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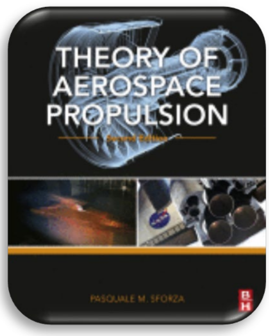

Quiz ASQ1
Reviewed Solutions to Sforza's *Theory of Aerospace Propulsion*, 2nd Ed.

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Problem Distribution

Chapter	Problems Covered
1	1.1, 1.2, 1.3, 1.4, 1.5, 1.11
2	2.2, 2.7
3	3.1, 3.2, 3.3, 3.4, 3.5, 3.6
10	10.2, 10.5, 10.6
11	11.3, 11.4, 11.7
12	12.2, 12.4
13	13.1, 13.2, 13.4, 13.5, 13.6

Problems

■ Chapter 1 – Propulsion Principles and Engine Classification

Problem 1.1

A jet airplane is producing 10 kN of net thrust flying at a constant speed of 500 m/s at 18 km altitude (ρ_0 = 0.1 kg/m^3, T_0 = 216 K). Determine the drag of the airplane.

Problem 1.2

A large turbofan engine has a bypass ratio of 10 and produces a static thrust of 500 kN with a jet exit velocity of 800 m/s and a fan exit velocity of 120 m/s. Assuming ideal conditions, what is the total mass flow $\dot{m}$ passing through the engine?

Problem 1.3

A jet engine on a test stand is operating at its maximum thrust of 100 kN with an exit velocity V_T = 1000 m/s. What is the mass flow of the exhaust?

Problem 1.4

A rocket using a 4 to 1 mixture of LOX and LH_2 as a propellant has a specific impulse of I_{sp} = 400 s. The LH_2 mass flow rate is 10 kg/s. What is the thrust?

Problem 1.5

A ramjet provides net thrust F_n = 10 kN to drive a missile at V_0 = 1000 m/s at an altitude of 33.5 km (ρ_0 = 0.01 kg/m^3 and T_0 = 232 K). If the missile inlet captures a streamtube of air A_0 = 1 m^2 in area, the fuel-to-air ratio is 0.04, and the fuel heating value is 50 MJ/kg, what is the overall efficiency?

Problem 1.6

A 3.57 m diameter propeller on an airplane requires 581.6 kW at a flight speed of 100 m/s at an altitude of 6.7 km (ρ = 0.5 kg/m^3) while operating at a propulsive efficiency of 95%. What is the net thrust produced?

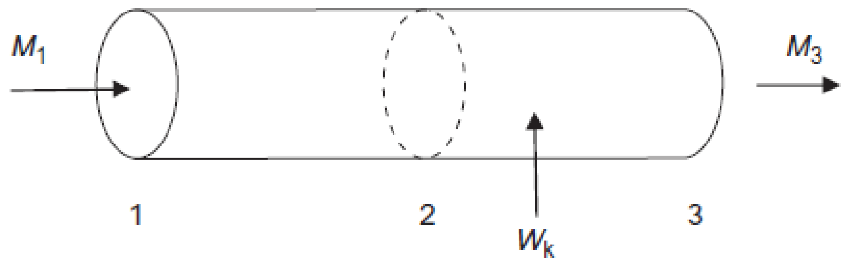
Problem 1.11

Define each of the following terms and indicate, whenever possible, what each term is a function of: **(a)** effective exhaust velocity, **(b)** ram drag, **(c)** specific fuel consumption, **(d)** gross thrust, **(e)** heating value of fuel, **(f)** additive drag, **(g)** specific impulse.

■ Chapter 2 – Quasi-One-Dimensional Flow Equations

Problem 2.2

Consider frictionless flow in the constant area duct shown in Fig. E2.1, in which work may be added between stations 2 and 3. **(a)** Using Table 2.1 of the text develop equations for the pressure, temperature, density, velocity, and work (per unit mass) done on the fluid as a function of Mach number in the duct, **(b)** for air entering station 1 at $p_1 = p_2 = 101$ kPa, $T_1 = T_2 = 288$ K, and $M_1 = M_2 = 0.4$ find the velocity, stagnation pressure, and stagnation temperature at station 2, **(c)** If 169.3 kJ/kg of work is done on the flow find the Mach number, pressure, temperature, density, velocity, stagnation pressure, and stagnation temperature at station 3.



Problem 2.7

Consider the flow of air through a constant area duct as shown in Figure E2.3. The gas constant is $R = 0.287$ kJ/kg·K and $\gamma = 1.4$. **(a)** Provide an analysis that determines the type of flow occurring, i.e. Fanno, Rayleigh, normal shock, isentropic, or some combination thereof. **(b)** Give the simplest set of equations that govern the model flow determined in the previous part. **(c)** Find the Mach number at station 2, M_2. **(d)** Determine as many pertinent quantitative physical factors of importance for this flow as possible, such as friction factor, duct length, amount of heat addition or removal, shock location, etc. **(e)** If the pressure at station 2 was reported to be $p_2 = 712$ kPa while the temperature T_2 was the same as found in the previous parts, what new answers would you give for parts (*a*) and (*c*)?

■ Chapter 3 – Idealized Cycle Analysis of Jet Propulsion Engines

Problem 3.1

A turbojet engine with a convergent nozzle is operating at 100% power at $M_0 = 0.9$ and an altitude of 11 km. The engine characteristics are such that the compressor, burner, and turbine efficiencies may be taken as 0.85, 0.96, and 0.90, respectively. The inlet pressure recovery may be taken as the MIL-E-5008B standard. The mass flow captured by the inlet is 29.17 kg/s, and the heating value of the fuel is 44 MJ/kg. The compressor pressure ratio is $p_{t,3}/p_{t,2} = 18$, the combustor pressure loss is $(p_{t,3} - p_{t,4}) = 0.02p_{t,3}$, and the turbine inlet temperature is $T_{t,4} = 1330$ K. Calculate the properties of the flow at each station in the engine, the gross thrust, and the net thrust.

Problem 3.2

A turbofan engine with the same component efficiencies as the turbojet engine in Problem 3.1 is operating at the same flight conditions. The total mass flow, core flow plus fan flow, is 50 kg/s and the bypass ratio $\beta = 1$. The fan operates in a bypass duct without mixing with the core flow. The fan pressure ratio $p_{t,3F}/p_{t,2} = 2.2$ and its efficiency is 0.85. The nozzle of the fan bypass duct may be assumed to operate critically, that is, with an exit $M_{3F} = 1$. Calculate the properties of the flow at each station in the engine, the gross thrust, and the net thrust.

Problem 3.3

Use the results of problems 3.1 and 3.2 to compare the performance of the two engines in terms of net thrust, specific fuel consumption, overall efficiency, and total mass flow. Discuss the significance of the comparison.

Problem 3.4

Consider adding an afterburner to the turbojet engine of Problem 3.1 with the additional information that the afterburner combustion efficiency is 0.85 and frictional losses in the afterburner yield $(p_{t,5} - p_{t,5b}) = 0.05p_{t,5}$. In the afterburner the fuel flow is $\dot{m}_f$ = 1.17 kg/s. Calculate the properties of the flow at each station in the engine, the gross thrust, and the net thrust.

Problem 3.5

Consider adding an afterburner to the turbofan engine of Problem 3.2 with the additional information that the afterburner combustion efficiency is 0.85 and frictional losses in the afterburner yield $(p_{t,5} - p_{t,5b}) = 0.05p_{t,5}$. In the afterburner the fuel flow is $\dot{m}_{f,AB}$ = 1.0 kg/s. The bypass duct flow is assumed to mix internally with the turbine exit flow such that the resulting stagnation pressure is the average of the two. Calculate the properties of the flow at each station in the engine, the gross thrust, and the net thrust.

Problem 3.6

Use the results of problems 3.4 and 3.5 to compute the performance of the two engines in terms of net thrust, specific fuel consumption, overall efficiency, and thrust augmentation ratio. Discuss the significance of the comparison.

■ Chapter 10 – Propellers

Problem 10.2

A three-bladed propeller with AF = 140, diameter of 15 ft, and design lift coefficient of $C_{L,d}$ = 0.5 is driven by a free turbine engine at standard sea level conditions. The engine delivers 2000 hp to the propeller at a free turbine rotational speed of 15,000 rpm through a transmission with a gear reduction of 12:1. The power delivered decreases linearly to 1800 hp as the rotational speed is decreased to 10,000 rpm. Using Figure 10.16, plot the variation of the static thrust delivered by the propeller as a function of propeller rpm. For $C_{Pc} < 0.12$, you may use the relation $C_{Tc}/C_{Pc} = 0.62C_{Pc}^{-0.5}$.

Problem 10.3

Consider the use of 3-bladed, AF = 140, 4.72 m diameter propellers for a VTOL application. Two integrated design lift coefficient choices are possible for the proposed airplane: $C_{L,d}$ = 0.3 or 0.5. In takeoff (V_0 = 0), the engine supplies 4030 kW at a propeller speed of 1230 rpm and in cruise (V_0 = 250 kts = 128.7 m/s) the engine supplies 750 kW at a propeller speed of 900 rpm. Determine **(a)** the static thrust at takeoff for each proposed propeller and the figure of merit for each, **(b)** The efficiency η_P in cruise of the propeller with $C_{L,d}$ = 0.5, **(c)** Using these results and the information that the $C_{L,d}$ = 0.3 case yields an efficiency in cruise of η_P = 0.9 discuss the design implications of the magnitude of the integrated lift coefficient $C_{L,i}$.

Problem 10.6

Estimate the static thrust and rpm of a 3 m diameter propeller with an input power of 746 kW as a function of the number of blades. Assume that the propeller is limited to a tip Mach number of 0.8 and that the propeller operates at standard sea-level conditions. Discuss the results in terms of the blade loading (the fraction of the thrust carried by each blade) and its relationship to the drag losses incurred.

■ Chapter 11 – Liquid Propellant Rocket Motors

Problem 11.3

A liquid propellant rocket motor using a JP-4/RFNA propellant combination must produce 19.6 kN of thrust at sea level. hrust at sea level. The characteristics of the combustion products are combustion chamber temperature T_c = 3000 K, average specific heat ratio γ = 1.2, molecular weight W = 27, and chamber pressure p_c = 19.34 atmospheres. Calculate the following: **(a)** the throat and exit areas, A_t and A_e for matched nozzle exit flow at sea level assuming a nozzle efficiency η_n = 95%; **(b)** the characteristic velocity c*, the propellant mass flow rate, and the specific impulse of the engine at sea level, **(c)** the thrust developed at an altitude of 11.5 km where the pressure is p_0 = 0.2 atm, **(d)** for comparison purposes calculate the thrust and exit area at this altitude for matched nozzle exit operation.

Problem 11.4

The Rocketdyne RS-X engine was a developmental outgrowth of the RS-27 engine used on Delta launchers. It is an RP-1/LOX engine that develops an I_{sp} = 264 s at sea level and an $I_{sp,vac}$ = 299 s in space. The engine O/F = 2.25 and the chamber pressure of p_c = 775 psia feeds a nozzle with an expansion ratio ε = 10. The engine weighs approximately 4500 lbs and is 134 in long and 72.6 in wide (nozzle exit diameter). The nozzle has an angular divergence α = 12° at the exit plane. The combustion chamber temperature may be taken as T_c = 6400 R while the molecular weight and ratio of specific heats are considered constants at the values of 21.9 and 1.135, respectively. Experiments have shown that the nozzle discharge and velocity coefficients are c_d = 0.95 and c_V = 0.903, respectively. Under these conditions and assuming sea level static test conditions determine: **(a)** the exit and throat areas A_e and A_t, **(b)** the mass flow of the propellants, **(c)** the thrust produced F, **(d)** the thrust coefficient c_F, **(e)** the nozzle exit Mach number M_e, **(f)** the nozzle exit pressure p_e, **(g)** the nozzle exit velocity, V_e, **(h)** the effective exhaust velocity V_e, **(i)** the characteristic velocity c*.

Problem 11.7

Consider a rocket motor operating with a chamber pressure and temperature of p_c = 2.07 MPa and T_c = 2860 K, respectively, and a nozzle which produces the optimum thrust of 1334 kN when exhausting into the atmosphere under standard sea level conditions. The exhaust gases are characterized by a mean molecular weight and isentropic exponent W = 21.87 kg/kg-mol and γ = 1.229, respectively. Determine the following properties of the rocket motor.

(a) Thrust coefficient c_F

(b) Characteristic exhaust velocity c*

(c) Nozzle minimum area A_t

(d) Nozzle exit area A_e

■ Chapter 12 – Solid Propellant Rocket Motors

Problem 12.2

The Space Shuttle's integrated solid rocket boosters, shown firing during a launch in Fig. E12.1, use a propellant mixture of ammonium perchlorate (NH_4ClO_4), polybutadiene acrylonitrile copolymer (PBAN), aluminum powder, and iron oxide catalyst. The characteristics of the propellant are given as follows: burn rate r = 0.434 in./s at 1000 psia, burn rate exponent n = 0.35, propellant density ρ_p = 0.063 lb/in.^{3}, temperature coefficient of pressure $(\pi_p)_k$ = 0.11 (%/F), characteristic exhaust velocity c* = 5053 ft/s, combustion temperature T_c = 6092°F, effective ratio of specific heats γ_c = 1.138 and γ_e = 1.147. The nozzle throat diameter is 53.86 in., the exit diameter is 152.6 in., and the nozzle produces an average thrust of 2.59 million pounds at an ambient pressure of 60°F. The average chamber pressure is 662 psia during operation. For operation at sea level liftoff determine: **(a)** the specific impulse, **(b)** the chamber molecular weight, **(c)** exit Mach number, **(d)** exit pressure, **(e)** exit velocity, **(f)** the exit temperature, and **(g)** the percent change in average thrust if the ambient temperature drops to 32°F.

Problem 12.4

Consider an end burning grain solid rocket motor with the following characteristics: ρ_p = 1770 kg/m^3, T_p = 15°C, $c^* = 1051p^{0.015}$ in m/s when p is in atmospheres, n = 0.745, a = 0.001 m/s when p is in atmospheres. The thrust developed is F = 4900 N at a chamber pressure p_c = 13.74 MPa over a burning time t_b = 15 s where the combustion gases have γ = 1.27 and a molecular weight W = 30. Determine: **(a)** the chamber temperature T_c, **(b)** the thrust coefficient of the nozzle c_F, **(c)** the throat area A_t, **(d)** the burning area A_b, **(e)** the length L of the grain, and **(f)** the specific impulse I_{sp}.

Note: The number highlighted in yellow was mistakenly given as 1.051 in some printings of the book.

■ Chapter 13 – Space Propulsion

Problem 13.1

An electrostatic thruster produces 0.5 N thrust for 3 hours and consumes a total of 0.5 kg using a power of 4 kW. **(a)** Find the specific impulse I_{sp} of the thruster, **(b)** find η', the efficiency of the propulsive jet.

Problem 13.2

A spacecraft with a solar electric power plant with a specific mass α = 15 kg/kW is considered for a space mission that requires 4.8 months. **(a)** Estimate the optimum specific impulse to carry out this mission. **(b)** Assuming that the efficiency of the propulsive jet η' = 50%, determine the maximum thrust that can be developed if the solar array power delivered to the engine is 25 kW, **(c)** find the total impulse delivered over the entire mission, **(d)** calculate the mass of the powerplant and of the propellant required for the mission.

Problem 13.4

Determine the operating time required for an ion engine with a specific impulse I_{sp} = 10,000 s to accelerate a vehicle through a velocity increment ΔV = 5 km/s if the thrust-to-weight ratio of the vehicle is 10^{-4}. If an MPD engine with I_{sp} = 1000 s is substituted with the vehicle thrust-to-weight ratio now equal to 10^{-3} how long would the acceleration take? Estimate the propellant mass needed in each case.

Problem 13.5

A nuclear thermal rocket using hydrogen as the propellant has a specific impulse $I_{sp,NUKE}$ = 900 s and a liquid rocket using *LH2-LOX* as the propellant has a specific impulse $I_{sp,LOX}$ = 450 s when both use the same nozzle and operate under the same ambient pressure conditions with the nozzle matched. The liquid rocket engine operates with a chamber temperature T_c = 3600 K. Calculate a reasonable estimate of the chamber temperature at which the nuclear rocket is operating.

Problem 13.6

A fast trip to Mars, including landing and return, in a spacecraft with initial and final mass M_i and M_f, respectively, requires a spacecraft velocity increment

$$\Delta V = gI_{sp}\ln\left(\frac{M_i}{M_f}\right) = 40{,}000 \text{ m/s}$$

The ideal mission would require 22,000 m/s, about half that of the fast mission. (*a*) For both missions compare the required propellant mass fraction M_f/M_i required for chemical, solid-core nuclear, gas-core nuclear, and electric propulsion devices having I_{sp} = 450 s, 1000, 2000, and 10,000 s, respectively.

SOLUTIONS

■ P1.1

At constant-speed horizontal flight, the net thrust and drag are equal, hence $D = F_n$ = 10 kN. Everything else is junk information.

■ P1.2

The total mass flow we're looking for equals the sum of the jet and fan mass flows; noting that the bypass ratio $\beta = \dot{m}_{fan}/\dot{m}_{jet}$ = 10, we may write

$$\dot{m} = \dot{m}_{jet} + \dot{m}_{fan} = \dot{m}_{jet}\left(1 + \underbrace{\frac{\dot{m}_{fan}}{\dot{m}_{jet}}}_{=10}\right) = 11\dot{m}_{jet} \quad \text{(I)}$$

Noting that the static thrust equals 500 kN, we can solve for $\dot{m}_{jet}$ and obtain

$$F = F_{jet} + F_{fan} = \dot{m}_{jet}V_{jet} + \dot{m}_{fan}V_{fan}$$

$$\therefore F = \dot{m}_{jet}(800 + 10 \times 120) = 500 \text{ kN}$$

$$\therefore \dot{m}_{jet} = \frac{500{,}000}{800 + 10 \times 120} = 250 \text{ kg/s}$$

Substituting in (I) gives the total mass flow:

$$\dot{m} = 11 \times 250 = \boxed{2750 \text{ kg/s}}$$

■ P1.3

On a static test stand the thrust developed is the gross thrust and the mass flow becomes

$$F_g = \dot{m}_7 V_7 \rightarrow \dot{m}_7 = \frac{F_g}{V_7}$$

$$\therefore \dot{m}_7 = \frac{100{,}000}{1000} = \boxed{100\,\text{kg/s}}$$

■ P1.4

We first compute the total mass flow of propellant:

$$\dot{m} = \dot{m}_o + \dot{m}_f = \dot{m}_f\left(1 + \frac{\dot{m}_o}{\dot{m}_f}\right) = 10 \times (1+4) = 50\,\text{kg/s}$$

Then, noting that $F_n = \dot{m}gI_{sp}$, we obtain

$$F_n = \dot{m}gI_{sp} = 50 \times 9.81 \times 400 = 196{,}000\,\text{N}$$

$$\therefore \boxed{F_n = 196\,\text{kN}}$$

■ P1.5

The efficiency of the ramjet is given by the usual ratio

$$\eta_o = \frac{F_n V_0}{\dot{m}_f \Delta Q} = \frac{10{,}000 \times 1000}{\dot{m}_f \times \left(50 \times 10^6\right)} = \frac{0.2}{\dot{m}_f} \quad \text{(I)}$$

To find the mass flow of fuel, we write, using the given fuel-to-air ratio and thermophysical properties for flight at 33.5 km,

$$\dot{m}_f = \frac{\dot{m}_f}{\dot{m}_o}\dot{m}_o = 0.04 \times \rho_0 A_0 V_0 = 0.04 \times 0.01 \times 1.0 \times 1000 = 0.4\,\text{kg/s}$$

Substituting in (I) yields

$$\eta_o = \frac{0.2}{0.4} = 0.5 = \boxed{50\%}$$

■ P1.6

Solving from the definition of propeller efficiency brings to

$$\eta_p = \frac{F_n V_0}{P} \rightarrow F_n = \frac{\eta_p P}{V_0}$$

$$\therefore F_n = \frac{0.95 \times \left(581.6 \times 10^3\right)}{100} = 5530\,\text{N} = \boxed{5.53\,\text{kN}}$$

■ P1.11

Part a: *Effective exhaust velocity* is the velocity a jet exiting at station 7 would reach once its pressure is equilibrated with that of the surroundings, station 0. It is a function of V_7, A_7, p_7, p_0 and the mass flow according to

$$V_e = V_7 + \frac{A_7}{\dot{m}}\left(p_7 - p_0\right)$$

Part b: *Ram drag* is the momentum penalty incurred by an airbreathing jet by taking onboard air from the surroundings. It is a function of the capture mass flow and the flight velocity according to $F_r = (\rho_0 A_0 V_0) V_0$.

Part c: *Specific fuel consumption* is the ratio of the mass flow of fuel consumed to the thrust produced. It is a function of the average velocity $(V_e + V_0)/2$, the burner efficiency η_b, the thermal efficiency η_{th}, and the heating value of the fuel Q_f according to

$$c_j = \frac{V_{\text{avg}}}{\eta_{th}\eta_b Q_f}$$

Part d: *Gross thrust* is the thrust produced by the exiting jet alone; it does not include ram drag. The equation for gross thrust is

$$F_g = \dot{m}V_e = \dot{m}V_7 + A_7\left(p_7 - p_0\right)$$

Part e: The *heating value* Q_f of a fuel is the heat energy released per unit mass in a complete combustion process. It depends upon the chemical composition of the fuel.

Part f: *Additive drag* is the force arising from pressure on the streamtube entering the engine. It is of particular concern in the design of inlets for supersonic flight.

Part g: *Specific impulse* is the ratio of thrust force produced to the mass flow of propellants. It is essentially the reciprocal of the specific fuel consumption with one difference: it is the mass flow of propellants rather than fuel alone. Using this definition for rockets properly accounts for the fact that rockets carry both fuel and oxidizer whereas airbreathing engines carry only fuel, taking air on board in flight to serve as oxidizer.

■ P2.2

Part a: The equations for pressure, temperature, density, velocity and work per unit mass done on the fluid may be written using the influence coefficients of Table 2.1 as follows:

$$\frac{dp}{p}=\frac{-\gamma M^2}{1-M^2}\left(-\frac{dW_k}{c_pT}-\frac{dW}{W}\right)$$

$$\frac{dT}{T}=\frac{1-\gamma M^2}{1-M^2}\left(-\frac{dW_k}{c_pT}+\frac{(\gamma-1)M^2}{1-\gamma M^2}\frac{dW}{W}\right)$$

$$\frac{d\rho}{\rho}=\frac{-1}{1-M^2}\left(-\frac{dW_k}{c_pT}-\frac{dW}{W}\right)$$

$$\frac{dV}{V}=\frac{-1}{1-M^2}\left(-\frac{dW_k}{c_pT}-\frac{dW}{W}\right)$$

$$\frac{dM^2}{M^2}=\frac{1+\gamma M^2}{1-M^2}\left(-\frac{dW_k}{c_pT}-\frac{dW}{W}\right)-\frac{d\gamma}{\gamma}$$

The equation for *M* can be rearranged as

$$\frac{d\left(\gamma M^2\right)}{\gamma M^2}=\frac{1+\gamma M^2}{1-M^2}\left(\frac{-dW_k}{c_pT}-\frac{dW}{W}\right) \quad \text{(I)}$$

Using this form of the equation brings to

$$\frac{dp}{p}=\frac{-d\left(\gamma M^2\right)}{1+\gamma M^2}$$

$$\frac{d\rho}{\rho}=\frac{-d\left(\gamma M^2\right)}{\gamma M^2\left(1+\gamma M^2\right)}$$

Using these two equations and the equation of state to find the temperature, we get

$$\frac{dT}{T}=\frac{1-kM^2}{1+kM^2}\frac{d\left(kM^2\right)}{kM^2}+\frac{dW}{W}$$

Integrating the three foregoing equations leads to the following results:

$$\frac{p_3}{p_2}=\frac{1+\gamma_2M_2^2}{1+\gamma_3M_3^2} \quad \text{(II.1)}$$

$$\frac{\rho_3}{\rho_2}=\frac{\gamma_2M_2^2\left(1+\gamma_3M_3^2\right)}{\gamma_3M_3^2\left(1+\gamma_2M_2^2\right)} \quad \text{(II.2)}$$

$$\frac{T_3}{T_2}=\frac{\gamma_2M_2^2\left(1+\gamma_3M_3^2\right)}{\gamma_3M_3^2\left(1+\gamma_2M_2^2\right)}\frac{W_3}{W_2} \quad \text{(II.3)}$$

Note that these are the same equations derived in Chapter 4 for the constant-area combustor, namely eqs. (4.15), (4.16), and (4.17). The equation for the velocity follows from the conservation of mass, giving

$$\frac{V_3}{V_2} = \frac{\rho_2}{\rho_3}$$

We assume that *W* and γ remain constant through the compression process, which should be quite accurate because the change in pressure will not cause high enough temperatures to change the chemical composition of the gas, but this can be checked *a posteriori*. Then, equation (I) can be restated as

$$-dW_k = c_p T\left(\frac{1-M^2}{1+\gamma M^2}\frac{dM^2}{M^2}\right) \quad \text{(III)}$$

Equation (II.3) is applicable to any point in the flow between 2 and 3 so that

$$T = C\frac{\gamma M^2}{\left(1+\gamma M^2\right)^2} \quad \text{(IV)}$$

where constant *C* is

$$C = \frac{\left(1+\gamma M_2^2\right)^2}{\gamma M_2^2} T_2 \quad \text{(V)}$$

so that equation (III) becomes

$$-dW_k = c_p C\frac{\left(1-M^2\right)dM^2}{\left(1+kM^2\right)^3} \quad \text{(VI)}$$

The equation can be integrated, noting that $dW_{k,2} = 0$, to yield

$$-dW_{k,3} + dW_{k,2} = -dW_{k,3} = c_p C\left[\frac{1+\gamma\left(2M_3^2-1\right)}{2\gamma\left(1+\gamma M_3^2\right)^2} - \frac{1+\gamma\left(2M_2^2-1\right)}{2\gamma\left(1+\gamma M_2^2\right)^2}\right] \quad \text{(VII)}$$

Part b: Conditions at station 2 and station 1 are identical since the area is constant and neither heat nor work is transferred across the boundaries between these two locations. Using $\gamma = 1.4$ the stagnation pressure, stagnation temperature, and velocity are given by

$$T_{t,2} = T_2\left(1+\frac{\gamma-1}{2}M_2^2\right) = 288\times\left(1+\frac{1.4-1}{2}\times 0.4^2\right) = 297\,\text{K}$$

$$p_{t,2} = p_2\left(1+\frac{\gamma-1}{2}M_2^2\right)^{\frac{\gamma}{\gamma-1}} = 101\times\left(1+\frac{1.4-1}{2}\times 0.4^2\right)^{\frac{1.4}{1.4-1}} = 113\,\text{kPa}$$

$$V_2 = M_2 a_2 = \sqrt{\gamma R T_2}M_2 = \sqrt{1.4\times 287\times 288}\times 0.4 = 136\,\text{m/s}$$

Part c: If the work added is $-dW_{k,3}$ = 169.3 kJ/kg and c_p = 1 kJ/kg·K, we first compute constant *C* using equation (V):

$$C = \frac{\left(1+\gamma M_2^2\right)^2}{\gamma M_2^2}T_2 = \frac{\left(1+1.4\times 0.4^2\right)^2}{1.4\times 0.4^2}\times 288 = 1930\,\text{K}$$

Then, we substitute in (VII):

$$-169{,}300 = 1000\times 1930\times\left[\frac{1+1.4\times\left(2M_3^2-1\right)}{2\times 1.4\times\left(1+1.4\times M_3^2\right)^2} - \frac{1+1.4\times\left(2\times 0.4^2-1\right)}{2\times 1.4\times\left(1+1.4\times 0.4^2\right)^2}\right]$$

We can easily solve the equation above for M_3 using Mathematica's *FindRoot* command:

```
In[327]:= FindRoot[-169 300 + 1000 * 1930 * (  (1 + 1.4 * (2 * M3^2 - 1)) / (2 * 1.4 * (1 + 1.4 * M3^2)^2) - (1 + 1.4 * (2 * 0.4^2 - 1)) / (2 * 1.4 * (1 + 1.4 * 0.4^2)^2) ),
    {M3, 0.5}]

Out[327]= {M3 → 0.610207}
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Thus, $M_3 = 0.610$. We proceed to compute pressure p_3 using equation (II.1):

$$\frac{p_3}{p_2} = \frac{1+\gamma_2 M_2^2}{1+\gamma_3 M_3^2} \rightarrow \frac{p_3}{101} = \frac{1+1.4\times 0.4^2}{1+1.4\times 0.610^2}$$

$$\therefore p_3 = 0.805\times 101 = 81.3\,\text{kPa}$$

Then, using equation (IV) gives temperature T_3:

$$T_3 = C\frac{\gamma M_3^2}{\left(1+\gamma M_3^2\right)^2} = 1930\times\frac{1.4\times 0.610^2}{\left(1+1.4\times 0.610^2\right)^2} = 435\,\text{K}$$

The velocity is

$$V_3 = a_3 M_3 = \sqrt{\gamma R T_3}M_3 = \sqrt{1.4\times 287\times 435}\times 0.610 = 255\,\text{m/s}$$

The density can be found from the equation of state, namely

$$\rho_3 = \frac{p_3}{RT_3} = \frac{81{,}300}{287\times 435} = 0.651\,\text{kg/m}^3$$

Lastly, the stagnation temperature and pressure can be found from the isentropic flow equations

$$T_{t,3} = T_3\left(1+\frac{\gamma-1}{2}M_3^2\right) = 435\times\left(1+\frac{1.4-1}{2}\times 0.610^2\right) = 467\,\text{K}$$

$$p_{t,3} = p_3\left(1+\frac{\gamma-1}{2}M_3^2\right)^{\frac{\gamma}{\gamma-1}} = 81.3\times\left(1+\frac{1.4-1}{2}\times 0.610^2\right)^{\frac{1.4}{1.4-1}} = 105\,\text{kPa}$$

■ P2.7

Part a: The pressure of flow in the duct rises from 69 kPa to 226 kPa. The speed of sound at the entrance is $a_1 = (\gamma R T_1)^{1/2} = (1.4 \times 287 \times 297)^{1/2} = 345$ m/s, so that $V_1 = M_1 a_1 = 3 \times 345 = 1040$ m/s. Since $V_2 = 850$ m/s $< V_1$, the flow velocity is decreasing down the duct. The temperature T_2 at the exit is, noting that $\rho_1 V_1 = \rho_2 V_2$ (conservation of mass),

$$T_2 = \left(\frac{p_2}{p_1}\right)\left(\frac{V_2}{V_1}\right)T_1 = \left(\frac{226}{69}\right)\times\left(\frac{850}{1040}\right)\times 297 = 795\,\text{K}$$

Then, the speed of sound at the exit is determined as

$$a_2 = \sqrt{\gamma R T_2} = \sqrt{1.4\times 287\times 795} = 565\,\text{m/s}$$

so that

$$M_2 = \frac{V_2}{a_2} = \frac{850}{565} = 1.50$$

To summarize, pressure p and temperature T are increasing along the duct, while velocity V is decreasing; further, Mach number M is shifting from a greater value to a lower (but still supersonic) one. Therefore, the flow cannot be a normal shock flow but can be either a Fanno and Rayleigh flow. In a Fanno flow the process is adiabatic so the stagnation temperature must be constant and such that $T_t = [1 + (\gamma + 1)M^2/2]T$. Carrying out this calculation for the entry and exit gives $T_{t,1} =$ 832 K and $T_{t,2} = 1160$ K; since $T_{t,1} \neq T_{t,2}$, Fanno flow is ruled out as well. The only remaining possibility is a Rayleigh flow with heat addition $Q = c_{p,2}T_{t,2} - c_{p,1}T_{t,1} = 1.17 \times 1160 - 1.11 \times 832 = 434$ kJ/kg assuming appropriate c_p values drawn from Appendix B of the text.

Part b: The equations of motion for Rayleigh flow are

$$\begin{cases} \rho V = \text{const.} \\ dp + \rho V dV = 0 \\ c_p dT + d\left(u^2/2\right) = Q \end{cases}$$

Part c: The Mach number at station 2 was determined in part (*a*) and equals 1.50.

Part d: Because this is a flow with simple heat addition the pertinent flow property is Q = 434 kJ/kg, as determined in part *a*.

Part e: If p_2 is now 712 kPa, it appears likely that *the flow would have to be a normal shock flow*. *i.e. an adiabatic flow with* Q = 0. Conservation of mass leads to

$$p_1 M_1 \left(\frac{\gamma}{RT_1}\right)^{\frac{1}{2}} = p_2 M_2 \left(\frac{\gamma}{RT_2}\right)^{\frac{1}{2}} \rightarrow 69 \times 3 \times \left(\frac{1.4}{287 \times 297}\right)^{\frac{1}{2}} = 712 \times M_2 \times \left(\frac{1.4}{287 \times 795}\right)^{\frac{1}{2}}$$

$$\therefore M_2 = 0.476$$

which is subsonic. To determine if the stated pressure p_2 = 712 kPa is the pressure behind a normal shock at upstream Mach number M_1 = 3 and upstream pressure p_1 = 69 kPa, we may refer to the normal shock table in Appendix A of the textbook and find that p_2/p_1 = 712/69 = 10.3 indeed corresponds to a normal shock at entrance Mach number M_1 = 3.0. Also from the table, the temperature ratio is T_2/T_1 = 2.679, giving T_2 = 2.679 × 297 = 796 K.

[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
3.00	.2722	−1	.7623	−1	.3571	2.828	.1715	4.235	1.96396	49.757	19.47	.4752	10.33	3.857
3.01	.2682	−1	.7541	−1	.3556	2.839	.1701	4.275	1.96629	49.950	19.40	.4746	10.40	3.866
3.02	.2642	−1	.7461	−1	.3541	2.850	.1687	4.316	1.96861	50.142	19.34	.4740	10.47	3.875
3.03	.2603	−1	.7382	−1	.3526	2.860	.1673	4.357	1.97091	50.333	19.27	.4734	10.54	3.884
3.04	.2564	−1	.7303	−1	.3511	2.871	.1659	4.399	1.97319	50.523	19.20	.4729	10.62	3.893

■ P3.1

Free stream, Station 0: With reference to the 1976 US standard atmosphere, we find that σ = 0.224, giving p_0 = 0.224 × 101,325 = 22.7 kPa. Also, T_0 = 217 K, a_0 = 295 m/s, and $V_0 = a_0 M_0$ = 295 × 0.9 = 266 m/s. For air, γ = 1.4 and the stagnation-to-static pressure ratio reads

$$\frac{p_t}{p} = \left(1 + \frac{\gamma - 1}{2} M_0^2\right)^{\frac{\gamma}{\gamma - 1}}$$

$$\therefore \frac{p_t}{p} = \left(1 + \frac{1.4 - 1}{2} \times 0.9^2\right)^{\frac{1.4}{1.4 - 1}} = 1.69$$

$$\therefore p_{t,0} = 1.69 \times p_0 = 1.69 \times 22.7 = 38.4\,\text{kPa}$$

The stagnation-to-static temperature ratio, in turn, is

$$\frac{T_t}{T} = 1 + \frac{\gamma - 1}{2} M_0^2$$

$$\therefore \frac{T_t}{T} = 1 + \frac{1.4 - 1}{2} \times 0.9^2 = 1.16$$

$$\therefore T_{t,0} = 1.16 \times T_0 = 1.16 \times 217 = 252\,\text{K}$$

Inlet, Stations 1-2: The MIL-E-5008B standard pressure recovery for M_0 = 0.9 is $p_{t,2}/p_{t,0}$ = 1.0 and the inlet flow is assumed to be adiabatic so that $T_{t,2}/T_{t,0}$ = 1. Therefore,

$$p_{t,2} = 38.4\,\text{kPa}$$
$$T_{t,2} = 252\,\text{K}$$

Compressor, Stations 2-3: The pressure ratio is given as $p_{t,3}/p_{t,2}$ = 18 and assuming a constant effective specific heat $c_{p2} = c_{p3} = c_{p,e}$ and γ = 1.4 the work done is given by the relation

$$W_C = \frac{c_p T_{t,2}\left[\left(p_{t3}/p_{t2}\right)^{0.286} - 1\right]}{\eta_c} \quad \text{(I)}$$

The approximate equation for the equivalent specific heat of air is given as

$$c_{p,e} = 0.94 + 0.2\left(T_{\text{avg}}/1000\right)$$

Likewise, the ratio of specific heats for air can be approximated as

$$\gamma = 1.4 - 0.006\left(T_{\text{avg}}/1000\right)$$

In the two previous correlations, $T_{avg} = (T_{t,3} + T_{t,2})/2$, but temperature $T_{t,3}$ is missing. This can be found from the compressor pressure ratio, recalling that

$$\left(\frac{T_{t,3}}{T_{t,2}}\right) = \left(\frac{p_{t,3}}{p_{t,2}}\right)^{(\gamma-1)/\gamma} \rightarrow T_{t,3} = T_{t,2}\left(\frac{p_{t,3}}{p_{t,2}}\right)^{(\gamma-1)/\gamma}$$

$$\therefore T_{t,3} = 252 \times (18)^{(1.4-1)/1.4} = 576\,\text{K}$$

so that $T_{avg} = (576 + 252)/2 = 414$ K and

$$c_{p,e} = 0.94 + 0.2 \times \left(414/1000\right) = 1.02\,\text{kJ/kg}\cdot\text{K}$$

Also,

$$\gamma = 1.4 - 0.006 \times \left(414/1000\right) = 1.398$$

Substituting in (I) brings to

$$W_C = \frac{1.02 \times 252 \times \left[18^{0.286} - 1\right]}{0.85} = 389\,\text{kJ/kg}$$

We can obtain a better approximation for the actual compressor exit temperature $T_{t,3}$ by using the relationship

$$W_C = c_p\left(T_{t,3} - T_{t,2}\right) \rightarrow T_{t,3} = \frac{W_C}{c_p} + T_{t,2}$$

$$\therefore T_{t,3} = \frac{389}{1.02} + 252 = 633\,\text{K}$$

This is about 10% greater than the 576-K result obtained just now.

Combustor, Stations 3-4: We first establish an updated average temperature T_{avg}, namely

$$T_{t,\text{avg}} = \frac{\left(T_{t,4} + T_{t,3}\right)}{2} = \frac{1330 + 633}{2} = 982\,\text{K}$$

Now, the specific heat is approximated by the slightly modified correlation

$$c_{pe} = 0.94 + 0.215\left(T_{t,\text{avg}}/1000\right)$$

$$\therefore c_{pe} = 0.94 + 0.215 \times \left(982/1000\right) = 1.15\,\text{kJ/kg}\cdot\text{K}$$

Likewise for the ratio of specific heats,

$$\gamma = 1.4 - 0.0667\left(T_{\text{t,avg}}/1000\right)$$

$$\therefore \gamma = 1.4 - 0.0667 \times \left(982/1000\right) = 1.34$$

Now, assuming that the liquid fuel enthalpy is negligible with respect to the gas enthalpy, the energy balance across the combustor is

$$\left[1 + (f/a)\right]c_{p,e}T_{t,4} = c_{p,e}T_{t,3} + (f/a)\eta_b Q_f$$

$$\therefore \left[1 + \left(\frac{f}{a}\right)\right] \times 1.15 \times 1330 = 1.15 \times 633 + \left(\frac{f}{a}\right) \times 0.96 \times 44{,}000$$

$$\therefore \frac{f}{a} = 0.0197$$

so that

$$\frac{\dot{m}_f}{\dot{m}_0} = \frac{f}{a} \rightarrow \dot{m}_f = (f/a)\dot{m}_0$$

$$\therefore \dot{m}_f = 0.0197 \times 29.17 = 0.575\,\text{kg/s}$$

We were told that the stagnation pressure loss in the combustor is ($p_{t,3}$ – $p_{t,4}$) = 0.02$p_{t,3}$, so

$$p_{t,4} = (1.0 - 0.02)\,p_{t,3} = 0.98\,p_{t,3}$$

$$\therefore\ p_{t,4} = 0.98 \times (18 \times 38.4) = 677\,\text{kPa}$$

Turbine, Stations 4-5: The matching condition for the turbine is that the work it extracts is equivalent to the work required by the compressor so $W_C = (1 + (f/a))W_T$. This expression accounts for the additional mass flow arising from the combustion of the fuel. However, it is common to bleed air from the compressor for various vehicle requirements like air conditioning, etc., and engines are designed to provide such airflow with the bleed rate typically kept to less than 5% of the total air flow. As a result, it is necessary to keep this in mind for detailed calculations. Keeping the increased mass flow due to the addition of fuel we obtain

$$W_T = c_{p,e}\left(T_{t,4} - T_{t,5}\right) = 1.15 \times \left(1330 - T_{t,5}\right)$$

$$W_T = \frac{W_C}{1 + \left(\frac{f}{a}\right)} = \frac{389}{1 + 0.0197} = 381\,\text{kJ/kg}$$

Combining the two previous equations and solving for $T_{t,5}$, we obtain $T_{t,5} \approx$ 999 K.

```
In[150]:= Solve[1.15 * (1330 - Tt5) == 381, Tt5]

Out[150]= {{Tt5 → 998.696}}
```

Now, we can improve upon the calculations above by taking $T_{t,avg}$ = (1330 + 999)/2 = 1160 K and computing $c_{p,e}$ = 0.94 + 0.215 × (1160/1000) = 1.19 kJ/kg·K and γ = 1.4 – 0.0667 × (1160/1000) = 1.32. Using this updated $c_{p,e}$ brings to

$$W_T = c_{p,e}\left(T_{t,4} - T_{t,5}\right) = 1.19 \times \left(1330 - T_{t,5}\right)$$

$$W_T = \frac{W_C}{1 + \left(\frac{f}{a}\right)} = \frac{389}{1 + 0.0197} = 381\,\text{kJ/kg}$$

```
In[154]:= Solve[1.19 * (1330 - Tt5) == 381, Tt5]

Out[154]= {{Tt5 → 1009.83}}
```

That is, $T_{t,5}$ = 1010 K. The stagnation pressure at the turbine exit may be found from the turbine work equation

$$W_T = \eta_T c_{p,e} T_{t,4}\left[1 - \left(\frac{p_{t,4}}{p_{t,5}}\right)^{-0.244}\right]$$

$$\therefore 381 = 0.90 \times 1.19 \times 1330\left[1 - \left(\frac{677}{p_{t,5}}\right)^{-0.244}\right]$$

```
In[155]:= Solve[381 == 0.90 * 1.19 * 1330 * (1 - (677 / pt5)^-0.244), pt5]

   Solve: Inverse functions are being used by Solve, so some solutions may n
   solution information.

Out[155]= {{pt5 → 189.052}}
```

As shown in the Mathematica code, $p_{t,5}$ = 189 kPa.

Nozzle, Stations 5 – 6: The nozzle pressure ratio p_{t5}/p_0 = 189/22.7 = 8.33, so the nozzle is clearly operating in a supercritical state and therefore will be choked with M_6 = 1.0. The exit pressure will be the critical pressure p^* and the ratio of specific heats is chosen to be γ = 1.33, consistent with an expected average temperature in the nozzle of about 900 K. The pressure and temperature at any point in the nozzle are then given by the isentropic relationships

$$\frac{p}{p_{t,5}} = \left(1 + 0.165M^2\right)^{-4.03}$$

$$\frac{T}{T_{t,5}} = \left(1 + 0.165M^2\right)^{-1}$$

Setting M = 1 and substituting $p_{t,5}$ = 189 kPa in the former equation brings to

$$\frac{p}{p_{t,5}} = \left(1 + 0.165M^2\right)^{-4.03} \rightarrow p = \left(1 + 0.165 \times 1.0^2\right)^{-4.03} p_{t,5}$$

$$\therefore p_6 = p^* = \left(1 + 0.165 \times 1.0^2\right)^{-4.03} \times 189 = 102\,\text{kPa}$$

Substituting in the latter equation, in turn, gives

$$\frac{T}{T_{t,5}} = \left(1 + 0.165M^2\right)^{-1} \rightarrow T_6 = \left(1 + 0.165 \times 1.0^2\right)^{-1} T_{t,5}$$

$$\therefore T_6 = \left(1 + 0.165 \times 1.0^2\right)^{-1} \times 1010 = 867\,\text{K}$$

The exit velocity is determined next,

$$V_6 = a^* = \left(\gamma R T^*\right)^{1/2} = \left(1.33 \times 287 \times 867\right)^{1/2} = 575\,\text{m/s}$$

The density is found from the equation of state

$$\rho^* = \frac{p^*}{RT^*} = \frac{102{,}000}{287 \times 867} = 0.410\,\text{kg/m}^3$$

Lastly, the nozzle exit area required can be obtained from conservation of mass:

$$\dot{m}_6 = \rho_6 A_6 V_6 \rightarrow A_6 = \frac{\dot{m}_6}{\rho_6 V_6}$$

$$\therefore A_6 = \frac{\dot{m}_6}{\rho_6 V_6} = \frac{\left(1.0197 \times 29.17\right)}{0.410 \times 575} = 0.126\,\text{m}^2$$

Performance: The gross thrust is given by

$$F_g = \dot{m}_6 V_6 + A_6\left(p_6 - p_0\right) = 29.17 \times 575 + 0.124 \times \left(102{,}000 - 22{,}700\right)$$

$$\therefore F_g = 26{,}800\,\text{N}$$

$$\therefore F_g = 26.8\,\text{kN}$$

The ram drag is

$$F_r = \dot{m}_0 V_0 = 29.17 \times 266 = 7760\,\text{N}$$

The net thrust is

$$F_n = 26.8 - 7.76 = 19.0\,\text{kN}$$

The specific fuel consumption is

$$c_j = \frac{\dot{m}_f}{F_n} = \frac{0.575}{19{,}000} = 3.03 \times 10^{-5}\,\text{kg/s/N} = 0.109\,\text{kg/hr/N}$$

Lastly, the overall efficiency of the engine is calculated to be

$$\eta_o = \frac{F_n V_0}{\dot{m}_f Q_f} = \frac{19.0 \times 266}{0.575 \times 44{,}000} = 0.200$$

$$\therefore \eta_o = 20.0\%$$

■ P3.2

Free Stream, Station 0: We were told that the turbofan craft is operating under the same conditions as the turbojet investigated in the previous problem. Thus, p_0 = 22.7 kPa, T_0 = 217 K, a_0 = 295 m/s, and V_0 = 266 m/s. Also in accordance with the previous problem, $p_{t,0}$ = 38.4 kPa and $T_{t,0}$ = 252 K. The total mass flow, core flow plus fan flow, is 50 kg/s, and the bypass ratio β = 1, hence the mass flow consists of 25 kg/s through the core engine and 25 kg/s through the fan.

Inlet, Stations 1 – 2: The MIL-E-5008B standard pressure recovery for M_0 = 0.9 is $p_{t,2}/p_{t,0}$ = 1.0 and the inlet flow is assumed to be adiabatic, giving $T_{t,2}/T_{t,0}$ = 1. It follows that $p_{t,2}$ = 38.4 kPa and $T_{t,2}$ = 252 K.

Compressor, Stations 2 – 3: The pressure ratio is given as $p_{t,3}/p_{t,2}$ = 18 and, assuming the calculations applied there also apply here, the specific heat and γ ratio may be taken as $c_{p,2} = c_{p,3} = c_{p,e}$ = 1.02 kJ/kg·K and γ = 1.398, respectively. The work done by the compressor remains unchanged at 389 kJ/kg, and temperature $T_{t,3}$ can be updated to 633 K in much the same way as in the turbojet problem. The power absorbed by the compressor is

$$P_C = \dot{m}W_C = 25\times 389 = 9725\,\text{kW} = 9.73\,\text{MW}$$

Fan, Stations 2 – 2.5: The fan pressure ratio is given as $p_{t,2.5}/p_{t,2}$ = 2.2 and, because of the low pressure ratio we assume constant specific heat $c_{p,2} = c_{p,2.5}$ = 1.02 kJ/kg·K and constant γ = 1.4. The work done is given by either $W_F = c_pT_{t,2}[(p_{t,2.5}/p_{t,2})^{0.286} - 1]/\eta_F$ or $W_F = c_p(T_{t,2.5} - T_{t,2})$. We first compute pressure $p_{t,2.5}$,

$$p_{t,2.5} = 2.2p_{t,2} = 2.2\times 38.4 = 84.5\,\text{kPa}$$

so that

$$W_F = \frac{1.02\times 252\times\left[2.2^{0.286}-1\right]}{0.85} = 76.5\,\text{kJ/kg}$$

Temperature $T_{t,2.5}$ easily follows,

$$T_{t,2.5} = \frac{W_F}{c_p} + T_{t,2} = \frac{76.5}{1.02} + 252 = 327\,\text{K}$$

The power absorbed by the fan is

$$P_F = \dot{m}W_F = 25\times 76.5 = 1912\,\text{kW} = 1.91\,\text{MW}$$

Combustor, Stations 3 – 4: As in the turbojet problem, we first compute an updated average temperature T_{avg}, namely

$$T_{t,\text{avg}} = \frac{\left(T_{t,4}+T_{t,3}\right)}{2} = \frac{1330+633}{2} = 982\,\text{K}$$

whence we can find specific heat $c_{p,e}$ = 1.15 kJ/kg·K and ratio γ = 1.34. Assuming that the liquid fuel enthalpy is negligible with respect to the gas enthalpy the energy balance across the combustor gives

$$\left[1+\left(f/a\right)\right]c_{p,e}T_{t,4} = c_{p,e}T_{t,3} + \left(f/a\right)\eta_b Q_f$$

$$\therefore \left[1+\left(\frac{f}{a}\right)\right]\times 1.15\times 1330 = 1.15\times 633 + \left(\frac{f}{a}\right)\times 0.96\times 44{,}000$$

$$\therefore \frac{f}{a} = 0.0197$$

so that

$$\dot{m}_f = 0.0197\times 25 = 0.493\,\text{kg/s}$$

We were told that the stagnation pressure loss in the combustor is $(p_{t,3} - p_{t,4}) = 0.02p_{t,3}$, so

$$p_{t,4} = \left(1.0-0.02\right)p_{t,3} = 0.98p_{t,3}$$
$$\therefore p_{t,4} = 0.98\times\left(18\times 38.4\right) = 677\,\text{kPa}$$

Turbine, Stations 4 – 5: The same reasoning used in the turbojet problem also applies here. The turbine work may be expressed as

$$W_T = c_{p,e}\left(T_{t,4}-T_{t,5}\right) = 1.15\times\left(1330-T_{t,5}\right)\ \text{(I)}$$

Alternatively, for a turbofan:

$$W_T = \frac{\left(W_C+\beta W_F\right)}{1+\left(\dfrac{f}{a}\right)} = \frac{\left(389+1.0\times 76.5\right)}{1+0.0197} = 457\,\text{kJ/kg}$$

Substituting in (I) and solving for $T_{t,5}$

$$W_T = 1.15 \times (1330 - T_{t,5}) = 457$$
$$\therefore T_{t,5} = 933 \text{ K}$$

Using this temperature, we may compute an updated $T_{t,avg}$ = (1330 + 933)/2 = 1130 K, leading to an approximate specific heat $c_{p,e}$ = 0.94 + 0.215 × 1130/1000 = 1.18 kJ/kg·K. Substituting in (I) brings to

$$W_T = c_{p,e}(T_{t,4} - T_{t,5}) = 1.18 \times (1330 - T_{t,5}) = 457$$
$$\therefore T_{t,5} = 943 \text{ K}$$

The stagnation pressure at the turbine exit may be found from the turbine work equation. Using an average value γ = 1.4 – 0.0667 × (1130/1000) = 1.325, we have

$$W_T = \eta_t c_{p,e} T_{t,4}\left[1 - (p_{t,4}/p_{t,5})^{-0.251}\right] = 457 \text{ kJ/kg}$$
$$\therefore W_T = 0.90 \times 1.18 \times 1330 \times \left[1 - (677/p_{t,5})^{-0.251}\right] = 457 \text{ kJ/kg}$$
$$\therefore p_{t,5} = 143 \text{ kPa}$$

Core Nozzle, Stations 5 – 6: The nozzle pressure ratio $p_{t,5}/p_0$ = 143/22.7 = 6.30, so the nozzle is clearly operating in a supercritical state and therefore will be choked with M_6 = 1.0. The exit pressure will be the critical pressure p^* and the ratio of specific heats is chosen to be γ = 1.33, consistent with an expected average temperature in the nozzle of about 900 K. The pressure and temperature at any point in the nozzle are then given by the isentropic relationships

$$\frac{p}{p_{t,5}} = (1 + 0.165M^2)^{-4.03}$$
$$\frac{T}{T_{t,5}} = (1 + 0.165M^2)^{-1}$$

Setting M = 1 and substituting $p_{t,5}$ = 189 kPa in the former equation brings to

$$\frac{p}{p_{t,5}} = (1 + 0.165M^2)^{-4.03} \rightarrow p = (1 + 0.165 \times 1.0^2)^{-4.03} p_{t,5}$$
$$\therefore p_6 = p^* = (1 + 0.165 \times 1.0^2)^{-4.03} \times 143 = 77.3 \text{kPa}$$

Substituting in the latter equation, in turn, gives

$$\frac{T}{T_{t,5}} = (1 + 0.165M^2)^{-1} \rightarrow T_6 = (1 + 0.165 \times 1.0^2)^{-1} T_{t,5}$$
$$\therefore T_6 = (1 + 0.165 \times 1.0^2)^{-1} \times 943 = 809 \text{K}$$

The exit velocity is determined next,

$$V_6 = a^* = (\gamma R T^*)^{1/2} = (1.33 \times 287 \times 809)^{1/2} = 556 \text{m/s}$$

The density is found from the equation of state

$$\rho^* = \frac{p^*}{RT^*} = \frac{77,300}{287 \times 809} = 0.333 \text{kg/m}^3$$

Finally, the nozzle exit area required can be obtained from conservation of mass:

$$\dot{m}_6 = \rho_6 A_6 V_6 \rightarrow A_6 = \frac{\dot{m}_6}{\rho_6 V_6}$$
$$\therefore A_6 = \frac{\dot{m}_6}{\rho_6 V_6} = \frac{(1.0197 \times 25)}{0.333 \times 556} = 0.138 \text{m}^2$$

Fan Nozzle, Stations 2.5 – 3F: The fan nozzle pressure ratio $p_{t,2.5}/p_0$ = 84.5/22.7 = 3.72, so the nozzle is clearly operating in a supercritical state and therefore will be choked with exit Mach number $M_{5f} = 1.0$. Because the fan operates at relatively cool temperatures we take c_p = const. = 1.004 kJ/kg·K and γ = 1.4. The pressure and temperature at any point in the nozzle are given by the isentropic relations $p/p_{t,2.5} = (1 + 0.2M^2)^{-3.5}$ and $T/T_{t,2.5} = (1 + 0.2M^2)^{-1}$. The fan exit pressure is then $p_{3F} = p^* = 0.528p_{t,2.5} = 0.528 \times 84.5 = 44.6$ kPa and the fan exit temperature is $T_{3F} = T^* = T_{t,2.5}/1.2 = 327/1.2 = 273$ K. The exit velocity is $V_{3F} = a^* = (\gamma RT^*)^{1/2} = (1.4 \times 287 \times 273)^{1/2} = 331$ m/s. The density is found from the equation of state

$$\rho^* = \frac{p^*}{RT^*} = \frac{44,600}{287 \times 273} = 0.569\,\text{kg/m}^3$$

Noting that the mass flow rate into the fan was given as 25 kg/s, the exit area required becomes

$$\dot{m}_{3F} = \rho_{3F} A_{3F} V_{3F} \rightarrow A_{3F} = \frac{\dot{m}_{3F}}{\rho_{3F} V_{3F}}$$

$$\therefore A_{3F} = \frac{25}{0.569 \times 331} = 0.133\,\text{m}^2$$

Performance: The gross thrust is given by

$$F_g = \dot{m}_6 V_6 + A_6 \left(p_6 - p_0\right) + \dot{m}_F V_{3F} + A_{3F}\left(p_{3F} - p_0\right)$$

$$\therefore F_g = \begin{bmatrix} (1.0197 \times 25) \times 556 + 0.138 \times (77,300 - 22,700) \\ +25 \times 331 + 0.133 \times (44,600 - 22,700) \end{bmatrix} = 32,900\,\text{N}$$

$$\therefore F_g = 32.9\,\text{kN}$$

The ram drag is

$$F_r = \dot{m}_0 V_0 = 50 \times 266 = 13,300\,\text{N} = 13.3\,\text{kN}$$

The net thrust is

$$F_n = 32.9 - 13.3 = 19.6\,\text{kN}$$

The specific fuel consumption is

$$c_j = \frac{\dot{m}_f}{F_n} = \frac{0.493}{19,600} = 2.55 \times 10^{-5}\,\text{kg/s/N} = 0.0918\,\text{kg/hr/N}$$

Finally, the overall efficiency of the engine is determined to be

$$\eta_o = \frac{F_n V_0}{\dot{m}_f Q_f} = \frac{19.6 \times 266}{0.493 \times 44,000} = 0.240$$

$$\therefore \eta_o = 24.0\%$$

■ P3.3

The pertaining quantities are compared below.

Variable	Turbojet (Problem 3.1)	Turbofan (Problem 3.2)
Net thrust, F_n	19.0 kN	19.6 kN
Specific fuel cons., c_j	0.109 kg/hr/N	0.0918 kg/hr/N
Overall efficiency, η_o	20.0%	24.0%
Air flow, $\dot{m}$	29.17 kg/s	50 kg/s

As can be seen, the turbofan offers about the same net thrust as the turbojet, but at a 16% lower SFC. What's more, the overall efficiency of the turbofan is 4 percentage points greater than that of the turbojet.

■ P3.4

The results obtained from stations 1 to 5 remain unchanged relatively to those of Problem 3.1, so we may begin the calculations with the afterburner.

Afterburner, Stations 5-5b: The total pressure at the exit of the afterburner, station 5b, may be found from the specified loss equation $(p_{t,5} - p_{t,5b}) = 0.05p_{t,5}$ so that $p_{t,5b} = 0.95p_{t,5} = 0.95 \times 189 = 180$ kPa. The total temperature at the afterburner exit $T_{t,5b}$ is given by the energy balance

$$\left(\dot{m}_o + \dot{m}_f\right) c_{p,e} T_{t,5} + \dot{m}_{f,AB} \eta_{AB} Q_{eff} = \left(\dot{m}_0 + \dot{m}_f + m_{f,AB}\right) c_{p,e} T_{t,5b}$$

The heating value in the afterburner, Q_{eff}, is lower than Q_f because the flow into the afterburner has had some oxygen depletion due to fuel burn in the main combustor. This can be approximated by Equation (3.26) in the text:

$$Q_{eff} = \left[1 - 14.7\left(f/a\right)\right]Q_f$$

The afterburner fuel flow is given as 1.17 kg/s, so that $(f/a)_{AB}$ = 1.17/29.17 = 0.0401. Then, using the specific heat $c_{p,e}$ = 1.19 kJ/kg·K obtained in the analysis of the turbine in Prob. 3.1, the energy balance becomes

$$\left[1+\left(\frac{f}{a}\right)\right]c_{p,e}T_{t,5} + \left(\frac{f}{a}\right)_{AB}\eta_{AB}\left[1-14.7\left(\frac{f}{a}\right)\right]Q_f = \left[1+\left(\frac{f}{a}\right)+\left(\frac{f}{a}\right)_{AB}\right]c_{p,e}T_{t,5b}$$

$$\therefore \left[1+0.0197\right]\times 1.19\times 1010 + 0.0401\times 0.85\times\left[1-14.7\times 0.0197\right]\times 44{,}000$$
$$= \left[1+0.0197+0.0401\right]\times 1.19\times T_{t,5b}$$

```
In[218]:= Solve[ (1 + 0.0197) * 1.19 * 1010 + 0.0401 * 0.85 * (1 - 14.7 * 0.0197) * 44 000 ==
             (1 + 0.0197 + 0.0401) * 1.19 * Tt5b, Tt5b]
Out[218]= {{Tt5b → 1816.58}}
```

Then, as a first approximation, $T_{t,5b}$ = 1820 K, so we may compute the average temperature $T_{t,avg}$ = (1010 + 1820)/2 = 1420 K and update the specific heat as $c_{p,e}$ = 0.94 + 0.215 × (1420/1000) = 1.25 kJ/kg·K. Substituting this $c_{p,e}$ into the foregoing equation and solving for $T_{t,5b}$ a second time yields $T_{t,5b}$ = 1780 K, as shown in the following code snippet.

```
In[221]:= Solve[ (1 + 0.0197) * 1.25 * 1010 + 0.0401 * 0.85 * (1 - 14.7 * 0.0197) * 44 000 ==
             (1 + 0.0197 + 0.0401) * 1.25 * Tt5b, Tt5b]
Out[221]= {{Tt5b → 1776.03}}
```

Nozzle, Stations 5 – 6: The nozzle pressure ratio $p_{t,5b}/p_0$ = 180/22.7 = 7.93, so the nozzle is clearly operating in a supercritical state and therefore will be choked with M_6 = 1.0. The exit pressure will be the critical pressure p^* and the ratio of specific heats is chosen to be γ = 1.4 – 0.0667$T_{t,avg}$ = 1.29, consistent with an average temperature in the nozzle of about 1600 K. The pressure and temperature at any point in the nozzle are given by the isentropic relations $p/p_{t,5}$ = $(1 + 0.145M^2)^{-4.45}$ and $T/T_{t,5}$ = $(1 + 0.145M^2)^{-1}$. Therefore the exit pressure is, substituting M = 1,

$$\frac{p}{p_{t,5}} = \left(1+0.145\times 1^2\right)^{-4.45} = 0.547$$
$$\therefore p_6 = 0.547p^* = 0.547\times 180 = 98.5\,\text{kPa}$$
$$\frac{T}{T_{t,5}} = \left(1+0.145\times 1^2\right)^{-1} = 0.873$$
$$\therefore T_6 = 0.873T^* = 0.873\times 1780 = 1550\,\text{K}$$

The exit velocity is, in turn,

$$V_6 = a^* = \left(\gamma RT^*\right)^{1/2} = \left(1.29\times 287\times 1550\right)^{1/2} = 758\,\text{m/s}$$

The density is found from the equation of state

$$\rho^* = \frac{p^*}{RT^*} = \frac{98{,}500}{287\times 1550} = 0.221\,\text{kg/m}^3$$

Finally, the nozzle exit area required can be obtained from conservation of mass:

$$\dot{m}_6 = \rho_6 A_6 V_6 \rightarrow A_6 = \frac{\dot{m}_6}{\rho_6 V_6}$$

$$\therefore A_6 = \frac{\dot{m}_6}{\rho_6 V_6} = \frac{\left[\left(1+0.0197+0.0401\right)\times 29.17\right]}{0.221\times 758} = 0.185\,\text{m}^2$$

Performance: The gross thrust is given by

$$F_g = \dot{m}_6 V_6 + A_6(p_6 - p_0) = [(1+0.0197+0.0401)\times 29.17]\times 758 + 0.185\times(95{,}800 - 22{,}700)$$
$$\therefore F_g = 37.0\,\text{kN}$$

The ram drag is

$$F_r = \dot{m}_0 V_0 = 29.17\times 266 = 7760\,\text{N}$$

The net thrust is

$$F_n = 37.0 - 7.76 = 29.2\,\text{kN}$$

The total mass flow of fuel is (0.0197 + 0.0401) × 29.17 = 1.74 kg/s. The specific fuel consumption is

$$c_j = \frac{\dot{m}_f}{F_n} = \frac{1.74}{29{,}200} = 5.96\times 10^{-5}\,\text{kg/s/N} = 0.215\,\text{kg/hr/N}$$

Lastly, the overall efficiency of the engine is calculated to be

$$\eta_o = \frac{F_n V_0}{\dot{m}_f Q_f} = \frac{29.2\times 266}{1.74\times 44{,}000} = 0.101$$
$$\therefore \eta_o = 10.1\%$$

■ P3.5

The results obtained from stations 1 to 6 remain unchanged relatively to those of Problem 3.2, so we may begin the calculations with the afterburner section. The bypass ratio was given as $\beta = 1$, and results from Prob. 3.2 include $(f/a) = 0.0197$, $p_{t,5} = 143$ kPa, $T_{t,5} = 943$ K, and $T_{t,F} = 327$ K.

Afterburner, Stations 5 – 5b: The fan flow is assumed to mix internally with the core flow and the mass-averaged stagnation pressure entering the afterburner is now $p_{t,5} = 0.5 \times (84.5 + 143) = 114$ kPa. The energy balance across the afterburner requires the sum of the total enthalpies of the fan flow (subscript F) and the core flow (subscript C) plus the energy released by combustion of the afterburner fuel to be equal to the total enthalpy passing through the nozzle and is given by

$$\dot{m}_{0,F} h_{t,5F} + [1 + (f/a)]\dot{m}_{0,C} h_{t,5} + \dot{m}_{f,AB}\eta_b Q_{eff} = \dot{m}_{5,b} h_{t,5b}$$

Note that the ratio of the fuel flow in the afterburner to the core air flow may be written as

$$\frac{\dot{m}_{f,AB}}{\dot{m}_{0,C}} = \frac{\dot{m}_{f,AB}}{\dot{m}_{0,C} + \dot{m}_{0,F}}(1+\beta) = 2(f/a)_{AB} = 2\times\left(\frac{1}{50}\right) = 0.04$$

Similarly, the ratio of the mass flow entering the nozzle to the core airflow is

$$\frac{\dot{m}_{5,b}}{\dot{m}_{0,C}} = \frac{\dot{m}_{0,F} + [1+(f/a)]\dot{m}_{0,C} + \dot{m}_{f,AB}}{\dot{m}_{0,C}} = \beta + 1 + (f/a) + 2(f/a)_{AB}$$
$$\therefore \frac{\dot{m}_{5,b}}{\dot{m}_{0,C}} = 1 + 1 + 0.0197 + 2\times 0.02 = 2.06$$

The effective heating value may be estimated on the basis that the core mass flow has combustion products while the bypass flow that mixes with it is pure air. The effective heating value for such flows is

$$Q_{eff} = \left[1 - 14.7\frac{(f/a)}{1+\beta}\right]Q_f$$

Assuming a constant effective specific heat for the process, the energy balance equation becomes

$$\beta c_{p,e} T_{t,F} + 1.0197 c_{p,e} T_{t,5} + 0.04\times 0.85\times\left[1 - 14.7\times\frac{0.0197}{1+1}\right]\times 44{,}000$$
$$= 2.06 c_{p,e} T_{t,5b}$$

Dividing through by $c_{p,e}$ gives

$$1\times 327+1.0197\times 943+\frac{0.04\times 0.85\times\left[1-14.7\times\dfrac{0.0197}{1+1}\right]\times 44,000}{c_{p,e}}=2.06\times T_{t,5b}$$

$$\therefore 1290+\frac{1280}{c_{p,e}}=2.06\times T_{t,5b}$$

$$\therefore 626+\frac{621}{c_{p,e}}=T_{t,5b}$$

Assume $c_{p,e}$ = 1.16 kJ/kg to estimate $T_{t,5b}$ = 1160 K. Averaging temperatures, we have $T_{t,avg}$ = (327 + 943 + 1160)/3 = 810 K, and the specific heat becomes $c_{p,e}$ = 0.94 + 0.215($T_{t,avg}$/1000) = 1.11 kJ/kg. The corresponding temperature is

$$T_{t,5b}=626+\frac{621}{1.11}=1190\,\text{K}$$

Stagnation pressure losses in the afterburner are such that $p_{t,5}-p_{t,5b}=0.05p_{t,5}$, so that

$$p_{t,5b}=0.95p_{t,5}=0.95\times 143=136\,\text{kPa}$$

Nozzle, Stations 5 – 6: The exit pressure will be the critical pressure p^* and the ratio of specific heats is chosen to be γ = 1.33, consistent with an average temperature in the nozzle of about 1100 K. The pressure and temperature at any point in the nozzle are given by the isentropic relations $p/p_{t,5b}=[1+0.165M^2]^{-4.03}$ and $T/T_{t,5b}=[1+0.165M^2]^{-1}$. Therefore, the exit pressure is

$$p_6=p^*=\left(1+0.165\times 1.0^2\right)^{-4.03}p_{t,5b}=0.540\times 136$$

$$\therefore p_6=p^*=73.4\,\text{kPa}$$

The exit temperature is

$$T_6=T^*=\left(1+0.165\times 1.0^2\right)^{-1}T_{t,5b}=0.858\times 1190$$

$$\therefore T_6=T^*=1020\,\text{K}$$

The exit velocity is

$$V_6=a^*=\sqrt{\gamma RT^*}=\sqrt{1.33\times 287\times 1020}=624\,\text{m/s}$$

Then, the required nozzle exit area is found from conservation of mass, namely

$$\dot{m}_6=\rho_6A_6V_6=\left[1+\left(f/a\right)\right]\dot{m}_{0,C}+\dot{m}_{0,F}+\dot{m}_{f,AB}$$

$$\therefore \dot{m}_6=\left[1+\left(f/a\right)+\beta\right]\dot{m}_{0,C}+\left(f/a\right)_{AB}\left(1+\beta\right)\dot{m}_{0,C}$$

$$\therefore \dot{m}_6=\left[1+\beta+\left(f/a\right)+\left(1+\beta\right)\left(f/a\right)_{AB}\right]\dot{m}_{0,C}$$

$$\therefore \dot{m}_6=2.06\times\dot{m}_{0,C}$$

$$\therefore \dot{m}_6=2.06\times 25=51.5\,\text{kg/s}$$

The density is found from the equation of state,

$$\rho^*=\frac{p^*}{RT^*}=\frac{73,400}{287\times 1020}=0.251\,\text{kg/m}^3$$

The exit area required follows as

$$\dot{m}_6=\rho_6A_6V_6\rightarrow A_6=\frac{\dot{m}_6}{\rho_6V_6}$$

$$\therefore A_6=\frac{51.5}{0.251\times 624}=0.329\,\text{m}^2$$

Performance: The fan is internally mixed, so the gross thrust can be calculated as

$$F_g = \dot{m}_6 V_6 + A_6(p_6 - p_0)$$

$$\therefore F_g = 51.5 \times 624 + 0.329 \times (73,400 - 22,700) = 48,800\,\text{N}$$

$$\therefore F_g = 48.8\,\text{kN}$$

The ram drag is

$$F_r = \dot{m}_0 V_0 = 51.5 \times 266 = 13,700\,\text{N} = 13.7\,\text{kN}$$

The net thrust is

$$F_n = 48.8 - 13.7 = 35.1\,\text{kN}$$

Noting that the fuel mass flow is 0.493 kg/s (determined in Prob. 3.2) for the core engine plus 1.0 kg/s for the afterburner, the specific fuel consumption becomes

$$c_j = \frac{\dot{m}_f}{F_n} = \frac{1.493}{35,100} = 4.25 \times 10^{-5}\,\text{kg/s/N} = 0.153\,\text{kg/hr/N}$$

Lastly, the overall efficiency of the engine is determined to be

$$\eta_o = \frac{F_n V_0}{\dot{m}_f Q_f} = \frac{35.1 \times 266}{1.493 \times 44,000} = 0.142$$

$$\therefore \eta_o = 14.2\%$$

■ P3.6

The two afterburner-equipped engines are compared in the following table. The net thrust of the afterburning turbofan is 20.2% greater than that of the afterburning turbojet, while its specific fuel consumption is 28.8% less than that of the turbojet. Both afterburning engines achieve a 50 to 70% greater net thrust than the corresponding non-afterburning craft examined in problems 3.1 and 3.2.

Variable	**Turbojet w/ A.B. (Problem 3.4)**	**Turbofan w/ A.B. (Problem 3.5)**
Net thrust, F_n	29.2 kN	35.1 kN
Specific fuel cons., c_j	0.215 kg/hr/N	0.153 kg/hr/N
Overall efficiency, η_o	10.1%	14.2%
Air flow, $\dot{m}$	29.17 kg/s	50 kg/s
Thrust augmentation ratio due to afterburning	29.2/19.0 = 1.54	35.1/19.6 = 1.79

■ P10.2

The power coefficient in terms of English units reads

$$C_{P,c} = \frac{(\text{BHP}/2000)}{\sigma\left(\frac{N}{1000}\right)^3\left(\frac{d}{10}\right)^5} \quad \text{(I)}$$

The thrust, in turn, is given by

$$F = \frac{C_T}{C_P}\frac{P}{nd}$$

If the engine delivers 2000 hp to the propeller when operating at 15,000 rpm and 1800 hp when operating at 10,000 rpm, a linear fit to the two data points gives

$$BHP = 1800 + 40\left(\frac{N_s}{1000} - 10\right) \quad \text{(II)}$$

The propeller is driven through a 12:1 reduction gearbox so that $N = N_s/12$. Substituting in (I) brings to

$$C_{P,c} = \frac{0.9 + 0.02(12N/1000 - 10)}{\sigma\left(\frac{N}{1000}\right)^3\left(\frac{15}{10}\right)^5} \quad \text{(III)}$$

The curve of $C_{T,c}/C_{P,c}$ as a function of $C_{P,c}$, Fig. 10.16 of the text, can be used to determine $C_{T,c}/C_{P,c}$ as a function of propeller speed N. Then, the static thrust (in lb) may be obtained using

$$F = \frac{C_{T,c}}{C_{P,c}} \times \frac{33{,}000 BHP}{Nd} \quad \text{(IV)}$$

Firstly, let N_s = 12,000 rpm, so that N = 12,000/12 = 1000 rpm. Substituting in (III) brings to

$$C_{P,c} = \frac{0.9 + 0.02 \times (12 \times 1000/1000 - 10)}{1 \times \left(\frac{1000}{1000}\right)^3 \times \left(\frac{15}{10}\right)^5} = 0.124$$

Mapping this coefficient into Fig. 10.16, we read $C_{T,c}/C_{P,c}$ = 1.70. Substituting N_s in (II), we get

$$BHP = 1800 + 40 \times \left(\frac{12{,}000}{1000} - 10\right) = 1880\,\text{hp}$$

Then, substituting the pertaining data in (IV) gives

$$F = 1.70 \times \frac{33{,}000 \times 1880}{1000 \times 15} = 7030\,\text{lb}$$

Next, let N_s = 14,400 rpm, so that N = 14,400/12 = 1200 rpm. Substituting in (III) brings to

$$C_{P,c} = \frac{0.9 + 0.02 \times (12 \times 1200/1000 - 10)}{1 \times \left(\frac{1200}{1000}\right)^3 \times \left(\frac{15}{10}\right)^5} = 0.0753$$

Note that $C_{P,c} < 0.12$, which places us outside of the range encompassed by the chart in Fig. 10.16; in such a case, we may use the correlation provided in the problem statement:

$$\frac{C_{T,c}}{C_{P,c}} = 0.62 C_{P,c}^{-0.5} = 0.62 \times 0.0753^{-0.5} = 2.26$$

Substituting N_s in (II),

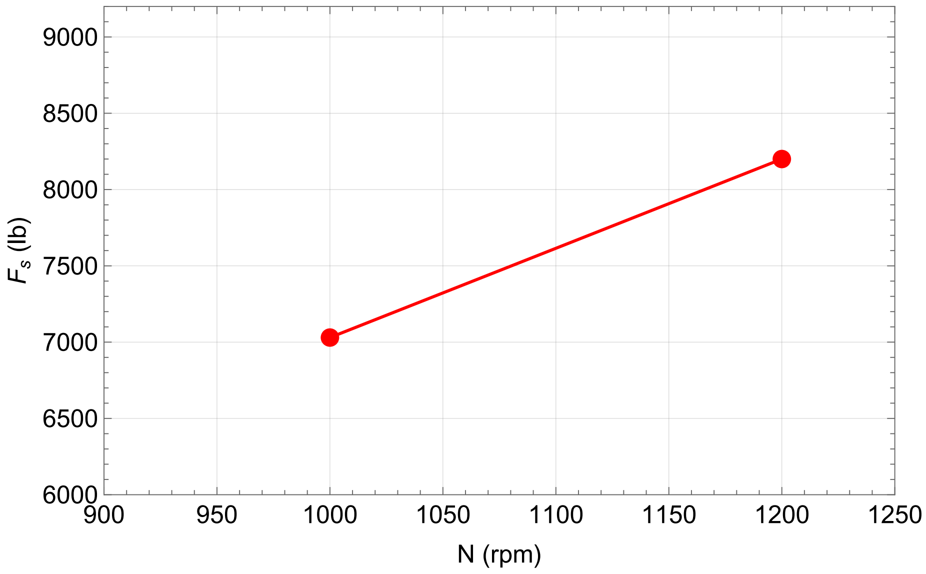
$$BHP = 1800 + 40 \times \left(\frac{14{,}400}{1000} - 10\right) = 1980\,\text{hp}$$

Then, substituting the pertaining data in (IV) gives

$$F = 2.26 \times \frac{33{,}000 \times 1980}{1200 \times 15} = 8200\,\text{lb}$$

We have two data points, as summarized below; the reader is invited to compute values for other propeller speeds.

***N* (rpm)**	***F*$_s$ (lb)**
1000	7030
1200	8200



■ P10.3

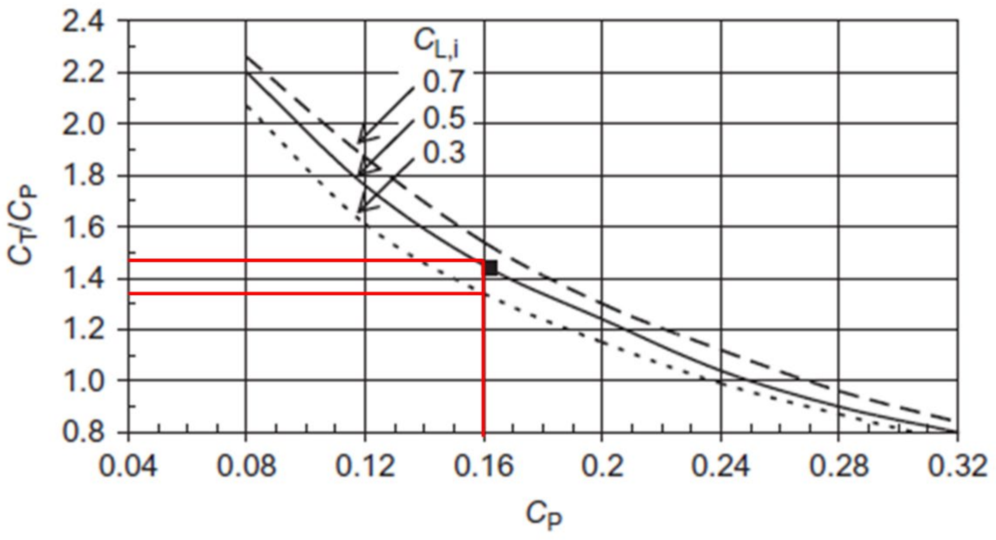
Parts a and b: The power coefficient in English units is given by

$$C_{P,c} = \frac{(\text{BHP}/2000)}{\sigma\left(\frac{N}{1000}\right)^3\left(\frac{d}{10}\right)^5}$$

For the situation at hand, *BHP* = 4030 kW = 5400 hp, *d* = 4.72 m = 15.5 ft, and σ = 1 at sea level, giving

$$C_{P,c} = \frac{(5400/2000)}{1\times\left(\frac{1230}{1000}\right)^3\times\left(\frac{15.5}{10}\right)^5} = 0.162$$

Entering this power coefficient into the chart in Figure 10.16, we read C_T/C_P = 1.33 for $C_{L,I}$ = 0.3 and C_T/C_P = 1.47 for $C_{L,I}$ = 0.5, as shown.



Now, the static thrust is given by eq. (10.45), so that, for $C_{L,I}$ = 0.3,

$$F = \frac{C_T}{C_P}\frac{BHP}{Nd}\times 33{,}000 = 1.33\times\frac{5400}{1230\times 15.5}\times 33{,}000 = \boxed{12{,}400\ \text{lb}}$$

or 55.2 kN. Similarly, for $C_{L,I}$ = 0.5,

$$F = 1.47\times\frac{5400}{1230\times 15.5}\times 33{,}000 = \boxed{13{,}700\ \text{lb}}$$

or 60.9 kN. We proceed to compute the figure of merit Φ, namely

$$\Phi = \sqrt{\frac{2}{\pi}}\frac{C_T^{3/2}}{C_P} = \sqrt{\frac{2C_P}{\pi}}\left(\frac{C_T}{C_P}\right)^{3/2}$$

$$\therefore \Phi = \sqrt{\frac{2\times 0.162}{\pi}}\times\left(\frac{C_T}{C_P}\right)^{3/2}$$

$$\therefore \Phi = 0.321\left(\frac{C_T}{C_P}\right)^{3/2}$$

It follows that, for $C_{L,I}$ = 0.3,

$$\Phi = 0.321\times 1.33^{3/2} = \boxed{0.492}$$

while for $C_{L,I}$ = 0.5,

$$\Phi = 0.321\times 1.47^{3/2} = \boxed{0.572}$$

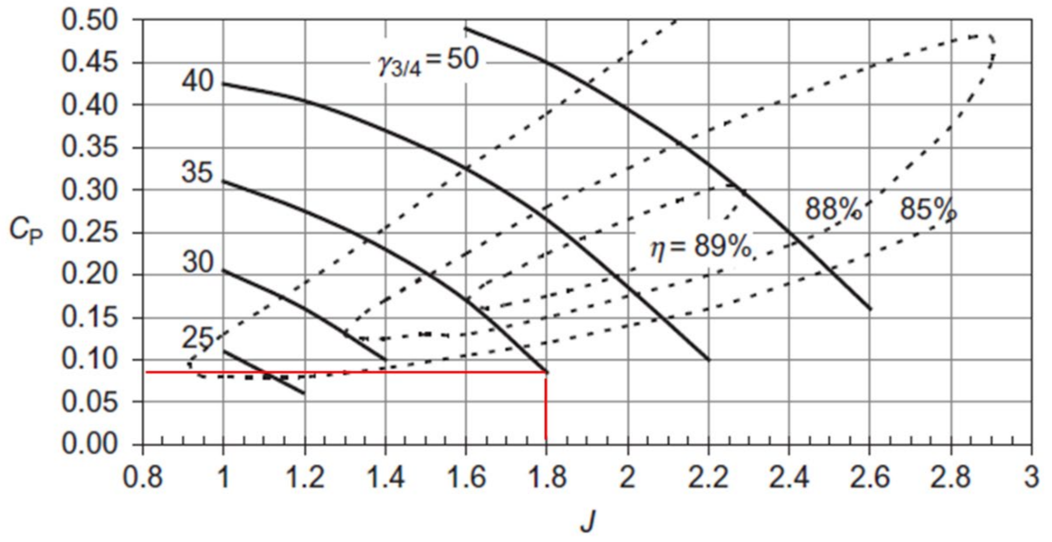
Now, the engine supplies a power of 750 kW, or 1005 hp, to the propeller, which operates at 900 rpm. Using the equation at the beginning of this solution, the power coefficient becomes

$$C_P = \frac{(1005/2000)}{1\times\left(\frac{900}{1000}\right)^3\times\left(\frac{15.5}{10}\right)^5} = 0.0770$$

The advance ratio is, in turn,

$$J = 101.4\frac{V}{Nd} = 101.4\times\frac{250}{900\times15.5} = 1.82$$

Entering the two foregoing quantities into the chart in Figure 10.12, we estimate an efficiency η = 83% at a blade pitch angle $\gamma_{3/4}$ = 35°.



Part c: It is stated that for an integrated design lift coefficient in forward flight of $C_{L,I}$ = 0.3 the efficiency becomes 90%, which is 7 percentage points greater than the efficiency at $C_{L,I}$ = 0.5. This indicates the difficulty in designing for vertical flight, for which a high value of $C_{L,I}$ is favorable, and designing for horizontal flight, where a low value of $C_{L,I}$ is desirable. A propeller with variable camber of its constituent airfoil elements would be a great asset in this regard, but variable geometry solutions often end up being too heavy and not sufficiently reliable.

■ P10.6

Using the equation for estimating C_P given in Section 10.6.2, we have

$$C_P = \left(\frac{6.44\times10^{-4}}{\sigma\theta^{3/2}}\right)\frac{P_a}{M_{tip}^3 d^2} = \left(\frac{6.44\times10^{-4}}{1\times1^{3/2}}\right)\times\frac{746}{0.8^3\times3^2} = 0.104$$

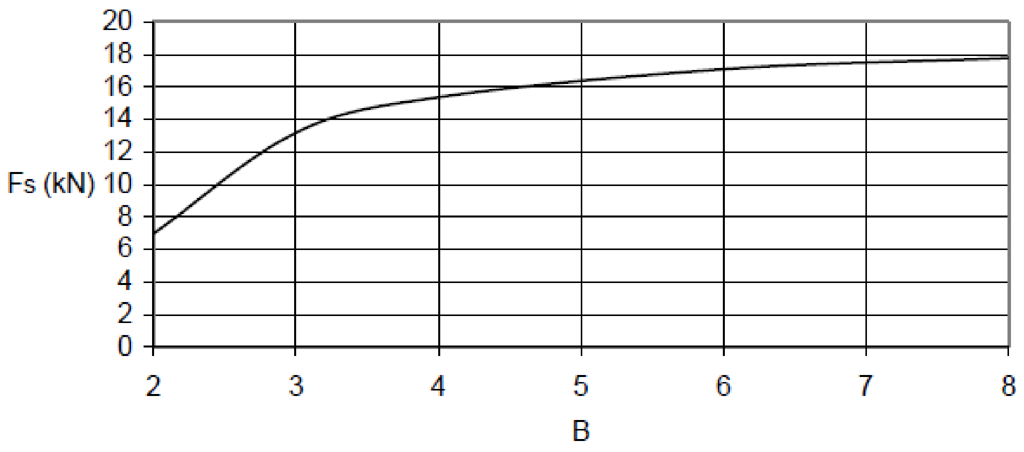
The estimate for the figure of merit from the same section is

$$\Phi = 0.85 - 18B^{-3/2}C_P$$

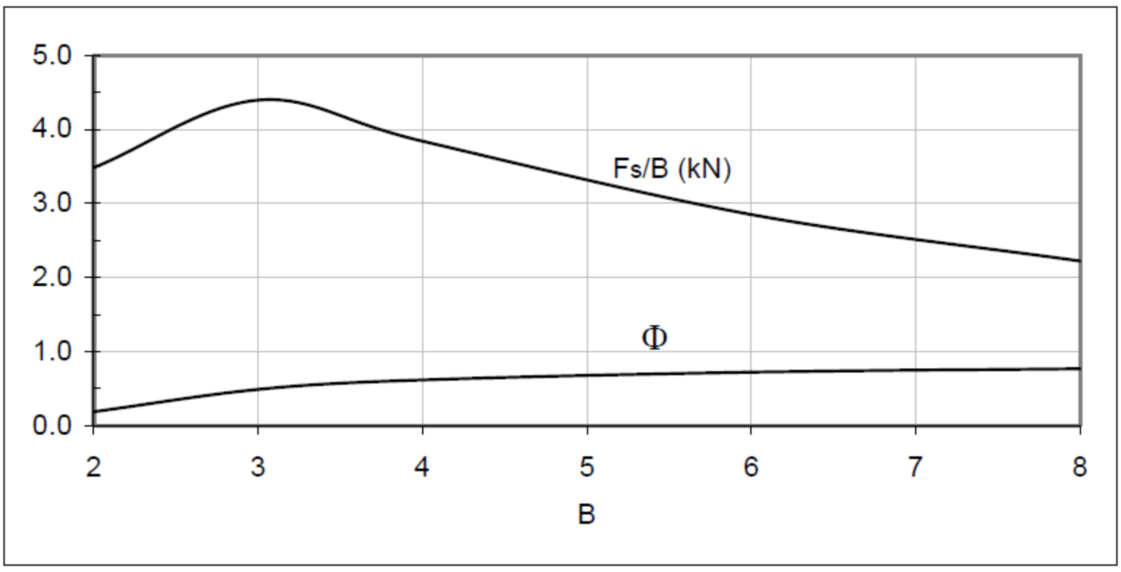
Using this result we may find the static thrust F_s from the equation given in Section 10.6.2:

$$F_s = 124\left[\left(\sqrt{\frac{2}{\pi}}\frac{C_T^{3/2}}{C_P}\right)\sqrt{\sigma}P_{s,a}d\right]^{2/3} = 124\left[\Phi\sqrt{\sigma}P_{s,a}d\right]^{2/3}$$

The results are shown in the following figure, where it is seen that the static thrust produced is seen to drop off rapidly for $B < 4$.



The blade loading and the figure of merit are shown in the following figure where it is seen that the thrust carried by each blade is a maximum at $B = 3$ and the figure of merit slowly increases with the number of blades.



Reducing the loading on each blade makes it easier for the boundary layer to stay attached thereby reducing the drag and improving performance. The propeller speed may be found by using the specified tip Mach number as follows:

$$N = \frac{60\omega}{2\pi} = \frac{30}{\pi}\left(\frac{u_{\text{tip}}}{r_{\text{tip}}}\right) = \frac{30}{\pi}\left(\frac{\sqrt{\gamma R T_{sl}\theta}}{d/2} M_{\text{tip}}\right)$$

$$\therefore N = \frac{30}{\pi}\left(\frac{\sqrt{1.4 \times 287 \times 298\theta}}{d} M_{\text{tip}}\right) = 6610\frac{\sqrt{\theta} M_{\text{tip}}}{d}$$

$$\therefore N = 6610 \times \frac{\sqrt{1} \times 0.8}{3} = \boxed{1760\,\text{rpm}}$$

■ P11.3

Part a: For a nozzle efficiency $\eta_n = 0.95$ the nozzle velocity coefficient $c_V = \eta_n^{1/2} = 0.975$ and, with a molecular weight $W = 27$ the gas constant $R = R_u/W = 8.314/27 = 0.308$ kJ/kg·K. Given the specific heat ratio $\gamma = 1.2$, the constant-pressure specific heat $c_p = \gamma R/(\gamma - 1) = 1.2 \times 0.308/(1.2 - 1) = 1.85$ kJ/kg·K. Also,

$$\Gamma \approx 0.192\gamma + 0.417 = 0.192 \times 1.2 + 0.417 = 0.647$$

The thrust coefficient is given by Eq. (11.17) of the text, namely

$$c_F = \lambda c_d c_V \Gamma \left\{ \frac{2\gamma}{\gamma - 1}\left[1 - \left(\frac{p_e}{p_c}\right)^{\frac{\gamma-1}{\gamma}}\right]\right\}^{1/2} + \frac{A_e}{A_t}\left(\frac{p_e}{p_c} - \frac{p_0}{p_c}\right)$$

Since the nozzle exit angle is not provided, we may assume a perfectly contoured nozzle providing parallel flow at the nozzle exit, hence we may take $\lambda = 1$. In a similar manner, in the absence of a specification of the discharge coefficient we may take $c_d = 1$ as well. The thrust coefficient is then calculated to be

$$c_F = 1 \times 1 \times 0.975 \times 0.647 \times \left\{ \frac{2 \times 1.2}{1.2 - 1} \times \left[1 - \left(\frac{1.0}{19.34}\right)^{\frac{1.2-1}{1.2}}\right]\right\}^{1/2} = 1.36$$

However, the thrust coefficient is also given by

$$c_F = \frac{F}{p_c A_t}$$

so that, solving for throat area,

$$c_F = \frac{F}{p_c A_t} = 1.36 \rightarrow A_t = \frac{F}{p_c \times 1.36}$$

$$\therefore A_t = \frac{19{,}600}{(19.34\times 101{,}325)\times 1.36} = \boxed{7.35\times 10^{-3}\ \mathrm{m}^2}$$

or 73.5 cm². With a discharge coefficient c_d = 1, the mass flow through the throat is

$$\dot{m} = \frac{p_c A_t \Gamma}{\sqrt{RT_c}} = \frac{(19.34\times 101{,}325)\times(7.35\times 10^{-3})\times 0.647}{\sqrt{308\times 3000}} = 9.70\,\mathrm{kg/s}$$

The specific impulse I_p is, in turn,

$$I_P = \frac{F}{\dot{m}g} = \frac{19{,}600}{9.70\times 9.81} = \boxed{206\,\mathrm{s}}$$

The exit velocity follows as

$$I_P = \frac{V_e}{g} = 206 \rightarrow V_e = 206g$$

$$\therefore V_e = 206\times 9.81 = \boxed{2020\,\mathrm{m/s}}$$

Assuming a constant c_p, the nozzle exit temperature may be estimated as

$$T_e = T_c - \frac{V_e^2}{2c_p} = 3000 - \frac{2020^2}{2\times 1850} = 1900\,\mathrm{K}$$

so that, from the equation of state,

$$\rho_e = \frac{p_e}{RT_e} = \frac{101{,}325}{308\times 1900} = 0.173\,\mathrm{kg/m^3}$$

Finally, the exit area is determined as

$$A_e = \frac{\dot{m}}{\rho_e V_e} = \frac{9.70}{0.173\times 2020} = \boxed{0.0278\,\mathrm{m}^2}$$

or 278 cm².

Part b: We have already determined the mass flow to be 9.70 kg/s and the specific impulse to be 206 sec; it remains to compute the characteristic velocity (equation (11.20) in the textbook),

$$c^* = \frac{\sqrt{RT_c}}{\Gamma} = \frac{\sqrt{308\times 3000}}{0.647} = \boxed{1490\,\mathrm{m/s}}$$

Part c: The thrust developed at an altitude of 11.5 km where the ambient pressure is p_0 = 0.2 atm can be determined as

$$F = \dot{m}V_e + A_e(p_e - p_0) = 19.6 + 0.0278\times[(1.0-0.2)\times 101.325]$$

$$\therefore \boxed{F = 21.9\,\mathrm{kN}}$$

Part d: If the nozzle were matched at an altitude of 11.5 km the pressure ratio would become p_e/p_c = 0.2/19.34 = 0.0103, leading to a thrust coefficient such that

$$c_F = 1\times 1\times 0.975\times 0.647\times\left\{\frac{2\times 1.2}{1.2-1}\times\left[1-0.0103^{\frac{1.2-1}{1.2}}\right]\right\}^{1/2} = 1.60$$

The thrust would then be

$$F = c_F p_c A_t = 1.60\times(19.34\times 101{,}325)\times(7.35\times 10^{-3}) = 23{,}000\,\mathrm{N}$$

$$\therefore \boxed{F = 23.0\,\mathrm{kN}}$$

The pressure ratio obtained just now is related to the exit Mach number by the isentropic-flow expression

$$\frac{p_e}{p_c} = \left(1+\frac{\gamma-1}{2}M_e^2\right)^{\frac{-\gamma}{\gamma-1}}$$

In principle, we could obtain Mach number M_e by entering pressure ratio = 0.0103 into an isentropic flow table and reading M_e; however, most such tables are created for a specific heat ratio of 1.4, which applies to air. As a workaround, we may take γ = 1.2, as given in the problem statement, and solve the ensuing nonlinear equation with MATLAB's *fzero* function:

```
gm = 1.2;
f = @(Me) (0.0103-(1+(gm-1)/2*Me^2)^(-gm/(gm-1)));
Me0 = 1;
fzero(f,Me0)
ans =

    3.3821
```

Thus, M_e = 3.38. Substituting this Mach number into the following equation gives the updated area ratio:

$$\frac{A_e}{A_t}=\frac{1}{M_e}\left[\frac{2+(\gamma-1)M_e^2}{\gamma+1}\right]^{\frac{\gamma+1}{2(\gamma-1)}}=\frac{1}{3.38}\times\left[\frac{2+(1.2-1)\times 3.38^2}{1.2+1}\right]^{\frac{1.2+1}{2\times(1.2-1)}}$$

$$\therefore \frac{A_e}{A_t}=11.6$$

For a throat area of 73.5 cm^2, this gives an exit area A_e = 11.6 × 73.5 = 853 cm^2, a three-fold increase relatively to the matched operation at sea level, which we found to be A_e = 278 cm^2.

■ P11.4

Part a: The nozzle exit diameter is 72.6 in. The corresponding area is then

$$A_e=\frac{\pi d_e^2}{4}=\frac{\pi\times 72.6^2}{4}=4140\,\text{in.}^2$$

$$\therefore \boxed{A_e=28.8\,\text{ft}^2}$$

For an expansion ratio ε = 10, the throat area can only be

$$A_t=\frac{28.8}{\varepsilon}=\boxed{2.88\,\text{ft}^2}$$

which corresponds to a throat diameter of approximately 23 in.

Part b: To compute the ideal mass flow of propellant, first note that the gas constant for the propellant at hand is $R = R_u/W$ = 1545/21.9 = 70.5 lb-ft/lbm = 2270 ft^2/s^2·R. Also, we approximate Γ as

$$\Gamma\approx 0.192\gamma+0.417=0.192\times 1.135+0.417=0.635$$

so that

$$\dot{m}_i=\frac{p_cA_t\Gamma}{\sqrt{T_c}}\sqrt{\frac{W_c}{R_u}}=\frac{(775\times 144)\times 2.88\times 0.635}{\sqrt{2270\times 6400}}\times 32.2=1720\,\text{lbm/s}$$

In view of the nozzle discharge coefficient, we obtain $\dot{m}$ = 0.95 × 1720 = 1630 lbm/s.

Part c: The thrust developed may be obtained from the specific impulse, which at sea level equals 264 s,

$$I_{sp}=\frac{F}{\dot{m}g}=\frac{F\times 32.2}{1630\times 32.2}=264\,\text{s}$$

$$\therefore \boxed{F=430{,}000\,\text{lb}}$$

Part d: The thrust coefficient equals

$$c_F=\frac{F}{p_cA_t}=\frac{430{,}000}{111{,}600\times 2.88}=\boxed{1.34}$$

Part e: The ideal nozzle exit Mach number may be found from the area ratio A^*/A_e = 0.1:

$$\frac{A^*}{A_e} = 0.1 = M_e\left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M_e^2\right)\right]^{\frac{-(\gamma+1)}{2(\gamma-1)}}$$

```
gm = 1.135;
f = @(Me) (0.1 - Me*(2/(gm+1)*(1+(gm-1)/2*Me^2))^(-(gm+1)/(2*(gm-1))));
x0 = 2;
fzero(f,x0)
ans =

   3.0982
```

As shown, MATLAB outputs $M_e \approx 3.10$.

Part f: The value of the ideal pressure ratio p_e/p_c is

$$\frac{p_e}{p_c} = \left(1+\frac{\gamma-1}{2}M_e^2\right)^{\frac{-\gamma}{\gamma-1}} = \left(1+\frac{1.135-1}{2}\times 3.10^2\right)^{\frac{-1.135}{1.135-1}} = 0.0149$$

so that

$$p_e = 0.0149 p_c = 0.0149\times 775 = 11.6\,\text{psia} = \boxed{1670\,\text{lb/ft}^2}$$

Part g: The ideal exit velocity is given by

$$V_{e,i} = \sqrt{\gamma_e R T_{e,i}}\,M_e \quad \text{(I)}$$

We first compute the temperature ratio

$$\frac{T_e}{T_c} = \left(1+\frac{\gamma-1}{2}M_e^2\right)^{-1} = \left(1+\frac{1.135-1}{2}\times 3.10^2\right)^{-1} = 0.607$$

so that

$$T_{e,i} = 0.607 T_c = 0.607\times 6400 = 3880\,\text{R}$$

The speed of sound is

$$\sqrt{\gamma_e R T_{e,i}} = \sqrt{1.135\times 2270\times 3880} = 3160\,\text{ft/s}$$

so that, substituting in (I),

$$V_{e,i} = 3160\times 3.10 = 9800\,\text{ft/s}$$

With a velocity coefficient $c_V = 0.903$, we obtain

$$V_e = c_V V_{e,i} = 0.903\times 9800 = \boxed{8850\,\text{ft/s}}$$

Part h: The effective exhaust velocity is

$$V_{eff} = V_e + \frac{A_e}{\dot{m}}(p_e - p_0)$$

$$\therefore V_{eff} = 8850 + \frac{28.8\times 32.2}{1720}(1670-2120) = \boxed{8610\,\text{ft/s}}$$

Part i: The characteristic velocity equals

$$c^* = \frac{\sqrt{RT_c}}{\Gamma} = \frac{\sqrt{2270\times 6400}}{0.635} = \boxed{6000\,\text{ft/s}}$$

■ P11.7

Part a: Assuming ideal operation, the thrust coefficient is given by

$$c_F = \Gamma\left\{\frac{2\gamma}{\gamma-1}\left[1-\left(\frac{p_e}{p_c}\right)^{\frac{\gamma-1}{\gamma}}\right]\right\}^{\frac{1}{2}} - \frac{A_e}{A_t}\left[\left(\frac{p_e}{p_c}\right)-\left(\frac{p_0}{p_c}\right)\right]$$

The nozzle is said to produce optimum thrust so we may assume that the nozzle is matched with $p_e = p_0$; as a result, the rightmost term in the equation above equals zero. We also have the pressure ratio p_e/p_c = 101.325/2070 = 0.0490, and

$$\Gamma \approx 0.192\gamma + 0.417 = 0.192 \times 1.229 + 0.417 = 0.653$$

so that

$$c_F = 0.653 \times \left\{ \frac{2 \times 1.229}{1.229 - 1} \left[1 - 0.0490^{\frac{1.229-1}{1.229}} \right] \right\}^{\frac{1}{2}} = \boxed{1.40}$$

Part b: The characteristic velocity is given by

$$c^* = \frac{\sqrt{RT_c}}{\Gamma}$$

where R = 8.314/21.87 = 0.380 kJ/kg·K and T_c = 2860 K, giving

$$c^* = \frac{\sqrt{380 \times 2860}}{0.653} = \boxed{1600\,\text{m/s}}$$

Part c: The minimum nozzle area may be obtained from the definition of thrust coefficient,

$$c_F = \frac{F}{p_c A_t} \rightarrow A_t = \frac{F}{p_c c_F}$$

$$\therefore A_t = \frac{1334 \times 10^3}{\left(2.07 \times 10^6\right) \times 1.40} = \boxed{0.460\,\text{m}^2}$$

Part d: The nozzle area ratio is expressed as

$$\frac{A_e}{A_t} = \frac{1}{M_e}\left[\frac{2}{\gamma+1}\left(1 + \frac{\gamma-1}{2}M_e^2\right)\right]^{\frac{\gamma+1}{2(\gamma-1)}} \quad \text{(I)}$$

This of course requires the exit Mach number M_e, which can be obtained from the equation for pressure ratio in isentropic flow,

$$\frac{p_c}{p_e} = \left(1 + \frac{\gamma-1}{2}M_e^2\right)^{\frac{\gamma}{\gamma-1}} \rightarrow \frac{1}{0.0490} = \left(1 + \frac{1.229-1}{2} \times M_e^2\right)^{\frac{1.229}{1.229-1}}$$

```
In[116]:= Solve[1/0.0490 == (1 + (1.229 - 1)/2 * Me^2)^(1.229/(1.229-1)), Me]

Solve: Inverse functions are being used by Solve, so some sol
solution information.

Out[116]= {{Me → -2.56634}, {Me → 2.56634}}
```

Using Mathematica, we get $M_e \approx 2.57$; lastly, we substitute in (I) to obtain

$$\frac{A_e}{0.460} = \frac{1}{2.57} \times \left[\frac{2}{1.229+1}\left(1 + \frac{1.229-1}{2} \times 2.57^2\right)\right]^{\frac{1.229+1}{2\times(1.229-1)}} = 3.56$$

$$\therefore A_e = 3.56 \times 0.460 = \boxed{1.64\,\text{m}^2}$$

■ P12.2

Part a: From equation (11.19) of the text the thrust is given by $F = \dot{m} c_F c^*$ and the throat area is $A_t = \pi \times 53.86^2/4$ = 2280 in.^{2} so the specific impulse is calculated to be

$$I_{sp} = \frac{c^* c_F}{g} = \frac{c^*}{g}\frac{F}{p_e A_t}$$

$$\therefore I_{sp} = \frac{5053}{32.2} \times \frac{\left(2.59 \times 10^6\right)}{662 \times 2280} = \boxed{269\,\text{s}}$$

Part b: The chamber ratio of specific heats is specified as γ_c = 1.138, which corresponds to a Γ_c parameter such that (see equation 11.3 in the text):

$$\Gamma_c = \sqrt{\gamma_c}\left(\frac{2}{\gamma_c+1}\right)^{\frac{\gamma_c+1}{2(\gamma_c-1)}} = \sqrt{1.138}\times\left(\frac{2}{1.138+1}\right)^{\frac{1.138+1}{2\times(1.138-1)}}$$

$$\therefore \Gamma_c = 0.636$$

We may then rearrange the equation for the characteristic velocity to obtain (note the conversion 6092°F ≈ 6550 R:

$$R_c = \frac{R_u}{W_c} = \frac{(c^*\Gamma)^2}{T_c} = \frac{(5053\times 0.636)^2}{6550} = 1580\,\text{ft}^2/\text{s}^2\cdot\text{R}$$

Accordingly, the chamber molar mass is $W_c = R_u/R_c$ = 49,720/1580 = 31.5.

Part c: From equation (5.25) of the text, the nozzle area ratio is expressed as

$$\frac{A^*}{A_e} = M_e\left[\frac{\frac{\gamma+1}{2}}{\left(1+\frac{\gamma-1}{2}M_e^2\right)}\right]^{\frac{\gamma+1}{2(\gamma-1)}}$$

From the given information, γ_e = 1.147 and A^*/A_e = (53.86/152.6)^2 = 0.125. Substituting in the formula above and applying Mathematica's *FindRoot* command, we obtain M_e = 2.99.

```
In[375]:= FindRoot[0.125 - Me * ((1.147+1)/2 / (1 + (1.147-1)/2 * Me^2))^((1.147+1)/(2*(1.147-1))), {Me, 1.5}]

Out[375]= {Me → 2.99268}
```

Part d: The (average) exit pressure may be found from the isentropic relation using γ_e = 1.147:

$$p_e = p_c\left(1+\frac{\gamma_e-1}{2}M_e^2\right)^{-\gamma_e/(\gamma_e-1)} = 662\times\left(1+\frac{1.147-1}{2}\times 2.99^2\right)^{-1.147/(1.147-1)}$$

$$\therefore \boxed{p_e = 12.9\,\text{psi}}$$

Part e: The exit velocity may be found with equation (11.21) of the text:

$$I_{sp} = \frac{V_e}{g}\left[1+\left(1-\frac{p_0}{p_e}\right)\frac{1}{\gamma_e M_e^2}\right] = 270\,\text{s}$$

$$\therefore I_{sp} = \frac{V_e}{32.2}\left[1+\left(1-\frac{14.7}{12.9}\right)\times\frac{1}{1.147\times 2.99^2}\right] = 270$$

$$\therefore 0.0306 V_e = 270$$

$$\therefore V_e = \frac{270}{0.0306} = \boxed{8820\text{ ft/s}}$$

Part f: As in the case of exit velocity, the exit temperature may be found from the pertaining isentropic flow relation:

$$T_e = T_c\left(1+\frac{\gamma-1}{2}M_e^2\right)^{-1} = 6550\times\left(1+\frac{1.147-1}{2}\times 2.99^2\right)^{-1}$$

$$\therefore \boxed{T_e = 3950\text{ R}}$$

Part g: The percent change in thrust due to a drop in ambient pressure of 32°F may be found using the sensitivity coefficient defined by equation (12.17) of the text, which shows

$$(\pi_p)_k = \frac{1}{p_c}\left(\frac{dp_c}{dT_i}\right)_k = 0.0011/^{\circ}\text{F} \quad \text{(I)}$$

This describes the change in chamber pressure as a function of the change in temperature of the propellant for a fixed value of *k*, the burner area to throat area. The thrust $F = c_F p_c A_t$ and, because the area ratio A_c/A^* is also fixed, $dF/F = dp_c/p_c + dc_F/c_F$. Furthermore, because the area ratio A_e/A^* is also fixed, the actual pressure ratio p_e/p_c is likewise fixed, independent of the change in the chamber pressure. Threfore, the only change in the thrust coefficient, as given by equation (11.17) in the text, is due to the change in actual exit pressure through the nozzle pressure mismatch term, so that

$$\frac{dc_F}{c_F} = \frac{1}{c_F} d\left[\frac{A_e}{A_t}\left(\frac{p_e}{p_c} - \frac{p_0}{p_c}\right)\right] = \frac{-1}{c_F}\left(\frac{A_e}{A_t}\right) d\left(\frac{p_0}{p_c}\right) = \frac{1}{c_F}\left(\frac{A_e}{A_t}\right)\left(\frac{p_0}{p_c}\right)\frac{dp_c}{p_c}$$

Using equation (I) above brings to

$$\frac{dc_F}{c_F} = \frac{p_0 A_e}{c_F p_c A_t} \frac{1}{p_c}\left(\frac{dp_c}{dT_i}\right)\bigg|_k dT_i = \frac{p_0 A_e}{c_F p_c A_t}\left(\pi_p\right)_k dT_i$$

For the solid rocket motor under investigation, we have

$$\frac{p_0 A_e}{c_F p_c A_t} = \frac{14.7 \times \left(\pi \times 152.6^2/4\right)}{2.59 \times 10^6} = 0.104$$

so that

$$\frac{dF}{F} = \frac{dp_c}{p_c} + \frac{dc_F}{c_F} = \left[1 + \underbrace{\frac{1}{c_F}\left(\frac{A_e}{A_t}\right)\left(\frac{p_0}{p_c}\right)}_{=0.104}\right]\left(\pi_p\right)_k dT_i$$

$$\therefore \frac{dF}{F} = 1.104 \times 0.0011 \times \left(32^\circ\mathrm{F} - 60^\circ\mathrm{F}\right) = \boxed{-0.0340}$$

Thus, $\Delta F/F = -3.4\%$ and though this modern solid propellant has a relatively low temperature sensitivity, the actual thrust loss is still 0.034 × (2.59×10^6) ≈ 88.000 lb.

■ P12.4

Part a: The characteristic velocity is determined with the given formula

$$c^* = 1051 \times \left(\frac{13.74 \times 10^6}{101.3 \times 10^3}\right)^{0.015} = 1130 \text{ m/s}$$

Now, the chamber temperature is related to c^* by the equation

$$c^* = \frac{\sqrt{RT_c}}{\Gamma} = \sqrt{RT_c}\left[\sqrt{\gamma}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}}\right]^{-1}$$

$$\therefore c^* = \sqrt{\left(\frac{8314}{30}\right) \times T_c}\left[\sqrt{1.27} \times \left(\frac{2}{1.27+1}\right)^{\frac{1.27+1}{2\times(1.27-1)}}\right]^{-1} = 1130$$

$$\therefore 25.2\sqrt{T_c} = 1130$$

$$\therefore \boxed{T_c = 2010\,\mathrm{K}}$$

Part b: The thrust coefficient is given by

$$c_F = \frac{F}{p_c A_t} = \frac{\dot{m} V_e}{p_c A_t} = \frac{V_e}{c^*} = \frac{1}{c^*}\sqrt{2 c_p T_c\left[1 - \left(\frac{p_e}{p_c}\right)^{\gamma/(\gamma-1)}\right]}$$

We were not given specific heat c_p, so we have to calculate it:

$$c_p = \frac{\gamma}{\gamma - 1}\frac{R_u}{W} = \frac{1.27}{1.27-1} \times \frac{8.314}{30} = 1.304\,\mathrm{kJ/kg\cdot K}$$

Finally,

$$c_F = \frac{1}{1130} \times \sqrt{2 \times 1304 \times 2010 \times \left[1 - \left(\frac{101.3 \times 10^3}{13.74 \times 10^6}\right)^{1.27/(1.27-1)}\right]} = \boxed{2.02}$$

Part c: We first compute the mass flow using the thrust equation

$$\dot{m} = \frac{F}{V_e} = \frac{F}{c_F c^*} = \frac{4900}{2.02 \times 1130} = 2.15\,\text{kg/s}$$

Then, the throat area follows as

$$A_t = \frac{F}{c_F p_c} = \frac{\dot{m} c^*}{p_c} = \frac{2.15 \times 1130}{13.74 \times 10^6} = 1.77 \times 10^{-4}\,\text{m}^2 = \boxed{1.77\,\text{cm}^2}$$

Part d: The burning area is calculated to be

$$A_b = \frac{p_c A_t \Gamma}{r \rho_p \sqrt{RT_c}}$$

$$\therefore A_b = \frac{\left(13.74 \times 10^6\right) \times \left(1.77 \times 10^{-4}\right) \times 0.662}{\left[0.001 \times \left(\frac{13.74 \times 10^6}{101.3 \times 10^3}\right)^{0.745}\right] \times 1770 \times \sqrt{278 \times 2010}} = \boxed{0.0314\,\text{m}^2}$$

Part e: Note first that

$$r_b = 0.001 \times \left(\frac{13.74 \times 10^6}{101.3 \times 10^3}\right)^{0.745} = 0.0388\,\text{m/s}$$

The length of the grain then becomes

$$L = r_b t_b = 0.0388 \times 15 = \boxed{0.582\,\text{m}}$$

Part f: The specific impulse is given by

$$I_{sp} = \frac{F}{\dot{m} g} = \frac{4900}{2.15 \times 9.81} = \boxed{232\,\text{s}}$$

■ P13.1

Part a: The specific impulse is

$$I_{sp} = \frac{F}{\dot{m} g} = \frac{0.5}{\left(\frac{0.5}{3 \times 3600}\right) \times 9.81} = \boxed{1100\,\text{s}}$$

Part b: It remains to compute the efficiency of the propulsive jet:

$$\eta' = \frac{P_e}{P} = \frac{\left(1/2\right) \dot{m} V_e^2}{P} = \frac{1}{2} \frac{F V_e}{P}$$

$$\therefore \eta' = \frac{1}{2} \times \frac{0.5 \times (1100 \times 9.81)}{4000} = 0.674$$

$$\therefore \boxed{\eta' = 67.4\%}$$

■ P13.2

Part a: Using the estimate for optimal exhaust velocity given as equation (14.13) in the text, we write

$$I_{sp} = \frac{V_e}{g} = \frac{1}{g} \sqrt{1000 \frac{t_p}{\alpha}}$$

The planned mission time is 4.8 months, which amounts to 146 days or 1.26×10^7 sec. Then, with α = 15 kg/kW the estimated optimum specific impulse is

$$I_{sp} = \frac{1}{9.81}\sqrt{1000 \times \frac{1.24 \times 10^7}{15}} = \boxed{2950\,\mathrm{s}}$$

Part b: The thrust F is directly proportional to the propulsive jet efficiency η' and the power delivered to the engine ηP and inversely proportional to the jet exhaust velocity; in mathematical terms,

$$F = 2000\eta' \frac{\eta P}{V_e} = 2000 \times 0.5 \times \frac{25}{(2950 \times 9.81)} = \boxed{0.864\,\mathrm{N}}$$

Part c: The total impulse delivered is obtained by integrating the thrust over the duration of the mission:

$$I = \int_0^{t_p} F dt = Ft_p = 0.864 \times \left(1.26 \times 10^7\right) = \boxed{1.09 \times 10^7\ \mathrm{N \cdot sec}}$$

Part d: The mass of the powerplant is $m_{eng} = \alpha(\eta P)$ = 15 kg/kW × 25 kW = 375 kg. The mass of propellant may be obtained by integrating the mass flow over the duration of the mission:

$$m_p = \int_0^{t_p} \dot{m}_p dt = \dot{m}_p t_p = \frac{F}{gI_{sp}} t_p = \frac{I}{gI_{sp}}$$

$$\therefore m_p = \frac{1.09 \times 10^7}{9.81 \times 2950} = \boxed{377\,\mathrm{kg}}$$

■ P13.4

The thrust is equal to the product of the mass flow of propellant and the exhaust velocity of the propellant jet; alternatively, the thrust can be determined as the product of the mass of the vehicle and its acceleration:

$$F = \dot{m}_p V_e = m\dot{V}$$

The rocket equation relates the change in velocity imparted to a space vehicle to the rocket's specific impulse and overall mass ratio and is obtained by rearranging the above equation and noting that the rate of consumption of propellant is equal to the negative of the rate of change of the mass of the vehicle:

$$\dot{V} = V_e \frac{\dot{m}_p}{m} = -V_e \frac{\dot{m}}{m} = -gI_{sp} \frac{\dot{m}}{m}$$

For a constant specific impulse, we may integrate the equation to obtain

$$\frac{dV}{dt} = -gI_{sp} \frac{1}{m}\frac{dm}{dt} \rightarrow \Delta V = gI_{sp} \ln\left(\frac{m_{\text{initial}}}{m_{\text{final}}}\right)$$

$$\therefore \Delta V = -gI_{sp} \ln\left(1 - \frac{m_p}{m_{\text{final}}}\right)$$

The burn time is often more important for low-thrust space propulsion systems, so we instead solve our equation for elapsed time

$$dt = \frac{m}{F} dV$$

so that, by integrating,

$$t_{\text{final}} - t_{\text{initial}} = \Delta t = \int \frac{m}{F} dV = \int \frac{m}{F}\left(-gI_{sp} \frac{dm}{m}\right) = -\frac{gI_{sp}}{F}\left(m_{\text{final}} - m_{\text{initial}}\right)$$

$$\therefore \Delta t = \frac{I_{sp}}{F/(m_{\text{initial}} g)}\left[1 - \exp\left(-\frac{\Delta V}{gI_{sp}}\right)\right]$$

For the ion rocket with initial thrust-to-weight ratio $F/m_{initial}g = 10^{-4}$, the equation above becomes

$$\Delta t = \frac{I_{sp}}{10^{-4}}\left[1-\exp\left(-\frac{5000}{9.81\times I_{sp}}\right)\right]$$

$$\therefore \Delta t = 10,000 I_{sp}\left[1-\exp\left(-\frac{510}{I_{sp}}\right)\right]$$

Accordingly, for an ion rocket with I_{sp} = 10,000 s, we get

$$\Delta t = 10,000\times 10,000\times\left[1-\exp\left(-\frac{510}{10,000}\right)\right] = 4.97\times 10^{6}\text{ s}$$

$$\therefore \boxed{\Delta t = 57.5\,\text{days}}$$

For the MPD rocket with thrust-to-initial weight ratio $F/m_{initial}g = 10^{-3}$ and specific impulse I_{sp} = 1000 s, it follows that

$$\Delta t = \frac{1000}{10^{-3}}\left[1-\exp\left(-\frac{5000}{9.81\times 1000}\right)\right] = 399,000\,\text{s}$$

$$\therefore \boxed{\Delta t = 4.62\,\text{days}}$$

The mass fraction of propellant required is given by the rocket equation as

$$\frac{m_p}{m_i} = 1-\exp\left(-\frac{\Delta V}{gI_{sp}}\right)$$

Substituting ΔV = 5000 m/s and I_{sp} = 10,000 sec for the ion rocket gives

$$\frac{m_p}{m_i} = 1-\exp\left(-\frac{5000}{9.81\times 10,000}\right) = \boxed{0.0497}$$

Substituting I_{sp} = 1000 sec and the same ΔV for the MPD rocket gives

$$\frac{m_p}{m_i} = 1-\exp\left(-\frac{5000}{9.81\times 1000}\right) = \boxed{0.399}$$

The propellant mass can be obtained by multiplying the fractions above by the initial mass of the spacecraft; for instance, a 1000-kg spacecraft would require 49.7 kg of propellant in the case of an ion rocket or 399 kg of propellant in the case of a MPD rocket.

■ P13.5

The solution is started by rearranging the definition of specific impulse:

$$I_{sp} = \frac{F}{mg} = \frac{c_F p_e A_t}{mg} = \frac{c_F p_e A_t}{\frac{p_e A_t}{\sqrt{RT_e}}\Gamma g} = \frac{c_F\sqrt{R_u}}{\Gamma g}\sqrt{\frac{T_e}{W_e}}$$

The quantities highlighted in red should be the same for matched nozzles under the same ambient pressure conditions. Then, noting that $W_e \approx 2$ g/mol for hydrogen and $W_e \approx 18$ g/mol for LOX-LH2, we may write the ratio

$$\frac{I_{sp,NUKE}}{I_{sp,LOX}} = \sqrt{\frac{T_{e,NUKE}}{T_{e,LOX}}\frac{W_{e,LOX}}{W_{e,NUKE}}} \approx \sqrt{\frac{T_{e,NUKE}}{T_{e,LOX}}\times\frac{18}{2}} = 3\sqrt{\frac{T_{e,NUKE}}{T_{e,LOX}}}$$

But $I_{sp,NUKE}/I_{sp,LOX}$ = 900/450 = 2 so that, substituting above and solving for $T_{e,NUKE}$, we obtain

$$3\sqrt{\frac{T_{e,NUKE}}{T_{e,LOX}}} = 2 \rightarrow T_{e,NUKE} = \left(\frac{2}{3}\right)^2 T_{e,LOX}$$

$$\therefore T_{e,NUKE} = \left(\frac{2}{3}\right)^2\times 3600 = \boxed{1600\,\text{K}}$$

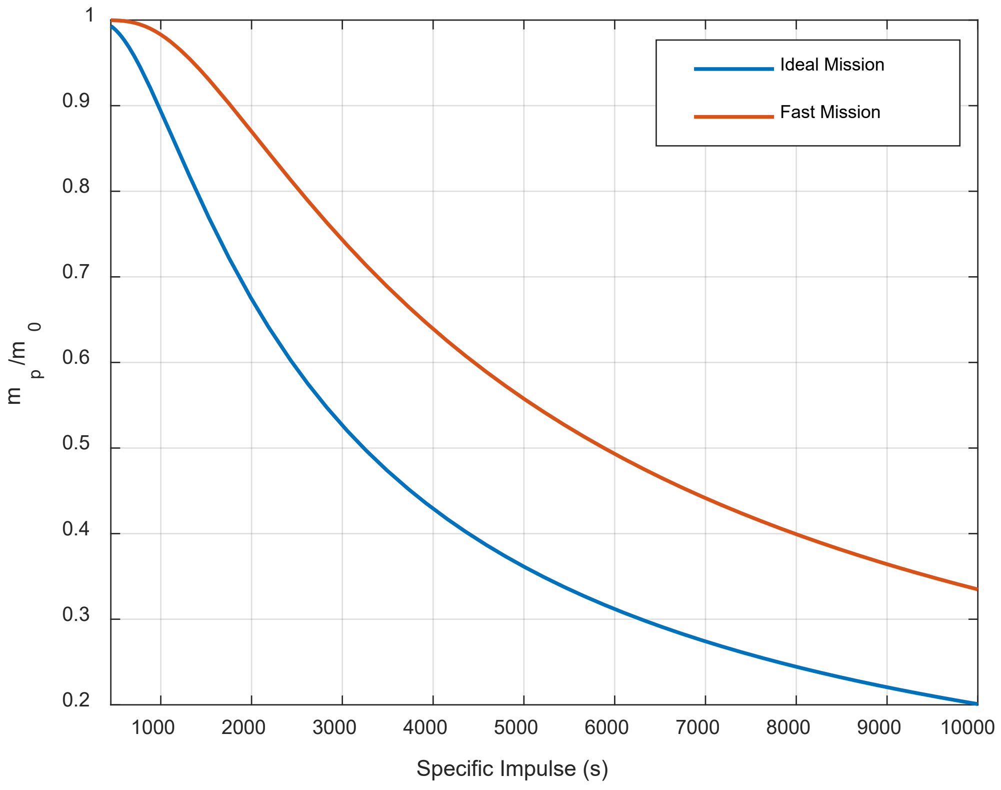
The chamber of the nuclear rocket is operating at a temperature of about 1600 K.

■ P13.6

The mass of propellant is the difference between the initial and final mass of the spacecraft so the propellant mass fraction required is

$$\frac{m_p}{m_i} = 1 - \exp\left(-\frac{\Delta V}{g I_{sp}}\right)$$

Here, ΔV = 22,000 m/s for the ideal Mars landing and 40,000 m/s for a fast mission. The corresponding mass fraction versus specific impulse curves are plotted below.



As shown, the higher the specific impulse the more practical the spacecraft will be. Of course, this assumes that the thrust-to-weight ratio will be sufficiently large to be able to provide sufficient acceleration for a fast trip. For total transfer times on the order of a year or more the low thrust, high specific impulse propulsion system will have a smaller initial mass, while for shorter times the opposite is true.

Some m_p/m_0 ratios for different specific impulses are summarized in the following table.

Rocket type	m_p/m_0 (Ideal mission)	m_p/m_0 (Fast mission)
Chemical (I_{sp} = 450 s)	0.9932	0.9999
Solid-core nuc. (I_{sp} = 1000 s)	0.8938	0.9830
Liq.-core nuc. (I_{sp} = 2000 s)	0.6741	0.8698
Electric (I_{sp} = 10,000 s)	0.2009	0.3349

REFERENCE

- SFORZA, P.M. (2016). *Theory of Aerospace Propulsion*. 2nd edition. Oxford: Butterworth-Heinemann.