# QUIZ GT107
## Shear Strength of Soil

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## ► PROBLEMS

### PROBLEM 1

Derive a general formula that gives the value of the major principal stress $\sigma_1$ as a function of the minor principal stress $\sigma_3$, the cohesion $c$, and the angle of internal friction $\phi$.

### PROBLEM 2

A sample of saturated clay is subjected to an unconfined compression test. The sample fails at a pressure of 40 kPa (that is to say, its unconfined compression strength is $q_u$ = 40 kPa). Determine the soil's undrained shear strength.

**A)** $s_u$ = 10 kPa
**B)** $s_u$ = 20 kPa
**C)** $s_u$ = 30 kPa
**D)** $s_u$ = 40 kPa

### PROBLEM 3

An unconfined compression test was carried out on a saturated clay sample. The maximum (peak) loading the clay sustained was 134 N and the vertical displacement was 3.8 mm. The size of the sample was 38 mm diameter × 76 mm long. Determine the undrained shear strength.

**A)** $s_u$ = 28.5 kPa
**B)** $s_u$ = 47.4 kPa
**C)** $s_u$ = 56.3 kPa
**D)** $s_u$ = 75.2 kPa

### PROBLEM 4

Direct shear tests were conducted on a dry sand. The samples used in the tests had a square base area of 2 in. × 2 in. and a height of 0.75 in. The test results are provided below. Determine the shear strength parameters $c'$ and $\phi'$.

| Test No. | Normal Load (lb) | Shear Force at Failure (lb) |
|---|---|---|
| 1 | 15 | 12 |
| 2 | 20 | 18 |
| 3 | 30 | 23 |
| 4 | 60 | 47 |
| 5 | 120 | 93 |

**A)** $c'$ = 0 and $\phi'$ = 26.8°
**B)** $c'$ = 0 and $\phi'$ = 37.9°
**C)** $c'$ = 539.6 lb/ft$^2$ and $\phi'$ = 26.8°
**D)** $c'$ = 539.6 lb/ft$^2$ and $\phi'$ = 37.9°

## PROBLEM 5

A triaxial test was performed on a dry, cohesionless soil under a confining pressure of 144.0 kN/m$^2$. If the sample failed when the deviator stress reached 395.8 kN/m$^2$, determine the soil's angle of internal friction.

**A)** $\phi'$ = 10.7°
**B)** $\phi'$ = 17.6°
**C)** $\phi'$ = 26.5°
**D)** $\phi'$ = 35.4°

## PROBLEM 6

The relationship between the relative density $D_r$ and the angle of friction $\phi'$ of a sand can be given as $\phi'$ = 28 + 0.18$D_r$ ($D_r$ in %). A drained triaxial test was conducted on the same sand with a chamber-confining pressure of 150 kN/m$^2$. The sand sample was prepared at a relative density of 68%. Calculate the major principal stress at failure.

**A)** $\sigma_1'$ = 300 kPa
**B)** $\sigma_1'$ = 400 kPa
**C)** $\sigma_1'$ = 500 kPa
**D)** $\sigma_1'$ = 600 kPa

## PROBLEM 7

Data obtained from a drained triaxial test are as follows. Determine the drained shear strength parameters $c'$ and $\phi'$.

| Test No. | Minor Princ. Stress (kPa) | Deviator Stress at Peak (kPa) |
|---|---|---|
| 1 | 50 | 191 |
| 2 | 100 | 226 |
| 3 | 150 | 261 |

**A)** $c'$ = 35 kPa and $\phi'$ = 15°
**B)** $c'$ = 35 kPa and $\phi'$ = 30°
**C)** $c'$ = 70 kPa and $\phi'$ = 15°
**D)** $c'$ = 70 kPa and $\phi'$ = 30°

## PROBLEM 8

A consolidated-undrained triaxial test was conducted on a normally consolidated clay. The results were a minor principal stress $\sigma_3'$ = 276 kN/m$^2$ and a deviatoric stress $\Delta\sigma_d$ = 276 kN/m$^2$ at failure. Determine the normal stress and the shear stress on the failure plane.

**A)** $\sigma_f'$ = 367.7 kPa and $\tau_f$ = 130.0 kPa
**B)** $\sigma_f'$ = 367.7 kPa and $\tau_f$ = 225.0 kPa
**C)** $\sigma_f'$ = 478.8 kPa and $\tau_f$ = 130.0 kPa
**D)** $\sigma_f'$ = 478.8 kPa and $\tau_f$ = 130.0 kPa

## PROBLEM 9

The shear strength of a normally consolidated soil can be given by the equation $\tau_f = \sigma' \tan 27°$. The results of a consolidated-undrained test on the soil were: chamber-confining pressure = 150 kN/m$^2$ and deviator stress at failure = 120 kN/m$^2$. Determine the consolidated-undrained friction angle and the porewater pressure developed in the specimen at failure.

**A)** $\phi$ = 16.6° and $u$ = 0
**B)** $\phi$ = 16.6° and $u$ = 77.7 kN/m$^2$
**C)** $\phi$ = 27.0° and $u$ = 0
**D)** $\phi$ = 27.0° and $u$ = 77.7 kN/m$^2$

## PROBLEM 10A

The results of three consolidated-undrained triaxial tests on identical specimens of a particular soil are given below. Determine the shear strength parameters related to the total stresses.

| Test No. | Minor Princ. Stess (kPa) | Deviator Stress at Peak (kPa) | *u* at Peak (kPa) |
|---|---|---|---|
| 1 | 200 | 244 | 55 |
| 2 | 300 | 314 | 107 |
| 3 | 400 | 384 | 159 |

**A)** $c$ = 20 kPa and $\phi$ = 15°
**B)** $c$ = 20 kPa and $\phi$ = 30°
**C)** $c$ = 40 kPa and $\phi$ = 15°
**D)** $c$ = 40 kPa and $\phi$ = 30°

## PROBLEM 10B

Using the data from the previous part, calculate the shear strength parameters related to the effective stresses.

**A)** $c'$ = 10 kPa and $\phi'$ = 12°
**B)** $c'$ = 10 kPa and $\phi'$ = 25°
**C)** $c'$ = 20 kPa and $\phi'$ = 12°
**D)** $c'$ = 20 kPa and $\phi'$ = 25°

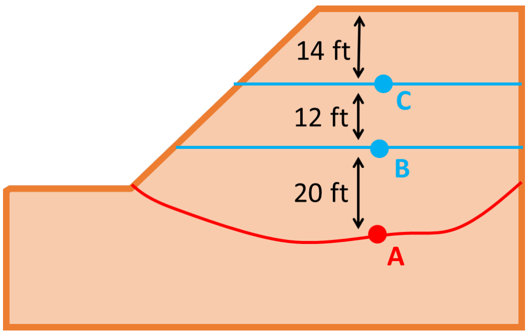
## PROBLEM 11

In an *in-situ* vane test on a saturated clay a torque of 35 N·m is required to shear the soil. The vane is 50 mm wide and 100 mm long. What is the undrained shear strength of the clay?

**A)** $c_u$ = 50 kPa
**B)** $c_u$ = 61 kPa
**C)** $c_u$ = 76 kPa
**D)** $c_u$ = 88 kPa

## PROBLEM 12

An engineer is evaluating the stability of the slope in the figure below, and considers that the potential for a shear failure occurs along the shear surface shown. The soil has an angle $\phi'$ = 30° and no cohesive strength. Compute the shear strength $\tau_1$ at point A along this surface when the groundwater table is at level B, then compute the new shear strength $\tau_2$ if it rises to level C. What is the absolute value of the difference $(\tau_2 - \tau_1)$? The unit weight of the soil is 120 lb/ft$^2$ above the WT and 123 lb/ft$^3$ below.




**A)** $|\tau_2 - \tau_1|$ = 200.8 lb/ft$^2$
**B)** $|\tau_2 - \tau_1|$ = 305.8 lb/ft$^2$
**C)** $|\tau_2 - \tau_1|$ = 411.5 lb/ft$^2$
**D)** $|\tau_2 - \tau_1|$ = 515.5 lb/ft$^2$

## PROBLEM 13 (Craig, 2004, w/ permission)

In a triaxial test a soil specimen was consolidated under an all-round pressure of 800 kN/m$^2$ and a back pressure of 400 kN/m$^2$. Thereafter, under undrained conditions, the all-round pressure was raised to 900 kN/m$^2$, resulting in a water pressure reading of 495 kN/m$^2$; then (with the all-round pressure remaining at 900 kN/m$^2$), an axial load was applied to give a principal stress difference of 585 kN/m$^2$ and a porewater pressure reading of 660 kN/m$^2$. Calculate the values of pore pressure coefficients $\bar{A}$ and $\bar{B}$.

**A)** $\bar{A}$ = 0.14 and $\bar{B}$ = 0.19
**B)** $\bar{A}$ = 0.14 and $\bar{B}$ = 0.38
**C)** $\bar{A}$ = 0.28 and $\bar{B}$ = 0.19
**D)** $\bar{A}$ = 0.28 and $\bar{B}$ = 0.38

## PROBLEM 14A

The following results were obtained from the undrained triaxial test on a compacted soil sample using a pressure of 300 kPa. Before the application of the cell pressure, the porewater pressure within the sample was zero. Determine the value of the pore coefficient *B* and state whether the soil was saturated or not.

| Strain (%) | $\sigma_1$ (kPa) | $u$ (kPa) |
|---|---|---|
| 0.0 | 300 | 120 |
| 2.5 | 500 | 150 |
| 5.0 | 720 | 150 |
| 7.5 | 920 | 120 |
| 10.0 | 1050 | 80 |
| 15.0 | 1200 | 10 |
| 20.0 | 1250 | – 6 |

**A)** $B$ = 0.4 and the soil was not saturated.
**B)** $B$ = 0.6 and the soil was not saturated.
**C)** $B$ = 0.8 and the soil was not saturated.
**D)** $B$ = 1.0 and the soil was saturated.

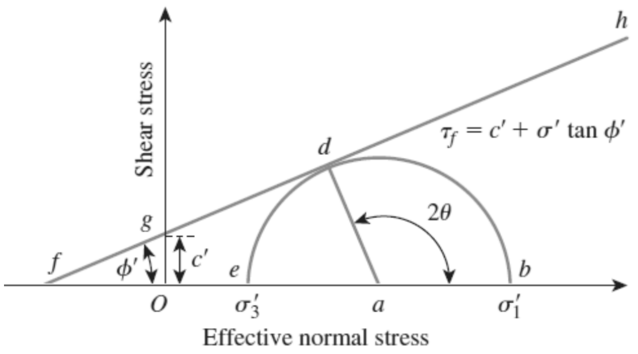
## PROBLEM 14B

Plot the variation of pore pressure coefficient *A* with strain.

# ▸ SOLUTIONS

## P.1 ■ Solution

Consider a hypothetical Mohr-Coulomb failure envelope and an accompanying stress circle. Although the tensional region of the envelope is of little importance in the mechanical behavior of soils, we show it as well to assess the geometry of the figure.




The failure envelope is *fgh*, and follows the equation $\tau_f = c' + \sigma' \tan \phi$.The radial line *ab* defines the major principal plane, and the radial line *ad* defines the failure plane. It can be shown that the angle $b\hat{a}d$ = $2\theta = 90 + \phi'$ or, equivalently, $\theta = 45° + \phi'/2$. (We take advantage of this result in some of the following problems). From the same figure, elementary trigonometry allows us to write $\sin\phi' = \overline{ad}/\overline{fa}$, where

$$\overline{fa} = \overline{fO} + \overline{Oa} = c'\cot\phi + \frac{\sigma_1' + \sigma_3'}{2}$$

and

$$\overline{ad} = \frac{\sigma_1' - \sigma_3'}{2}$$

Thus,

$$\sin\phi' = \frac{\dfrac{\sigma_1' - \sigma_3'}{2}}{c'\cot\phi' + \dfrac{\sigma_1' + \sigma_3'}{2}}$$

which becomes

$$\sigma_1' = \sigma_3'\left(\frac{1+\sin\phi'}{1-\sin\phi'}\right) + 2c'\left(\frac{\cos\phi'}{1-\sin\phi'}\right)$$

However, in view of the trigonometric identities

$$\frac{1+\sin\phi'}{1-\sin\phi'} = \tan^2\left(45^\circ + \frac{\phi'}{2}\right)$$
$$\frac{\cos\phi'}{1-\sin\phi'} = \tan\left(45^\circ + \frac{\phi'}{2}\right)$$

we can substitute into the previous expression and ultimately obtain

$$\boxed{\sigma_1' = \sigma_3'\tan^2\left(45^\circ + \frac{\phi'}{2}\right) + 2c'\tan\left(45^\circ + \frac{\phi'}{2}\right)}$$

We thus have a general relationship for the major principal stress as a function of minor principal stress, cohesion, and the internal friction angle of the soil. Although we have considered effective stresses all the while, a similar relationship could have been derived for principal stresses, namely,

$$\sigma_1 = \sigma_3\tan^2\left(45^\circ + \frac{\phi}{2}\right) + 2c\tan\left(45^\circ + \frac{\phi}{2}\right)$$

## P.2 ■ Solution

The unconfined compression test is a special type of compression test in which no radial stress is applied to the sample ($\sigma_3$ = 0). Consequently, the test is only suitable for soils capable of standing by themselves without lateral support, i.e., cohesive soils. It is also commonly employed in rock mechanics practice. Ordinarily, the principal stresses and cohesion would be related by

$$\sigma_1 = \sigma_3\tan^2\left(\frac{\phi}{2} + 45^\circ\right) + 2c\tan\left(\frac{\phi}{2} + 45^\circ\right)$$

In a UC test, however, we can apply the simplifications $\sigma_3$ = 0 and $\phi$ = 0. Then, the equation above simplifies to

$$\sigma_1 = \cancel{\sigma_3\tan^2\left(\frac{\phi}{2} + 45^\circ\right)} + 2c\underbrace{\tan\left(\frac{\phi}{2} + 45^\circ\right)}_{=1}$$

$$\therefore \sigma_1 = 2c$$

$$\therefore c = \frac{\sigma_1}{2}$$

$$\therefore s_u = \frac{q_u}{2}$$

where we have made the substitutions $c = s_u$ and $\sigma_1 = q_u$; the first is because the concept of undrained cohesion is equivalent to that of undrained shear strength, so $c = s_u$, and the latter is proposed because the normal stress at which a soil sample fails in the UC test is the so-called unconfined compressive strength, $q_u$. We now have a straightforward relationship between unconfined compressive strength, $s_u$, and $q_u$. Substituting $q_u$ = 40 kPa gives

$$s_u = \frac{40}{2} = \boxed{20 \text{ kPa}}$$

► The correct answer is **B**.

## P.3 ■ Solution

Since this is a UC test, $\sigma_3$ = 0 and $(\sigma_1)_p$ is the principal stress at failure. The initial base area of the sample is $A_o$, namely,

$$A_0 = \frac{\pi D_0^2}{4} = \frac{\pi \times \left(38 \times 10^{-3}\right)^2}{4} = 1.13 \times 10^{-3} \text{ m}^2$$

The vertical strain $\varepsilon_1$ is, in turn,

$$\varepsilon_1 = \frac{\Delta z}{H_0} = \frac{3.8}{76} = 0.05$$

The final area $A$ of the specimen then follows as

$$A = \frac{A_0}{1 - \varepsilon_1} = \frac{1.13 \times 10^{-3}}{1 - 0.05} = 1.19 \times 10^{-3} \text{ m}^2$$

The major principal stress at failure is given by the ratio of the peak loading, $P_z$ = 134 N, to the specimen base area, $A$; that is,

$$(\sigma_1)_p = \frac{P_z}{A} = \frac{134}{1.19 \times 10^{-3}} = 112.6 \text{ kPa}$$

Finally, the undrained shear strength is

$$s_u = \frac{(\sigma_1)_f - \cancel{(\sigma_3)_f}}{2} = \frac{(\sigma_1)_f}{2} = \frac{112.6}{2} = \boxed{56.3 \text{ kPa}}$$

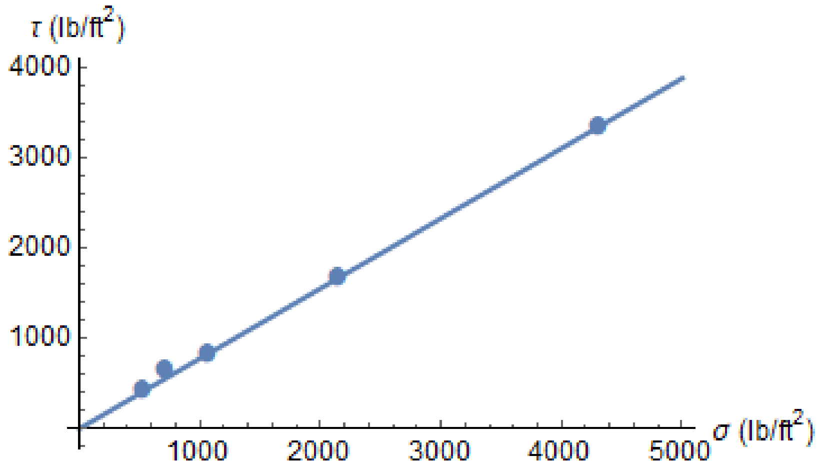
► The correct answer is **C**.

## P.4 ■ Solution

Since the soil is a sand, it goes without saying that $c'$ = 0 and, consequently, the failure envelope will begin at the origin of the $\sigma\tau$-plane. The cross-section area of the specimen is $(2/12) \times (2/12)$ = 0.0278 ft$^2$; dividing the normal loads and the shear forces at failure by this area yields the corresponding stresses, as shown below.

| Test No. | Normal Load (lb) | Shear Force at Failure (lb) | Normal Stress (lb/ft²) | Shear Stress at Failure (lb/ft²) |
|---|---|---|---|---|
| 1 | 15 | 12 | 539.6 | 432.0 |
| 2 | 20 | 18 | 720.0 | 648.0 |
| 3 | 30 | 23 | 1080.0 | 828.0 |
| 4 | 60 | 47 | 2160.0 | 1692.0 |
| 5 | 120 | 93 | 4320.0 | 3348.0 |

The abscissae are the normal stresses, taken from the blue column above, while the ordinates are the shear stresses at failure, taken from the red column. The line can be obtained in Mathematica by means of the *Fit* function. The resulting function for the envelope is $y = 0.779x$, which corresponds to a friction angle $\phi' = \arctan(0.779) = 37.9°$.




► The correct answer is **B**.

## P.5 ■ Solution

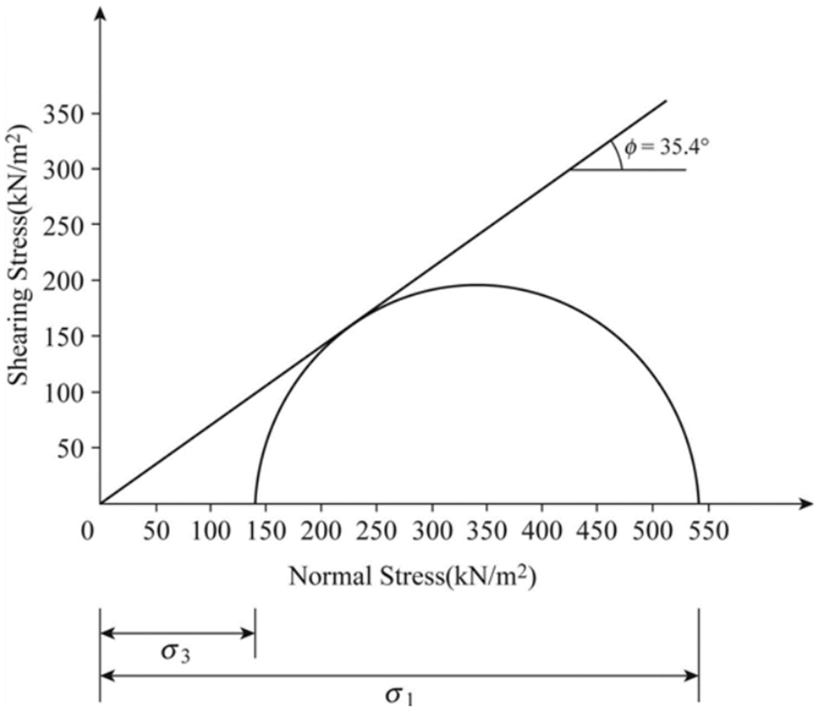
To obtain the major principal stress, we resort to the equation for deviator stress,

$$\Delta\sigma_d = \sigma_1 - \sigma_3 \rightarrow \sigma_1 = \Delta\sigma_d + \sigma_3$$

Substituting $\Delta\sigma_d$ = 395.8 kN/m² and $\sigma_3$ = 144 kN/m² gives

$$\sigma_1 = 395.8 + 144 = 539.8 \text{ kPa}$$

To draw the Mohr circle, we note that, from left to right, its first intercept with the horizontal axis is $\sigma_3$, whereas the second is $\sigma_1$. The radius of the circle is $(\sigma_1 - \sigma_3)/2$. To obtain the friction angle, we then draw a tangent to the circle; since the soil is cohesionless, we have $c'$ = 0 and the tangent begins at the origin, as shown. The inclination of this tangent is the friction angle we seek, i.e., $\phi'$ = 35.4°.




The problem can also be solved analytically, using the relation between principal stresses developed in Problem 1,

$$\sigma_1' = \sigma_3' \tan^2\left(45^\circ + \frac{\phi'}{2}\right) + \cancel{2c' \tan\left(45^\circ + \frac{\phi'}{2}\right)} \rightarrow \sigma_1' = \sigma_3' \tan^2\left(45^\circ + \frac{\phi'}{2}\right)$$

$$\therefore 539.8 = 144 \tan^2\left(45^\circ + \frac{\phi'}{2}\right)$$

$$\therefore 3.75 = \tan^2\left(45^\circ + \frac{\phi'}{2}\right)$$

$$\therefore \tan\left(45^\circ + \frac{\phi'}{2}\right) = 1.94$$

$$\therefore 45^\circ + \frac{\phi'}{2} = 62.7^\circ$$

$$\therefore \boxed{\phi' = 35.4^\circ}$$

► The correct answer is **C.**

## P.6 ■ Solution

The angle of friction can be determined with the relation we were given,

$$\phi' = 28 + 0.18D_r = 28 + 0.18 \times 68 = 40.2^\circ$$

Then, the major principal stress at failure follows from the equation

$$\sin\phi' = \frac{\sigma_1' - \sigma_3'}{\sigma_1' + \sigma_3'} \rightarrow \sin 40.2^\circ = \frac{\sigma_1' - 150}{\sigma_1' + 150}$$

where we have substituted the chamber-confining pressure $\sigma_3'$ = 150 kPa. Solving for $\sigma_1'$ gives

$$\sin 40.2^\circ = 0.60 = \frac{\sigma_1' - 150}{\sigma_1' + 150}$$

$$\therefore 0.60\sigma_1' + 90 = \sigma_1' - 150$$

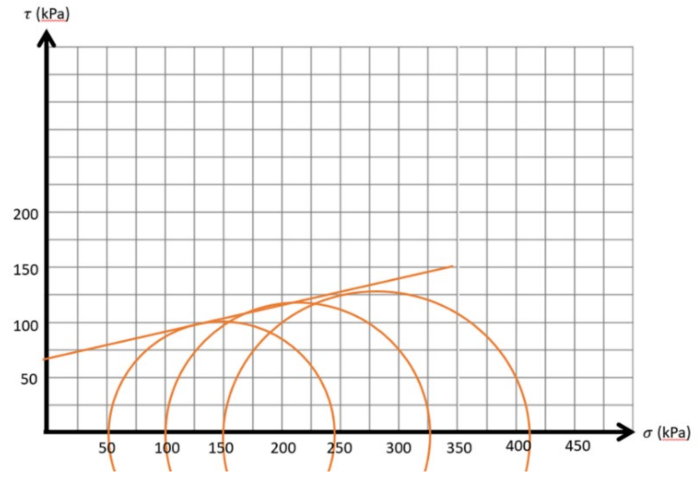
$$\therefore 240 = 0.40\sigma_1'$$

$$\therefore \sigma_1' = \frac{240}{0.40} = \boxed{600 \text{ kPa}}$$

► The correct answer is **D.**

## P.7 ■ Solution

Consider test 1. For a drained test, $\sigma_3' = \sigma_3$ = 50 kPa. The major principal stress can be determined from the deviatoric stress, i.e., $\sigma_1 = \sigma_3 + \Delta\sigma_d$ = 50 + 191 = 241 kPa. The first of Mohr's circles can then be obtained. Proceeding similarly, we have $\sigma_1$ = 326 kPa and $\sigma_3$ = 100 kPa for test 2, $\sigma_1$ = 411 kPa and $\sigma_3$ = 150 kPa for test 3. The Mohr's circles are drawn below, along with the failure envelope.




We thus obtain shear strength parameters $c'$ = 70 kPa and $\phi'$ = 15°.

► The correct answer is **C**.

## P.8 ■ Solution

For a normally consolidated soil in a consolidated-drained triaxial test, the effective major and minor principal stresses at failure are

$$\sigma_3' = \sigma_3 = 276 \text{ kPa}$$

$$\sigma_1' = \sigma_1 = \sigma_3 + \left(\Delta\sigma_d\right)_f = 276 + 276 = 552 \text{ kPa}$$

The friction angle can be computed with the following equation,

$$\sin\phi = \frac{\sigma_1' - \sigma_3'}{\sigma_1' + \sigma_3'} \rightarrow \sin\phi = \frac{552 - 276}{552 + 276} = 0.333$$

$$\therefore \phi = \arcsin 0.333 = 19.5^\circ$$

The inclination of the failure plane with respect to the horizontal is related to $\phi$ by the expression

$$\alpha = 45^\circ + \frac{\phi}{2} = 45^\circ + \frac{19.5^\circ}{2} = 54.8^\circ$$

In plane stress theory, the normal and shear stresses at the failure plane are, respectively,

$$\sigma_f' = \frac{\sigma_1' + \sigma_3'}{2} + \frac{\sigma_1' - \sigma_3'}{2}\cos 2\alpha$$

$$\tau_f = \frac{\sigma_1' - \sigma_3'}{2}\sin 2\alpha$$

Substituting $\sigma_1'$ = 276 kN/m$^2$, $\sigma_3'$ = 552 kN/m$^2$, and $\alpha$ = 54.8° gives

$$\sigma_f' = \frac{552 + 276}{2} + \frac{552 - 276}{2}\cos\left(2\times 54.8\right) = \boxed{367.7 \text{ kPa}}$$

$$\tau_f = \frac{552 - 276}{2}\sin\left(2\times 54.8\right) = \boxed{130.0 \text{ kPa}}$$

► The correct answer is **A**.

## P.9 ■ Solution

The chamber-confining pressure corresponds to the minor principal stress, i.e., $\sigma_3$ = 150 kN/m$^2$, and the major principal stress can be determined with the deviatoric stress $\Delta\sigma_d$,

$$\Delta\sigma_d = \sigma_1 - \sigma_3$$

$$\therefore \sigma_1 = \sigma_3 + \Delta\sigma_d = 150 + 120 = 270 \text{ kPa}$$

Since the clay is normally consolidated, the cohesion $c'$ = 0. Applying this simplification, the relation between the principal stresses simplifies to

$$\sigma_1 = \sigma_3 \tan^2\left(45^\circ + \frac{\phi}{2}\right) \quad \text{(I)}$$

Substituting $\sigma_1$ = 270 kPa and $\sigma_3$ = 150 kPa, we can solve for the friction angle $\phi$,

$$\sigma_1 = \sigma_3 \tan^2\left(45^\circ + \frac{\phi}{2}\right) \rightarrow 270 = 150\tan^2\left(45^\circ + \frac{\phi}{2}\right)$$

$$\therefore 1.8 = \tan^2\left(45^\circ + \frac{\phi}{2}\right)$$

$$\therefore \tan\left(45^\circ + \frac{\phi}{2}\right) = 1.34$$

$$\therefore 45^{\circ}+\frac{\phi}{2}=53.3^{\circ}$$

$$\therefore \boxed{\phi=16.6^{\circ}}$$

In order to compute the porewater pressure at failure, *u*, we must rewrite equation (I) in terms of effective stresses,

$$\sigma_1'=\sigma_3'\tan^2\left(45^{\circ}+\frac{\phi'}{2}\right)\rightarrow\left(\sigma_1-u\right)=\left(\sigma_3-u\right)\tan^2\left(45^{\circ}+\frac{\phi'}{2}\right)$$

Substituting $\sigma_1$ = 270 kPa, $\sigma_3$ = 150 kPa, and $\phi'$ = 16.6°, only one unknown remains,

$$\left(\sigma_1-u\right)=\left(\sigma_3-u\right)\tan^2\left(45^{\circ}+\frac{\phi'}{2}\right)\rightarrow(270-u)=(150-u)\times\underbrace{\tan^2\left(45^{\circ}+\frac{16.6^{\circ}}{2}\right)}_{=1.80}$$

$$\therefore\left(270-u\right)=(150-u)\times 1.80$$

$$\therefore 270-u=270-1.80u$$

$$\therefore\boxed{u=0}$$

That is, the porewater pressure developed in the specimen is zero.

► The correct answer is **A**.

## P.10 ■ Solution

**Part A**: The total stresses can be obtained with the data in the second and third columns only. The minor principal stresses are the data contained on the first column themselves, while the major principal stresses are obtained from the deviator stress. For test 1, for example, we have

$$\Delta\sigma_d=\sigma_1-\sigma_3\rightarrow\sigma_1=\Delta\sigma_d+\sigma_3=244+200=444\text{ kPa}$$

Proceeding similarly with tests 2 and 3, we obtain the data shown in the next table.

| Test No. | $\sigma_1$ (kPa) | $\sigma_3$ (kPa) |
|---|---|---|
| 1 | 444 | 200 |
| 2 | 614 | 300 |
| 3 | 784 | 400 |

The corresponding Mohr's circles are shown as the dotted lines (see the end of the next part). The failure parameters are found to be *c* = 40 kPa and $\phi$ = 15°.

► The correct answer is **C**.

**Part B**: The principal stresses can be determined from the second, third, and fourth columns. The minor effective principal stress for test 1, for example, is

$$\sigma_3'=\sigma_3-u=200-55=145\text{ kPa}$$

while the major principal stress is obtained from the deviatoric stress,

$$\sigma_1-\sigma_3=244$$

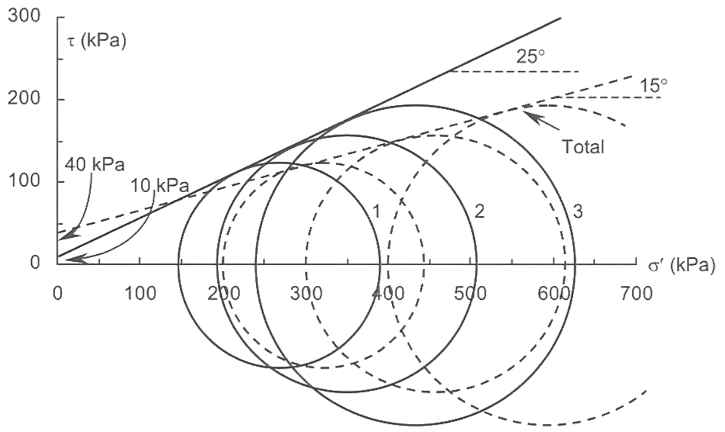
$$\therefore\sigma_1=244+\sigma_3=244+200=444\text{ kPa}$$

or, in terms of effective stresses,

$$\sigma_1'=\sigma_1-u=444-55=389\text{ kPa}$$

Equipped with $\sigma_1'$ and $\sigma_3'$, we can determine Mohr's circle for test 1. Proceeding similarly with tests 2 and 3, we obtain the values shown. The circles are illustrated as the solid lines below.

| Test No. | $\sigma_1'$ (kPa) | $\sigma_3'$ (kPa) |
|---|---|---|
| 1 | 389 | 145 |
| 2 | 507 | 193 |
| 3 | 625 | 241 |




The effective failure parameters are found to be $c'$ = 10 kPa and $\phi'$ = 25°.

▶ The correct answer is **B**.

## P.11 ■ Solution

The torque required to produce shear failure is given by

$$T = \pi h d c_u \frac{d}{2} + 2\int_0^{d/2} 2\pi r dr c_u r$$

$$\therefore T = \pi c_u \frac{d^2 h}{2} + 4\pi c_u \int_0^{d/2} r^2 dr$$

$$\therefore T = \pi c_u \left( \frac{d^2 h}{2} + \frac{d^3}{6} \right)$$

Substituting the available data gives

$$35 = \pi c_u \left( \frac{5^2 \times 10}{2} + \frac{5^3}{6} \right) \times 10^{-3}$$

$$\therefore \boxed{c_u = 76 \text{ kN/m}^2}$$

▶ The correct answer is **C**.

## P.12 ■ Solution

When the groundwater table is at B, the porewater pressure is

$$u = \gamma_w z_w = 62.4 \times 20 = 1248 \text{ lb/ft}^2$$

The vertical stress at *A*, in turn, is

$$\left(\sigma_z'\right)_1 = \Sigma \gamma H - u = 120 \times (12 + 14) + 123 \times 20 - 1248 = 4332 \text{ lb/ft}^2$$

The shear stress at point *A* follows as

$$\tau_1 = \sigma' \tan\phi' = 4332 \times \tan 30^\circ = 2501.1 \text{ lb/ft}^2$$

Suppose, now, that the groundwater table is at *C*. The new porewater pressure is

$$u = \gamma_w z_w = 62.4 \times (20 + 12) = 1996.8 \text{ lb/ft}^2$$

while the normal stress at point *A* is now

$$\left(\sigma_z'\right)_2 = \Sigma \gamma H - u = 120 \times 14 + 123 \times (20 + 12) - 1996.8 = 3619.2 \text{ lb/ft}^2$$

and the corresponding shear is

$$\tau_2 = 3619.2 \times \tan 30^\circ = 2089.6 \text{ lb/ft}^2$$

Notice that the rise of the groundwater table translates into a *decrease* in the shear stress imparted on point *A*. Finally, the absolute value of the difference $|\tau_2 - \tau_1|$ is

$$|\tau_2 - \tau_1| = |2089.6 - 2501.1| = \boxed{411.5 \text{ lb/ft}^2}$$

► The correct answer is **C**.

## P.13 ■ Solution

The total major principal stress increases from 900 to (900 + 585) kN/m², with a corresponding increase in pore pressure from 495 to 660 kN/m². Coefficient $\bar{A}$ is then

$$\bar{A} = \frac{\Delta u_1}{\Delta \sigma_1} = \frac{660 - 495}{585} = \boxed{0.28}$$

The overall increase in pore pressure is from 400 to 660 kN/m² and corresponds to an increase in major principal stress from 800 to 800 + (100 + 585) kN/m². Accordingly, coefficient $\bar{B}$ is determined to be

$$\bar{B} = \frac{\Delta u}{\Delta \sigma_1} = \frac{660 - 400}{100 + 585} = \boxed{0.38}$$

► The correct answer is **D**.

## P.14 ■ Solution

**Part A:** The *B* coefficient is the ratio of pore pressure $\Delta u_a$ to the cell pressure $\Delta\sigma_3$. Substituting $\Delta u_a$ = 120 kPa and $\Delta\sigma_3$ = 300 kPa, we get

$$B = \frac{\Delta u_a}{\Delta \sigma_3} = \frac{120}{300} = \boxed{0.4}$$

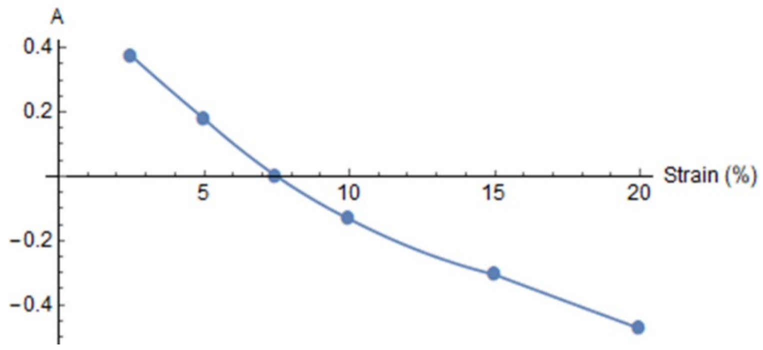
Consequently, the soil is not saturated.

► The correct answer is **A**.

**Part B:** Instead of obtaining *A* coefficients directly, we propose first determining coefficient $\bar{A} = AB$, where $\bar{A} = \Delta u_d/(\Delta\sigma_1 - \Delta\sigma_3)$, then establishing the value *A*. The following table is prepared.

| Strain | $\Delta u_d$ | $(\Delta\sigma_1 - \Delta\sigma_3)$ | $\bar{A}$ | $A = \frac{\bar{A}}{B}$ |
|---|---|---|---|---|
| [1] | [2] | [3] | [4] = [2] / [3] | [5] = [4] / 0.4 |
| 2.5 | 30 | 200 | 0.150 | 0.375 |
| 5.0 | 30 | 420 | 0.071 | 0.179 |
| 7.5 | 0 | 620 | 0.000 | 0.000 |
| 10.0 | -40 | 750 | -0.053 | -0.133 |
| 15.0 | -110 | 900 | -0.122 | -0.306 |
| 20.0 | -180 | 950 | -0.189 | -0.474 |

Bear in mind that the porewater pressure and principal stress variations are established with reference to the initial (zero strain) value, not the previous tabulated value; a common mistake is to compute the variables with respect to the latter instead of the former. The graph we aim for is a plot of the values in the fifth (red) column versus the values in the first (blue) column. The following graph is obtained.



## ► ANSWER SUMMARY

| | | |
|---|---|---|
| **Problem 1** | | Open-ended pb. |
| **Problem 2** | | **B** |
| **Problem 3** | | **C** |
| **Problem 4** | | **B** |
| **Problem 5** | | **D** |
| **Problem 6** | | **D** |
| **Problem 7** | | **C** |
| **Problem 8** | | **A** |
| **Problem 9** | | **A** |
| **Problem 10** | **10A** | **C** |
| | **10B** | **B** |
| **Problem 11** | | **C** |
| **Problem 12** | | **C** |
| **Problem 13** | | **D** |
| **Problem 14** | **14A** | **A** |
| | **14B** | Open-ended pb. |

## ► REFERENCES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