is the terminal fall velocity of sediment, a is the reference depth, u_* is the shear velocity, and y is the flow depth. Factor P_C is a factor given in Figure 1 as a function of w_s/u_* with $n/y^{1/6}$ as a parameter. (n is the Manning roughness coefficient. In the chart, y is given in inches.)

13 → *Einstein (1950) equation for suspended-load transport rate:*

$$q_b = \frac{C_a u'_* a}{\kappa}\left(P_E J_1 + J_2\right)$$

where C_a is the reference concentration, u'_* is the shear velocity due to particle roughness, a is the reference depth, κ = 0.41 is the Von Kármán constant, and P_E is a coefficient given by

$$P_E = \ln\left(\frac{30.2y}{\Delta_k}\right)$$

in which Δ_k is the apparent roughness. Factors J_1 and J_2 are coefficients given by J_1 = I_1/0.216 and J_2 = I_2/0.216, where I_1 and I_2 are integrals whose values are plotted in Figures 2 and 3 as functions of the dimensionless reference level $\tilde{a}$, with the Rouse number ζ (eq. 11) as a parameter.

14 → *Brooks (1963) equation for suspended-load transport rate:*

$$\frac{q_b}{qC_{0.5h}} = T_B\left(\frac{\kappa U}{u_*},\zeta,\tilde{a}\right)$$

where q is the unit water discharge, $C_{0.5h}$ is the reference concentration at mid-depth, κ = 0.41 is the Von Kármán constant, U is the flow velocity, u_* is the shear velocity, ζ is the Rouse number (eq. 11), and $\tilde{a}$ is the dimensionless reference depth. Factor T_B is taken from Figure 4, which provides this quantity as a function of ζ, with $\kappa U/u_*$ as a parameter.

15 → *Chang et al. (1965) equation for suspended-load transport rate:*

$$q_b = C_a y\left(UI_3 - \frac{2u_*}{\kappa}I_4\right)$$

where C_a is the reference concentration, y is the flow depth, U is the flow velocity, u_* is the shear velocity, and κ = 0.41 is the Von Kármán constant. Factors I_3 and I_4 are integrals given in Figures 4 and 5 as functions of the dimensionless reference depth $\tilde{a}$, with a slightly modified Rouse number as a parameter,

$$\zeta_1 = \frac{2w_s}{\beta\kappa u_*}$$

16 → *Julien and Klaassen (1995) approximations for average dune height and length:*

$$\overline{\eta}_d = 2.5y^{0.7}d_{50}^{0.3} \;\; ; \;\; \overline{\lambda}_d = 6.5y$$

where y is the flow depth and d_{50} is the median grain size.

17 → *Van Rijn (1984) equations for dune height and length:*

$$\eta_d = 0.11y\left(\frac{d_{50}}{y}\right)^{0.3}\left[1-\exp\left(-0.5\frac{\tau_0'-(\tau_0)_{cr}}{(\tau_0)_{cr}}\right)\right]\left(25-\frac{\tau_0'-(\tau_0)_{cr}}{(\tau_0)_{cr}}\right) \; ; \; \lambda_d = 7.3y$$

where d_{50} is median grain size, y is the flow depth, τ_0' is the bed shear due to particle roughness, and $(\tau_0)_{cr}$ is the threshold bed shear stress.

18 → *Watanabe (1989) equation for dune height:*

$$\eta_d = 2000y\left(1-\mathrm{Fr}^2\right)\left(\theta-\theta_{cr}\right)^{1.5}$$

where y is the flow depth, Fr is the Froude number, θ is the Shields parameter, and θ_{cr} is the critical Shields parameter.

Table 1 Values of coefficients W and B for use with the Koelzer and Lara equation

Reservoir operation	Sand		Silt		Clay	
	W_1 (kg/m^3)	B_1	W_2 (kg/m^3)	B_2	W_3 (kg/m^3)	B_3
Sediment always submerged or nearly submerged	1490	0	1040	91.3	480	256.3
Normally a moderate reservoir drawdown	1490	0	1185	43.3	737	171.4
Normally considerable reservoir drawdown	1490	0	1265	16.0	961	96.1
Reservoir normally empty	1490	0	1315	0	1250	0

Table 2 Bedform prediction scheme according to van Rijn (1993)

Flow regime	Transport stage parameter	Particle parameter and corresponding bedforms	
		$1 \le D_* \le 10$	$D_* > 10$
Lower	$0 < T_* \le 3$	Small ripples	Dunes
	$3 < T_* \le 10$	Large ripples and dunes	Dunes
	$10 < T_* \le 15$	Dunes	Dunes
Transition	$15 < T_* < 25$	Washed-out dunes and sand waves (asymmetrical)	
Upper	$T_* \ge 25$, $Fr < 0.8$	Sand waves (symmetrical)	
	$T_* \ge 25$, $Fr \ge 0.8$	Plane bed and/or antidunes	

Figure 1 Variation of P_C factor for use with the Lane and Kalinske (1941) formula.

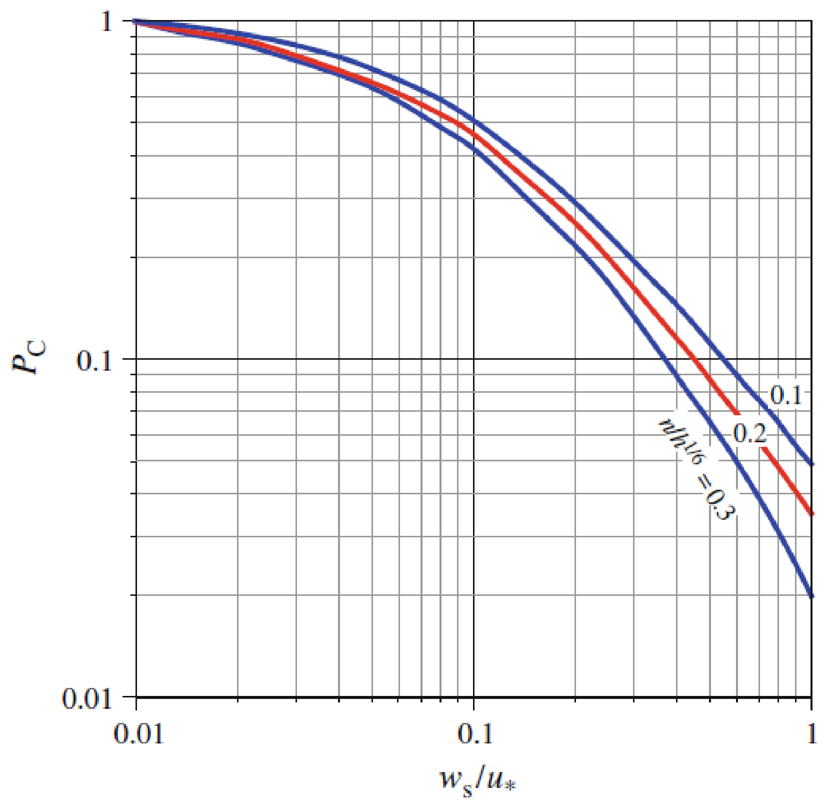


Figure 2 Variation of integral I_1 for use with the Einstein (1950) formula.

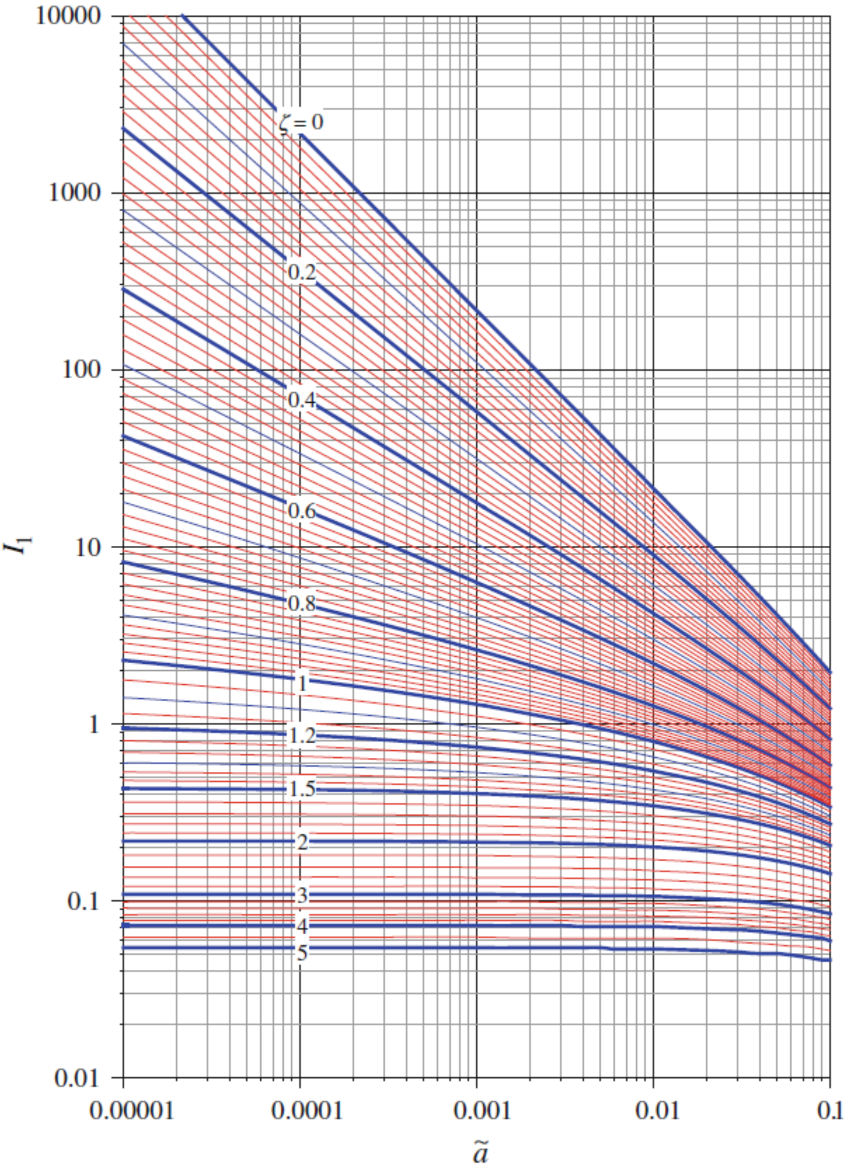


Figure 3 Variation of integral $-I_2$ for use with the Einstein (1950) formula.

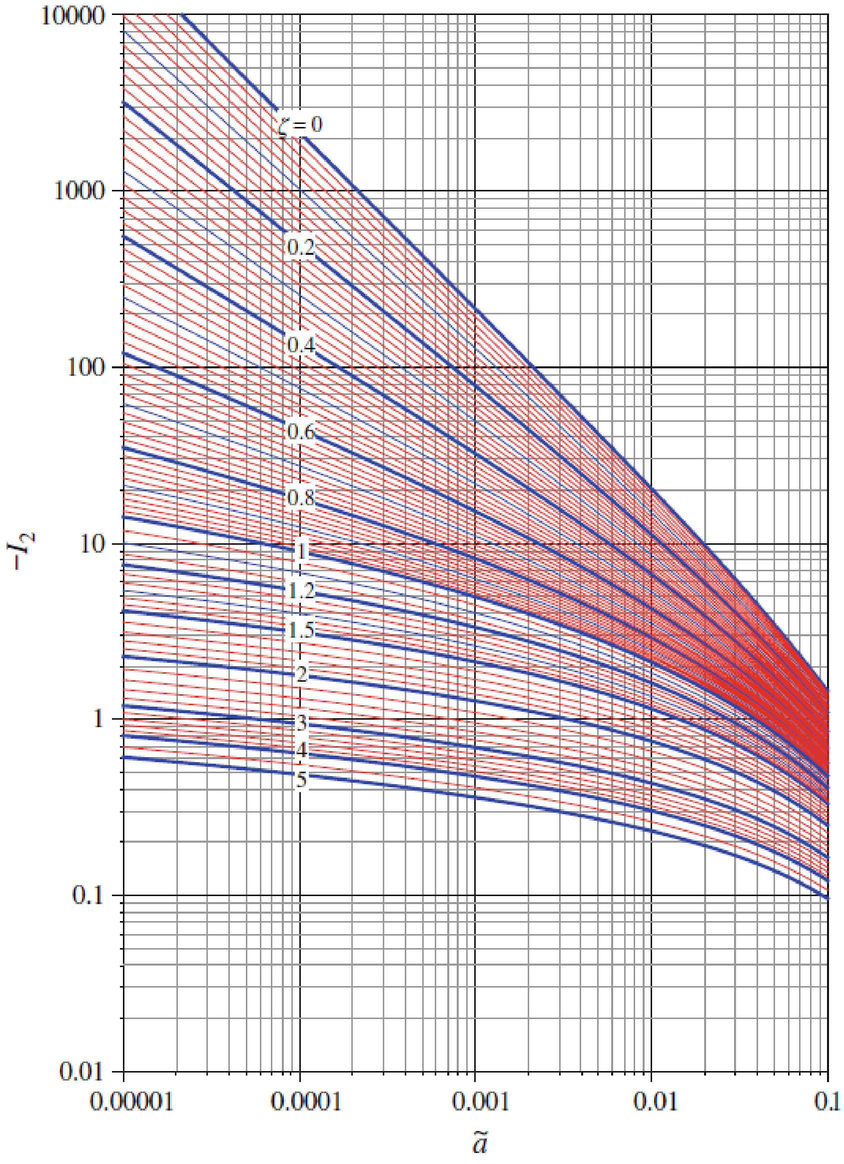


Figure 4 Variation of suspended load transport function for use with the Brooks (1963) formula.

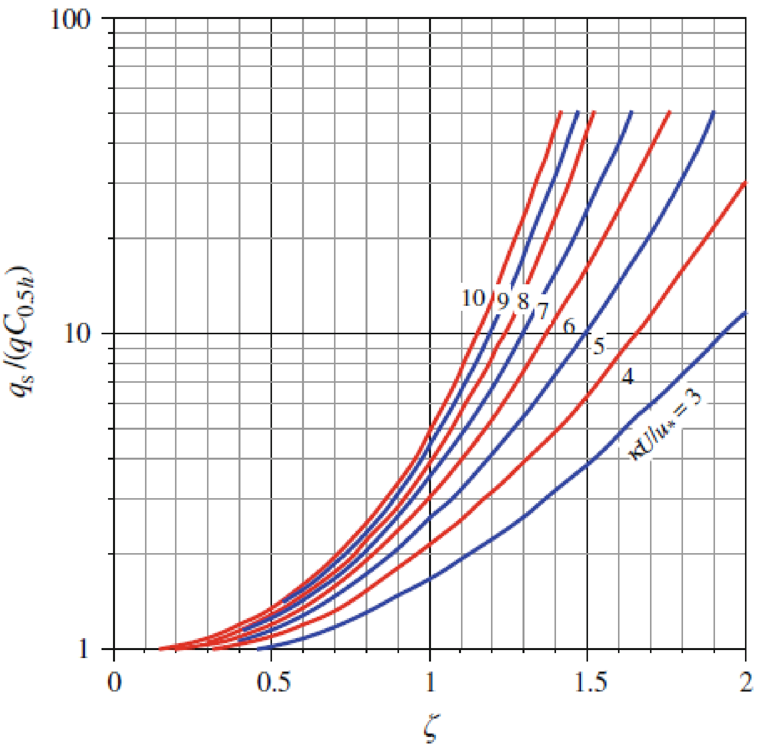


Figure 5 Variation of integral I_3 for use with the Chang et al. (1965) formula.

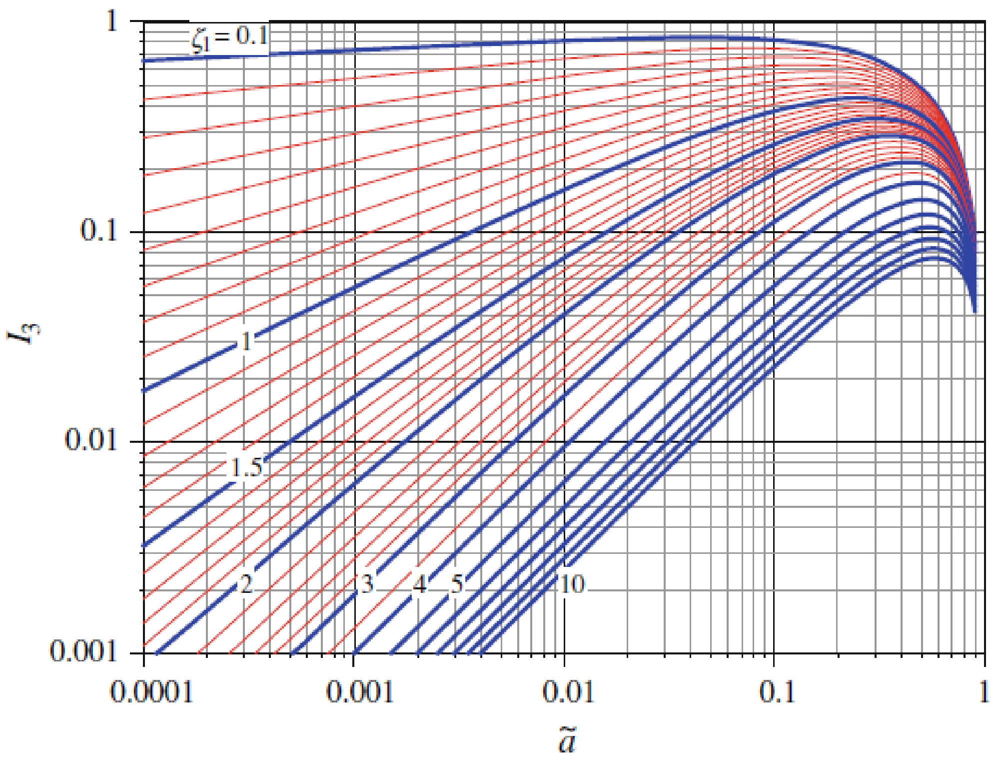


Figure 6 Variation of integral I_4 for use with the Chang et al. (1963) formula.

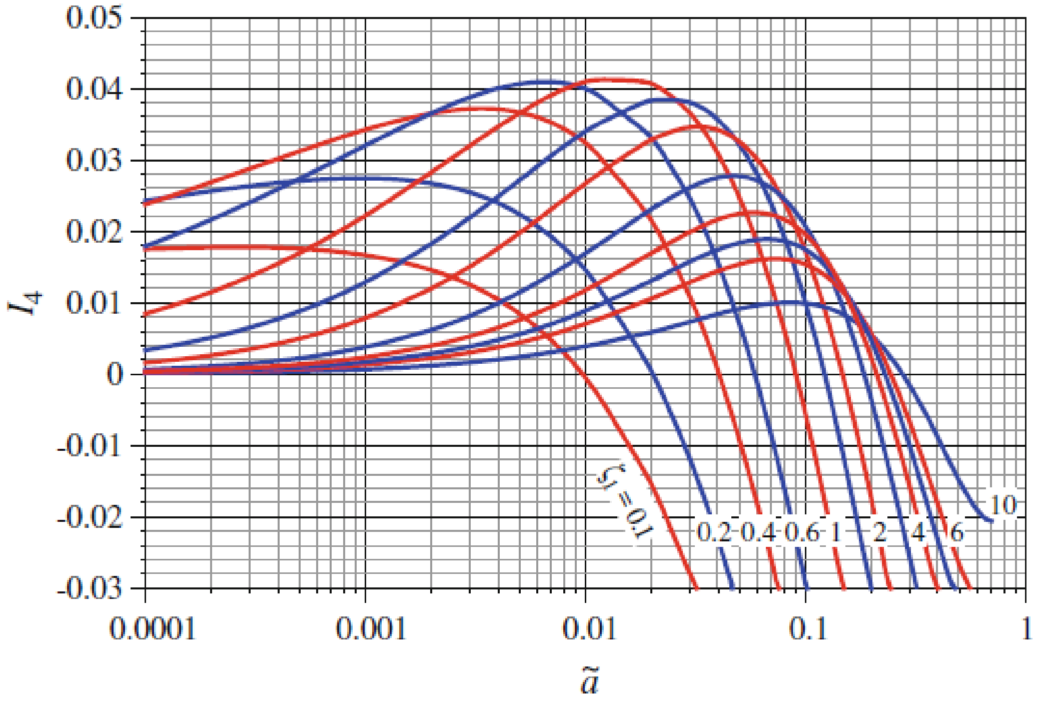


Figure 7 Bedform predictor as per Simons and Richarson (1961, 1966).

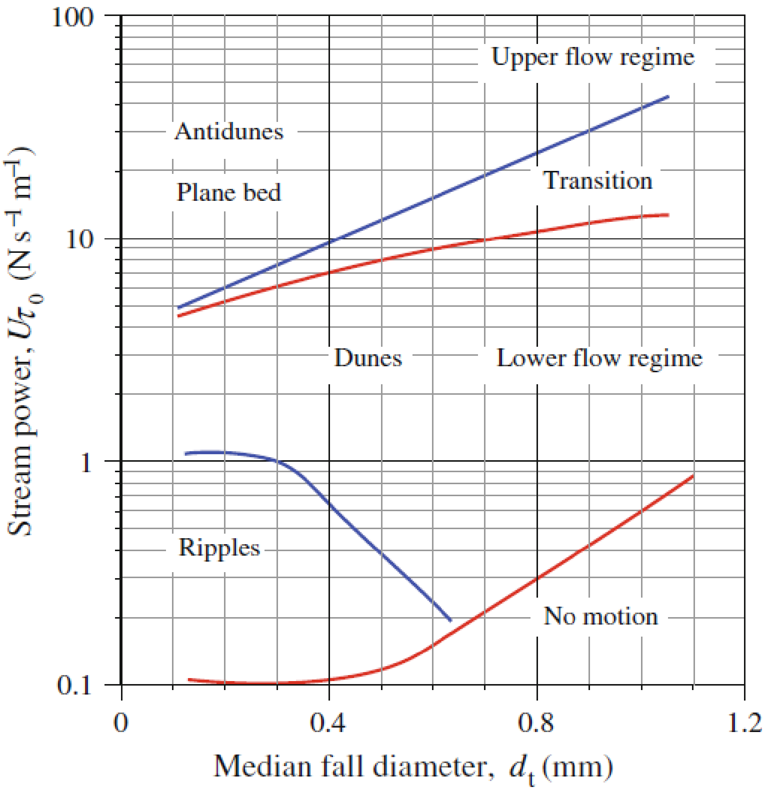
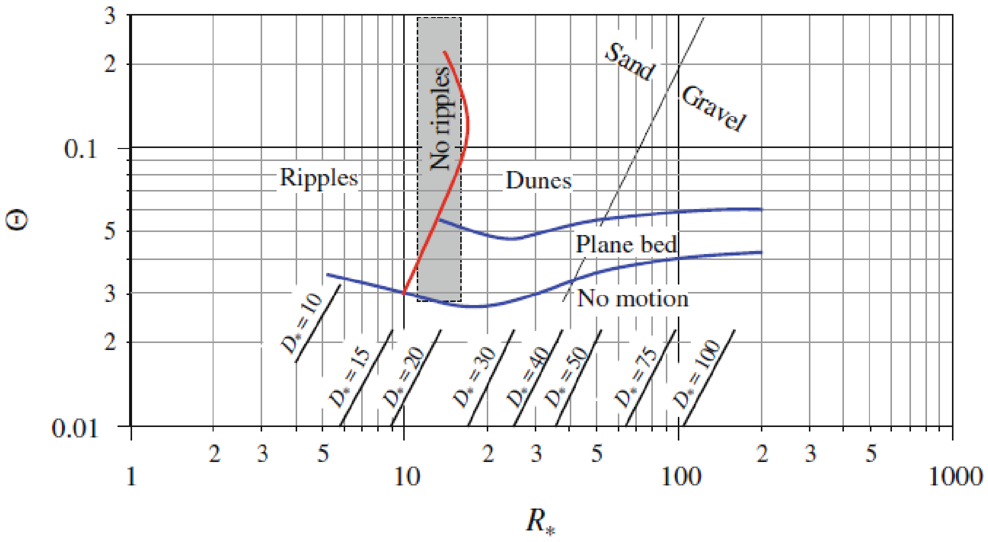


Figure 8 Bedform predictor as per Chabert and Chauvin (1963).



Solutions

P.1 ■ Solution

Since the reservoir will lose 30% of its initial capacity, the final capacity should be C_f = 200 × 0.35 = 70 Mm³, signifying a loss of 15 Mm³. This loss is processed in 3 steps of equal capacity loss, each with ΔC = 5.0 Mm³. The calculations are tabulated below.

1	2	3	4	5	6	7
Capacity C (Mm³)	ΔC (Mm³)	*C/I*	Trap Eff. (%)	Average TE (%)	Vol. of Sediment Deposited per Year (Mm³/year)	Time to fill ΔC (years)
50		0.714	100			
45	5	0.643	98.80	99.40	0.1491	33.53
40	5	0.571	98.09	98.44	0.1477	33.86
35	5	0.500	97.28	97.68	0.1465	34.12
					Sum =	**101.5**

Column 1 is the capacity of the reservoir in each time step. Column 2 is the variation in capacity from one time step to the next. Column 3 is $C/I = C/70$ is the capacity-inflow ratio. Column 4 is the trap efficiency η_f, a percentage given by

$$\eta_t = 6.064 \ln\left(\frac{C}{I}\right) + 101.48 \; ; \; 0.10 < \frac{C}{I} < 0.70$$

or simply η_f = 100 if C/I > 0.70. Column 5 is the average trap efficiency between two consecutive time steps. Column 6 is the volume of sediment deposited in the reservoir per year, given by 0.15 (annual sediment inflow) × avg. trap efficiency (column 5) ÷ 100. Column 7 is the time in years required to fill the capacity represented by the step; it is given by the ratio of column 2, the variation in capacity for the time step, to column 6, the rate of sediment deposition per year. The quantity we are looking for, the time required for the loss of 30% of the initial capacity of the reservoir, is the sum of the values in column 7. Accordingly, t_{30} = 101.5 ≈ 102 years.

★ The correct answer is **D**.

P.2 ■ Solution

A 50% reduction in volume corresponds to a loss of 0.50 × 90,000 = 45,000 ha-m. Since the reduction in volume is to be processed in five time steps, each step brings a loss of 45,000/5 = 9000 ha-m of sediment. The calculations are summarized below.

1	2	3	4	5	6	7
Capacity C (ha-m)	ΔC (ha-m)	*C/I*	Trap Eff. (%)	Average TE (%)	Vol. of Sediment Deposited per Year (ha-m/year)	Time to fill ΔC (years)
90000		0.225	92.43			
81000	9000	0.203	91.80	92.12	552.7	16.28
72000	9000	0.180	91.08	91.44	548.6	16.40
63000	9000	0.158	90.27	90.68	544.1	16.54
54000	9000	0.135	89.34	89.80	538.8	16.70
45000	9000	0.113	88.23	88.78	532.7	16.89
					Sum =	**82.8**

The ratio in column 3 is $C/I = C/600$. As recommended by the guidelines of the problem, the trap efficiency in column 4 is calculated with the usual formula $\eta_t = 6.064 \ln(C/I) + 101.48$. The average η_f in the two adjacent time steps is computed in column 5. The volume of sediment deposited per year is 600 (the annual sediment load) × col. 5 (the average η_t) ÷ 100. Finally, the time required to fill the variation in capacity ΔC is obtained if we divide col. 2 by col. 6. The quantity we aim for, the time required for the loss of 50% of the initial capacity of the reservoir, is the sum of the values in column 7. That is, t_{50} = 82.8 ≈ 83 years.

★ The correct answer is **C**.

P.3 ■ Solution

The water yield is 2660 × 0.40 = 1064 Mm^3/year and the total sediment inflow per year is 360 × 2660 = 0.9576 Mt/year. As an initial trial, consider a dry unit weight of 1.1 t/m^3. The corresponding volume of sediment inflow per year is then 0.9576/1.1 = 0.871 Mm^3/year. Taking the trap efficiency to be equal to unity, the approximate time for 40% loss of capacity is 0.40 × 360/0.871 = 165 years. Referring to the coefficients given in Table 1 for a moderate reservoir drawdown, the initial unit weight W_1 of the sediment is approximated as

$$W_1 = 1490 \times 0.20 + 1185 \times 0.35 + 737 \times 0.45 = 1040 \text{ kg/m}^3 = 1.04 \text{ tonnes/m}^3$$

The weighted B_w coefficient follows as

$$B_w = \frac{p_{\text{sand}} \times B_1 + p_{\text{silt}} \times B_2 + p_{\text{clay}} \times B_3}{100} = 0 + 0.35 \times 43.3 + 0.45 \times 171.4 = 92.3$$

The average unit weight of deposit material for a period of 165 years is then

$$\overline{W} = W_1 + 0.4343 B_w \left[\left(\frac{T}{T-1} \right) \ln T - 1 \right]$$

$$\therefore \overline{W} = 1.04 + 0.4343 \times 92.3 \times \left[\left(\frac{165}{165-1} \right) \ln(165) - 1 \right] = 1205.8 \text{ kg/m}^3 = 1.206 \text{ tonnes/m}^3$$

The calculations for estimating the time to fill 40 percent (0.4 × 360 = 144 Mm^3) of the reservoir capacity are tabulated below.

1	2	3	4	5	6	7
Capacity C (Mm^3)	ΔC (Mm^3)	*C/I*	Trap Eff. (%)	Average TE (%)	Vol. of Sediment Deposited per Year (Mm^3/year)	Time to fill ΔC (years)
360		0.338	94.90			
320	40	0.301	94.20	94.55	0.748	53.50
280	30	0.273	93.61	93.90	0.743	40.40
240	25	0.249	93.05	93.33	0.738	33.88
200	25	0.226	92.46	92.76	0.733	34.08
160	24	0.203	91.81	92.14	0.729	32.94
					Sum =	**194.8**

The time required to fill the 144 Mm^3 capacity of the reservoir has been determined to be t_{40} = 194.8 ≈ 195 years. Since this differs from the assumed value of 165 years, another trial is in order. Let t = 195 years be the refined time value. The updated average unit weight $\overline{W}$ is

$$\overline{W} = 1040 + 0.4343 \times 92.3 \times \left[\left(\frac{195}{195-1} \right) \ln(195) - 1 \right] = 1212.4 \text{ kg/m}^3 = 1.212 \text{ tonnes/m}^3$$

The new calculations are summarized below.

1	2	3	4	5	6	7
Capacity C (Mm^3)	ΔC (Mm^3)	*C/I*	Trap Eff. (%)	Average TE (%)	Vol. of Sediment Deposited per Year (Mm^3/year)	Time to fill ΔC (years)
360		0.338	94.90			
320	40	0.301	94.20	94.55	0.745	53.72
280	30	0.273	93.61	93.90	0.739	40.57
240	25	0.249	93.05	93.33	0.735	34.02
200	25	0.226	92.46	92.76	0.730	34.23
160	24	0.203	91.81	92.14	0.726	33.08
					Sum =	**195.6**

The time required to fill the reservoir up to specified limit is now t_{40} = 195.6 years. Since this is less than 5 percent away from our assumed value of t, we can take this value as our solution. Accordingly, the time required for 40% of the reservoir capacity to be lost to sedimentation is t_{40} = 195 years.

★ The correct answer is **B**.

P.4 ■ Solution

The first step is to determine the normal depth in the reach. For the combination of parameters in the case at hand, the normal depth is y_0 = 0.52 m and the hydraulic radius is R = 0.454 m. The average shear stress across the channel cross-section is calculated as

$$\tau_0 = \gamma R S_0 = 9800 \times 0.454 \times 0.003 = 13.3 \text{ Pa}$$

Both the Reynolds number and the Shields parameter depend on the particle diameter, d. Use d = 2 mm as an initial assumption. The particle Reynolds number is

$$\text{Re}_* = \frac{d}{\mu}\sqrt{\rho \tau_0} = \frac{0.001}{1.307 \times 10^{-3}} \times \sqrt{1000 \times 13.3} = 88.2$$

The corresponding Shields parameter is approximated as

$$\theta = 0.118 \times 88.2^{-0.979} + 0.056 \times \exp\left(-5.31 \times 88.2^{-0.679}\right) = 0.0449$$

However, from the definition of θ, we have

$$\theta = \frac{\tau_0}{\gamma(G_s-1)d} = \frac{13.3}{9800\times(2.65-1)\times 0.001} = 0.823$$

Since there is a mismatch between the two calculated values of θ, the assumed particle diameter is incorrect. In a second attempt, let d = 10 mm. In this case, the particle Reynolds number is Re = 882, the Shields parameter calculated from the correlation is 0.0533, and the Shields parameter calculated from the definition is 0.0823. The gap between the two computed values of θ has narrowed, but the solution is still not correct. Proceeding similarly in additional trials, we ultimately obtain the solution d = 15.3 mm. Further, the particle Reynolds number is Re_* = 1350 and the Shields parameter is θ = 0.0538. It should be noted that the Shields parameter approaches a constant of θ = 0.056 for large Reynolds numbers, and the actual value calculated in this problem, 0.0538, is only slightly less than this. The flow conditions in this problem are in the turbulent region of the Shields diagram.

★ The correct answer is **C**.

P.5 ■ Solution

As a sample calculation, consider the size fraction of 2 to 4 mm. The geometric mean of this size fraction is

$$d_{2.0\to4.0} = \sqrt{0.002\times0.004} = 0.00283 \text{ m}$$

As established in the previous problem, the average shear stress was 13.3 Pa. Accordingly, the Shields parameter is

$$\theta = \frac{\tau_0}{\gamma(G_s-1)d_{2.0\to4.0}} = \frac{13.3}{9800\times(2.65-1)\times0.00283} = 0.291$$

The dimensionless bed load transport rate in the Einstein-Brown approach, Φ_g, is given by

$$\Phi_g = \frac{g_b}{\gamma G_s d_{50} w_s}$$

where g_b is the unit weight transport rate per unit time and w_s is the terminal fall velocity. The value of Φ depends on the Shields parameter being used and, in the Einstein-Brown approach, can be approximated as

$$\Phi_g = 0 \;\; ; \;\; \theta \le 0.056$$
$$\Phi_g = 0.425\sqrt{\theta} - 0.1 \;\; ; \;\; 0.056 \le \theta \le 0.08$$
$$\Phi_g = 40\theta^3 \;\; ; \;\; \theta > 0.08$$

In the ongoing problem, we have

$$\Phi_g = 40\times0.291^3 = 0.986$$

The fall velocity, in turn, can be estimated with the Rubey formula,

$$w = F\times\sqrt{g(G_s-1)d}$$

where F is a factor given by

$$F = \sqrt{\frac{2}{3}+\frac{36\nu^2}{g(G_s-1)d^3}} - \sqrt{\frac{36\nu^2}{g(G_s-1)d^3}}$$

In the case at hand, F is such that

$$F = \sqrt{\frac{2}{3}+36\times\frac{(1.006\times10^{-6})^2}{9.81\times(2.65-1)\times0.00283^3}} - \sqrt{\frac{36\times(1.006\times10^{-6})^2}{9.81\times(2.65-1)\times0.00283^3}} = 0.807$$

so

$$w = 0.807 \times \sqrt{9.81 \times (2.65 - 1) \times 0.00283} = 0.173 \text{ m/s}$$

The value of g_b is determined next,

$$\Phi = \frac{g_b}{\gamma G_s dw} \rightarrow g_b = \Phi \gamma G_s dw$$

$$\therefore g_b = 0.986 \times 9800 \times 2.65 \times 0.00283 \times 0.173 = 12.5 \text{ N/sm}$$

For the 4 mm sieve opening, 25% passes, while, for 2 mm, 18% passes. Consequently, there is 7% of the overall material that resides in this size fraction, so the incremental bed load is

$$p_i \times g_{b,i} = 0.07 \times 12.5 = 0.875 \text{ N/m} \cdot \text{s}$$

For an 8-m wide channel, the bed load transport rate in this size fraction is

$$p_i \times g_{b,i} \times b = 1.63 \times 8 = 13.0 \text{ N/s}$$

The calculations are summarized in the following table.

1	2	3	4	5	6	7	8
Size Range (mm)	Fraction (%)	d_i (m)	1/ψ	Φ	w (m/s)	$g_{b,i}$ [N/(m-s)]	$p_i g_{b,i}$ [N/(m-s)]
13.2 → 16.0	20	0.0145	0.0566	0.00111	0.395	0.000165	0.0830
11.2 → 13.2	10	0.0122	0.0676	0.0105	0.362	1.20	0.108
8.0 → 11.2	16	0.00947	0.0869	0.0262	0.319	2.06	0.329
6.3 → 8.0	13	0.0071	0.116	0.0624	0.276	3.18	0.413
4.75 → 6.3	8	0.00547	0.150	0.1350	0.242	4.64	0.371
4.0 → 4.75	8	0.00436	0.189	0.270	0.215	6.57	0.526
2.0 → 4.0	7	0.00283	0.291	0.986	0.172	12.5	0.875
1.0 → 2.0	6	0.00141	0.583	7.93	0.118	34.2	2.05
< 1 → 1.0	4	0.0005	1.65	180	0.0594	138.6	5.54
						Sum =	**10.3**

Finally, the overall sediment transport, g_s, is simply the product of the unit bed load, calculated in column 8 above, and the channel width of 8 m; that is,

$$g_s = \Sigma\left(p_i g_{b,i}\right) \times b = 10.3 \times 8 = \boxed{82.4 \text{ N/s}}$$

★ The correct answer is **A**.

P.6 ■ Solution

Part A: Sediment transport begins when $\tau_0 > \tau_{cr}$ = 13.5 Pa. Invoking the equation for bed shear, we have

$$\tau_0 = \gamma R S_0 \rightarrow R = \frac{\tau_0}{\gamma S_0}$$

$$\therefore R = \frac{13.5}{9800 \times 0.0025} = 0.551 \text{ m}$$

For a trapezoidal channel,

$$R = \frac{(b + my)y}{b + 2y\sqrt{1 + m^2}} = \frac{(7.5 + 3 \times y)y}{7.5 + 2 \times y\sqrt{1 + 3^2}} = 0.551$$

$$\therefore y = 0.682 \text{ m}$$

Then, the cross-sectional area is A = (7.5 + 3×0.682) × 0.682 = 6.51 m^2. The corresponding discharge can be obtained from the Manning equation,

$$q_{\min} = \frac{1}{n} A R^{2/3} S^{1/2} = \frac{1}{0.064} \times 6.51 \times 0.551^{2/3} \times 0.0025^{1/2} = \boxed{3.42 \text{ m}^3\text{/s}}$$

Thus, there will be no sediment transport if the discharge is below 3.42 m^3/s.

★ The correct answer is **B**.

Part B: Consider, for instance, the fourth time step (3 → 4 h). Here, we have a discharge of 10 m^3/s, which corresponds to a normal depth of 1.23 m and a hydraulic radius of 0.90 m. The average shear stress is

$$\tau_0 = \gamma R S_0 = 9800 \times 0.90 \times 0.0025 = 22.1 \text{ Pa}$$

Because this shear stress exceeds the critical shear stress of 13.5 N/m^2, we conclude that there is sediment transport during hour 3. That transport rate is

$$g_s = 0.0019 \times 10^{1.2} \times (22.1 - 13.5) = 0.259 \text{ N/s}$$

The sediment load for the entire 1-hour period is then

$$L = g_s \times \Delta t = 0.259 \times 3600 = 932 \text{ N} = 0.932 \text{ kN}$$

The remaining calculations are summarized in the table below.

1	2	3	4	5	6	7
Hour	q (m^3/s)	y_0 (m)	R (m)	τ (N/m^2)	g_s (N/s)	Load (kN)
0 → 1	1	0.337	0.298	7.3	0	0
1 → 2	2	0.503	0.424	12.3	0	0
2 → 3	4.5	0.795	0.627	15.4	0.0219	0.0788
3 → 4	10	1.23	0.9	22.1	0.259	0.932
4 → 5	15	1.52	1.07	26.2	0.622	2.24
5 → 6	25	1.97	1.33	32.6	1.73	6.23
6 → 7	22	1.85	1.26	30.9	1.35	4.86
7 → 8	18.5	1.67	1.16	28.4	0.939	3.38
8 → 9	9.5	1.19	0.877	21.5	0.227	0.817
9 → 10	3	0.634	0.518	12.7	0	0
					Sum =	**18.5**

The cumulative sediment load for the flood and sediment rating curve is the sum of the values in column 7, or L = 18.5 kN.

★ The correct answer is **A**.

P.7 ■ Solution

Part A: One of the formulas proposed by van Rijn to estimate the (dimensionless) bed load transport rate is

$$\Phi_b = \frac{0.053}{D_*^{0.3}}\left(\frac{\theta}{\theta_{cr}} - 1\right)^{2.1}$$

For the channel under consideration, the bed shear stress is

$$\tau_0 = \gamma y S_0 = 9800 \times 2.8 \times 0.001 = 27.4 \text{ Pa}$$

The Shields parameter is

$$\theta = \frac{\tau_0}{\gamma (G_s - 1) d_{50}} = \frac{27.4}{9800 \times (2.65 - 1) \times 0.0015} = 1.13$$

In order to compute the critical Shields parameter, we must first determine D_*,

$$D_* = \left[\frac{(G_s - 1) g}{\nu^2}\right]^{1/3} d_{50} = \left[\frac{(2.65 - 1) \times 9.81}{(1.006 \times 10^{-6})^2}\right]^{1/3} \times 0.0015 = 37.8$$

The formula to use for θ_{cr}, then, is

$$\theta_{cr}(20 < D_* \le 150) = 0.013 D_*^{0.29} = 0.013 \times 37.8^{0.29} = 0.0373$$

and difference $\theta/\theta_{cr} - 1 = 29.3$. Inserting our data into the van Rijn equation gives

$$\Phi_b = \frac{0.053}{37.8^{0.3}} \times 29.3^{2.1} = 21.5$$

Resorting to the definition of Φ_b, we obtain

$$\Phi_b = \frac{q_b}{\sqrt{g(G_s - 1)d_{50}^3}} \rightarrow q_b = \Phi_b \sqrt{g(G_s - 1)d_{50}^3}$$

$$\therefore q_b = 21.5 \times \sqrt{9.81 \times (2.65 - 1) \times 0.0015^3} = 0.00503 \text{ m}^2/\text{s}$$

Finally, for a channel width of 45 m,

$$q_s = q_b \times b = 0.00503 \times 45 = \boxed{0.226 \text{ m}^3/\text{s}}$$

★ The correct answer is **B**.

Part B: The dimensionless form of the Yalin equation for bed load transport is

$$\Phi_b = 0.635\theta^{0.5}\left(\frac{\theta}{\theta_{\text{cr}}} - 1\right)\left\{1 - \frac{\theta_{\text{cr}}}{a_1(\theta - \theta_{\text{cr}})}\ln\left[1 + a_1\left(\frac{\theta}{\theta_{\text{cr}}} - 1\right)\right]\right\}$$

where

$$a_1 = \frac{2.45\theta_{\text{cr}}^{0.5}}{G_s^{0.4}}$$

In the present case,

$$a_1 = \frac{2.45\theta_{\text{cr}}^{0.5}}{G_s^{0.4}} = \frac{2.45 \times 0.0373^{0.5}}{2.65^{0.4}} = 0.320$$

Backsubstituting into the equation for Φ_b gives

$$\Phi_b = 0.635 \times 1.13^{0.5} \times 29.3 \times \left[1 - \frac{1}{0.320 \times 29.3}\ln(1 + 0.320 \times 29.3)\right] = 14.8$$

Accordingly,

$$\Phi_b = \frac{q_b}{\sqrt{g(G_s - 1)d_{50}^3}} \rightarrow q_b = \Phi_b \sqrt{g(G_s - 1)d_{50}^3}$$

$$\therefore q_b = 14.8 \times \sqrt{9.81 \times (2.65 - 1) \times 0.0015^3} = 0.00346 \text{ m}^2/\text{s}$$

and, for a bottom width of 45 m,

$$q_s = q_b \times b = 0.00346 \times 45 = \boxed{0.156 \text{ m}^3/\text{s}}$$

★ The correct answer is **B**.

Part C: In the Engelund & Fredsøe approach, the dimensionless bed load transport rate is given by

$$\Phi_b = \frac{9.3}{\mu_d}(\theta - \theta_{\text{cr}})(\theta^{0.5} - 0.7\theta_{\text{cr}}^{0.5})$$

where μ_d is the dynamic coefficient of friction of the bed particles. Here, μ_d = $\tan 20^o$ = 0.364. Substituting our data in the formula above gives

$$\Phi_b = \frac{9.3}{0.364}(1.13 - 0.0373)(1.13^{0.5} - 0.7 \times 0.0373^{0.5}) = 25.9$$

Note that, for $\theta \gg \theta_{\text{cr}}$, this expression can be simplified as

$$\Phi_b \approx \frac{9.3\theta^{1.5}}{\mu_d}$$

In the ongoing problem, $\theta/\theta_{\text{cr}} \approx 30$ and, if we had recourse to the approximation above, the result would be

$$\Phi_b \approx \frac{9.3\theta^{1.5}}{\mu_d} = \frac{9.3 \times 1.13^{1.5}}{0.364} = 30.7$$

This is 18.5% greater than the result from the original formula. For the sake of precision, let's maintain the original result of 25.9. Thus,

$$\Phi_b = \frac{q_b}{\sqrt{g(G_s-1)d_{50}^3}} \rightarrow q_b = \Phi_b \sqrt{g(G_s-1)d_{50}^3}$$

$$\therefore q_b = 25.9 \times \sqrt{9.81 \times (2.65-1) \times 0.0015^3} = 0.00605 \text{ m}^2/\text{s}$$

A bottom width of 45 m brings to

$$q_s = q_b \times b = 0.00605 \times 45 = \boxed{0.272 \text{ m}^3/\text{s}}$$

★ The correct answer is **A**.

Part D: **1. True.** The saltation length in van Rijn's formulation is given by

$$\frac{\lambda_b}{d_{50}} = 3D_*^{0.6}\left(\frac{\theta}{\theta_{cr}}-1\right)^{0.9} \rightarrow \lambda_b = 0.0015 \times 3 \times 37.8^{0.6} \times 29.3^{0.9} = 0.832 \text{ m}$$

2. False. The saltation height, as per the van Rijn approach, is given by

$$\frac{h_s}{d_{50}} = 0.3D_*^{0.7}\left(\frac{\theta}{\theta_{cr}}-1\right)^{0.5} \rightarrow h_s = 0.0015 \times 0.3 \times 37.8^{0.7} \times 29.3^{0.5} = 0.031 \text{ m}$$

3. True. For a saltating particle, van Rijn estimated the mean particle velocity as a function of nondimensional particle diameter and Shields parameter as

$$\frac{\bar{u}_b}{u_*} = 9 + 2.6\log D_* - 8\left(\frac{\theta_{cr}}{\theta}\right)^{0.5}$$

With $u_* = \sqrt{27.4/1000}$ = 0.166 m/s, we get

$$\bar{u}_b = 0.166 \times \left[9 + 2.6\log(37.8) - 8 \times \left(\frac{0.0373}{1.13}\right)^{0.5}\right] = 1.93 \text{ m/s}$$

It is worth noting that van Rijn also proposed an approximation to the relation above,

$$\frac{\bar{u}_b}{\sqrt{(G_s-1)gd_{50}}} = 1.5\left(\frac{\theta}{\theta_{cr}}-1\right)^{0.6}$$

Applying this formula leads to

$$\bar{u}_b = \sqrt{(2.65-1) \times 9.81 \times 0.0015} \times 1.5 \times 29.3^{0.6} = 1.77 \text{ m/s}$$

There is a 9% disparity between the two results, but both of them are nevertheless greater than 1.2 m/s.

P.8 ■ Solution

Part A: The Rouse number is given by

$$\zeta = \frac{w_s}{\beta \kappa u_*}$$

where w_s is the terminal fall velocity, u_* is the shear velocity, and β and κ are constants. The former is often taken as unity, and the latter, κ = 0.41, is the Von Kármán constant. As was done in Problem 5, the value of w_s can be estimated with the Rubey formula

$$w_s = F \times \sqrt{g(G_s-1)d}$$

where

$$F = \sqrt{\frac{2}{3} + \frac{36\nu^2}{g(G_s-1)d^3}} - \sqrt{\frac{36\nu^2}{g(G_s-1)d^3}}$$

Inserting the data for the ongoing problem, we obtain F = 0.675 and w_s = 0.0576 m/s. The value of the shear velocity u_*, in turn, is calculated according to

$$u_* = \sqrt{\frac{\tau_0}{\rho}}$$

where τ_0 is the bed shear stress, given by

$$\tau_0 = \gamma y S_0 = 9800 \times 1.5 \times 0.0026 = 38.2 \text{ Pa}$$

and ρ = 1000 kg/m^3 as given. Accordingly,

$$u_* = \sqrt{\frac{38.2}{1000}} = 0.195 \text{ m/s}$$

We are now in position to compute ζ,

$$\zeta = \frac{0.0576}{1.0 \times 0.41 \times 0.195} = \boxed{0.720}$$

★ The correct answer is **C**.

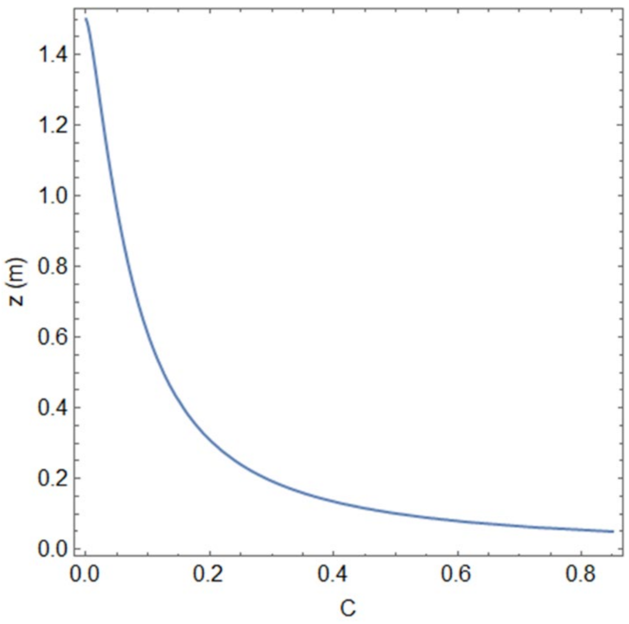
Part B: The sediment concentration distribution is described by the relation

$$C(z) = C_0 \left(\frac{y-z}{z} \times \frac{a}{y-a} \right)^{\zeta}$$

where C_0 is the reference concentration, y is the flow depth, z is the vertical distance from the bed, a is the reference depth, and ζ is the Rouse number. Substituting the available data gives

$$C(z) = 0.85 \times \left(\frac{1.5-z}{z} \times \frac{0.05}{1.5-0.05} \right)^{0.720} = 0.0752 \left(\frac{1.5-z}{z} \right)^{0.720}$$

The sediment concentration distribution curve is plotted below.



P.9 ■ Solution

Part A: The bed shear stress is

$$\tau_0 = \gamma y S_0 = 9800 \times 2.8 \times 0.001 = 27.4 \text{ Pa}$$

The shear velocity is

$$u_* = \sqrt{\frac{\tau_0}{\rho}} = \sqrt{\frac{27.4}{1000}} = 0.166 \text{ m/s}$$

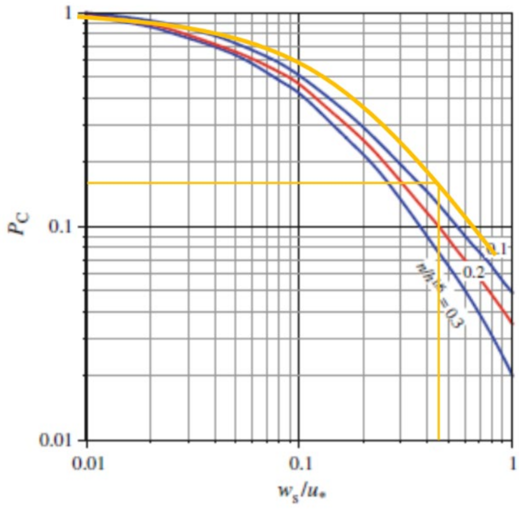
The Rouse number follows as

$$\zeta = \frac{w_s}{\kappa u_*} = \frac{0.075}{0.41 \times 0.166} = 1.10$$

In the Lane and Kalinske approach, the suspended-load transport rate is given by

$$q_b = qC_aP_C\exp\left(\frac{15w_sa}{u_*y}\right)$$

where q is the discharge, C_a is the reference concentration, P_C is a function of $n/y^{1/6}$ and w_s/u_* given by Figure 1, w_s is the terminal fall velocity, a is the reference depth, u_* is the shear velocity, and y is the flow depth. Note that, in the ratio $n/y^{1/6}$, the depth y is in inches. Here, y = 2.8 m = 110 in. Thus, $n/y^{1/6}$ = 0.15/110$^{1/6}$ = 0.0685. Since there is no curve for this particular $n/y^{1/6}$ ratio in Figure 1, we draw an additional line as an extrapolation, as shown in yellow below.



Entering a ratio w_s/u_* = 0.075/0.166 = 0.45 into this graph, we find that P_C = 0.17. We can then compute the factor

$$\frac{15w_sa}{u_*y} = \frac{15\times0.075\times0.3}{0.166\times2.8} = 0.726$$

The converted unit discharge is q = 12/0.3048^2 = 129 ft^2/s. The suspended-load transport rate follows as

$$q_b = 129\times10^{-3}\times0.17\times\exp(0.726) = 0.0453 \text{ ft}^2/\text{s}$$
$$\therefore q_b = 4.21\times10^{-3} \text{ m}^2/\text{s}$$

The suspended-load transport rate in weight per unit time and width is then

$$g_b = q_bG_s\gamma = (4.21\times10^{-3})\times2.65\times9800 = 109 \text{ N/sm}$$

or, multiplying by the width of the channel,

$$g_s = g_b\times b = 109\times45 = \boxed{4900 \text{ N/s}}$$

★ The correct answer is **C**.

Part B: In the Einstein formulation, the suspended-load transport rate is given by

$$q_b = \frac{C_au_*'a}{\kappa}(P_EJ_1 + J_2)$$

where C_a is the reference concentration, u_*' is the shear velocity due to particle roughness, a is the reference depth, κ = 0.41 is the Von Kármán constant, and P_E is a factor given by

$$P_E = \ln\left(\frac{30.2y}{\Delta_k}\right)$$

where Δ_k is the apparent roughness. Finally, J_1 and J_2 are factors derived from the so-called Einstein integrals, I_1 and I_2, in accordance with the relations

$$J_1 = \frac{I_1}{0.216}\ ;\ J_2 = \frac{I_2}{0.216}$$

Terms I_1 and I_2 can be obtained from Figures 2 and 3 in terms of $\tilde{a}$ for different values of ζ. Referring to these charts with $\tilde{a} = a/y$ = 10^-3^/2.8 = 3.57 × 10^-4^ and ζ = 1.1, we read I_1 = 1.0 and I_2 = −6.0. Factors J_1 and J_2 are determined next,

$$J_1 = \frac{I_1}{0.216} = \frac{1.0}{0.216} = 4.63$$

$$J_2 = \frac{I_2}{0.216} = -\frac{6.0}{0.216} = -27.8$$

We also require P_E, which is given by

$$P_E = \ln\left(\frac{30.2\times 2.8}{10^{-4}}\right) = 13.6$$

As an approximation, we let the shear velocity due to particle roughness be equal to u_* = 0.166 m/s. Inserting our data in the equation for q_b gives

$$q_b = \frac{0.40\times 0.166\times 10^{-3}}{0.41}\times\left(13.6\times 4.63 - 27.8\right) = 0.00570\text{ m}^2/\text{s}$$

It remains to convert this quantity to a weight transport rate,

$$g_b = q_b G_s \gamma = \left(6.32\times 10^{-4}\right)\times 2.65\times 9800 = 148\text{ N/sm}$$

$$\therefore g_s = g_b\times b = 148\times 45 = \boxed{6660\text{ N/s}}$$

★ The correct answer is **B**.

Part C: In the Brooks approach, the suspended-load transport rate is given by

$$\frac{q_b}{qC_{0.5y}} = T_B\left(\frac{\kappa U}{u_*},\zeta,\tilde{a}\right)$$

where q is the flow rate, $C_{0.5y}$ is the concentration at one-half of the flow depth, and T_B is a factor extracted from Figure 4. In the present case, the average flow velocity is U = 12/2.8 = 4.29 m/s and $\kappa U/u_*$ is such that

$$\frac{\kappa U}{u_*} = \frac{0.41\times 4.29}{0.166} = 10.6$$

Entering this quantity and ζ = 1.10 into Figure 4, we read $q_b/qC_{0.5y}$ = 7. With $\tilde{a} = a/y$ = 4×10^-4^/2.8 = 1.43×10^-4^, mid-depth concentration $C_{0.5y}$ is determined next,

$$C_{0.5y} = 0.4\times\left(\frac{1-\tilde{z}}{\tilde{z}}\cdot\frac{\tilde{a}}{1-\tilde{a}}\right)^{\zeta} \rightarrow C_{0.5y} = 0.4\times\left[\frac{1-0.5}{0.5}\times\frac{\left(1.43\times 10^{-4}\right)}{1-\left(1.43\times 10^{-4}\right)}\right]^{1.10} = 2.36\times 10^{-5}$$

The suspended-load transport rate q_b is calculated as

$$\frac{q_b}{qC_{0.5y}} = 7 \rightarrow q_b = 7\times 12\times\left(2.36\times 10^{-5}\right) = 1.98\times 10^{-3}\text{ m}^2/\text{s}$$

Lastly, performing the necessary conversions, we obtain

$$g_b = q_b G_s \gamma = \left(1.93\times 10^{-3}\right)\times 2.65\times 9800 = 51.5\text{ N/sm}$$

$$g_s = g_b\times b = 51.5\times 45 = \boxed{2320\text{ N/s}}$$

★ The correct answer is **B**.

Part D: In the Chang et al. approach, the suspended-load transport rate is given by

$$q_b = C_a y\left(UI_3 - \frac{2u_*}{\kappa}I_4\right)$$

where C_a is the reference concentration, y is the flow depth, U is the average flow velocity, u_* is the shear velocity, κ = 0.41 is the Von Kármán constant, and I_3 and I_4

are integrals given as functions of the dimensionless reference depth $\tilde{a}$ and factor ζ_1 in Figures 5 and 6. In these charts, ζ_1 is a slightly modified Rouse number,

$$\zeta_1 = \frac{2w_s}{\beta \kappa u_*}$$

Since w_s/u_* = 0.075/0.166 = 0.452 ∈ [0.1, 1.0], coefficient β can be approximated as

$$\beta\left(0.1 < \frac{w_s}{u_*} < 1.0\right) = 1 + 2\left(\frac{w_s}{u_*}\right)^2 = 1 + 2\times 0.452^2 = 1.41$$

with the result that

$$\zeta_1 = \frac{2\times 0.075}{1.41\times 0.41\times 0.166} = 1.56$$

Referring to Figures 5 and 6 with this value of ζ_1 and the dimensionless reference depth $\tilde{a} = a/y$ = 5.6 × 10^{-4}/2.8 = 2 × 10^{-4}, we read I_3 = 0.005 and I_4 = 0.004. Inserting the pertaining data in the equation for q_b gives

$$q_b = 0.40\times 2.8\times\left(4.29\times 0.005 - \frac{2\times 0.166}{0.41}\times 0.004\right) = 0.0365 \text{ m}^2/\text{s}$$

Our final task is to convert this to a weight transport rate,

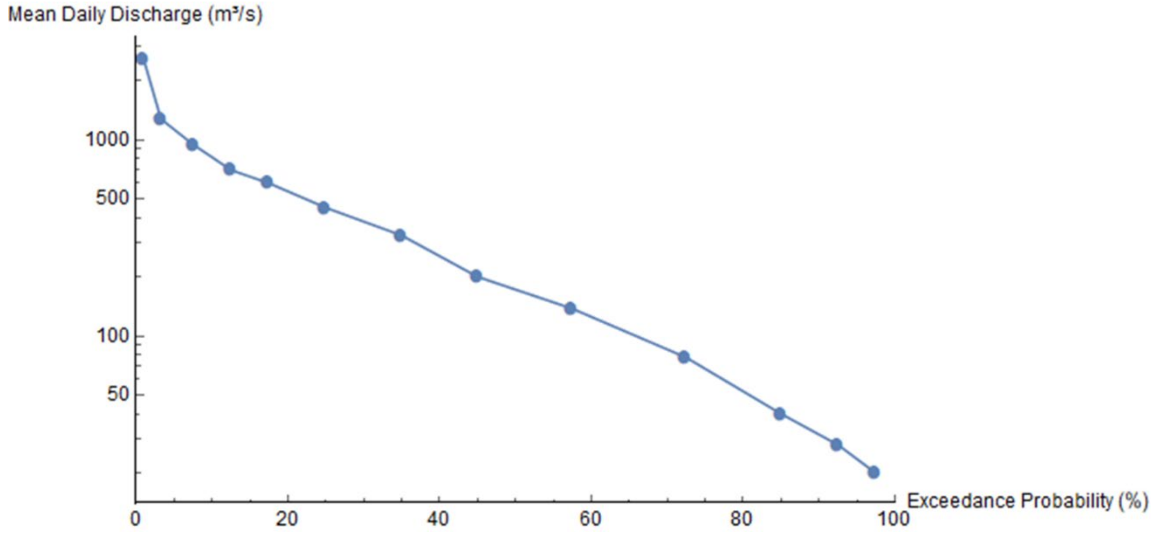
$$g_b = q_b G_s \gamma = 0.0365\times 2.65\times 9800 = 948 \text{ N/sm}$$

$$\therefore g_s = g_b \times b = 948\times 45 = \boxed{42,660 \text{ N/s}}$$

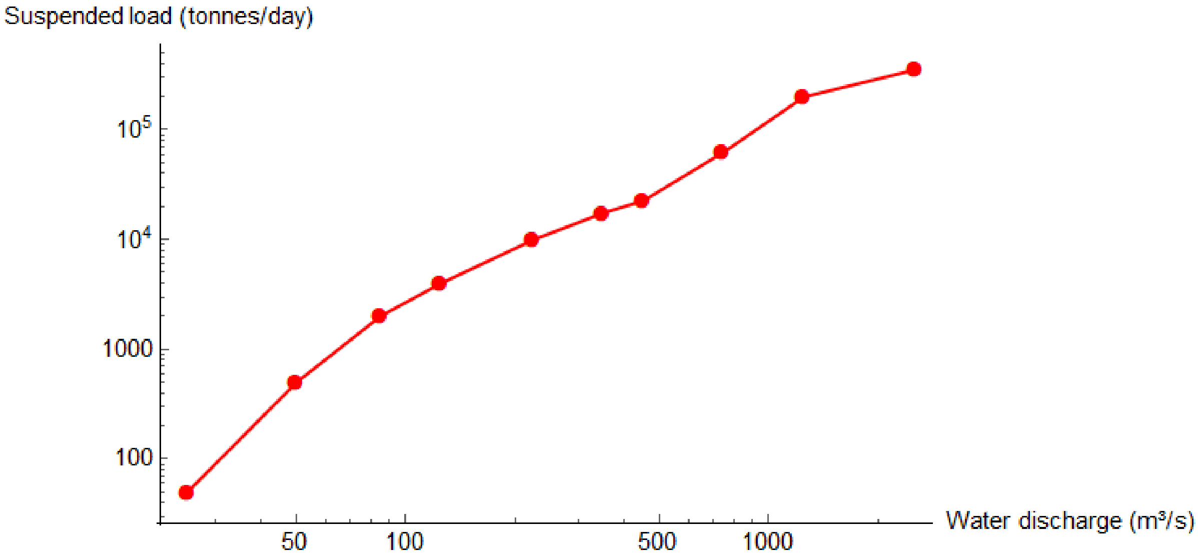
★ The correct answer is **D**.

P.10 ■ Solution

Part A: The flow duration curve is plotted on semi-log paper, as shown below.



The sediment rating curve is plotted on log-log paper, as shown below.



With reference to these plots, the following table is prepared.

1	2	3	4	5	6
Exceedance Frequency Range (%)	Interval (%)	Midpoint (%)	Mean Daily Discharge (m^3/s)	Sediment Discharge (tonnes/day)	Volume of Water Flow (cumec/day)
0.5 - 1.5	1	1	2550	3500	25.5
1.5 - 5.0	3.5	3.25	1275	5133.6	44.6
5.0 - 10.0	5	7.5	945	5287.2	47.3
10.0 - 15.0	5	12.5	700	2722	35
15.0 - 20.0	5	17.5	600	1999.8	30
20.0 - 30.0	10	25	450	2249.8	45
30.0 - 40.0	10	35	325	1593.4	32.5
40.0 - 50.0	10	45	200	832.3	20
50.0 - 65.0	15	57.5	137.5	696.2	20.6
65.0 - 80.0	15	72.5	77.5	235.6	11.6
80.0 - 90.0	10	85	40	24.1	4
90.0 - 95.0	5	92.5	27.8	3.6	1.4
95.0 - 99.9	4	97.45	20	1	0.8
			Sum =	**24279**	**318**

The sediment discharge is the sum of the values in column 5, or 24,279 tonnes/day. Over the course of a year, this amounts to 365 × 24,279 = 8.86 × 10^6 tonnes. To account for the sediment yield due to bed load transport, we increase this quantity by 10 percent and obtain m_s = 1.1 × 8.86 × 10^6 = 9.75 × 10^6 tonnes.

★ The correct answer is **D**.

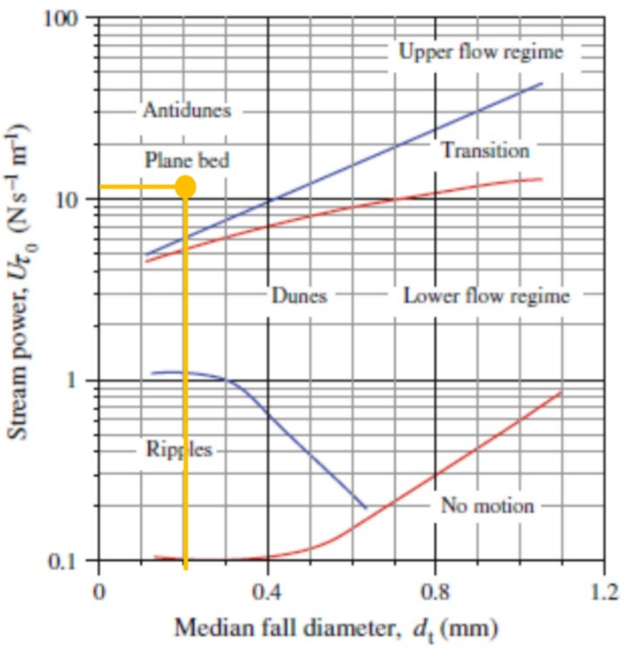
Part B: From the above table, we find that the mean daily yield of water is 318 m^3/day, which corresponds to an annual load of 318 × 365 × 24 × 60 × 60/10^6 = 10,028 × 10^6 tonnes. The average concentration of suspended load follows as

$$C_s = \frac{8.86\times10^6}{10,028\times10^6 + 8.86\times10^6}\times10^6 = \boxed{883\text{ ppm}}$$

★ The correct answer is **D**.

P.11 ■ Solution

Part A: The bed shear is $\tau_0 = \gamma y S_0$ = 9800 × 3.4 × 0.0002 = 6.66 Pa. The Simons and Richardson chart requires the median fall diameter d_t, which we assume to be approximately equal to d_{50} = 0.2 mm, and the stream power $U\tau_0$ = 1.4 × 6.66 = 12 N/sm. Mapping these quantities onto the chart (Figure 7), we verify that the channel will have a plane bed form.



★ The correct answer is **A**.

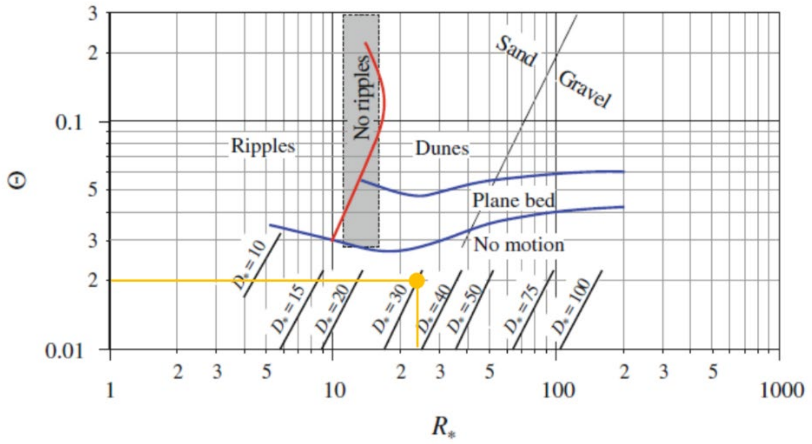
Part B: The Shields parameter is calculated as

$$\theta = \frac{\tau_0}{\gamma(G_s - 1)d_{50}} = \frac{6.66}{9800\times(2.65-1)\times(0.2\times10^{-3})} = 2.06$$

The shear velocity is $u_* = \sqrt{\tau_0/\rho} = \sqrt{6.66/1000} = 0.0816$ m/s and the particle Reynolds number is

$$\mathrm{Re}_* = \frac{u_* d_{65}}{\nu} = \frac{0.0816\times(0.3\times10^{-3})}{1.006\times10^{-6}} = 24.3$$

Inserting this Shields parameter/shear Reynolds number combination into the Chabert and Chauvin chart (Figure 8), we find that there should be no bed motion.



★ The correct answer is **D**.

Part C: The van Rijn approach is based on the ratio $[\tau_0 - (\tau_0)_{cr}]/(\tau_0)_{cr}$, known as the *transport stage parameter*, T_*. In the present case, with a particle parameter

$$D_* = \left[\frac{(G_s-1)g}{\nu^2}\right]^{1/3} d_{50} = \left[\frac{(2.65-1)\times9.81}{(1.006\times10^{-6})^2}\right]^{1/3} \times(0.2\times10^{-3}) = 5.04$$

the equation to use for the threshold Shields parameter is

$$\theta_{cr}(4 < D_* \le 20) = 0.04D_*^{-0.1} = 0.04\times5.04^{-0.1} = 0.034$$

and the threshold bed shear is

$$(\tau_0)_{cr} = \theta_{cr}\times\gamma(G_s-1)d_{50} = 0.034\times9800\times(2.65-1)\times(0.2\times10^{-3}) = 0.11\text{ Pa}$$

The value of T_* is then

$$T_* = \frac{\tau_0 - (\tau_0)_{cr}}{(\tau_0)_{cr}} = \frac{6.66-0.11}{0.11} = 59.5$$

We also require the Froude number Fr,

$$\mathrm{Fr} = \frac{U}{\sqrt{gy}} = \frac{1.8}{\sqrt{9.81\times3.4}} = 0.312$$

Referring to Table 2 with this information, we conclude that the bed form of the river is sand waves (symmetrical).

★ The correct answer is **A**.

P.12 ■ Solution

1. True. The particle parameter D_* is

$$D_* = \left[\frac{(G_s-1)g}{\nu^2}\right]^{1/3} d_{50} = \left[\frac{(2.65-1)\times9.81}{(1.006\times10^{-6})^2}\right]^{1/3} \times(0.6\times10^{-3}) = 15.1$$

which fits in the interval [3, 70]. In order to compute the shear Reynolds number Re_*, we require the shear velocity $u_* = \sqrt{\tau_0/\rho}$. With a bed shear $\tau_* = \gamma y S_0$ = 9800 × 2.0 × 0.00035 = 6.86 Pa, we have $u_* = \sqrt{6.86/1000}$ = 0.0828 m/s. Accordingly, Re_* follows as

$$\mathrm{Re}_* = \frac{u_* d_{65}}{\nu} = \frac{0.0828\times\left(0.75\times10^{-3}\right)}{1.006\times10^{-6}} = 61.7$$

which happens to be within the interval [11.6, 70]. Finally, we must verify whether the bed shear satisfies the inequality

$$\tau_0' \approx \tau_0 < \frac{1}{D_*\kappa}\ln\left(\frac{y}{20d}\right) = \frac{1}{15.1\times0.41}\times\ln\left[\frac{2.0}{20\times\left(0.6\times10^{-3}\right)}\right] = 0.826$$

Since τ_0 = 6.86 Pa > 0.826, the inequality is not true. The river in question meets two of Julien's three requirements for the formation of dunes.

2. False. The Julien and Klaassen approximation for mean dune height is

$$\overline{\eta}_d = 2.5y^{0.7}d_{50}^{0.3} = 2.5\times2.0^{0.7}\times\left(0.6\times10^{-3}\right)^{0.3} = 0.439\ \mathrm{m}$$

3. False. As per the van Rijn formula, the dune height is given by

$$\eta_d = 0.11y\left(\frac{d_{50}}{y}\right)^{0.3}\left[1-\exp\left(-0.5\frac{\tau_0'-(\tau_0)_{\mathrm{cr}}}{(\tau_0)_{\mathrm{cr}}}\right)\right]\left(25-\frac{\tau_0'-(\tau_0')_{\mathrm{cr}}}{(\tau_0')_{\mathrm{cr}}}\right)$$

In order to evaluate the expression above, we must first establish the threshold shear $(\tau_0)_{\mathrm{cr}}$. This, in sequence, requires the critical Shields parameter θ_{cr},

$$\theta_{\mathrm{cr}}\left(4<D_*\le20\right) = 0.04D_*^{-0.1} = 0.04\times15.1^{-0.1} = 0.0305$$

so that

$$(\tau_0)_{\mathrm{cr}} = \theta_{\mathrm{cr}}\times\gamma\left(G_s-1\right)d_{50} = 0.0305\times9800\times(2.65-1)\times\left(0.6\times10^{-3}\right) = 0.296\ \mathrm{Pa}$$

Substituting the pertaining variables in the equation for η_d gives

$$\eta_d = 0.11\times2.0\times\left(\frac{0.6\times10^{-3}}{2.0}\right)^{0.3}\times\left[1-\exp\left(-0.5\times\frac{6.86-0.296}{0.296}\right)\right]\times\left(25-\frac{6.86-0.296}{0.296}\right) = 54.5\ \mathrm{mm}$$

4. True. The Watanabe expression for dune height is

$$\eta_d = 2000d_{50}\left(1-\mathrm{Fr}^2\right)\left(\theta-\theta_{\mathrm{cr}}\right)^{1.5}$$

The Froude number is given by

$$\mathrm{Fr} = \frac{U}{\sqrt{gy}} = \frac{1.0}{\sqrt{9.81\times2.0}} = 0.226$$

The Shields parameter is determined next,

$$\theta = \frac{\tau_0}{\gamma\left(G_s-1\right)d_{50}} = \frac{6.86}{9800\times(2.65-1)\times\left(0.6\times10^{-3}\right)} = 0.707$$

Finally, η_d is computed as

$$\eta_d = 2000\times\left(0.6\times10^{-3}\right)\times\left(1-0.226^2\right)\times(0.707-0.0305)^{1.5} = 0.634\ \mathrm{m}$$

5. True. According to van Rijn, the dune length is taken as 7.3 times the flow depth; that is,

$$\lambda_d = 7.3y = 7.3\times2.0 = 14.6\ \mathrm{m}$$

Answer Summary

Problem 1		D
Problem 2		C
Problem 3		B
Problem 4		C
Problem 5		A
Problem 6	6A	B
	6B	A
Problem 7	7A	B
	7B	B
	7C	A
	7D	T/F
Problem 8	8A	C
	8B	Open-ended pb.
Problem 9	9A	C
	9B	B
	9C	B
	9D	D
Problem 10	10A	D
	10B	D
Problem 11	11A	C
	11B	B
	11C	A
Problem 12		T/F

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