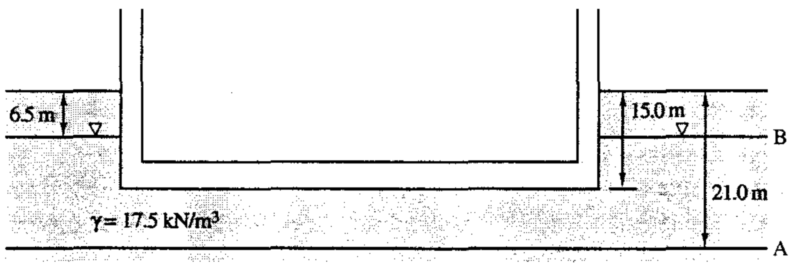
QUIZ GT201
Shallow Foundations – Basic Problems

Lucas Montogue

► PROBLEMS

PROBLEM 1 (Coduto, 2000, w/ permission)

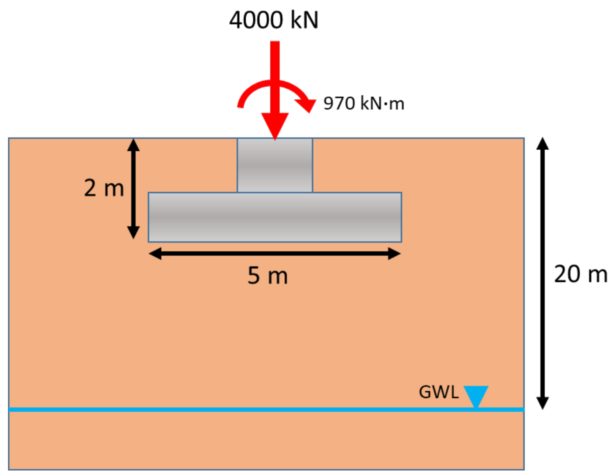
The mat foundation shown is 45 m wide and 90 m long. It has a weight of 140 MN. The sum of the applied structural loads is 1300 MN. Compute the average bearing pressure with the groundwater table at position A, then repeat your calculations with the groundwater table at position B.



A) q_A = 288.4 kN/m² and q_B = 205.0 kN/m²
B) q_A = 288.4 kN/m² and q_B = 272.2 kN/m²
C) q_A = 355.6 kN/m² and q_B = 205.0 kN/m²
D) q_A = 355.6 kN/m² and q_B = 272.2 kN/m²

PROBLEM 2

A 5-m square, 2-m deep spread footing is subjected to a concentric vertical load of 4000 kN and an overturning moment of 970 kN·m. The overturning moment acts parallel to one of the sides of the footing, the top of the footing is flush with the ground surface, and the groundwater table is at a depth of 20 m below the ground surface. Determine the minimum and maximum bearing pressures. Use 23.6 kN/m³ for the unit weight of concrete.

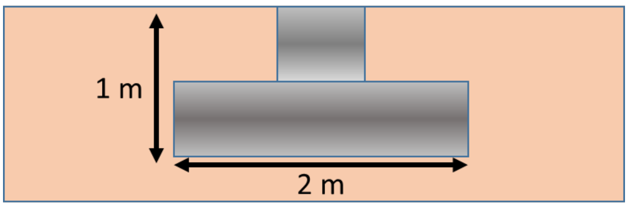


A) $q_{\min}$ = 88.0 kN/m² and $q_{\max}$ = 254.4 kN/m²
B) $q_{\min}$ = 88.0 kN/m² and $q_{\max}$ = 343.2 kN/m²
C) $q_{\min}$ = 160.0 kN/m² and $q_{\max}$ = 254.4 kN/m²
D) $q_{\min}$ = 160.0 kN/m² and $q_{\max}$ = 343.2 kN/m²

PROBLEM 3A

Determine the allowable gross vertical load bearing capacity of the foundation. The factor of safety is FS = 4. Use Terzaghi's equation and assume general shear failure in soil.

B (Width)	D_f (Depth)	ϕ' (Friction Angle)	c' (Cohesion)	γ (Unit Weight)	Foundation Type
2 m	1 m	30°	0	17.5 kN/m^3	Continuous

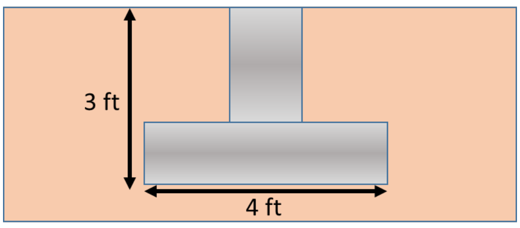


A) q_{all} = 115.8 kN/m^2
B) q_{all} = 176.8 kN/m^2
C) q_{all} = 237.8 kN/m^2
D) q_{all} = 298.8 kN/m^2

PROBLEM 3B

Determine the allowable gross vertical load-bearing capacity of the foundation. The factor of safety is FS = 4. Use Terzaghi's equation and assume general shear failure in soil.

B (Width)	D_f (Depth)	ϕ' (Friction Angle)	c' (Cohesion)	γ (Unit Weight)	Foundation Type
4 ft	3 ft	25°	600 lb/ft^2	110 lb/ft^3	Continuous

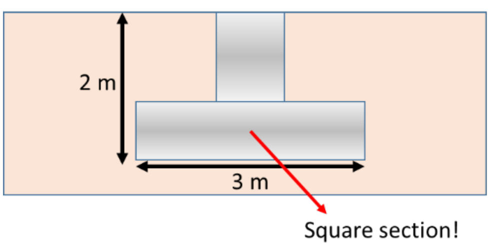


A) q_{all} = 2.2 kip/ft^2
B) q_{all} = 5.3 kip/ft^2
C) q_{all} = 8.4 kip/ft^2
D) q_{all} = 11.5 kip/ft^2

PROBLEM 3C

Determine the allowable gross vertical load-bearing capacity of the foundation. The factor of safety is FS = 4. Use Terzaghi's equation and assume general shear failure in soil.

B (Width)	D_f (Depth)	ϕ' (Friction Angle)	c' (Cohesion)	γ (Unit Weight)	Foundation Type
3 m	2 m	30°	0	16.5 kN/m^3	Square



A) q_{all} = 280.0 kN/m^2
B) q_{all} = 400.6 kN/m^2
C) q_{all} = 505.2 kN/m^2
D) q_{all} = 621.3 kN/m^2

PROBLEM 4

A square column foundation has to carry a gross allowable load of 1805 kN with a factor of safety *FS* = 3. Given the depth D_f = 1.5 m, the unit weight γ = 15.9 kN/m^3, the friction angle ϕ' = 34°, and the cohesion c' = 0, use Terzaghi's equation to determine the size of the foundation, *B*. Assume general shear failure.

A) *B* = 0.9 m
B) *B* = 2.0 m
C) *B* = 3.2 m
D) *B* = 4.5 m

PROBLEM 5

A bearing wall carries a dead load of 120 kN/m and a live load of 100 kN/m. It is to be supported on a 400-mm deep continuous footing. The underlying soils are medium sands with c' = 0, ϕ' = 37°, and γ' = 19.2 kN/m^3. The groundwater table is at great depth. Compute the minimum footing width required to maintain a factor of safety of at least 2 against a bearing capacity failure.

A) *B* = 0.57 m
B) *B* = 1.06 m
C) *B* = 1.58 m
D) *B* = 2.09 m

PROBLEM 6A

Use the general bearing capacity equation to obtain the allowable gross bearing capacity of the foundation in Problem 3A. Use the following mathematical expressions for Terzaghi's coefficients N_q, N_c and N_γ.

$$\begin{cases} N_q = \tan^2\left(45^\circ + \dfrac{\phi'}{2}\right) e^{\pi \tan\phi'} \\ N_c = \left(N_q - 1\right)\cot\phi' \\ N_\gamma = 2\left(N_q + 1\right)\tan\phi' \end{cases}$$

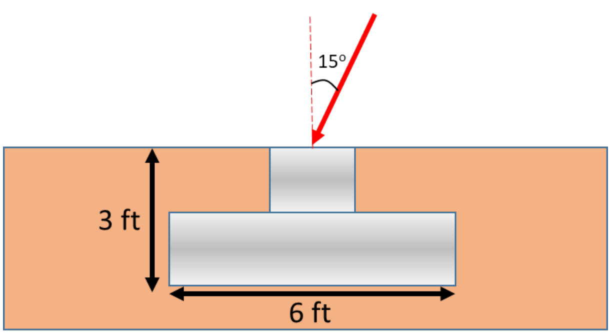
PROBLEM 6B

Use the general bearing capacity equation to obtain the allowable gross bearing capacity of the foundation in Problem 3B. Use the following mathematical expressions for Terzaghi's coefficients N_q, N_c and N_γ.

$$\begin{cases} N_q = \tan^2\left(45^\circ + \dfrac{\phi'}{2}\right) e^{\pi \tan\phi'} \\ N_c = \left(N_q - 1\right)\cot\phi' \\ N_\gamma = 2\left(N_q + 1\right)\tan\phi' \end{cases}$$

PROBLEM 7

The applied load on a shallow square foundation makes an angle of 15° with the vertical, as shown. Given *B* = 6 ft, D_f = 3 ft, ϕ' = 25°, and c' = 500 lb/ft^2, determine the gross allowable load for *FS* = 4.0. Use the general bearing capacity equation.



A) Q_{all} = 75.8 kips

B) Q_{all} = 157.5 kips

C) Q_{all} = 232.0 kips

D) Q_{all} = 302.1 kips

PROBLEM 8

A foundation measuring 8 ft × 8 ft has to be constructed in a granular soil deposit. Given: D_f = 5 ft and γ = 110 pcf. The following are the results of a standard penetration test in that soil.

Depth (ft)	Field standard penetration number (N_{60})
5	11
10	14
15	16
20	21
25	24

Estimate an average friction angle for this soil, using p_a = 14.7 psi as the atmospheric pressure. Then, using the general bearing capacity equation, determine the gross ultimate load capacity the foundation can carry.

A) Q_{all} = 2880 kips

B) Q_{all} = 4550 kips

C) Q_{all} = 6573 kips

D) Q_{all} = 8101 kips

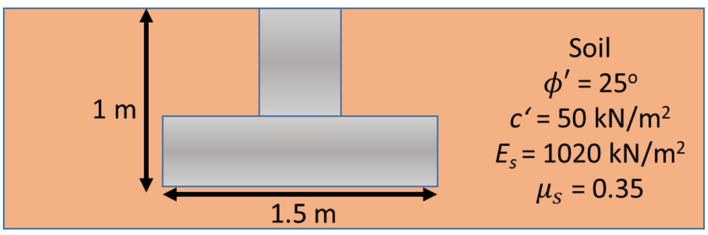
PROBLEM 9

For the design of a shallow foundation, the following information is given:

Soil	
Friction angle	ϕ' = 25°
Cohesion	c' = 50 kN/m^2
Unit weight	γ = 17 kN/m^3
Modulus of elasticity	E_s = 1020 kN/m^2
Poisson's ratio	μ_s = 0.35

Foundation	
Width	B = 1 m
Length	L = 1.5 m
Depth	D_f = 1 m

Calculate the ultimate bearing capacity while accounting for soil compressibility, i.e., Vesic's equation.



A) q_{ult} = 318 kN/m^2

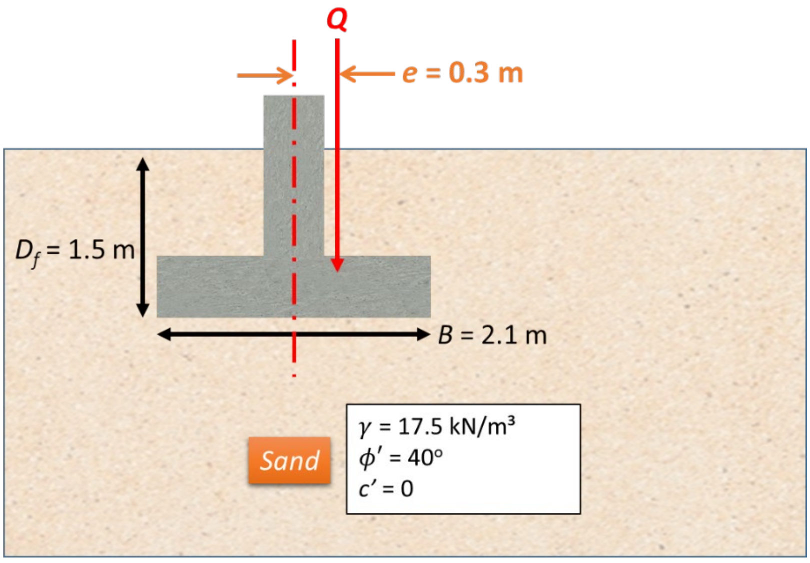
B) q_{ult} = 532 kN/m^2

C) q_{ult} = 717 kN/m^2

D) q_{ult} = 915 kN/m^2

PROBLEM 10A

A continuous foundation is shown in the figure below. If the load eccentricity is 0.3 m, determine the ultimate load Q_{ult} per unit length of the foundation using Meyerhof's effective area method.



A) Q_{ult} = 2022 kN
B) Q_{ult} = 3044 kN
C) Q_{ult} = 4055 kN
D) Q_{ult} = 5067 kN

PROBLEM 10B

Solve the previous problem using Prakash & Saran's theory, i.e., the shallow foundation under eccentric load approach based on the modified Terzaghi bearing capacity equation

$$Q_{\text{ult}} = B\left[c'N_{c,e} + qN_{q,e} + \frac{1}{2}\gamma BN_{\gamma,e}\right]$$

A) Q_{ult} = 2076 kN
B) Q_{ult} = 3086 kN
C) Q_{ult} = 4097 kN
D) Q_{ult} = 5108 kN

PROBLEM 10C

Solve Problem 9A by using Purkayastha & Char's stability analysis of eccentrically loaded continuous foundations supported by a layer of sand. Recall that, in this approach, the bearing capacity for an eccentrically loaded footing is related to the corresponding *centric* bearing capacity by an equation of the form

$$q_{u,\text{eccentric}} = q_{u,\text{centric}}\left(1 - a\left[\frac{e}{B}\right]^k\right)$$

A) Q_{ult} = 2178 kN
B) Q_{ult} = 3289 kN
C) Q_{ult} = 4390 kN
D) Q_{ult} = 5401 kN

▸ ADDITIONAL INFORMATION

Table 1 Terzaghi's bearing capacity factors

ϕ'	N_c	N_q	N_γ	ϕ'	N_c	N_q	N_γ
0	5.70	1.00	0.00	**26**	27.085	14.210	9.84
1	6.00	1.10	0.01	**27**	29.236	15.896	11.60
2	6.30	1.22	0.04	**28**	31.612	17.808	13.70
3	6.62	1.35	0.06	**29**	34.242	19.981	16.18
4	6.97	1.49	0.10	**30**	37.162	22.456	19.13
5	7.34	1.64	0.14	**31**	40.411	25.282	22.65
6	7.73	1.81	0.20	**32**	44.036	28.517	26.87
7	8.15	2.00	0.27	**33**	48.090	32.230	31.94
8	8.60	2.21	0.35	**34**	52.637	36.504	38.04
9	9.09	2.44	0.44	**35**	57.754	41.440	45.41
10	9.60	2.69	0.56	**36**	63.528	47.156	54.36
11	10.16	2.98	0.69	**37**	70.067	53.799	65.27
12	10.76	3.29	0.85	**38**	77.495	61.546	78.61
13	11.41	3.63	1.04	**39**	85.966	70.614	95.03
14	12.11	4.02	1.26	**40**	95.663	81.271	115.31
15	12.86	4.45	1.52	**41**	106.807	93.846	140.51
16	13.68	4.92	1.82	**42**	119.669	108.750	171.99
17	14.56	5.45	2.18	**43**	134.580	126.498	211.56
18	15.52	6.04	2.59	**44**	151.950	147.736	261.60
19	16.56	6.70	3.07	**45**	172.285	173.285	325.34
20	17.69	7.44	3.64	**46**	196.219	204.191	407.11
21	18.92	8.26	4.31	**47**	224.549	241.800	512.84
22	20.27	9.19	5.09	**48**	258.285	287.855	650.67
23	21.75	10.23	6.00	**49**	298.718	344.636	831.99
24	23.36	11.40	7.08	**50**	347.509	415.146	1072.80
25	25.13	12.72	8.34				

Figure 1 Variation of bearing capacity factor $N_{c,e}$ with friction angle ϕ' for different values of eccentricity ratio e/B.

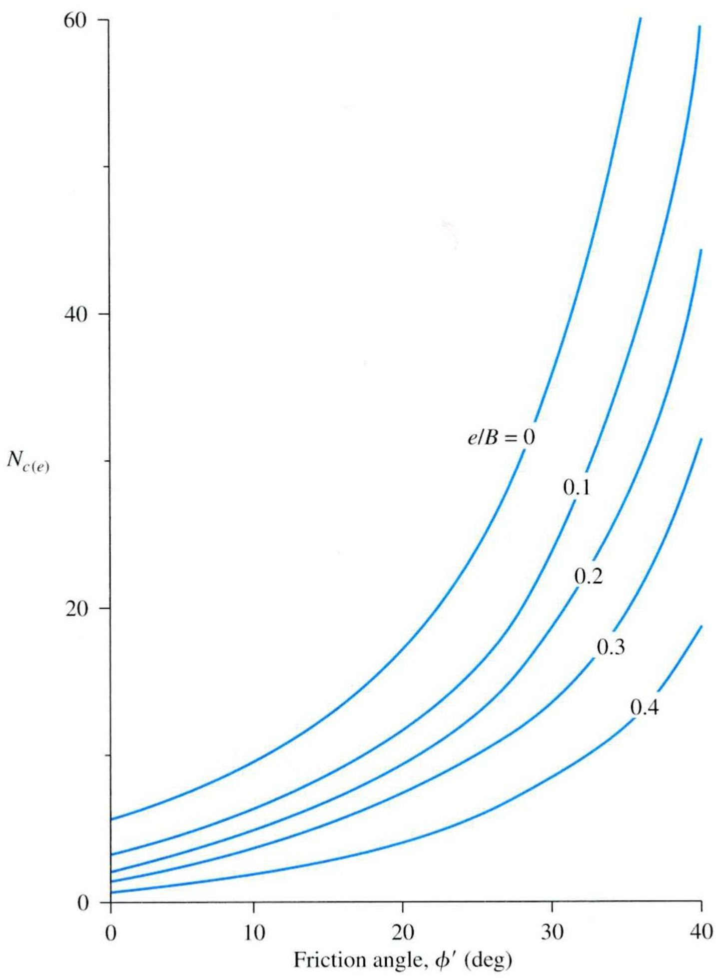


Figure 2 Variation of bearing capacity factor $N_{q,e}$ with friction angle ϕ' for different values of eccentricity ratio e/B.

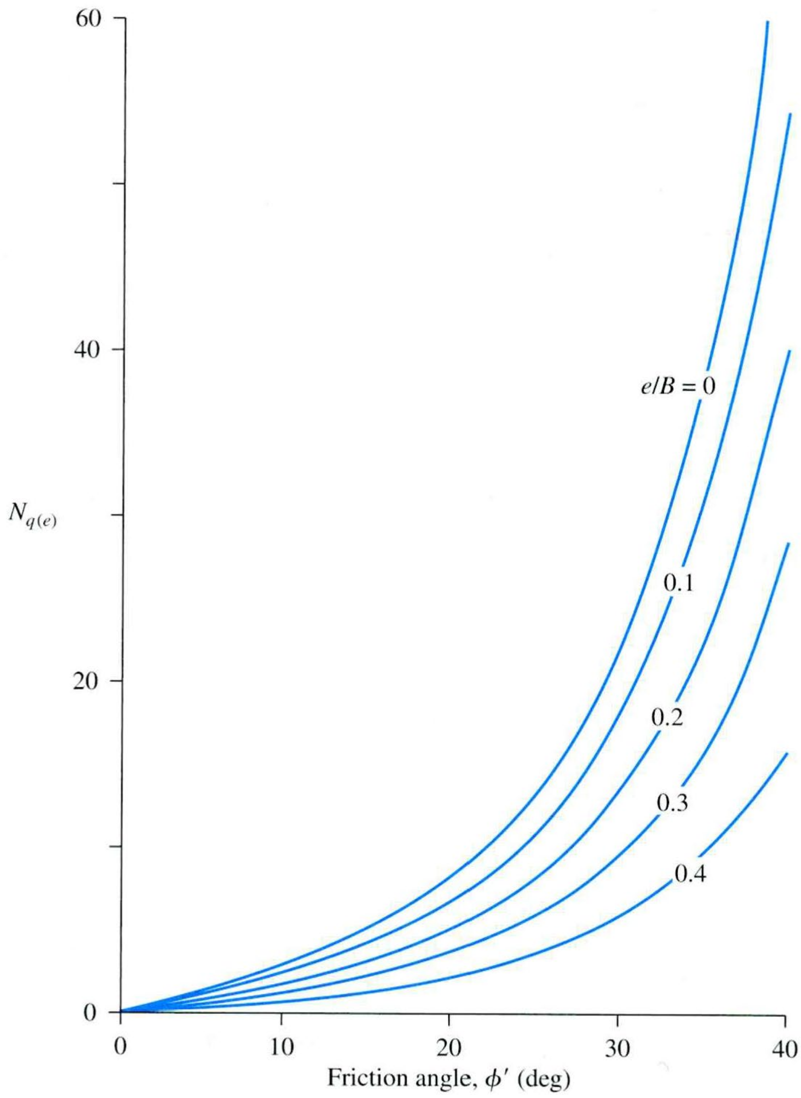


Figure 3 Variation of bearing capacity factor $N_{\gamma,e}$ with friction angle ϕ' for different values of eccentricity ratio e/B.

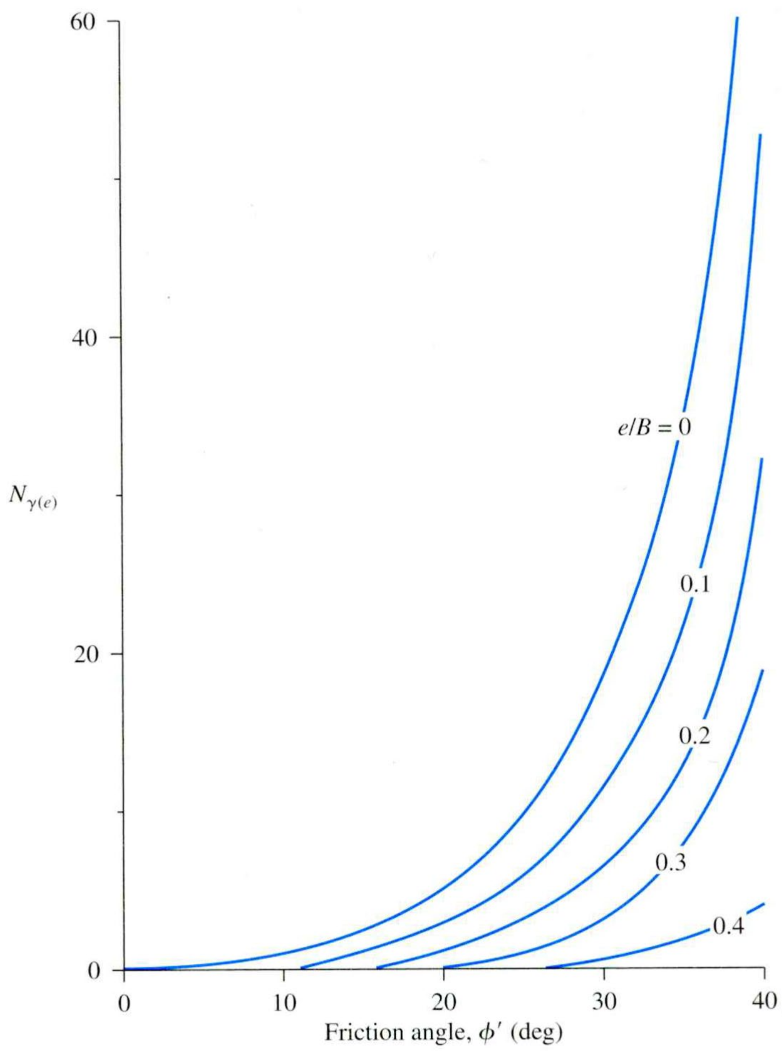


Table 2 Variations of *a* and *k* in Purkayastha & Char's eccentric load reduction factor.

D_f/B	a	k
0.00	1.862	0.73
0.25	1.811	0.785
0.50	1.754	0.80
1.00	1.820	0.888

→ Factors for use with the general bearing capacity equation

→ Shape Factors

$$F_{cs} = 1 + \left(\frac{B}{L}\right)\left(\frac{N_q}{N_c}\right)$$

$$F_{qs} = 1 + \left(\frac{B}{L}\right)\tan\phi'$$

$$F_{\gamma s} = 1 - 0.4\left(\frac{B}{L}\right)$$

→ Depth Factors

$$F_{cd} = 1 + 0.4\left(\frac{D_f}{B}\right)$$

$$F_{qd} = 1 + 2\tan\phi'(1-\sin\phi')^2\frac{D_f}{B}$$

$$F_{\gamma d} = 1$$

The equations for F_{cd} and F_{qd} are valid if $D_f/B \leq 1.0$. For a depth-of-embedment-to-foundation-width ratio greater than unity, the equations have to be modified to

$$F_{cd} = 1 + 0.4\tan^{-1}\left(\frac{D_f}{B}\right)$$

$$F_{qd} = 1 + 2\tan\phi'(1-\sin\phi')^2\tan^{-1}\left(\frac{D_f}{B}\right)$$

→ Inclination Factors

$$F_{ci} = F_{qi} = \left(1 - \frac{\beta^\circ}{90^\circ}\right)^2$$

$$F_{\gamma i} = \left(1 - \frac{\beta^\circ}{\phi'}\right)^2$$

where β = inclination of the load on the foundation with respect to the vertical.

→ Effect of soil compressibility – Vesic's formulation

Vesic (1973) introduced three additional factors, F_{cc}, F_{qc}, and $F_{\gamma c}$, to be used in the bearing capacity equation when the effect of soil compressibility is to be taken into account. According to Vesic's theory, in order to calculate these factors, the following steps should be taken.

Step 1: *Calculate the rigidity index*. This is given by

$$I_r = \frac{E_s}{2(1+\mu_s)(c' + q'\tan\phi')}$$

where E_s is Young's modulus, μ_s is Poisson's ratio, c' is the soil cohesion, q' is the effective overburden pressure at a depth of D_f + *B*/2, and the ϕ' is the soil's friction angle.

Step 2: *Compute the critical rigidity index*, $I_{r(cr)}$. This is given by

$$I_{r(\text{cr})} = \frac{1}{2}\exp\left[\left(3.30 - 0.45\frac{B}{L}\right)\cot\left(45 - \frac{\phi'}{2}\right)\right]$$

Step 3: *Compute the compressibility bearing capacity factors.* If $I_r \geq I_{r(\text{cr})}$, then we have simply

$$F_{cc} = F_{qc} = F_{\gamma c} = 1$$

whereas if $I_r < I_{r(cr)}$, then

$$F_{\gamma c} = F_{qc} = \exp\left\{\left(-4.4 + 0.6\frac{B}{L}\right)\tan\phi' + \left[\frac{(3.07\sin\phi')(\log 2I_r)}{1+\sin\phi'}\right]\right\}$$

and, for ϕ' = 0,

$$F_{cc} = 0.32 + 0.12\frac{B}{L} + 0.60\log I_r$$

or, if ϕ' > 0,

$$F_{cc} = F_{qc} - \frac{1 - F_{qc}}{N_q\tan\phi'}$$

► SOLUTIONS

P.1 ■ Solution

The bearing pressure can be computed with the equation

$$q = \frac{P + W_f}{A} - u = \frac{P + W_f}{B \times L} - u$$

where *P* is the applied load on the footing, W_f is the weight of the footing, *A* = *B*×*L* is the area of the footing, with *B* and *L* being its width and length, and *u* is the porewater pressure. Consider first that the water table is at position A, namely, 21 m below the surface; since the water table is substantially below the bottom of the footing, we can take *u* = 0 in the computation of the average bearing pressure. We have *P* = 1300 MN, W_f = 140 MN, *B* = 45 m, and *L* = 90 m. Substituting in the relation above gives

$$q_A = \frac{(1300\times10^3) + (140\times10^3)}{45\times90} - 0 = \boxed{355.6\ \text{kN/m}^2}$$

Next, suppose that the groundwater level is moved to position B, 6.5 m below the surface. Since the water table level is now less than the depth of the footing, porewater pressure will affect the bearing pressure calculations; that is, *u* is such that

$$u = \gamma_w\left(D_{\text{footing}} - D_{\text{water table}}\right)$$

in which γ_w = 9.81 kN/m^3 is the unit weight of water, $D_{footing}$ = 15.0 m is the depth of the footing, and $D_{water\ table}$ = 6.5 m is the depth of the groundwater. Substituting, we obtain

$$u = 9.81 \times (15.0 - 6.5) = 83.4\ \text{kN/m}^2$$

The updated bearing pressure q_B is such that

$$q_B = \underbrace{\frac{P + W_f}{A}}_{=355.6} - u = 355.6 - 83.4 = \boxed{272.2\ \text{kN/m}^2}$$

► The correct answer is **D**.

P.2 ■ Solution

The minimum bearing pressure for a foundation with trapezoidal pressure distribution (that is, such that $e \leq B/6$) can be determined with the equation

$$q_{\min} = \left(\frac{P+W}{A} - u\right)\left(1 - \frac{6e}{B}\right)$$

where *P* is the applied load on the footing, *W* is the weight of the footing, *A* is the area of the footing, *u* is the porewater pressure, *e* is the eccentricity of the bearing pressure distribution, and *B* is the width of the footing. We were given *P* = 4000 kN; the weight *W* of the footing is the product of its volume *V* = *B*×*L*×*D* and the unit weight of concrete γ_c = 23.6 kN/m^3, i.e.,

$$W = \gamma_c \times V = \gamma_c \times B \times L \times D = 23.6 \times 5 \times 5 \times 2 = 1180 \text{ kN}$$

We also require geometric parameters *B* = 5 m and *A* = *B*×*L* = 5×5 = 25 m^2. Since the groundwater level is substantially below the bottom of the footing, we can take *u* = 0. The eccentricity *e* is the ratio of the applied moment to the sum of the applied forces,

$$e = \frac{M}{P+W} = \frac{970}{4000+1180} = 0.19 \text{ m}$$

Note that *e* = 0.19 m ≤ *B*/6 = 0.83 m, thus corroborating our assumption of a trapezoidal pressure distribution. It remains the substitute each variable in the expression for q_{min},

$$q_{\min} = \left[\frac{4000+1180}{(5\times 5)} - 0\right]\left(1 - \frac{6\times 0.19}{5}\right) = \boxed{160.0 \text{ kN/m}^2}$$

The maximum bearing pressure, in turn, is obtained with the similar formula

$$q_{\max} = \left(\frac{P+W}{A} - u\right)\left(1 + \frac{6e}{B}\right)$$

$$\therefore q_{\max} = \left[\frac{4000+1180}{(5\times 5)} - 0\right]\left(1 + \frac{6\times 0.19}{0.5}\right) = \boxed{254.4 \text{ kN/m}^2}$$

► The correct answer is **C**.

P.3 ■ Solution

Part A: Use Terzaghi's equation.

$$q_{\text{ult}} = c'N_c + qN_q + \frac{1}{2}B\gamma N_\gamma$$

We were given *c'* = 0 (the soil is cohesionless) and *B* = 3 m. From Table 1, we read coefficients N_q = 22.46, N_c = 37.16, and $N\gamma$ = 19.13. Substituting in the foregoing equation gives

$$q_{\text{ult}} = 0\times 37.16 + (1\times 17)\times 22.46 + \frac{1}{2}\times 2\times 17\times 19.13 = 707.03 \text{ kN/m}^2$$

Given a factor of safety *FS* = 4, the allowable bearing capacity q_{all} is

$$q_{\text{all}} = \frac{q_{\text{ult}}}{FS} = \frac{707.03}{4} = \boxed{176.8 \text{ kN/m}^2}$$

The bearing capacity of the foundation under service-load conditions is about 177 kilonewtons per square meter.

► The correct answer is **B**.

Part B: The bearing capacity for a continuous foundation such as the one in the present problem can be computed with Terzaghi's equation,

$$q_{\text{ult}} = c'N_c + qN_q + \frac{1}{2}B\gamma N_\gamma$$

We were given c' = 600 lb/ft^3 and B = 4 ft. From Table 1, we read coefficients N_q = 12.72, N_c = 25.13 and $N\gamma$ = 8.34. Substituting in the foregoing equation gives

$$q_{\text{ult}} = 600\times 25.13 + (3\times 110)\times 12.72 + \frac{1}{2}\times 4\times 110\times 8.34 = 21.1\ \text{kip/ft}^2$$

Given a factor of safety FS = 4, the allowable bearing capacity q_{all} is

$$q_{\text{all}} = \frac{q_{\text{ult}}}{FS} = \frac{21.1}{4} = \boxed{5.3\ \text{kip/ft}^2}$$

The bearing capacity of the foundation under service-load conditions is about 5 ksf.

► The correct answer is **B**.

Part C: In this case, we use Terzaghi's equation for a square foundation, which is slightly different from that for a continuous foundation,

$$q_{\text{ult}} = c'N_c + qN_q + 0.4B\gamma N_\gamma$$

We were given c' = 0 (the soil is cohesionless) and B = 3 m. From Table 1, we read coefficients N_q = 22.46, N_c = 37.16, and $N\gamma$ = 19.13. Substituting in the foregoing equation gives

$$q_{\text{ult}} = 0\times 37.16 + (2\times 16.5)\times 22.46 + 0.4\times 3\times 16.5\times 19.13 = 1120\ \text{kN/m}^2$$

Given a factor of safety FS = 4, the allowable bearing capacity q_{all} is

$$q_{\text{all}} = \frac{1120}{4} = \boxed{280\ \text{kN/m}^2}$$

The bearing capacity of the foundation under service-load conditions is about 280 kilonewtons per square meter.

► The correct answer is **A**.

P.4 ■ Solution

The allowable bearing capacity is given by the ratio of the applied load Q to the cross-section area of the footing, which is the square of its size B ; that is,

$$q_{\text{all}} = \frac{Q}{B^2}$$

Since the gross allowable load is 1805 kN, it follows that

$$q_{\text{all}} = \frac{1805}{B^2}$$

It remains to compute the allowable bearing capacity, q_{all}, which is related to the ultimate bearing capacity q_u by the factor of safety, FS, namely,

$$q_{\text{all}} = \frac{q_u}{\text{FS}} = \frac{q_u}{3} \rightarrow q_u = 3q_{\text{all}}$$

Substituting in the foregoing equation gives

$$q_u = 3q_{\text{all}} = 3\times\frac{1805}{B^2} = \frac{5415}{B^2} \quad \text{(I)}$$

We have thus attained an expression that provides the dimension of the footing as a function of the ultimate bearing capacity q_u only. This latter variable is to be calculated with Terzaghi's equation,

$$q_u = 1.3c'N_c + qN_q + 0.4\gamma BN_\gamma$$

$$\therefore q_u = 1.3c'N_c + \gamma D_f N_q + 0.4\gamma BN_\gamma \quad \text{(II)}$$

Equating expressions (I) and (II), we get

$$\frac{5415}{B^2} = 1.3c'N_c + \gamma D_f N_q + 0.4\gamma B N_\gamma$$

Then, we substitute c' = 0 (cohesionless soil), D_f = 1.5 m, γ = 15.9 kN/m^3, as well as coefficients N_c = 52.64, N_q = 36.50, and $N\gamma$ = 38.04 from Table 1, so that

$$\frac{5415}{B^2} = 1.3 \times 0 \times 52.64 + 15.9 \times 1.5 \times 36.50 + 0.4 \times 15.9 \times 38.04B$$

$$\therefore \frac{5415}{B^2} = 0 + 870.53 + 241.93B$$

$$\therefore 5415 = 870.53B^2 + 241.93B^3$$

$$\therefore 241.93B^3 + 870.53B^2 - 5415 = 0$$

The result is a third-degree equation in the dimension B. One way to solve it is by means of the *Solve* command in Mathematica,

$$\text{Solve}[241.93B^3 + 870.53B^2 - 5415 == 0, B]$$

This yields two imaginary solutions and B = 2 m. Hence, a square foundation of capable of withstanding this allowable load with the prescribed factor of safety has a size of 2 meters.

► The correct answer is **B**.

P.5 ■ Solution

The design working load can be determined with the equation

$$\frac{P}{b} = P_D + P_L$$

where P_D = 120 kN/m is the dead load on the column and P_L = 100 kN/m is the live load on the column. The vertical effective stress is determined with the effective stress formula,

$$\sigma'_{zD} = \gamma D - u$$

where γ = 19.2 kN/m^3 is the unit weight of the soil, D = 0.4 m is the foundation depth, and u is the porewater pressure, which is taken as zero due to the depth of the groundwater table. Thus, σ'_{zD} is

$$\sigma'_{zD} = 19.2 \times 0.4 - 0 = 7.68 \text{ kN/m}^2$$

The ultimate bearing capacity is obtained with Terzaghi's equation,

$$q_{\text{ult}} = c'N_c + \sigma'_{zD} N_q + \frac{1}{2}\gamma' B N_\gamma$$

The N factors for ϕ' = 37° are N_c = 70.1, N_q = 53.8, and N_γ = 68.1 (Table 1). Substituting the available data in the equation above gives

$$q_{\text{ult}} = c'N_c + \sigma'_{zD} N_q + \frac{1}{2}\gamma' B N_\gamma = 0 \times 70.1 + 7.68 \times 53.8 + \frac{1}{2} \times 19.2 \times B \times 68.1$$

$$\therefore q_{\text{ult}} = 413.18 + 653.76B$$

For a factor of safety equal to 2, the allowable bearing capacity q_{all} follows as

$$q_{\text{all}} = \frac{413.18 + 653.76B}{2} = 206.59 + 326.88B \quad \text{(I)}$$

The weight of the foundation is the product of the volume of soil, V, and the unit weight of soil, γ,

$$W_f = V\gamma = B^2 D\gamma \rightarrow \frac{W_f}{b} = BD\gamma$$

Here, we substitute D = 0.4 m and γ = 19.2 kN/m^3,

$$\frac{W_f}{b} = B \times 0.4 \times 19.2 = 7.68B \text{ (II)}$$

Next, we equate the formula for bearing capacity to that for the allowable bearing capacity,

$$q_{\text{all}} = q = \frac{P/b + W_f/b}{A} - u$$

We have already obtained an expression for the allowable bearing capacity, namely, equation (I). Further, we can substitute *P*/*b* = 220 kN/m, W_f/b = 7.68 from equation (II), *A* = B^2 due to the geometry of the footing, and *u* = 0 because of the position of the groundwater table, giving

$$206.59 + 326.88B = \frac{220 + 7.68B}{B} - 0$$

$$\therefore (206.59 + 326.88B)B = 220 + 7.68B$$

$$\therefore 326.88B^2 + 206.59B = 220 + 7.68B$$

$$\therefore 326.88B^2 + 206.59B - 7.68B - 220 = 0$$

$$\therefore 326.88B^2 + 198.91B - 220 = 0$$

The ensuing equation is a second-degree polynomial in *B*. and one way to solve it is by means of the *Solve* command in Mathematica,

$$\text{Solve}[326.88B^2 + 198.91B - 220 == 0, B]$$

This a negative root and *B* = 0.57 m, which is the one feasible solution.

► The correct answer is **A**.

P.6 ■ Solution

Part A: As the engineer should know, the general bearing capacity equation referenced in the problem statement is

$$q_u = c'N_c F_{cs} F_{cd} F_{ci} + qN_q F_{qs} F_{qd} F_{qi} + \frac{1}{2}\gamma B N_\gamma F_{\gamma s} F_{\gamma d} F_{\gamma i}$$

in which N_c, N_q, and N_γ are the familiar bearing capacity factors for cohesion, self-weight of the soil and effective overburden pressure, respectively, and are all dependent on the angle of internal friction. As demanded by the problem statement, these are to be obtained with the prescribed mathematical equations rather than tabulated values. First, we have N_q,

$$N_q = \tan^2\left(45^\circ + \frac{\phi'}{2}\right)e^{\pi \tan\phi'} = \tan^2\left(45^\circ + \frac{30^\circ}{2}\right)e^{\pi \tan 30^\circ} = 18.40$$

and then we compute N_c and N_γ,

$$N_c = (N_q - 1)\cot\phi' = (18.40 - 1)\cot 30^\circ = 30.14$$

$$N_\gamma = 2(N_q + 1)\tan\phi' = 2 \times (18.40 + 1) \times \tan 30^\circ = 22.40$$

We also have shape factors F_{cs}, F_{qs}, and $F_{\gamma s}$, depth factors F_{cd}, F_{qd}, and $F_{\gamma d}$, and load inclination factors F_{ci}, F_{qi}, and $F_{\gamma i}$. Since the foundation is continuous, the shape factors are all equal to unity. As the load is vertical, the inclination factors are all 1 as well. Finally, the expressions for the depth factors are dependent on the depth-to-width ratio, D_f/B, which in this case is D_f/B = ½ = 0.5 < 1.0. Thus, depth factor F_{qd} is

$$F_{qd} = 1 + 2\tan\phi'(1 - \sin\phi')^2 \frac{D_f}{B} = 1 + 2 \times \tan 30^\circ (1 - \sin 30^\circ)^2 \times \frac{1}{2} = 1.14$$

while depth factor F_{cd} can be ignored, since it is multiplied by the cohesion *c'*. Lastly, we have $F_{\gamma d}$ = 1.0. The ultimate bearing capacity is now

$$q_{\text{ult}} = c'N_c \underbrace{\prod F_c}_{=1} + qN_q \underbrace{\prod F_q}_{=1.14} + \frac{1}{2}\gamma BN_\gamma \underbrace{\prod F_\gamma}_{=1}$$

$$q_{\text{ult}} = 0\times30.14 + (1\times17)\times18.40\times1.14 + \frac{1}{2}\times2\times17\times22.40 = 737.4\text{ kN/m}^2$$

The corresponding allowable bearing capacity is

$$q_{\text{all}} = \frac{q_{\text{ult}}}{FS} = \frac{737.4}{4} = \boxed{184.4\text{ kN/m}^2}$$

This value is about 4.3% higher than that predicted with the elementary Terzaghi theory.

Part B: As before, we begin with bearing capacity factor N_q,

$$N_q = \tan^2\left(45^\circ + \frac{\phi'}{2}\right)e^{\pi\tan\phi'} = \tan^2\left(45^\circ + \frac{25^\circ}{2}\right)e^{\pi\times\tan25^\circ} = 10.66$$

Thereafter, we establish factors N_c and N_γ,

$$N_c = (N_q - 1)\cot\phi' = (10.66-1)\times\cot25^\circ = 20.72$$

$$N_\gamma = 2(N_q+1)\tan\phi' = 2\times(10.66+1)\times\tan25^\circ = 10.87$$

There are also the shape factors F_{cs}, F_{qs}, and $F_{\gamma s}$, the depth factors F_{cd}, F_{qd}, and $F_{\gamma d}$, and the load inclination factors F_{ci}, F_{qi}, and $F_{\gamma i}$. Since the foundation is continuous, the shape factors are all equal to unity, and, as the load is vertical, load inclination factors are also equal to 1. Depth factors depend on the ratio D_f/B, which in this case is ¾ = 0.75. Therefore, depth factor F_{qd} is

$$F_{qd} = 1 + 2\tan\phi'(1-\sin\phi')^2\frac{D_f}{B} = 1 + 2\times\tan25^\circ\times(1-\sin25^\circ)^2\times\frac{3}{4} = 1.23$$

Depth factor F_{cd}, in turn, is

$$F_{cd} = F_{qd} - \left(\frac{1-F_{qd}}{N_c\tan\phi'}\right) = 1.23 - \left(\frac{1-1.23}{20.72\times\tan25^\circ}\right) = 1.25$$

Finally, we have depth factor $F_{\gamma d}$ = 1.0. Thereafter, the ultimate bearing capacity can be computed with the equation

$$q_{\text{ult}} = c'N_c\prod F_c + qN_q\prod F_q + \frac{1}{2}\gamma BN_\gamma\prod F_\gamma$$

$$\therefore q_{\text{ult}} = 600\times20.72\times(1.25) + (3\times110)\times10.66\times(1.23) + \frac{1}{2}\times110\times4\times10.87\times(1)$$

$$\therefore q_{\text{ult}} = 22.3\text{ kip/ft}^2$$

The corresponding allowable bearing capacity is

$$q_{\text{all}} = \frac{22.3}{4} = \boxed{5.6\text{ kip/ft}^2}$$

This value is about 5.7% greater than the allowable bearing capacity predicted with the Terzaghi equation.

P.7 ■ Solution

We begin by computing the Terzaghi bearing capacity factors,

$$N_q = \tan^2\left(45^\circ + \frac{\phi'}{2}\right)e^{\pi\tan\phi'} = \tan^2\left(45^\circ + \frac{25^\circ}{2}\right)e^{\pi\tan25^\circ} = 10.66$$

$$N_c = (N_q - 1)\cot\phi' = (10.66-1)\times\cot25^\circ = 20.72$$

$$N_\gamma = 2(N_q+1)\tan\phi' = 2\times(10.66+1)\times\tan25^\circ = 10.87$$

We proceed to determine the shape factors, noting that $B = L = 6$ ft,

$$F_{cs} = 1 + \underbrace{\left(\frac{B}{L}\right)}_{=1}\left(\frac{N_q}{N_c}\right) = 1 + \frac{10.66}{20.72} = 1.51$$

$$F_{qs} = 1 + \underbrace{\left(\frac{B}{L}\right)}_{=1}\tan\phi' = 1 + \tan 25^\circ = 1.47$$

$$F_{\gamma s} = 1 - 0.4\underbrace{\left(\frac{B}{L}\right)}_{=1} = 1 - 0.4 = 0.6$$

Next, noting that the load makes an inclination $\beta = 15^\circ$ with the vertical and that the soil friction angle is $\phi' = 25^\circ$, we can establish the inclination factors,

$$F_{ci} = F_{qi} = \left(1 - \frac{\beta^\circ}{90^\circ}\right)^2 = \left(1 - \frac{15^\circ}{90^\circ}\right)^2 = 0.69$$

$$F_{\gamma i} = \left(1 - \frac{\beta}{\phi'}\right)^2 = \left(1 - \frac{15^\circ}{25^\circ}\right)^2 = 0.16$$

Then, noting that $D_f/B = 3/6 = 0.5 < 1$, the proper equations to calculate the depth factors are the following,

$$F_{qd} = 1 + 2 \times \tan 25^\circ \times \left(1 - \sin 25^\circ\right)^2 \times \left(\frac{3}{6}\right) = 1.16$$

$$F_{cd} = F_{qd} - \frac{1 - F_{qd}}{N_c \tan\phi'} = 1.16 - \frac{1 - 1.16}{20.72 \times \tan 25^\circ} = 1.18$$

$$F_{\gamma d} = 1$$

The ultimate bearing capacity of the foundation, q_{ult}, is determined to be

$$q_{\text{ult}} = c'N_c F_{cs} F_{cd} F_{ci} + qN_q F_{qs} F_{qd} F_{qi} + \frac{1}{2}\gamma B N_\gamma F_{\gamma s} F_{\gamma d} F_{\gamma i}$$

$$\therefore q_{\text{ult}} = 500 \times 20.72 \times 1.51 \times 1.18 \times 0.69 + (3 \times 115) \times 10.66 \times 1.47 \times 1.18 \times 0.69$$

$$+ \frac{1}{2} \times 115 \times 6 \times 10.87 \times 0.6 \times 1 \times 0.16 = 17{,}500 \text{ lb/ft}^2 = 17.5 \text{ kip/ft}^2$$

The corresponding load, Q_{ult}, is

$$Q_{\text{ult}} = q_{\text{ult}} B^2 = 17.5 \times 6^2 = 630 \text{ kips}$$

For a factor of safety $FS = 4$, the gross allowable load is such that

$$Q_{\text{all}} = \frac{Q_{\text{ult}}}{FS} = \frac{630}{4} = \boxed{157.5 \text{ kips}}$$

► The correct answer is **B**.

P.8 ■ Solution

The friction angle can be estimated with the relation

$$\phi' = \tan^{-1}\left[\frac{N_{60}}{12.2 + 20.3\left(\frac{\sigma'_o}{p_a}\right)}\right]^{0.34}$$

The values of ϕ' for each data point are tabulated below.

Depth (ft)	σ_0' (psi)	p_a (psi)	N_{60}	ϕ' (deg)
5	110 × 5 = 550 psf = 3.82 psi	14.7	11	40.5
10	7.64	14.7	14	40.3
15	11.5	14.7	16	39.6
20	15.3	14.7	21	40.5
25	19.1	14.7	24	40.4

For the first data point, for instance, we have

$$\phi' = \tan^{-1}\left[\frac{11}{12.2+20.3\times\left(\dfrac{3.82}{14.7}\right)}\right]^{0.34} = 40.5^\circ$$

The friction angle is taken as the average of the ϕ' values indicated in the blue column above, namely,

$$\phi' = \frac{40.5+40.3+39.6+40.5+40.4}{5} = 40.3^\circ$$

We can then proceed to determine the bearing capacity for the foundation. In this application of the general bearing capacity equation, the only relevant factors are the shape and depth factors, which are calculated as

$$F_{qs} = 1+\frac{B}{L}\tan\phi' = 1+\frac{8}{8}\times\tan 40.3^\circ = 1.85$$

$$F_{\gamma s} = 1-0.4\frac{B}{L} = 1-0.4\times\frac{8}{8} = 0.6$$

$$F_{qd} = 1+2\tan\phi'(1-\sin\phi')^2\frac{D_f}{B} = 1+2\tan 40.3^\circ\left(1-\sin 40.3^\circ\right)^2\times\frac{5}{8} = 1.13$$

$$F_{\gamma d} = 1.0$$

In addition, we have N_q = 64.2 and N_γ = 109.4 (Table 1). Substituting in the bearing capacity equation gives

$$Q_u = \frac{8\times 8}{1000}\times\left[(5\times 110)\times 64.2\times 1.85\times 1.13+\frac{1}{2}\times 110\times 8\times 109.4\times 0.6\times 1.0\right] = \boxed{6573\text{ kip}}$$

► The correct answer is **C**.

P.9 ■ Solution

Vesic's equation is distinct from the general bearing capacity equation in that it accounts for the change in failure mode of soil, which is due to soil compressibility. The modified equation in question is the following,

$$q_{\text{ult}} = c'N_cF_{cs}F_{cd}F_{cc} + qN_qF_{qs}F_{qd}F_{qc} + \frac{1}{2}\gamma BN_\gamma F_{\gamma s}F_{\gamma d}F_{\gamma c}$$

in which F_{cc}, F_{qc}, and $F_{\gamma c}$ are the soil compressibility factors. We begin by determining factors N_q, N_c, and N_γ, which are given by the usual formulas

$$N_q = \tan^2\left(45^\circ+\frac{\phi'}{2}\right)e^{\pi\tan\phi'} = \tan^2\left(45^\circ+\frac{25^\circ}{2}\right)e^{\pi\tan 25^\circ} = 10.66$$

$$N_c = (N_q-1)\cot\phi' = (10.66-1)\times\cot 25^\circ = 20.72$$

$$N_\gamma = 2(N_q+1)\tan\phi' = 2\times(10.66+1)\times\tan 25^\circ = 10.87$$

We proceed to determine the shape factors,

$$F_{qs} = 1 + \frac{B}{L} \times \tan\phi' = 1 + \frac{1}{1.5} \times \tan 25^\circ = 1.31$$

$$F_{cs} = 1 + \left(\frac{B}{L}\right)\left(\frac{N_q}{N_c}\right) = 1 + \left(\frac{1}{1.5}\right) \times \left(\frac{10.66}{20.72}\right) = 1.34$$

$$F_{\gamma s} = 1 - 0.4\left(\frac{B}{L}\right) = 1 - 0.4 \times \left(\frac{1}{1.5}\right) = 0.73$$

Then, we compute the depth factors. The set of expressions to be used is dictated by the magnitude of the ratio D_f/B = 1/1 = 1.0. Accordingly,

$$F_{qd} = 1 + 2\tan\phi'(1 - \sin\phi')^2\left(\frac{D_f}{B}\right) = 1 + 2 \times \tan 25^\circ \times (1 - \sin 25^\circ)^2 \times \left(\frac{1.0}{1.0}\right) = 1.31$$

$$F_{cd} = F_{qd} - \left(\frac{1 - F_{qd}}{N_c \tan\phi'}\right) = 1.31 - \left(\frac{1 - 1.31}{20.72 \times \tan 25^\circ}\right) = 1.34$$

$$F_{\gamma d} = 1$$

It remains to compute the soil compressibility factors. These, in turn, depend on the rigidity index I_r of the soil-foundation system, i.e.,

$$I_r = \frac{E_s}{2(1 + \mu_s)(c' + q'\tan\phi')}$$

where E_s = 1020 kN/m^2 is Young's modulus for the soil, μ_s = 0.35 is Poisson's ratio for the soil, c' = 50 kN/m^2 is the cohesion of the soil, $q' = \gamma(D_f + B/2)$ = 17×(1 + ½) = 25.5 kN/m^2 is the soil overburden pressure at a depth approximately $D_f + B/2$ below the bottom of the footing, and ϕ' = 25°. Substituting, we obtain

$$I_r = \frac{1020}{2 \times (1 + 0.35) \times (50 + 17 \times \tan 25^\circ)} = 6.10$$

The critical rigidity index, $I_{r,\text{crit}}$, is used to settle which set of values and expressions will be used for the soil compressibility factors; it is given by

$$I_{r,\text{crit}} = \frac{1}{2}\left\{\exp\left[\left(3.30 - 0.45\frac{B}{L}\right)\cot\left(45^\circ - \frac{\phi'}{2}\right)\right]\right\}$$

$$I_{r,\text{crit}} = \frac{1}{2}\left\{\exp\left[\left(3.30 - 0.45 \times \frac{1}{1.5}\right)\cot\left(45^\circ - \frac{25^\circ}{2}\right)\right]\right\} = 55.47$$

This is significantly larger than the I_r we have computed. Consequently, the soil compressibility factors are the following,

$$F_{\gamma c} = F_{qc} = \exp\left\{\left(-4.4 + 0.6\frac{B}{L}\right)\tan\phi' + \left[\frac{(3.07\sin\phi')(\log 2I_r)}{1 + \sin\phi'}\right]\right\}$$

$$\therefore F_{\gamma c} = F_{qc} = \exp\left\{\left(-4.4 + 0.6 \times \frac{1.0}{1.5}\right)\tan 25^\circ + \left[\frac{(3.07 \times \sin 25^\circ)(\log_{10} 2 \times 6.10)}{1 + \sin 25^\circ}\right]\right\} = 0.42$$

$$F_{cc} = F_{qc}\left(\frac{1 - F_{qc}}{N_q \tan\phi'}\right) = 0.42 - \left(\frac{1 - 0.42}{10.66 \times \tan 25^\circ}\right) = 0.30$$

We are now ready to substitute the available variables in Vesic's equation,

$$q_{\text{ult}} = c'N_cF_{cs}F_{cd}F_{cc} + qN_qF_{qs}F_{qd}F_{qc} + \frac{1}{2}\gamma BN_\gamma F_{\gamma s}F_{\gamma d}F_{\gamma c}$$

$$\therefore q_{\text{ult}} = 50 \times 20.72 \times 1.34 \times 1.34 \times 0.30 + (17 \times 1) \times 10.66 \times 1.31 \times 1.31 \times 0.42$$
$$+ \frac{1}{2} \times 17 \times 1 \times 10.87 \times 0.73 \times 1 \times 0.42 = 4258.22 = \boxed{717\ \text{kN/m}^2}$$

The ultimate bearing capacity accounting for soil compressibility is close to 700 kilonewtons per square meter.

► The correct answer is **C**.

P.10 ■ Solution

Part A: For $\phi' = 40°$, from Table 1 we read bearing capacity factors N_q = 64.20 and N_γ = 109.41. The modified width B' is such that

$$B' = B - 2e = 2.1 - 2 \times 0.3 = 1.5\,\text{m}$$

Because the foundation in question is continuous, B'/L' is zero. Also, shape bearing capacity factors $F_{qs} = F_{\gamma s} = 1$. Since the load is vertical, load inclination factors $F_{qi} = F_{\gamma i} = 1$. As for the depth factors, we have

$$F_{qd} = 1 + 2\tan\phi'(1-\sin\phi')^2 \frac{D_f}{B} = 1 + 2\times\tan 40° \times (1-\sin 40°)^2 \times \frac{1.5}{2.1} = 1.15$$

$$F_{\gamma d} = 1$$

We can then determine the ultimate bearing capacity with the complete formula

$$q'_{\text{ult}} = qN_q F_{qs} F_{qd} F_{qi} + \frac{1}{2}\gamma' B' N_\gamma F_{\gamma s} F_{\gamma d} F_{\gamma i}$$

where we have highlighted the modified width B'. We have $q = \gamma D_f = 17.5 \times 1.5 =$ 26.3 kN/m^2. Substituting these and other variables, we obtain

$$q'_{\text{ult}} = 26.3 \times 64.20 \times 1 \times 1.15 \times 1 + \frac{1}{2} \times 17.5 \times 1.5 \times 109.41 \times 1 \times 1 \times 1 = 3377.7\ \text{kN/m}^2$$

Consequently, the ultimate load Q_{ult} is

$$Q_{\text{ult}} = B' \times 1 \times q'_{\text{ult}} = 1.5 \times 1 \times 3377.7 = \boxed{5067\ \text{kN}}$$

► The correct answer is **D**.

Part B: For a cohesionless soil, we do not need $N_{c,e}$. As for $N_{q,e}$ and $N_{\gamma,e}$, the factors corresponding to an eccentricity ratio e/B = 0.3/2.1 = 0.143 can be obtained by interpolation in Figures 1, 2, and 3, given the factors corresponding to e/B = 0.1 and e/B = 0.2; this yields $N_{q,e}$ = 51.40 and $N_{\gamma,e}$ = 58.81. Therefore, the ultimate loading Q_{ult} is calculated as

$$Q_{\text{ult}} = 2.1 \times \left(26.3 \times 51.40 + \frac{1}{2} \times 17.5 \times 2.1 \times 58.81\right) = \boxed{5108\ \text{kN}}$$

► The correct answer is **D**.

Part C: In the Purkayastha & Char approach, we resort to a reduction factor R_k, such that

$$R_k = 1 - \frac{q_{u,\text{eccentric}}}{q_{u,\text{centric}}}$$

in which $q_{u,eccentric}$ is the ultimate bearing capacity of eccentrically loaded continuous foundations and $q_{u,centric}$ is the ultimate bearing capacity of centrally loaded continuous foundations. The magnitude of R_k can be expressed as

$$R_k = a\left(\frac{e}{B}\right)^k$$

where a and k are functions of the embedment ratio D_f/B as given in Table 2. Combining the previous expressions, we obtain the relation

$$q_{u,\text{eccentric}} = q_{u,\text{centric}}(1 - R_k) = q_{u,\text{centric}}\left[1 - a\left(\frac{e}{B}\right)^k\right]$$

wherein the bearing capacity for centric load on a granular soil can be obtained with the usual equation

$$q_{u,\text{centric}} = qN_q F_{qd} + \frac{1}{2}\gamma B N_\gamma F_{\gamma d}$$

From Table 1 we read N_q = 64.20 and N_γ = 109.41. Depth factor $F_{\gamma d}$ = 1, whereas its counterpart F_{qd} is, as computed in Problem 9A,

$$F_{qd} = 1 + 2\tan 40^{\circ}\left(1-\sin 40^{\circ}\right)^2\left(\frac{1.5}{2.1}\right) = 1.15$$

It follows that the centric bearing capacity is

$$q_{u,\text{centric}} = 26.3\times 64.20\times 1.15+\frac{1}{2}\times 17.5\times 2.1\times 109.41\times 1 = 3952\ \text{kN/m}^2$$

The eccentricity ratio *e/B* = 0.3/2.1 = 0.143. Given an embedment ratio D_f /*B* = 0.714, coefficients *a* and *k* can be obtained by interpolation from Table 2, from which we obtain *a* = 1.782 and k = 0.838. Finally, the eccentric bearing capacity is such that

$$q_{u,\text{eccentric}} = 3952.1\left[1-1.782\times 0.143^{0.838}\right] = 2572.0\ \text{kN/m}^2$$

The ultimate load per unit length of the foundation is then

$$Q_u = B\times 1\times q_{u,\text{eccentric}} = 2.1\times 1\times 2572.0 = \boxed{5401\ \text{kN}}$$

- The correct answer is **D**.

ANSWER SUMMARY

Problem 1		D
Problem 2		C
Problem 3	**3A**	B
	3B	B
	3C	A
Problem 4		B
Problem 5		A
Problem 6	**6A**	Open-ended pb.
	6B	Open-ended pb.
Problem 7		B
Problem 8		C
Problem 9		C
Problem 10	**10A**	D
	10B	D
	10C	D

REFERENCES

- CODUTO, D. (2001). *Foundation Design: Principles and Practices*. 2nd edition. Upper Saddle River: Pearson.
- DAS, B. (2007). *Principles of Foundation Engineering*. 6th edition. Toronto: Thomson.

Got any questions related to this quiz? We can help!
Send a message to contact@montogue.com and we'll
answer your question as soon as possible.