Quiz NA101
Ship Stability and Geometry – Part 1
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PROBLEMS

Problem 1

A vessel of length 90 m has equally spaced half ordinates of the waterplane as follows, commencing from the after perpendicular. Find the area of the waterplane.

Station	0	1	2	3	4	5	6
½ Ordinate (m)	0.1	2.4	2.7	2.8	2.8	2.2	0.2

A) A = 205 m^2

B) A = 307 m^2

C) A = 409 m^2

D) A = 504 m^2

Problem 2

A vessel of length 200 m has the half ordinates of waterplane values given in the next table, commencing at the after perpendicular (AP). True or false?

Station	0	1	2	3	4	5	6	7	8	9	10
½ Ordinate (m)	0	10.0	13.0	14.0	14.2	14.2	14.1	14.0	11.5	6.2	0.2

1.( ) The area of the waterplane is greater than 5000 m^2.

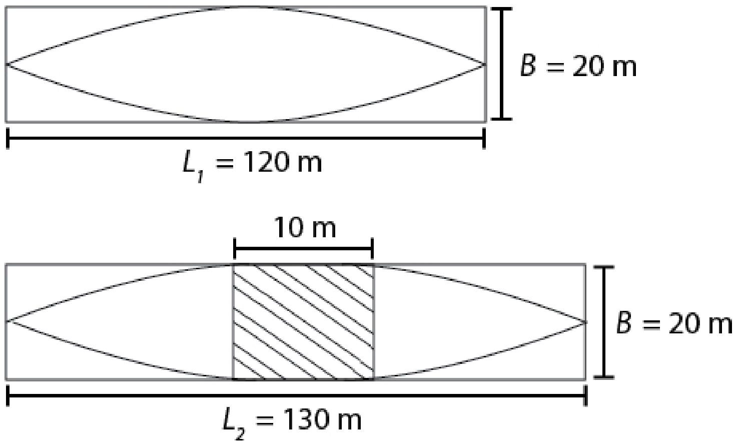
2.( ) The centroid is more than 90 m away from the after perpendicular.

3.( ) The moment of inertia about a transverse axis passing through the centroid is greater than 2.0×10^7 m^4.

4.( ) The moment of inertia about a longitudinal axis through the centerline is greater than 200,000 m^4.

Problem 3 (Barrass, 2001, w/ permission)

For a general cargo ship, the length between perpendiculars L_{BP} = 120 m, moulded breadth = 20 m, moulded draught = 8 m, displacement at 8 m draught = 14,000 t, midship area coefficient C_M = 0.985, and waterplane area coefficient C_{WP} = 0.808. Using "ship surgery," a midship section 10-m long is welded into the ship, as shown. True or false?



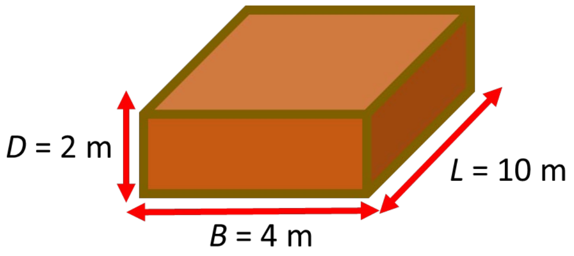
1.( ) The block coefficient of the modified ship is greater than 0.76.

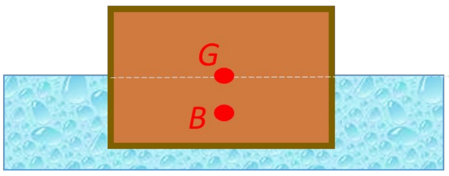
2.( ) The waterplane area coefficient of the modified ship is greater than 0.84.

3.( ) The prismatic coefficient of the modified ship is greater than 0.77.

Problem 4A

A solid block made of material with density $\rho_{\text{block}} = 0.5\rho_{\text{water}}$ floats on a body of calm water, as shown. Evaluate the block's initial transverse stability.





A) $GM > 0$ and the block is in stable equilibrium.

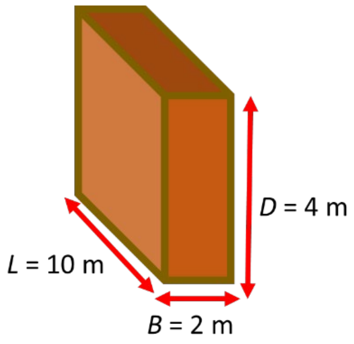
B) $GM = 0$ and the block is in neutral equilibrium.

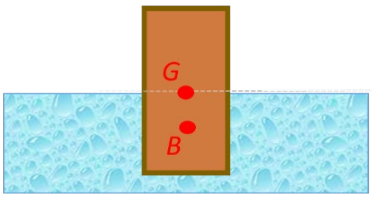
C) $GM < 0$ and the block is in unstable equilibrium.

D) There is not enough information to assess the stability of the block.

Problem 4B

Suppose the block were rotated and placed in calm water again, as shown. Evaluate the block's initial stability.





A) $GM > 0$ and the block is in stable equilibrium.

B) $GM = 0$ and the block is in neutral equilibrium.

C) $GM < 0$ and the block is in unstable equilibrium.

D) There is not enough information to assess the stability of the block.

Problem 5 (Lee, 2019, w/ permission)

A ship of mass displacement equal to 10,000 tonnes has its center of gravity 8 m above baseline. The following operations are carried out. Calculate the new VCG.

Operation No.	Operation Type	Mass (tonnes)	Position Change (m)
1	Load	500	3 m above baseline
2	Discharge	300	5 m AB
3	Load	200	12 m AB
4	Move	1000	Downward by 2 m

A) $\bar{z} = 7.40$ m

B) $\bar{z} = 7.73$ m

C) $\bar{z} = 8.27$ m

D) $\bar{z} = 8.61$ m

Problem 6 (Lee, 2019, w/ permission)

A box-like vessel 100 m × 30 m ($L \times B$) is operating at an even keel draft of 6 m. At a port, the loading/unloading operations listed below were carried out. Calculate the trim.

Operation No.	Operation Type	Mass (t)	Distance from Midship (m)
1	Load	100	10 fwd
2	Unload	50	40 aft
3	Load	200	30 aft
4	Unload	50	20 fwd

A) Trim = 8.56 cm by forward

B) Trim = 15.6 cm by forward

C) Trim = 8.56 cm by aft

D) Trim = 15.6 cm by aft

Problem 7 (Lee, 2019, w/ permission)

A container ship is operating at a level keel draft of 15 m in seawater with a displacement of 15,000 tonnes. The L_{BP} of the ship is 100 m. In this condition the vessel has the following characteristics:

Center of buoyancy at 10 m aft of midship and 5.5 m above baseline
VCG at 8 m above baseline
TPC = 35 tonnes/cm
LCF at 5 m aft of midship
I_L = 1,351,303 m^4

I_L, *TPC*, and *LCF* may be assumed constant for the range of drafts considered herein. The following loading/unloading operations were carried out.

Operation No.	Operation	Mass (tonnes)	Distance from Midship (m)	Above Baseline (m)
1	Load	150	20 m fwd	5
2	Unload	220	40 m aft	9
3	Unload	540	5 m aft	6
4	Load	325	25 m aft	10
5	Load	400	12 m aft	7
6	Load	100	35 m aft	9

True or false?

1.( ) After the last loading/unloading operation has been carried out, the *LCG* will be more than 8 m away from amidships.
2.( ) After the last loading/unloading operation has been carried out, the *LCB* will be more than 11 m away from amidships.
3.( ) After the last loading/unloading operation has been carried out, the height of the *VCG* will be greater than 7.5 m.
4.( ) After the last loading/unloading operation has been carried out, the height of the *VCB* will be greater than 6 m.
5.( ) The overall trim is greater than 1.25 m.

Problem 8

The cross-curves of stability for a vessel are given below.

Displacement (t)	Righting Arm (ft) with Pole Height = 15.0 ft ABL							
	10^o	20^o	30^o	40^o	50^o	60^o	70^o	80^o
1050	0	3.7	7.02	9.22	10.37	10.38	9.08	6.72
1100	0	3.66	6.94	9.15	10.25	10.28	8.85	6.62
1150	0	3.64	6.85	8.88	10.09	10.2	8.93	6.5

The vessel currently displaces 1150 tonnes of seawater, with *KG* = 18.0 ft. The righting arm at 30 degrees list is:

A) *GZ* = 5.85 ft

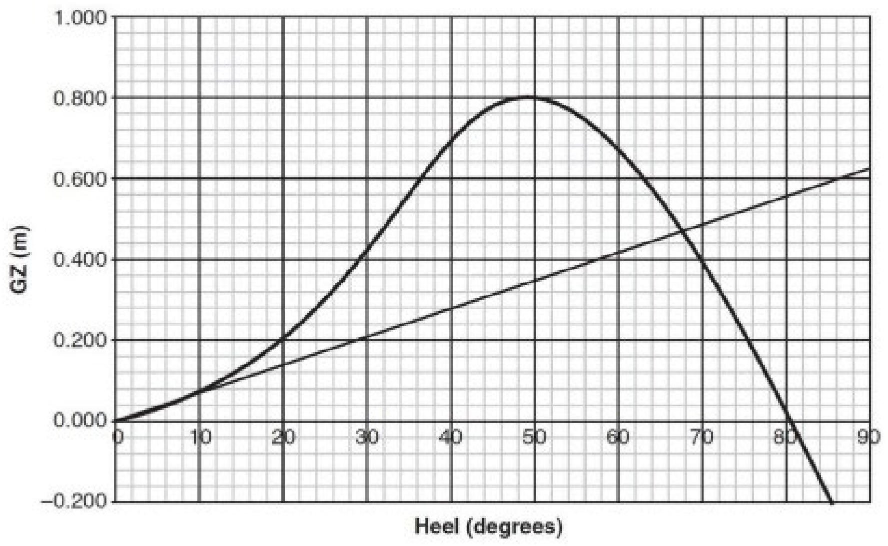
B) *GZ* = 6.51 ft

C) *GZ* = 7.38 ft

D) *GZ* = 8.88 ft

Problem 9

The GZ curve for a specific ship, along with its initial slope, are drawn below. Find the value of GM.



A) GM = 0.4 m

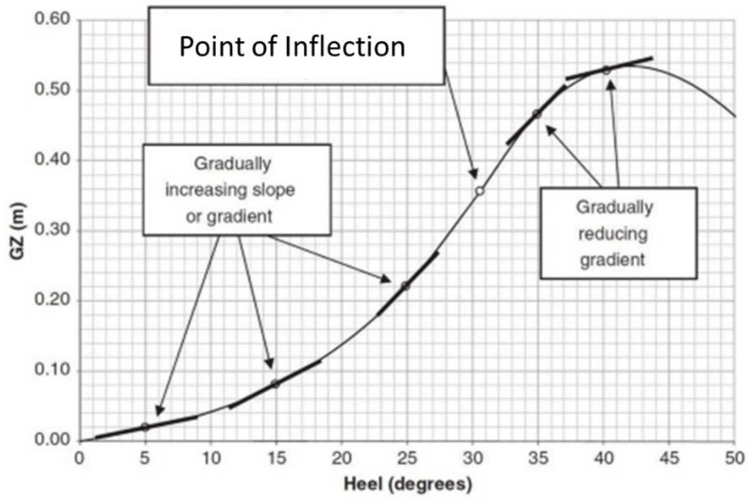
B) GM = 0.5 m

C) GM = 0.6 m

D) GM = 0.7 m

Problem 10 (Patterson & Ridley, 2014)

The figure below shows the ascending segment of a statical stability curve. The ascending portion of most curves features a point of inflection, in which the upward-concave curve becomes downward-concave. The angle at which this occurs is known as



A) Angle of vanishing stability.

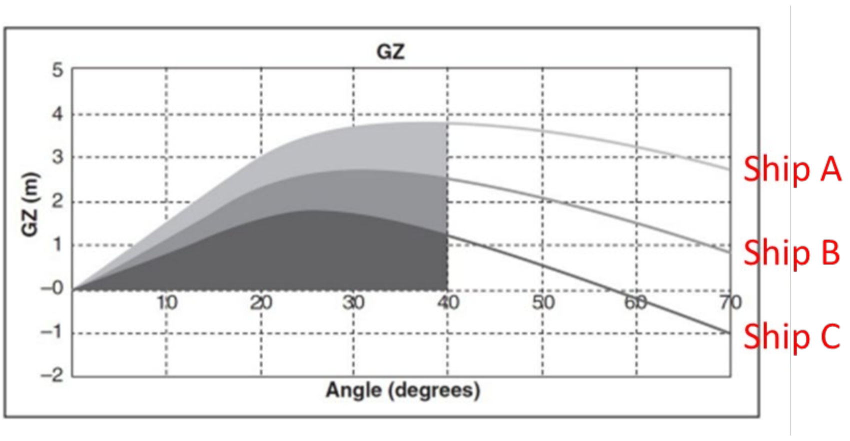
B) Angle of loll.

C) Deck edge immersion angle.

D) Down-flooding angle.

Problem 11 (Patterson & Ridley, 2014)

Below, we have the *GZ* curves for three different ships. Which of the vessels will require the most energy to be rolled to an angle of 40 degrees?



A) Ship A.

B) Ship B.

C) Ship C.

D) There is not enough information to determine which ship requires the most energy.

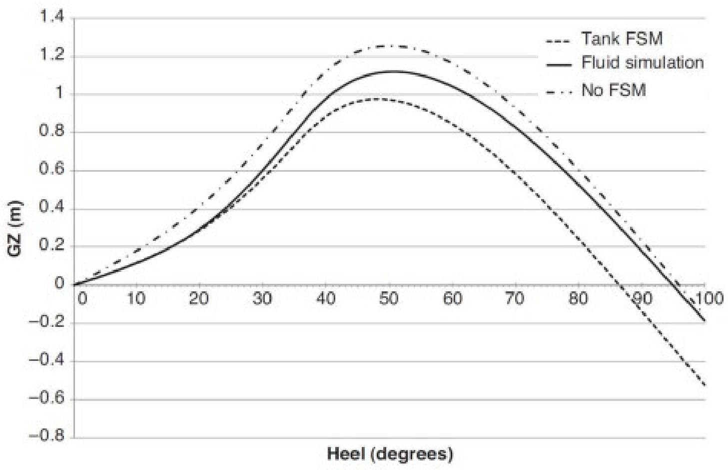
Problem 12

Below, we have four statements concerning changes in the righting arm and the *GZ* curve that occur as a consequence of certain events. True or false?

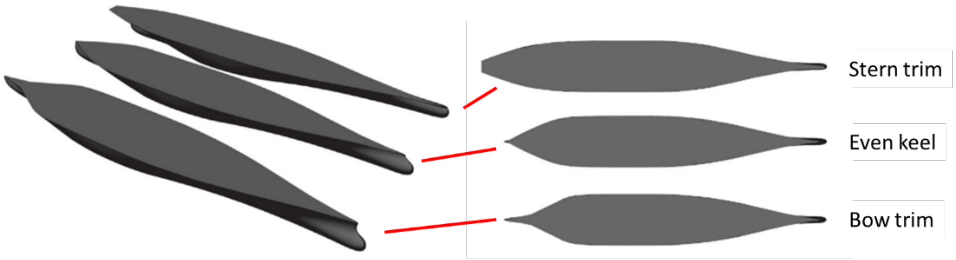
1.( ) Increasing the freeboard available to a vessel will increase *GZ* values, but the deck-edge immersion angle will remain the same.

2.( ) Increasing the beam of a vessel will increase *GZ* values. This, however, comes at the cost of a decreased deck-edge immersion angle.

3.( ) Calculating the effects of free surfaces is not always necessary for heel at small angles, but becomes an indispensable task for inclination at large angles. Evidently, there are different approaches to this problem, and each may lead to different *GZ* curves. In continuation, we have three *GZ* curves for a hypothetical ship, each obtained from a different FSE method. The dashed-dot line shows the *GZ* values obtained by ignoring any FSEs altogether; the dashed line shows the *GZ* values using the free-surface moment data from tank hydrostatics to determine the effective *KG*, and then *GZ*; finally, the solid line shows the *GZ* values based on the simulation of fluid moving in the tanks at each angle of heel, and therefore is the most accurate calculation of FSE. From the graph, it can be seen that, despite its accuracy, the fluid simulation approach does not yield the safest (or most pessimistic) *GZ* values.



4.( ) As a vessel trims, the waterplane area will vary. Though this effect is quite small for many watercraft, it may appreciable for ships with large overhanging sterns and large bow flare, such as offshore supply vessels. For these ships, the typical hull shape results in a widening of the waterplane area with stern trim or a shortening with bow trim, as shown in the following illustrations. In the stern trim case, the result is a decreased transverse waterplane inertia, with an ensuing decrease in the BM. Accordingly, there is a decrease in the *GM* and in the slope of the *GZ* curve.



Problem 13A (Tupper, 2004)

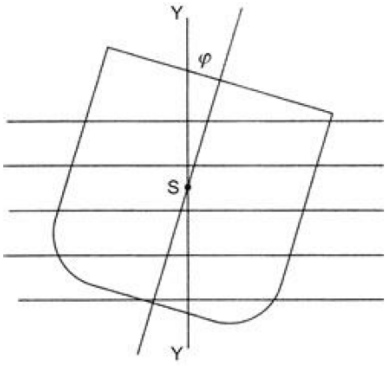
The angles of inclination and corresponding righting levers for a ship at an assumed *KS* = 6.6 m, with *S* being the arbitrary point on the centerline about which the ship is inclined, are given in the following table.

Inclination (°)	0	15	30	45	60	75	90
Righting Lever (m)	0	0.14	0.35	0.40	3.98	3.92	2.78

In a particular loading condition, the displacement mass is made up of the following items.

Item	Mass (tonnes)	*KG* (m)
Lightship	4000	6.0
Cargo	9000	7.0
Fuel	1500	1.0
Stores	200	7.5

Plot the curve of statical stability and determine the range of stability.



Problem 13B

Using the tabulated values of *GZ* from the previous problem, determine the dynamical stability of the vessel at 60° inclination.

A) U = 20.3 MN·m
B) U = 41.5 MN·m
C) U = 60.7 MN·m
D) U = 81.2 MN·m

Problem 14 (Zubaly, 2009)

A Mariner class ship is at a mean seawater draft of 22.25 ft and with *KG* = 28.0 ft. At this draft, the sail area (projected area of the above-water profile) is 16,150 ft^2 and the centroid of the sail area is 18.9 ft above the waterline. Plot the statical stability curve of this ship and superimpose on it the wind heel arm curve for a 100-knot beam wind. What angle of heel will this wind produce?

Angle (°)	0	10	20	30	40	50	60	70	80
GZ (m)	0	0.70	1.65	2.97	3.98	3.92	2.78	1.59	0.08

A) ϕ = 4.0°
B) ϕ = 7.5°
C) ϕ = 14.5°
D) ϕ = 20.2°

Solutions

P.1 ■ Solution

The area in question can be determined from Simpson's first rule. The calculations are prepared in the following table.

Station	1/2 Ordinate	Simpson's Multiplier	Product
0	0.1	1	0.1
1	2.4	4	9.6
2	2.7	2	5.4
3	2.8	4	11.2
4	2.8	2	5.6
5	2.2	4	8.8
6	0.2	1	0.2
		Σ*P* =	40.9

The spacing between stations is h = 90/6 = 15 m. The area we seek is determined to be

$$A = 2 \times \frac{15}{3} \times 40.9 = \boxed{409 \text{ m}^2}$$

★ The correct answer is **C**.

P.2 ■ Solution

1. False. The calculations are summarized in the following table.

Station	1/2 Ordinate	Simpson's Multiplier	F_1 (Area)	Lever (First Moment)	F_2 (First Moment)	Lever (Second Moment)	F_3 (Second Moment)
0	0	1	0.0	0	0.0	0	0.0
1	10	4	40.0	1	40.0	1	40.0
2	13	2	26.0	2	52.0	2	104.0
3	14	4	56.0	3	168.0	3	504.0
4	14.2	2	28.4	4	113.6	4	454.4
5	14.2	4	56.8	5	284.0	5	1420.0
6	14.1	2	28.2	6	169.2	6	1015.2
7	14	4	56.0	7	392.0	7	2744.0
8	11.5	2	23.0	8	184.0	8	1472.0
9	6.2	4	24.8	9	223.2	9	2008.8
10	0.2	1	0.2	10	2.0	10	20.0
		ΣF_1 =	339.4	ΣF_2 =	1628.0	ΣF_3 =	9782.4

The spacing between stations is h = 200/10 = 20 m. The area of the waterplane is then

$$A = 2 \times \frac{h}{3} \times \Sigma F_1 \rightarrow A = 2 \times \frac{20}{3} \times 339.4 = 4525 \text{ m}^2$$

2. True. The centroid about the after perpendicular is located by dividing the product of station spacing h and sum of area factors ΣF_2 by the sum of first moment factors ΣF_1; that is,

$$\text{Centroid about AP} = \frac{h \times \Sigma F_2}{\Sigma F_1} = \frac{20 \times 1628.0}{339.4} = 95.9 \text{ m forward of AP}$$

3. False. The moment of inertia about the after perpendicular is such that

$$I_{AP} = 2 \times \frac{1}{3} \times h^3 \times \Sigma F_3 = \frac{2}{3} \times 20^3 \times 9782.4 = 5.22 \times 10^7 \text{ m}^4$$

The moment of inertia we seek, however, is the moment about a transverse axis through the centroid, I_{NA}. This quantity can be obtained with the parallel-axis theorem,

$$I_{NA} = I_{AP} - Ax^2 \rightarrow I_{NA} = 5.22 \times 10^7 - 4525 \times 95.9^2 = 1.06 \times 10^7 \text{ m}^4$$

4. True. To determine the second moment of the waterplane area about the centerline, the ordinates are cubed and summed as follows.

Station	1/2 Ord.	1/2 Ord³	Simpson's Multiplier	F_4 (Second Moment)
0	0	0.0	1	0.0
1	10	1000.0	4	4000.0
2	13	2197.0	2	4394.0
3	14	2744.0	4	10976.0
4	14.2	2863.3	2	5726.6
5	14.2	2863.3	4	11453.2
6	14.1	2803.2	2	5606.4
7	14	2744.0	4	10976.0
8	11.5	1520.9	2	3041.8
9	6.2	238.3	4	953.3
10	0.2	0.0	1	0.0
			ΣF_4 =	57127.2

The moment of inertia we desire is then

$$I_{CL} = 2\times\frac{1}{3}\times\frac{1}{3}\times h\Sigma F_4 = 2\times\frac{1}{3}\times\frac{1}{3}\times 20\times 57{,}127.2 = 253{,}900 \text{ m}^4$$

P.3 ■ Solution

1. False. First, we calculate the volume of the added portion.

$$\text{Volume of added portion} = \delta\nabla = C_M \times B\times T\times 10 = 0.985\times 20\times 8\times 10 = 1576 \text{ m}^3$$

The variation in displacement, then, is the product of the volume of the added portion and the density of seawater,

$$\delta\Delta = \delta\nabla\times\rho_{sw} = 1576\times 1.025 = 1615 \text{ ton}$$

The updated displacement is

$$\Delta_2 = \Delta_1 + \delta\Delta = 14{,}000 + 1615 = 15{,}615 \text{ ton}$$

The new block coefficient follows as

$$C_B = \frac{\text{Volume of displacement 2}}{L_2\times B\times T} = \frac{(15{,}615/1.025)}{130\times 20\times 8} = 0.732$$

2. False. To find the new waterplane area coefficient, we first require the area of the modified waterplane,

$$\text{New WPA} = 0.808\times 120\times 20 + 10\times 20 = 2139 \text{ m}^2$$

The new waterplane area coefficient is then

$$C_{WP} = \frac{\text{New WPA}}{L_2\times B} = \frac{2139}{130\times 20} = 0.823$$

C_{WP} is raised because the addition of a rectangular segment increases the robustness of the hull.

3. False. The updated prismatic coefficient is

$$C_P = \frac{\text{Volume of displacement 2}}{L_2\times A_M} = \frac{(15{,}615/1.025)}{130\times(0.985\times 20\times 8)} = 0.744$$

Another way to obtain this quantity is to evoke the relation

$$C_B = C_P\times C_M \rightarrow C_P = \frac{C_B}{C_M}$$

$$\therefore C_P = \frac{0.732}{0.985} = 0.744$$

P.4 ■ Solution

Part A: Since the block is made of a material with density equal to half of the water density, we surmise that half of the block will be immersed. The distance from keel to the center of buoyancy is then *KB* = 1.0/2 = 0.5 m, while the distance from the keel to the center of gravity is KG = 2.0/2 = 1.0 m. The stability of the block is dictated by its metacentric height GM, which can be obtained from the relation

$$GM = KB + BM - KG$$

We already have KB and KG, and it remains to compute the metacentric radius. This is given by the ratio of the transverse moment of inertia to the volume of displacement; that is,

$$BM = \frac{I_T}{\nabla} = \frac{\left(LB^3/12\right)}{LBT}$$

$$\therefore BM = \frac{10\times 4^3/12}{10\times 4\times 1} = 1.33 \text{ m}$$

Hence,

$$GM = 0.5 + 1.33 - 1.0 = 0.83 \text{ m}$$

Since the metacentric height is greater than zero, we conclude that the block is in stable equilibrium.

★ The correct answer is **A**.

Part B: Needless to say, the material of which the block is made has not changed; as before, then, half of the block will be immersed. Accordingly, we compute distances KB = 2.0/2 = 1.0 m and KG = 4.0/2 = 2.0 m. The metacentric height is given by $GM = KB + BM - KG$, and the metacentric radius is now changed to

$$BM = \frac{I_T}{\nabla} = \frac{\left(LB^3/12\right)}{LBT}$$

$$\therefore BM = \frac{10\times 2^3/12}{10\times 2\times 2} = 0.17 \text{ m}$$

Finally, we have

$$GM = KB + BM - KG = 1.0 + 0.17 - 2.0 = -0.83 \text{ m}$$

Since the metacentric height is below zero, the block is in unstable equilibrium.

★ The correct answer is **C**.

P.5 ■ Solution

The new VCG can be easily obtained by means of a weighted sum,

$$\text{New VCG} = \bar{z} = \frac{10{,}000\times 8 + 500\times 3 - 300\times 5 + 200\times 12 - 1000\times 2}{10{,}000 + 500 - 300 + 200} = \boxed{7.73 \text{ m}}$$

As the number of operations increases, however, calculations are better performed in tabular form. See below.

Operation No.	Operation Type	Mass (t)	VCG (m)	dΔ (t)	Moment (t-m)
Ship		10000	8	10000	10000
1	Load	500	3 m AB	+500	+1500
2	Discharge	300	5 m AB	-300	-1500
3	Load	200	12 m AB	+200	+2400
4	Move	1000	Downward by 2 m	0	-2000
				Σm = 10400	ΣM = 80400

Thus,

$$\bar{z} = \frac{80{,}400}{10{,}400} = 7.73 \text{ m}$$

★ The correct answer is **B**.

P.6 ■ Solution

We know that $\nabla = LBT$ and that the change in draught is given by $\delta T = \Sigma m/LB$, where Σm is the added mass after all loading/unloading procedures are completed. The data we were given are processed in the following table.

Process	Mass Change (t)	Lever Arm (m)	Moment (t-m)
1	+100	+10	1000
2	-50	-40	2000
3	+200	-30	-6000
4	-50	+20	-1000
	Σm = 200		Σmx = -4000

The initial volume displacement was ∇_0 = 100×30×6 = 18,000 m^3, and the corresponding mass displacement was Δ_0 = 18,000×1.025 = 18,450 tonnes. The mass displacement after all loading/unloading processes are carried out is Δ_1 = 18,450 + 200 = 18,650 t, and the corresponding volume is ∇_1 = 18,650/1.025 = 18,195 m^3. The final draft is

$$T_1 = 6 + \frac{200}{100 \times 30 \times 1.025} = 6.07 \text{ m}$$

The moment of inertia of the vessel is I_L = 30×100^3/12 = 2.5×10^6 m^4, and the metacentric radius is $BM_L = I_L/\nabla_1$ = 2.5×10^6/18,195 = 137.4 m. The moment to change trim follows as

$$MCT = 18,650 \times \frac{137.4}{100 \times 100} = 256.3 \text{ tonne-m/cm}$$

The corresponding trim is then

$$\text{Trim} = -\frac{4000}{256.3} = \boxed{15.6 \text{ cm by aft}}$$

If need be, we could also compute the updated drafts forward and aft. Since the *LCF* is amidships, we have, owing to symmetry,

$$T_{\text{aft}} = T_1 + \frac{0.156}{2} = 6.15 \text{ m}$$

$$T_{\text{fwd}} = T_1 - \frac{0.156}{2} = 5.99 \text{ m}$$

★ The correct answer is **D**.

P.7 ■ Solution

1. True. The pertaining calculations are summarized in the following table.

Operation No.	Mass (t)	x_i (m)	$m_i x_i$ (t-m)	z_i (m)	$m_i z_i$ (t-m)
1	+150	20	3000	5	750
2	-220	-40	8800	9	-1980
3	-540	-5	2700	6	-3240
4	+325	-25	8125	10	3250
5	+400	-12	-4800	7	2800
6	+100	-25	3500	9	900
	Σm = 215		Σmx = 14,325		Σmz = 2480

The updated displacement is

$$\Delta = 15,000 + 215 = 15,215 \text{ tonnes}$$

The modified position of the longitudinal center of gravity, *LCG*, is found as

$$LCG_1 = \frac{15,000 \times (-10) + 14,325}{15,215} = -8.92 \text{ m (aft of midship)}$$

2. False. The modified position of the longitudinal center of buoyancy, *LCB*, is given by

$$LCB_1 = \frac{15{,}000 \times (-10) + 215 \times (-5)}{15{,}215} = -9.93 \text{ m (aft of midship)}$$

3. True. The modified vertical center of gravity, *VCG*, follows as

$$VCG_1 = \frac{15{,}000 \times 8 + 2480}{15{,}215} = 8.05 \text{ m}$$

4. False. The modified vertical center of buoyancy, *VCB*, is determined as

$$VCB_1 = \frac{15{,}000 \times 5.5 + 215 \times 15.03}{15{,}215} = 5.63 \text{ m}$$

5. False. The moment to change trim 1 cm is

$$MCT = \frac{\Delta GM_L}{100L}$$

Here, $GM_L = KB + BM_L - KG = 5.64 + I_L/\nabla - 8.05 = 88.6$ m, so that

$$MCT = \frac{15{,}215 \times 88.6}{100 \times 100} = 134.8 \text{ tonnes-m/cm}$$

The trimming moment is

$$\text{Trimming moment} = 15{,}215 \times (9.93 - 8.92) = 15{,}367 \text{ tonnes-m}$$

The overall trim is given by

$$\text{Trim} = \frac{15{,}367}{134.8} = 114 \text{ cm} = 1.14 \text{ m}$$

P.8 ■ Solution

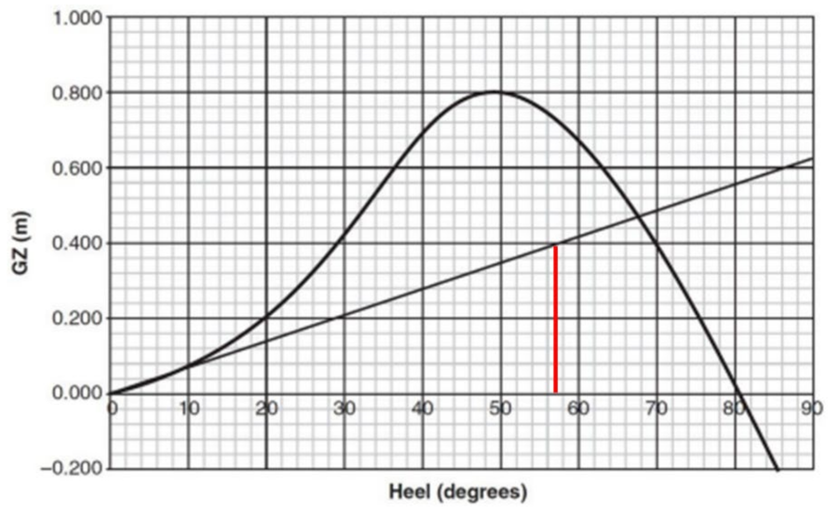
The difference between *KG* and the pole height is 18.0 – 15.0 = 3.0 ft. Using the table we were given, the righting arm for 1150 tonnes displacement and 30° list is read as 8.88 ft. The corrected righting arm *GZ* is then

$$GZ = 8.88 - 3 \times \sin 30^\circ = \boxed{7.38 \text{ ft}}$$

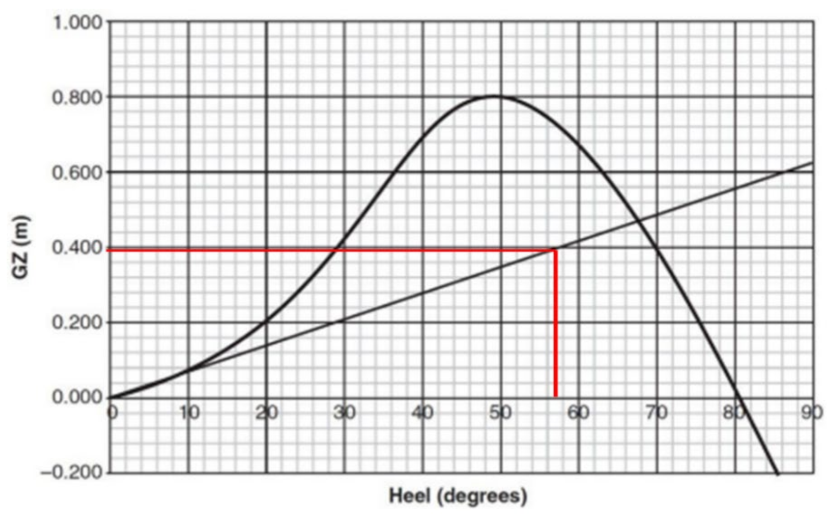
★ The correct answer is **C**.

P.9 ■ Solution

The first step to determine *GM* is to draw a vertical line that intersects the heel angle axis at 57.3 degrees, or 1 radian.



The next step is to trace a horizontal line that passes through the intersection of the previous line and the slope of the stability curve.



The value of GM we seek is the intersection of the horizontal line we just drew and the vertical axis. Clearly, GM = 0.40 m.

★ The correct answer is **A**.

Quick Proof: Determination of *GM* from the *GZ* curve.

The reader may wonder: why is it that reading the intercept of the horizontal line yields the GM value, not the righting arm GZ? This can be easily shown. Recall that, for small angles of heel, we can write

$$GZ = GM \sin\phi$$

If ϕ is in radians, then at small angles, $\sin\phi \approx \phi$ and

$$GZ = GM\phi$$

The initial slope of the stability curve, being a line, is described by an expression of the form $y = ax + b$, where y is the GZ, a is the gradient and x is the angle ϕ. As the line passes through the origin of the graph, b equals zero. We then surmise that

$$y = mx$$

equates to

$$GZ = GM\phi$$

The foregoing equation can be differentiated to find the slope,

$$\frac{d\left(GM\phi\right)}{d\phi} = GM$$

Therefore, the gradient of the equation $GZ = GM\phi$ is the initial GM of the vessel. If $d\phi$ is 1, then the resulting dGZ value must be equal to GM. Accordingly, reading the value of the line at one radian (where ϕ = 1) gives us the value of the metacentric height.

P.10 ■ Solution

The angle that occurs in the point of inflection of stability curves is known as the *deck edge immersion* (DEI) angle. As the name implies, it is the angle of heel at which the edge of the deck starts to be immersed. With most vessels, GZ gets progressively larger as the vessel starts to heel. This is seen as a gently increasing slope, or gradient, on the GZ curve. When the deck edge is immersed, the rate at which GZ grows reduces as the underwater geometry starts to change quickly. This is seen as a point of inflection on the curve, or the point at which the curve stops increasing in steepness. The DEI angle is important in that it has an influence on the angle of vanishing stability and on the range of stability.

★ The correct answer is **C**.

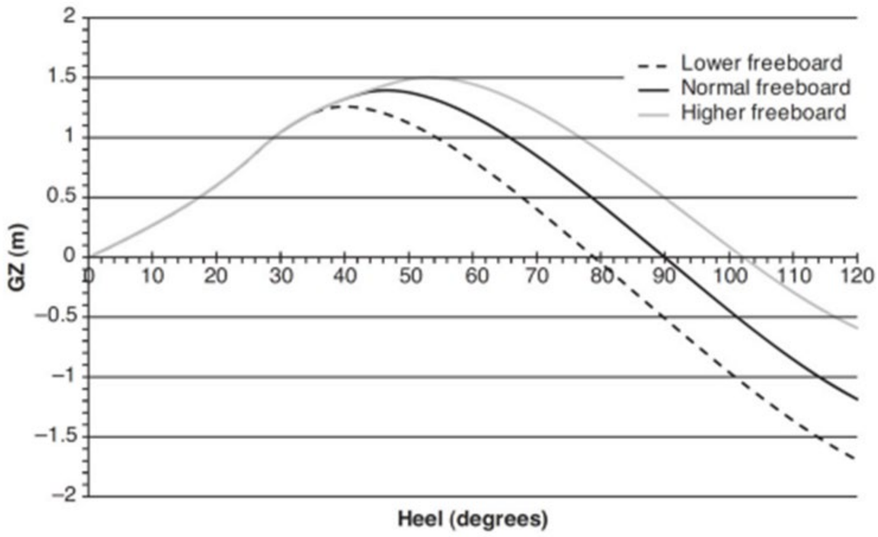
P.11 ■ Solution

One of the functions of the righting moment curve is related to energy. The area under the curve up to a specific angle is equal to the energy required to roll the vessel to that angle. Therefore, the greater the area under the *GZ* curve, the more energy it will take to roll the vessel to a given angle. In the present case, then, ship A is the one that requires the most energy to roll the specified amount.

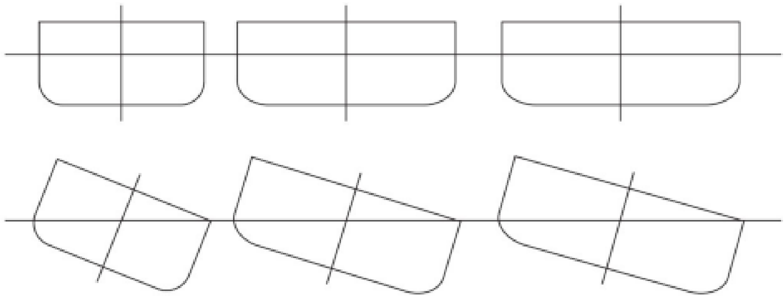
★ The correct answer is **A**.

P.12 ■ Solution

1. False. Below, we have the *GZ* curves for vessels at different freeboard levels. The freeboard is greatest for the vessel that corresponds to the gray line, intermediate for the vessel associated with the black line, and lowest for the vessel represented by the dotted line. Indeed, as a vessel with larger freeboard inclines, the center of buoyancy will move outboard at the same rate. It can be seen, furthermore, that the point of inflection, and therefore the DEI angle, will increase as well.



2. True. As the beam of a vessel increases, the *BM*, and hence *GM*, also increases. Therefore, for a similar *KG* value, the initial slope will be greater if the beam is larger. However, an increase in beam actually reduces the DEI angle, as shown in the cross-section sketches shown below. The point of inflection in the *GZ* curve will actually occur at a smaller angle.



3. True. All it takes is a quick inspection of the graph: the fluid simulation method produces a *GZ* curve that is essentially intermediate to the values obtained from the tank FSM method and with no FSE assessment at all. The safest, or most pessimistic, values come from the plot that produces the *GZ* curve with lowest righting arms for each angle of heel. In the present case, the curve in question is the one obtained with FSM data to determine *GZ* values indirectly, using the effective *KG*.

4. False. The chain of events described in the statement would've been correct if it referred to bow, not stern, trim. Indeed, inspection of the illustrations reveals that trimming the vessel in the stern direction leads to a more robust waterplane, and hence to a greater waterplane inertia. This implies an increased *BM*, with a corresponding increase in *GM* and in the slope of the *GZ* curve.

P.13 ■ Solution

Part A: To begin, we determine the height of the center of gravity by taking moments about the keel.

$$(4000+9000+1500+200)\times KG = 4000\times 6.0+9000\times 7.0+1500\times 1.0+200\times 7.5$$

$$\therefore KG = \frac{90,000}{14,700} = 6.12 \text{ m}$$

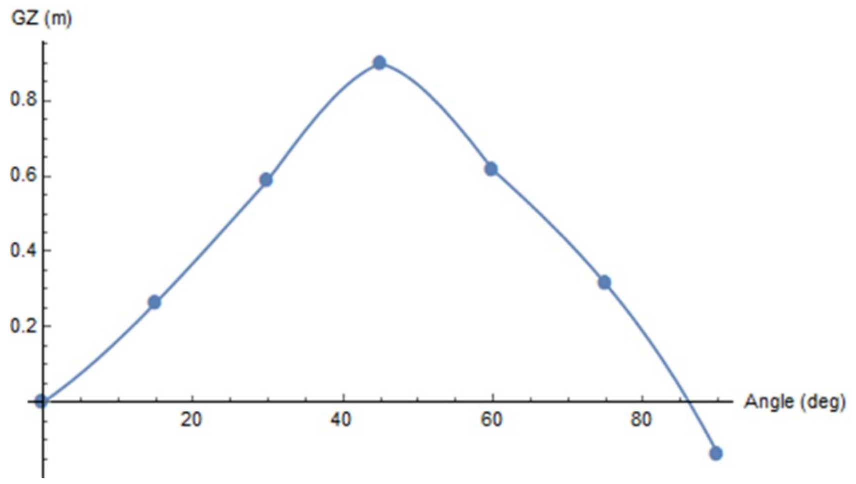
Since *G* is below *S*, the righting lever values are corrected as

$$GZ = SZ + SG\sin\phi$$

where *SG* = 6.6 – 6.12 = 0.48 m. The *GZ* values for various angles of inclination are determined in tabular form, as shown.

Angle (°)	sin ϕ	*SG* sin ϕ (m)	*SZ* (m)	*GZ* (m)
0	0.000	0.000	0	0.000
15	0.259	0.124	0.14	0.264
30	0.500	0.240	0.35	0.590
45	0.707	0.339	0.56	0.899
60	0.866	0.416	0.2	0.616
75	0.966	0.464	-0.15	0.314
90	1.000	0.480	-0.62	-0.140

The curve of statical stability is given below.



The range of stability *R* is the interval of angles over which *GZ* is positive. Inspecting the graph above, we see that *R* ≈ 86°.

Part B: As outlined by Tupper, the area under the *GZ* curve at a certain point is proportional to the energy needed to heel the ship to conditions of that point. It is a measure of the energy the ship can absorb from wind and waves. Mathematically, the dynamical stability is given by

$$U = \Delta\int GZ d\phi$$

In the present case, we want to assess the dynamical stability of the vessel when it is inclined 60 degrees. Since there are five data points in the interval going from zero to 60°, the integration may be performed via Simpson's 1,4,1-rule. The area products are listed and summed below.

Angle (°)	GZ (m)	Simpson's Multiplier	Area Product
0	0.000	1	0.000
15	0.264	4	1.057
30	0.590	2	1.180
45	0.899	4	3.598
60	0.616	1	0.616
		Sum	6.450

The area under the curve up to 60° is then

$$\text{Area} = \frac{15 \text{ deg}}{57.3 \dfrac{\text{deg}}{\text{rad}}}\times\left(\frac{1}{3}\times 6.45\right) = 0.563 \text{ m-rad}$$

and, knowing the displacement Δ = 14,700 tonnes, it follows that

$$U = 14,700 \times 9.81 \times 0.563 = \boxed{81.2\ \text{MN} \cdot \text{m}}$$

★ The correct answer is **D**.

P.14 ■ Solution

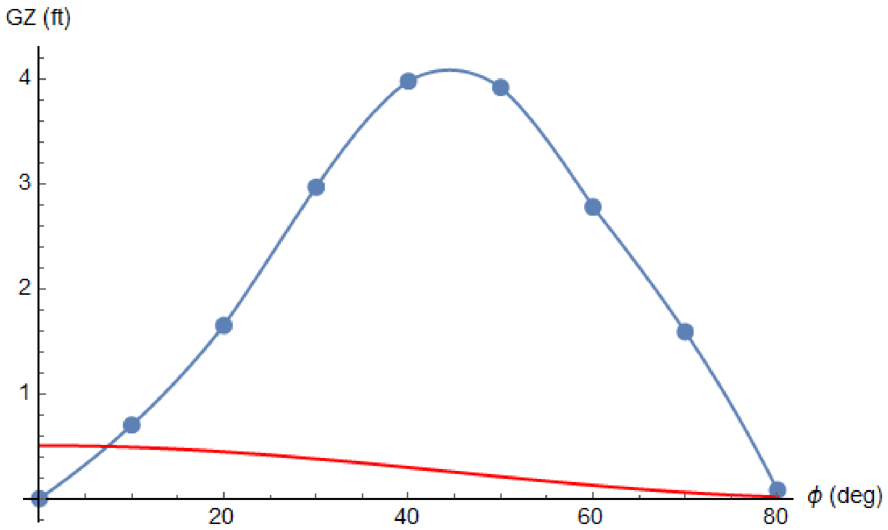
The center of lateral resistance is at half the draft, or 11.1 ft below the waterline, and hence the lever arm of the wind heel moment is $\ell = 18.9 + 11.1 = 30.0$ ft. Now, the wind heel arm is given by

$$\text{WHA} = \frac{0.0035 V_w^2 A \ell \cos^2 \phi}{2240\Delta}$$

where V_w is the wind velocity (in knots); A is the sail area (the projected area of the above-water profile, in ft^2); ϕ is the angle of heel; and Δ is the displacement (in long tons). Substituting the available data, we get

$$\text{WHA} = \frac{0.0035 \times 100^2 \times 16{,}150 \times 30 \times \cos^2 \phi}{2240 \times 15{,}000} = 0.5047 \cos^2 \phi$$

We can then plot this equation together with the *GZ* curve. The intersection between the two curves will be the angle of heel due to the wind.



The WHA curve, shown in red, intersects the *GZ* curve, shown in blue, at an angle of 7.5°. Hence, the wind will cause the ship to heel 7.5 degrees. This seven-and-a-half-degree heel angle is a rather modest angle of heel for a ship in hurricane-force winds of 100 knots. It should be noted, however, that this ship is a general dry cargo ship, which has a relatively small sail area. Wind heel can be much more severe on ships with higher profiles. For example, if the ship in question were to be converted to a container ship by removing the cargo-handling gear and stacking containers on the deck, the sail area would increase substantially, by 50 percent or more. Furthermore, the added sail area would be well above the deck. The wind heel angle experienced by such ships in strong beam winds can be significant.

★ The correct answer is **B**.

Answer Summary

Problem 1		**C**
Problem 2		T/F
Problem 3		T/F
Problem 4	**4A**	**A**
	4B	**C**
Problem 5		**B**
Problem 6		**D**
Problem 7		T/F
Problem 8		**C**
Problem 9		**A**
Problem 10		**C**
Problem 11		**A**
Problem 12		T/F
Problem 13	**13A**	Open-ended pb.
	13B	**D**
Problem 14		**B**

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