Quiz HT103
Shape Factors
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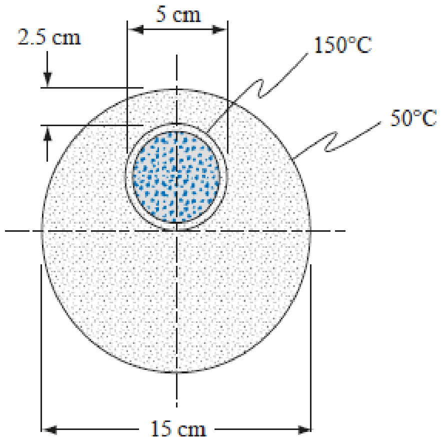
Problems

Problem 1 (Kreith et al., 2011, w/ permission)

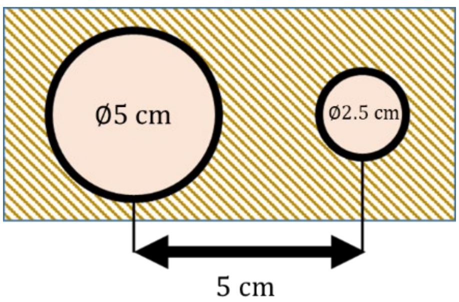
Determine the rate of heat transfer per meter length from a 5-cm-OD pipe at 150°C placed eccentrically within a larger cylinder of 85% magnesia wool (k = 0.06 W/mK) as shown in the sketch. The outside diameter of the larger cylinder is 15 cm and the surface temperature is 50°C.



A) $\dot{q}$ = 17.8 W
B) $\dot{q}$ = 28.9 W
C) $\dot{q}$ = 39.2 W
D) $\dot{q}$ = 46.5 W

Problem 2 (Kreith et al., 2011, w/ permission)

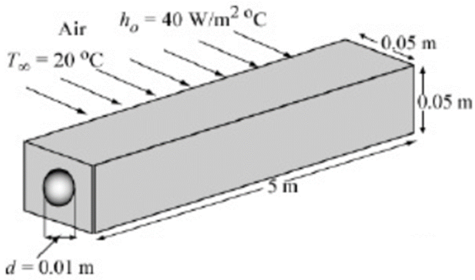
A 2.5-cm-OD hot steam line at 100°C runs parallel to a 5.0-cm-OD cold water line at 15°C. The pipes are 5 cm apart (center to center) and deeply buried in concrete with a thermal conductivity of 1.1 W/mK. What is the heat transfer rate per meter of pipe between the two pipes?



A) $\dot{q}$ = 198.7 W
B) $\dot{q}$ = 254.9 W
C) $\dot{q}$ = 300.8 W
D) $\dot{q}$ = 351.6 W

Problem 3

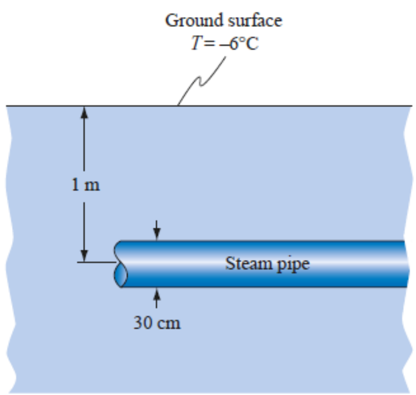
Water at 0.1 kg/s and a mean temperature of 80°C flows in a 50-mm wide, 50-mm high, and 5-m long rectangular stainless steel (k = 15 W/mK) block with a 10-mm diameter passage. The temperature of the surroundings is 20°C, and the heat transfer coefficient is 40 W/m^{2}K. Assuming that the surface heat transfer coefficient in the inner surface is very large, determine the heat transfer rate.



A) $\dot{q}$ = 2100 W
B) $\dot{q}$ = 3000 W
C) $\dot{q}$ = 3800 W
D) $\dot{q}$ = 4500 W

Problem 4A (Kreith et al., 2011, w/ permission)

A 30-cm-OD pipe with a surface temperature of 90°C carries steam over a distance of 100 m. The pipe is buried with its centerline at a depth of 1 m, the ground surface is −6°C, and the mean thermal conductivity of the soil is 0.7 W/mK. Calculate the heat transfer rate.



A) $\dot{q}$ = 6.91 kW
B) $\dot{q}$ = 10.6 kW
C) $\dot{q}$ = 16.3 kW
D) $\dot{q}$ = 20.8 kW

Problem 4B

Considering the configuration in the previous problem, determine the thickness of 85% magnesia insulation (k = 0.06 W/mK) necessary to achieve the same insulation provided by the soil with a total heat transfer coefficient of 23 W/m^{2}K on the outside of the pipe.

A) r_o = 1.5 cm
B) r_o = 3.4 cm
C) r_o = 5.1 cm
D) r_o = 6.0 cm

Problem 5 (Kreith et al., 2011, w/ permission)

A 15-cm-OD pipe is buried with its centerline 1.25 m below the surface of the ground (k of soil is 0.35 W/mK). An oil having a density of 800 kg/m^3 and a specific heat of 2.1 kJ/kg·K flows in the pipe at 5.6 liters/s. Assuming a ground surface temperature of 5°C and a pipe wall temperature of 95°C, estimate the length of pipe in which the soil temperature decreases by 5.5°C.

A) L = 319.5 m

B) L = 530.2 m

C) L = 725.1 m

D) L = 917.6 m

Problem 6 (Kreith et al., 2011, w/ permission)

A long, 1-cm-diameter electric copper cable is embedded in the center of a 25-cm-square concrete block (k = 0.128 W/mK). If the outside temperature of the concrete is 25°C and the rate of electrical energy dissipation in the cable is 150 W per meter length, determine the temperature at the surface of the cable.

A) T_s = 443.2°C

B) T_s = 638.5°C

C) T_s = 803.4°C

D) T_s = 906.4°C

Problem 7 (Bergman et al., 2011, w/ permission)

A double-glazed window consists of two sheets of glass (k = 1.4 W/mK) separated by a L = 0.2-mm-thick gap. The gap is evacuated, eliminating conduction and convection across the gap. Small cylindrical pillars, each L = 0.2 mm long and D = 0.15 mm in diameter, are inserted between the glass sheets to ensure that the glass does not break due to stresses imposed by the pressure difference across each glass sheet. A contact resistance of $R''_{t,c}$ = 1.5 × 10^{-6} m^{2}K/W exists between the pillar and the sheet. For nominal glass temperatures of T_1 = 20°C and T_2 = −10°C, determine the conduction heat transfer through an individual stainless steel pillar (k = 15 W/mK).

A) $\dot{q}$ = 5.28 mW

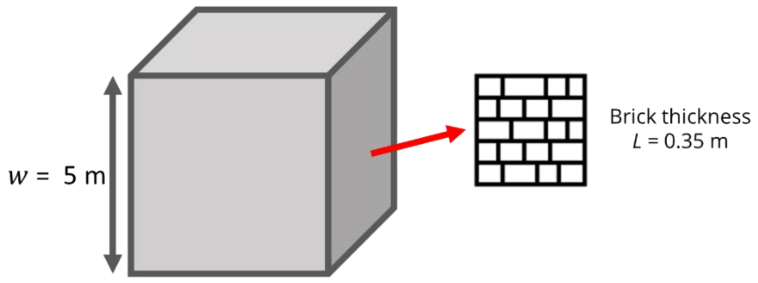
B) $\dot{q}$ = 12.6 mW

C) $\dot{q}$ = 20.3 mW

D) $\dot{q}$ = 27.4 mW

Problem 8 (Bergman et al., 2011, w/ permission)

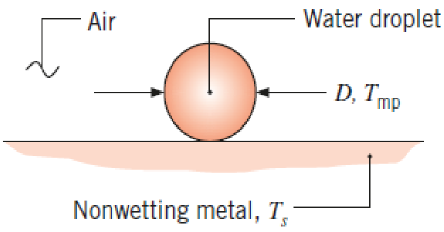
A cubical glass melting furnace has exterior dimension of width w = 5 m on a side and is constructed from refractory brick of thickness L = 0.35 m and thermal conductivity k = 1.4 W/mK. The sides and top of the furnace are exposed to ambient air at 25°C with free convection characterized by an average heat transfer coefficient of h = 5 W/m²K. The bottom of the furnace rests on a framed platform for which much of the surface is exposed to the ambient air, and a convection coefficient of h = 5 W/m²K may be assumed as a first approximation. Under operating conditions for which combustion gases maintain the inner surfaces of the furnace at 1100°C, what is the heat loss from the furnace?



A) $\dot{q}$ = 108.4 kW
B) $\dot{q}$ = 201.1 kW
C) $\dot{q}$ = 316.2 kW
D) $\dot{q}$ = 400.2 mW

Problem 9 (Bergman et al., 2011, w/ permission)

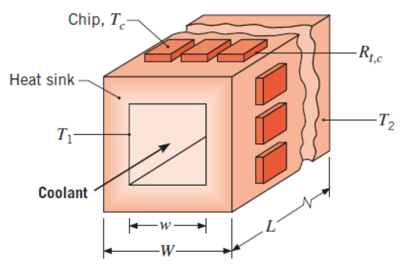
A small water droplet of diameter D = 100 μm and temperature T_{mp} = 0°C falls on a nonwetting metal surface that is at temperature T_s = –15°C. Determine how long it will take for the droplet to freeze completely. The thermal conductivity of air is 0.024 W/mK and the latent heat of fusion of water is h_{sf} = 334 kJ/kg.



A) Δt = 0.39 s
B) Δt = 0.48 s
C) Δt = 0.57 s
D) Δt = 0.66 s

Problem 10 (Bergman et al., 2011, w/ permission)

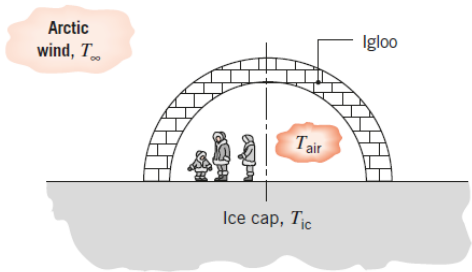
An aluminum heat sink (k = 240 W/mK), used to cool an array of electronic chips, consists of a square channel of inner width w = 25 mm, through which liquid flow may be assumed to maintain a uniform surface temperature of T_1 = 20°C. The outer width and length of the channel are w = 40 mm and L = 160 mm, respectively. If N = 120 chips attached to the outer surfaces of the heat sink maintain an approximately uniform surface temperature of T_2 = 50°C and all of the heat dissipated by the chips is assumed to be transferred to the coolant, determine the heat dissipation per chip. If the contact resistance between each chip and the heat sink is $R_{t,c}$ = 0.2 K/W, also determine the chip temperature.



A) $\dot{q}_c$ = 156.3 W and T_c = 72.1°C
B) $\dot{q}_c$ = 156.3 W and T_c = 81.3°C
C) $\dot{q}_c$ = 194.4 W and T_c = 72.1°C
D) $\dot{q}_c$ = 194.4 W and T_c = 81.3°C

Problem 11 (Bergman et al., 2011, w/ permission)

An igloo is built in the shape of a hemisphere, with an inner radius of 1.8 m and walls of compacted snow that are 0.5 m thick. On the inside of the igloo, the surface heat transfer coefficient is 6 W/m²K; on the outside, under normal wind conditions, it is 15 W/m²K. The thermal conductivity of compacted snow is 0.15 W/mK. The temperature of the ice cap in which the igloo sits is –20°C and has the same thermal conductivity as the compacted snow. Assuming that the occupants' body heat provides a continuous source of 320 W within the igloo, calculate the inside air temperature when the outside is at T_∞ = –40°C. Be sure to consider heat losses through the floor of the igloo.



A) $T_{\infty,i} = -17.4^{o}C$
B) $T_{\infty,i} = 1.2^{o}C$
C) $T_{\infty,i} = 5.1^{o}C$
D) $T_{\infty,i} = 8.2^{o}C$

Additional Information

Table 1 Shape factors for different geometries

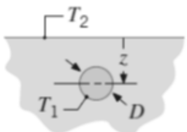
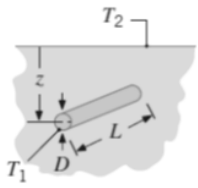
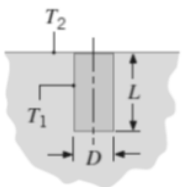
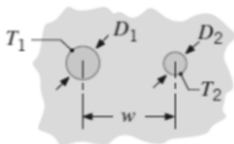
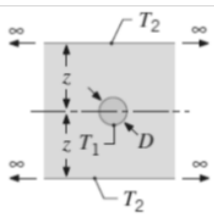
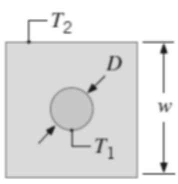
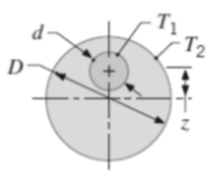
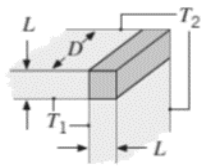
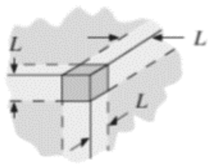
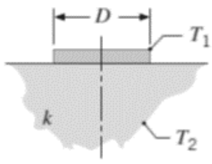
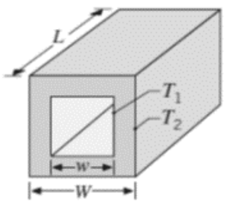
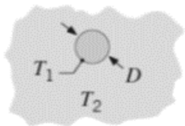
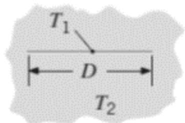
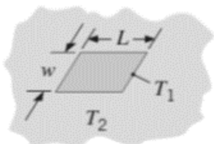
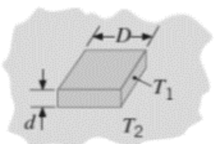
System	Schematic	Restrictions	Shape Factor
Case 1 Isothermal sphere buried in a semi-infinite medium		$z > D/2$	$\dfrac{2\pi D}{1 - D/4z}$
Case 2 Horizontal isothermal cylinder of length L buried in a semi-infinite medium		$L \gg D$ $L \gg D$, $z > 3D/2$	$\dfrac{2\pi L}{\cosh^{-1}(2z/D)}$ $\dfrac{2\pi L}{\ln(4z/D)}$
Case 3 Vertical cylinder in a semi-infinite medium		$L \gg D$	$\dfrac{2\pi L}{\ln(4L/D)}$
Case 4 Conduction between two cylinders of length L in infinite medium		$L \gg D_1, D_2$ $L \gg w$	$\dfrac{2\pi L}{\cosh^{-1}\left(\dfrac{4w^2 - D_1^2 - D_2^2}{2D_1 D_2}\right)}$
Case 5 Horizontal circular cylinder of length L midway between parallel planes of equal length and infinite width		$z \gg D/2$ $L \gg z$	$\dfrac{2\pi L}{\ln(8z/\pi D)}$
Case 6 Circular cylinder of length L centered in a square solid of equal length		$w > D$ $L \gg w$	$\dfrac{2\pi L}{\ln(1.08\, w/D)}$
Case 7 Eccentric circular cylinder of length L in a cylinder of equal length		$D > d$ $L \gg D$	$\dfrac{2\pi L}{\cosh^{-1}\left(\dfrac{D^2 + d^2 - 4z^2}{2Dd}\right)}$

Table 1 Shape factors for different geometries (*continued*)

Case 8 Conduction through the edge of adjoining walls		$D > 5L$	$0.54D$
Case 9 Conduction through corner of three walls with a temperature difference ΔT_{1-2} across the walls		$L \ll$ length and width of wall	$0.15L$
Case 10 Disk of diameter D and temperature T_1 on a semi-infinite medium of thermal conductivity k and temperature T_2		None	$2D$
Case 11 Square channel of length L		$\frac{W}{w} < 1.4$ $\frac{W}{w} > 1.4$ $L \gg W$	$\frac{2\pi L}{0.785 \ln (W/w)}$ $\frac{2\pi L}{0.930 \ln (W/w) - 0.050}$
Case 12 Isothermal sphere of diameter D and temperature T_1 in an infinite medium of temperature T_2		πD^2	1
Case 13 Infinitely thin, isothermal disk of diameter D and temperature T_1 in an infinite medium of temperature T_2		$\frac{\pi D^2}{2}$	$\frac{2\sqrt{2}}{\pi} = 0.900$
Case 14 Infinitely thin rectangle of length L, width w, and temperature T_1 in an infinite medium of temperature T_2		$2wL$	0.932
Case 15 Cuboid shape of height d with a square footprint of width D and temperature T_1 in an infinite medium of temperature T_2		$2D^2 + 4Dd$	d/D — q^*_{ss} 0.1 — 0.943 1.0 — 0.956 2.0 — 0.961 10 — 1.111

Solutions

P.1 Solution

The shape factor per unit length for an eccentric circular cylinder of length *L* in a cylinder of equal length is calculated as (Case 7 in Table 1)

$$S = \frac{2\pi}{\cosh^{-1}\left(\frac{D^2+d^2-4z^2}{2Dd}\right)}$$

Note that we have omitted the length *L* because we are assessing heat transfer on a unit-length basis. Substituting *D* = 0.15 m, *d* = 0.05 m, and *z* = 0.025 m, we obtain

$$S = \frac{2\pi}{\cosh^{-1}\left(\frac{D^2+d^2-4z^2}{2Dd}\right)} = \frac{2\pi}{\cosh^{-1}\left(\frac{0.15^2+0.05^2-4\times0.025^2}{2\times0.15\times0.05}\right)} = 6.53$$

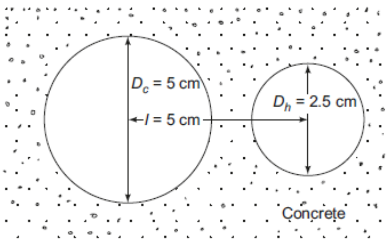
The heat transfer rate follows as

$$\dot{q} = kS\Delta T = 0.06\times6.53\times(150-50) = \boxed{39.2\text{ W}}$$

♦ The correct answer is **C**.

P.2 Solution

The system is illustrated below.



The shape factor for conduction between two cylinders of length *L* in an infinite medium is, per unit length, (Case 4 in Table 1)

$$S = \frac{2\pi}{\cosh^{-1}\left(\frac{4w^2-D_1^2-D_2^2}{2D_1D_2}\right)}$$

Substituting w = 0.05 m, D_1 = 0.05 m, and D_2 = 0.025 m, we obtain

$$S = \frac{2\pi}{\cosh^{-1}\left(\frac{4\times0.05^2-0.05^2-0.025^2}{2\times0.05\times0.025}\right)} = 3.76$$

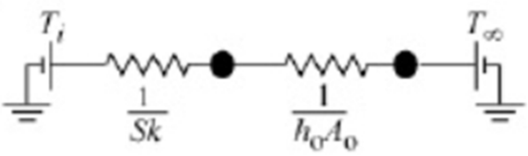
Accordingly, the rate of heat transfer per unit length is

$$\dot{q} = kS\Delta T = 1.1\times3.76\times(100-15) = \boxed{351.6\text{ W}}$$

♦ The correct answer is **D**.

P.3 Solution

The heat transfer rate can be computed with the thermal circuit shown below.



The conduction resistance is found through the appropriate conduction shape factor and added to the resistance associated with the inner surface. Referring to the circuit, we can write

$$\dot{q} = \frac{T_i - T_\infty}{\dfrac{1}{Sk} + \dfrac{1}{h_0 A_0}}$$

The shape factor *S* is obtained with the relation (Case 6 in Table 1)

$$S = \frac{2\pi L}{\ln\left(0.54w/R\right)}$$

Substituting L = 5 m, w = 0.05 m and R = 0.005 m gives

$$S = \frac{2\pi \times 5}{\ln\left(\dfrac{0.54 \times 0.05}{0.005}\right)} = 18.63 \text{ m}$$

Backsubstituting in the first equation, we obtain

$$\dot{q} = \frac{T_i - T_\infty}{\dfrac{1}{Sk} + \dfrac{1}{h_0 A_0}} = \frac{80 - 20}{\dfrac{1}{18.63 \times 15} + \dfrac{1}{40 \times \left(4 \times 0.05 \times 5\right)}} = \boxed{2100 \text{ W}}$$

♦ The correct answer is **A**.

P.4 Solution

Part A: The shape factor, in this case, has the form (Case 2 in Table 1)

$$S = \frac{2\pi L}{\cosh^{-1}\left(\dfrac{2z}{D}\right)}$$

and applies so long as the length L is substantially greater than the diameter D. Substituting L = 100 m, z = 1 m and D = 0.3 m gives

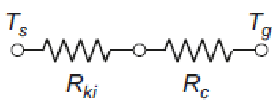
$$S = \frac{2\pi \times 100}{\cosh^{-1}\left(\dfrac{2 \times 1}{0.3}\right)} = 243.1 \text{m}$$

The rate of heat transfer follows as

$$\dot{q} = kS\Delta T = 0.7 \times 243.1 \times \left[90 - \left(-6\right)\right] = \boxed{16.3 \text{ kW}}$$

♦ The correct answer is **C**.

Part B: The thermal circuit for the pipe covered with insulation is shown below.



The rate of heat loss from the pipe is then

$$16,300 = \frac{2\pi L\left(T_s - T_g\right)}{\dfrac{1}{k_i}\ln\left(\dfrac{r_o}{r_i}\right) + \dfrac{1}{r_o h_c}} = \frac{2\pi \times 100 \times \left[90 - \left(-6\right)\right]}{\dfrac{1}{0.06}\ln\left(\dfrac{r_o}{0.15}\right) + \dfrac{1}{23r_o}}$$

This equation can be solved by trial-and-error to yield r_o = 0.184 m. Accordingly, the insulation thickness is r_0 = 0.184 – 0.15 = 0.034 = 3.4 cm.

♦ The correct answer is **B**.

P.5 Solution

The shape factor is, per unit length (Case 2 in Table 1),

$$S = \frac{2\pi}{\cosh^{-1}\left(\dfrac{2\times 1.25}{0.15}\right)} = 1.79$$

The rate of heat transfer per unit length is given by

$$\dot{q} = kS\Delta T = 0.35\times 1.79\times(95-5) = 56.39\ \text{W}$$

The total heat loss required to decrease the oil temperature by 5.5°C is

$$\dot{q}_t = \dot{m}C_p\Delta T = \left[\left(5.6\times 10^{-3}\right)\times 800\right]\times 2100\times 5.5 = 51,744\ \text{W}$$

We can estimate the length of pipe in which the oil temperature drops by 5.5°C by assuming the rate of heat loss from the pipe to be constant. Thus,

$$L = \frac{\dot{q}_t}{\dot{q}} = \frac{51,744}{56.39} = \boxed{917.6\ \text{m}}$$

♦ The correct answer is **D**.

P.6 Solution

The shape factor for this geometry is, per unit length (Case 6 in Table 1),

$$S = \frac{2\pi}{\ln\left(\dfrac{1.08\times 0.25}{0.01}\right)} = 1.91$$

The rate of heat flow per unit length is given by

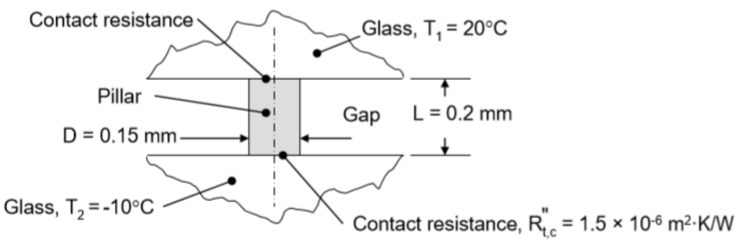
$$\frac{\dot{Q}_G}{L} = kS\Delta T = kS\left(T_s - T_o\right)$$

$$\therefore T_s = T_o + \frac{\dot{Q}_G/L}{kS} = 25 + \frac{150}{0.128\times 1.91} = \boxed{638.5^\circ\text{C}}$$

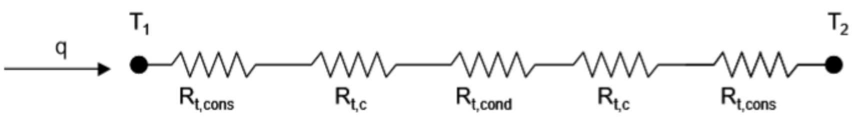
♦ The correct answer is **B**.

P.7 Solution

The system in question is illustrated below.



Conduction through the pillar results in a depression of the glass temperature adjacent to the pillar. This is associated with a *constriction resistance* within each glass sheet. Therefore, the resistance network consists of two constriction resistances, two contact resistances, and a conduction resistance through the pillar, as illustrated in continuation.



From Case 10 of Table 1, the shape factor is $S = 2D = 2\times(0.15\times 10^{-3}) = 0.3\times 10^{-3}$ m. The thermal resistances are given by

$$R_{t,\text{cons}} = \frac{1}{Sk_{\text{glass}}} = \frac{1}{\left(0.3\times10^{-3}\right)\times1.4} = 2381\ \text{K/W}$$

$$R_{t,c} = \frac{R''_{t,c}}{A_p} = \frac{1.5\times10^{-6}}{\frac{\pi\times\left(0.15\times10^{-3}\right)^2}{4}} = 84.9\ \text{K/W}$$

$$R_{t,\text{cond}} = \frac{L}{k_{\text{steel}}A_p} = \frac{0.2\times10^{-3}}{15\times\left[\frac{\pi\times\left(0.15\times10^{-3}\right)^2}{4}\right]} = 754.5\ \text{K/W}$$

Referring to the thermal circuit, the total resistance is determined as

$$R_{\text{tot}} = 2\times R_{\text{t,cons}} + 2\times R_{\text{t,c}} + R_{\text{t,cond}} = 2\times2381 + 2\times84.9 + 754.5 = 5686\ \text{K/W}$$

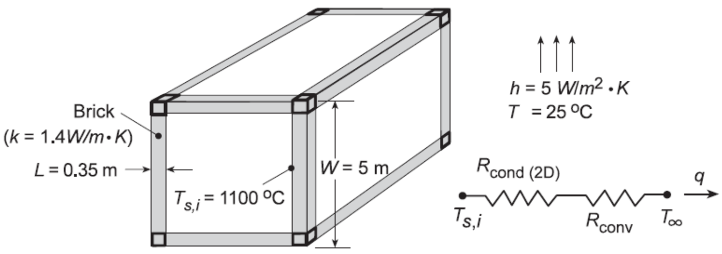
The rate of conduction heat transfer for an individual pillar is then

$$\dot{q} = \frac{(T_1 - T_2)}{R_{\text{tot}}} = \frac{\left[20-(-10)\right]}{5686} = 5.28\times10^{-3} = \boxed{5.28\ \text{mW}}$$

♦ The correct answer is **A**.

P.8 Solution

Consider the following schematic.



From the thermal circuit illustrated above, the heat loss is

$$\dot{q} = \frac{T_{s,\text{i}} - T_\infty}{R_{\text{cond}} + R_{\text{conv}}}$$

where $R_{conv} = 1/hA_0 = 1/(5 \times 6 \times 5^2) = 0.00133$ K/W. (A factor of 6 was included to account for the six faces of the furnace.) The two-dimensional conduction resistance, in turn, is given by $R_{cond(2D)} = 1/Sk$. The shape factor S must include the effects of conduction through the 8 corners (Case 9 in Table 1) and the 12 edges (Case 8 in Table 1). We must also consider the 6 plane walls. In mathematical terms,

$$S = 8\times0.15L + 12\times0.54\left(w-2L\right) + 6A_{s,\text{i}}/L$$

where $A_{s,i} = (w - 2L)^2 = (5 - 2 \times 0.35)^2 = 18.5$ m^2. Substituting the appropriate geometric quantities, we obtain

$$S = 8\left(0.15\times0.35\right) + 12\times0.54\left(5-2\times0.35\right) + 6\times18.5/0.35 = 345.4\ \text{m}$$

The resistance to conduction in the furnace is

$$R_{\text{cond}} = 1/(345.4\times1.4) = 0.00207\ \text{K/W}$$

Finally, the heat loss in the furnace is

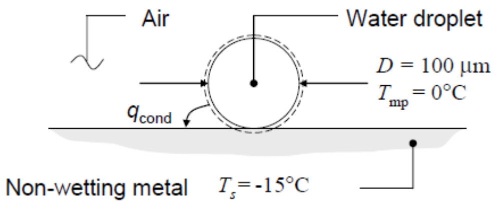
$$\dot{q} = \frac{(1100-25)}{0.00207+0.00133} = \boxed{316.2\ \text{kW}}$$

Since the heat loss is quite high, measures should be taken to insulate the furnace.

♦ The correct answer is **C**.

P.9 Solution

Consider the following schematic of the problem.



An energy balance on the droplet yields

$$\dot{q}_{\text{cond}}\Delta t = \Delta E$$

$$\therefore Sk\left(T_{\text{mp}} - T_s\right)\Delta t = mh_{sf} = V\rho_w h_{sf}$$

$$\therefore \Delta t = \frac{V\rho_w h_{sf}}{Sk\left(T_{\text{mp}} - T_s\right)}$$

The shape factor to use, in this case, is (Case 1 in Table 1)

$$S = \frac{2\pi D}{1 - D/4z}$$

Substituting *z* = *D/2* gives

$$S = \frac{2\pi D}{1 - \frac{D}{4\left(\frac{D}{2}\right)}} = \frac{2\pi D}{1 - \frac{\cancel{D}}{2\cancel{D}}} = \frac{2\pi D}{\frac{1}{2}} = 4\pi D$$

Substituting this result and the expression for volume of a sphere (= $4\pi R^3/3 = \pi D^3/6$), the result is

$$\Delta t = \frac{(\pi D^3/6)\rho_w h_{sf}}{(4\pi D)k\left(T_{\text{mp}} - T_s\right)} = \frac{D^2 h_{sf}\rho_w}{24k(T_{\text{mp}} - T_s)}$$

Notice that the time required for the droplet to freeze completely is sensibly proportional to its diameter and inversely proportional to the thermal conductivity. Substituting the pertaining variables, the time required to freeze the droplet is computed as

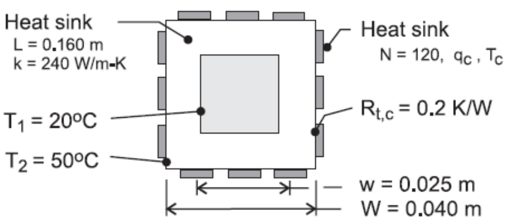
$$\Delta t = \frac{D^2 h_{sf}\rho_w}{24k(T_{\text{mp}} - T_s)} = \frac{(10^{-4})^2 \times \left(334\times 10^3\right)\times 1000}{24\times 0.024\times\left[0-(-15)\right]} = \boxed{0.39\,\text{s}}$$

It should be noted that, among the problems of this exercise set, this situation is probably the least realistic. For one, the solidification process might initiate in the lower region of the droplet, and the ice that forms would pose an additional resistance between the cold metal surface and the liquid water. Also, the air thermal conductivity in the vicinity of the contact point may be reduced by nanoscale effects.

♦ The correct answer is **A**.

P.10 Solution

The system in question is illustrated below.



The total heat rate, given by $q = (T_2 - T_1)/R_{t,\text{cond}}$, is determined by the two-dimensional conduction resistance of the channel wall, with the resistance governed by Case 11 of Table 1. Since *W/w* = 1.6 > 1.4, the shape factor is given by the relation

$$S = \frac{2\pi L}{0.930\ln(W/w) - 0.050}$$

and the thermal resistance is, accordingly,

$$R_{\text{t,cond}} = \frac{0.93\ln(W/w) - 0.05}{2\pi Lk} = \frac{0.93\ln(1.6) - 0.05}{2\pi \times 0.16 \times 240} = 0.0016\,\text{K/W}$$

The heat rate per chip, *N* being the total number of chips, is

$$\dot{q}_c = \frac{T_2 - T_1}{N \times R_{\text{t,cond}}} = \frac{(50-20)}{120 \times 0.0016} = \boxed{156.3\,\text{W}}$$

and, with $q_c = (T_c - T_2)/R_{t,\text{cond}}$, the chip temperature follows as

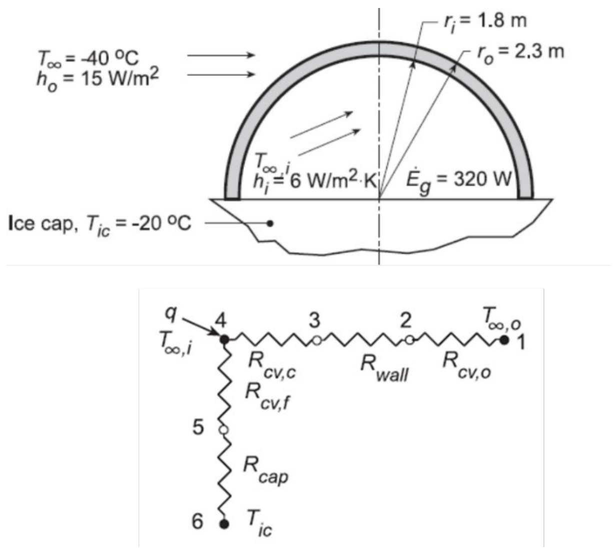
$$\dot{q}_c = \frac{(T_c - T_2)}{R_{\text{t,c}}} \rightarrow T_c = T_2 + R_{t,\text{cond}} q_c$$

$$\therefore T_c = 50 + 0.2 \times 156.3 = \boxed{81.3^\circ\text{C}}$$

♦ The correct answer is **B**.

P.11 Solution

Consider the following schematic for the present system and the thermal circuit that describes it.



The heat loss is given by the expression

$$\dot{q} = \frac{T_{\infty,\text{i}} - T_{\infty,\text{o}}}{R_{\text{cv,c}} + R_{\text{wall}} + R_{\text{cv,o}}} + \frac{T_{\infty,\text{i}} - T_{\text{ic}}}{R_{\text{cv,f}} + R_{\text{cap}}}$$

where the resistances are: the resistance to convection on the ceiling,

$$R_{\text{cv,c}} = \frac{2}{h_i\left(4\pi r_i^2\right)} = \frac{2}{6\times\left(4\times\pi\times 1.8^2\right)} = 0.0082\,\text{K/W}$$

The resistance to convection in the outside,

$$R_{\text{cv,o}} = \frac{2}{h_o\left(4\pi r_o^2\right)} = \frac{2}{15\times\left(4\times\pi\times 2.3^2\right)} = 0.0020\,\text{K/W}$$

The resistance to convection on the floor,

$$R_{\text{cv,f}} = \frac{1}{h_i\left(\pi r_i^2\right)} = \frac{1}{6\times\left(\pi\times 1.8^2\right)} = 0.0164\,\text{K/W}$$

The resistance to conduction on the wall,

$$R_{\text{wall}} = 2\left[\frac{1}{4\pi k}\left(\frac{1}{r_i}-\frac{1}{r_o}\right)\right] = 2\left[\frac{1}{4\pi\times 0.15}\left(\frac{1}{1.8}-\frac{1}{2.3}\right)\right] = 0.1281\text{ W/K}$$

And the resistance to conduction in the ice cap that underlies the igloo, where the shape factor $S = 2D = 4r_i$ is taken from Case 10 in Table 1,

$$R_{\text{cap}} = \frac{1}{kS} = \frac{1}{4kr_i} = \frac{1}{4\times 0.15\times 1.8} = 0.926\text{ W/K}$$

Substituting these resistances along with $T_{\text{ic}} = -20^{\text{o}}\text{C}$, $T_{\infty,o} = -40^{\text{o}}\text{C}$, and $\dot{q}$ = 320 W, we can solve the ensuing equation for the inside air temperature, $T_{\infty,\text{i}}$,

$$\dot{q} = 320 = \frac{T_{\infty,\text{i}}-(-40)}{0.0082+0.1281+0.002}+\frac{T_{\infty,\text{i}}-(-20)}{0.0164+0.926}$$

$$\therefore \boxed{T_{\infty,\text{i}} = 1.2^{\circ}\text{C}}$$

Note the significance of the resistance presented by the floor of the igloo: were the resistance due to the disk-shaped surface to be omitted from the equation we began with, the temperature obtained for the inside of the igloo would have been −17.4°C. This is more than 18 degrees lower than the true solution (and one of the wrong alternatives).

- The correct answer is **B**.

Answer Summary

Problem 1		C
Problem 2		D
Problem 3		A
Problem 4	4A	C
	4B	B
Problem 5		D
Problem 6		B
Problem 7		A
Problem 8		C
Problem 9		A
Problem 10		B
Problem 11		B

References

- BERGMAN, T., LAVINE, A., INCROPERA, F., and DEWITT, D. (2011). *Fundamentals of Heat and Mass Transfer.* 7th edition. Hoboken: John Wiley and Sons.
- KREITH, F., MANGLIK, R., and BOHN, M. (2011). *Principles of Heat Transfer*. 7th edition. Stamford: Cengage Learning.

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