## PROBLEMS

## PROBLEM 1

Consider a signalized intersection with the following characteristics:
$\rightarrow$ Cycle length $C=75$ s
$\rightarrow$ Green time $G=30$ s
$\rightarrow$ Yellow plus all-red time $Y=5 \mathrm{~s}$
$\rightarrow$ Saturation headway $h=3.0 \mathrm{~s} / \mathrm{veh}$
$\rightarrow$ Start-up lost time $\ell_{1}=2 \mathrm{~s}$
$\rightarrow$ Clearance lost time $\ell_{2}=2 \mathrm{~s}$
For these characteristics, what is the capacity (per lane) of this movement?
A) $c=288 \mathrm{veh} / \mathrm{h} / \mathrm{ln}$
B) $c=496 \mathrm{veh} / \mathrm{h} / \mathrm{ln}$
C) $c=675 \mathrm{veh} / \mathrm{h} / \mathrm{ln}$
D) $c=894 \mathrm{veh} / \mathrm{h} / \mathrm{ln}$

## PROBLEM $A_{\text {(Roess et al., 2010, w/ permission) }}$

Find the minimum number of lanes for the intersection illustrated below. Assume that all volumes shown have been converted to compatible "through-car equivalent" values for the conditions shown. Suppose that critical volumes reverse in the other daily peak hour. In the following figure, $h$ is the saturation headway, $t_{L}$ is the total lost time per phase, and $C$ is the cycle length. As your answer, provide the total number of lanes.

A) Minimum no. of lanes $=4$
B) Minimum no. of lanes $=6$
C) Minimum no. of lanes $=8$
D) Minimum no. of lanes $=10$

## PROBLEM 2 ㄹ

Find the minimum number of lanes for the intersection illustrated below. As before, suppose that critical volumes reverse in the other daily peak hour and provide the total number of lanes as your answer.

A) Minimum no. of lanes $=8$
B) Minimum no. of lanes $=10$
C) Minimum no. of lanes $=12$
D) Minimum no. of lanes $=14$

## PROBLEM (Roess et al., 2010, w/ permission)

What change and clearance intervals are recommended for an intersection with an average approach speed of $35 \mathrm{mi} / \mathrm{h}$, a grade of $-2 \%$, a crossstreet width of 50 feet, and 10 -foot crosswalks? Assume a standard vehicle length of 20 feet, a driver reaction time of 1.0 second, a deceleration rate of $10 \mathrm{ft} / \mathrm{s}^{2}$, and significant pedestrian movements.
A) $y=4.1 \mathrm{~s}$ and $a r=1.8 \mathrm{~s}$
B) $y=4.1 \mathrm{~s}$ and ar $=3.4 \mathrm{~s}$
C) $y=6.3 \mathrm{~s}$ and $a r=1.8 \mathrm{~s}$
D) $y=6.3 \mathrm{~s}$ and $a r=3.4 \mathrm{~s}$

## PROBLEM H/A (Roess et al., 2010, w/ permission)

An analysis of pedestrian needs at a signalized intersection is undertaken. Important parameters concerning pedestrian needs and the existing vehicular signal timing are given in the table below. Are pedestrians safely accommodated? Assume a standard lost time of 4.0 s .

| Phase | Green Time <br> $G(s)$ | Yellow + All-Red <br> $Y(s)$ | Pedestrian Requirement <br> $G_{p}(s)$ |
| :---: | :---: | :---: | :---: |
| A | 18 | 4.5 | 30 |
| B | 60 | 4 | 15 |

A) Pedestrians are safely accommodated in both phases.
B) Pedestrians are safely accommodated in phase A, but not in phase B.
C) Pedestrians are safely accommodated in phase $B$, but not in phase $A$.
D) Pedestrians are not safely accommodated in both phases.

## PROBLEM 4 B

Which of the following is the smallest cycle length so that pedestrians are safely accommodated and the ratio of effective green times maintained?
A) $L=102 \mathrm{~s}$
B) $L=118 \mathrm{~s}$
C) $L=133 \mathrm{~s}$
D) $L=150 \mathrm{~s}$

## PROBLEM SA (Mannering \& Washburn, 2013, w/ permission)

A left-turn movement has a maximum arrival rate of 200 veh/h. The saturation flow of this movement is 1400 veh/h. For this approach, the yellow time is 4 seconds, the red time is 2 seconds, and the total lost time is 3 seconds. The cycle length is 120 seconds. What minimum displayed green time must be provided to ensure that the queue in each cycle clears?
A) $G=14 \mathrm{~s}$
B) $G=20 \mathrm{~s}$
C) $G=27 \mathrm{~s}$
D) $G=34 \mathrm{~s}$

## PROBLEM 5 B

What is the total vehicle delay per cycle and the average delay per vehicle for the conditions specified in the previous part?
A) $D_{t}=185$ veh -s and $\bar{d}=51.4 \mathrm{~s}$
B) $D_{t}=185$ veh -s and $\bar{d}=80.1 \mathrm{~s}$
C) $D_{t}=344$ veh -s and $\bar{d}=51.4 \mathrm{~s}$
D) $D_{t}=344$ veh -s and $\bar{d}=80.1 \mathrm{~s}$

## PROBLEM (Mannering \& Washburn, 2013, w/ permission)

An isolated pretimed signalized intersection has an approach with a saturation flow rate of $1900 \mathrm{veh} / \mathrm{h}$. For this approach, the displayed red time is 58 seconds, the displayed yellow time is 3 seconds, the all-red time is 2 seconds, the effective green time is 28 seconds, and the total lost time is $4 \mathrm{~s} /$ phase. What is the average delay per vehicle when the approach flow rate is 550 veh/h?
A) $\bar{d}=10.7 \mathrm{~s}$
B) $\bar{d}=20.5 \mathrm{~s}$
C) $\bar{d}=30.1 \mathrm{~s}$
D) $\bar{d}=40.6 \mathrm{~s}$

## PROBLEM (Mannering \& Washburn, 2013, w/ permission)

An intersection approach has a saturation flow rate of 1500 veh/h, and vehicles arrive at the approach at a rate of $800 \mathrm{veh} / \mathrm{h}$. The approach is controlled by a pretimed signal with a cycle length of 60 seconds and $D / D / 1$ queueing holds. Local standards dictate that signals should be set such that all approach queues dissipate 10 seconds before the end of the effective green portion of the cycle. Assuming that approach capacity exceeds arrivals, determine the maximum length of effective red that will satisfy the local standards.
A) $r=18 \mathrm{~s}$
B) $r=23 \mathrm{~s}$
C) $r=28 \mathrm{~s}$
D) $r=33 \mathrm{~s}$

## PROBLEM (Mannering \& Washburn, 2013, w/ permission)

An observer notes that an approach to a pretimed signal has a maximum of eight vehicles in a queue in a given cycle. If the saturation flow rate is 1440 veh/h and the effective red time is 40 seconds, how much time will it take this queue to clear after the start of effective green (assuming that approach capacity exceeds arrivals and D/D/1 queueing applies)?
A) $t_{c}=10 \mathrm{~s}$
B) $t_{c}=20 \mathrm{~s}$
C) $t_{c}=30 \mathrm{~s}$
D) $t_{c}=40 \mathrm{~s}$

## PROBLEM (Mannering \& Washburn, 2013, w/ permission)

The saturation flow rate for an intersection approach is 3600 veh $/ \mathrm{h}$. At the beginning of a cycle (effective red) no vehicles are queued. The signal is timed so that when the queue (from the continuously arriving vehicles) is 13 vehicles long, the effective green begins. If the queue dissipates 8 seconds before the end of the cycle and the cycle length is 60 seconds, what is the arrival rate, assuming $D / D / 1$ queueing?
A) $v=1000 \mathrm{veh} / \mathrm{h}$
B) $v=1400 \mathrm{veh} / \mathrm{h}$
C) $v=1800 \mathrm{veh} / \mathrm{h}$
D) $v=2200 \mathrm{veh} / \mathrm{h}$

PROBLEM 1 (Mannering \& Washburn, 2013, w/ permission)
An approach to a signalized intersection has a saturation flow rate of 2640 veh/h. For one cycle, the approach has three vehicles in queue at the beginning of an effective red, and vehicles arrive at 1064 veh/h. The signal for the approach is timed such that the effective green starts eight seconds after the approach's vehicle queue reaches 10 vehicles, and lasts 15 seconds. What is the total vehicle delay for this signal cycle?
A) $D_{t}=198$ veh -sec
B) $D_{t}=379$ veh -sec
C) $D_{t}=542$ veh -sec
D) $D_{t}=705$ veh-sec

PROBLEM 1 [Mannering \& Washburn, 2013, w/ permission (modified)]
An approach to a pretimed signal with a 60-second cycle has nine vehicles in the queue at the beginning of the effective green. Four of the nine vehicles in the queue are left over from the previous cycle (at the end of the previous cycle's effective green). The saturation flow rate of the approach is $1500 \mathrm{veh} / \mathrm{h}$, the total delay for the cycle is 4.0 vehicle-minutes, and at the end of the effective green there are 2 vehicles left in the queue. Determine the arrival rate assuming that it is unchanged over the duration of the observation period (from the beginning to the end of the 4.0 -vehicle-minute delay cycle).
A) $\lambda=601 \mathrm{veh} / \mathrm{h}$
B) $\lambda=752 \mathrm{veh} / \mathrm{h}$
C) $\lambda=904 \mathrm{veh} / \mathrm{h}$
D) $\lambda=1050 \mathrm{veh} / \mathrm{h}$

## PROBLEM ${ }^{1 ?}$ (Mannering \& Washburn, 2013, w/ permission)

A signal approach has 20 seconds of displayed green, 4 seconds of yellow, and 3 seconds of all red (start-up lost time and clearance times are 4.0 s). The cycle length is 60 seconds. At the beginning of an effective red there are no vehicles in queue and vehicles arrive at three-quarters of the saturation flow rate for 30 seconds. Then there is zero flow for 10 seconds, and then one-half of the saturation flow rate from 40 seconds until the end of the cycle. What is the total delay for this cycle if the saturation flow rate is 1400 veh/h? (Assume D/D/1 queueing).
A) $D_{t}=178$ veh -sec
B) $D_{t}=342$ veh -sec
C) $D_{t}=508$ veh -sec
D) $D_{t}=651$ veh -sec

## PROBLEM 15 (Mannering \& Washburn, 2013, w/ permission)

At the start of the effective red at an intersection approach to a pretimed signal, vehicles begin to arrive at a rate of 800 veh $/ \mathrm{h}$ for the first 40 seconds and $500 \mathrm{veh} / \mathrm{h}$ from then on. The approach has a saturation flow rate of $1200 \mathrm{veh} / \mathrm{h}$ and an effective green of 20 seconds, and the cycle length is 40 seconds. What is the total vehicle delay for two cycles after the 800-veh/h arrival rate begins?
A) $D_{t}=68$ veh-sec
B) $D_{t}=124$ veh-sec
C) $D_{t}=182$ veh -sec
D) $D_{t}=230$ veh -sec

## SOLUTIONS

## P. 1 ■ Solution

Following the textbook by Roess et al., this problem can be approached in two different ways. In the first method, we can straightforwardly apply the relation

$$
c=s_{i}\left(\frac{g_{i}}{C}\right)
$$

where $s_{i}$ is the saturation flow rate for lane $i, g_{i}$ is the effective green time for lane $i$, and $C$ is the signal cycle length. Given the saturation headway $h=3.0 \mathrm{~s} / \mathrm{veh}, \mathrm{s}$ is computed as

$$
s=\frac{3600}{h}=\frac{3600}{3.0}=1200 \mathrm{veh} / \mathrm{h} / \ln
$$

The effective green time $g_{i}$, in turn, is determined as

$$
g_{i}=G_{i}+Y_{i}-t_{L, i}
$$

where $G_{i}=30 \mathrm{~s}$ is the actual green time for movement, $Y_{i}=5 \mathrm{~s}$ is the sum of yellow and all-red intervals, and $t_{L, I}=2+2=4 \mathrm{~s}$ is the total lost time for movement, with the result that

$$
g_{i}=G_{i}+Y_{i}-t_{L, i}=30+5-4=31 \mathrm{~s}
$$

Inserting these data into the foregoing equation, we obtain

$$
c=1200 \times \frac{31}{75}=496 \mathrm{veh} / \mathrm{h} / \ln
$$

As a second method of solution, we create a ledger of time within an hour. Once the amount of time per hour used by vehicles at the saturation flow rate is established, capacity can be found by assuming that this time is used at a rate of one vehicle every $h$ seconds. Because the characteristics are initially provided on a per phase basis, we should convert these data to a per hour basis. This can be easily done by considering the number of cycles that take place on an hourly rate, i.e. at each 3600 seconds. For a 75-s cycle, there are $m=3600 / 75=48$ cycles/h. The red time in this hour is computed as

Red time in hour $=(C-G-Y) \times m=(75-30-5) \times 48=1920 \mathrm{~s}$
The lost time over the course of an hour amounts to
Lost time in hour $=\left(\ell_{1}+\ell_{2}\right) \times m=(2+2) \times 48=192 \mathrm{~s}$
The remaining time, deducting these two intervals, is
Remaining time in hour $=3600-1920-192=1488 \mathrm{~s}$
The 1488 remaining seconds of time in the hour represent the amount of time that can be used at a rate of one vehicle every $h$ seconds. In the present case, $h=3.0 \mathrm{~s} / \mathrm{veh}$. This number was calculated by deducting the periods during which no vehicles (in the subject movements) are effectively moving, encompassing the red time and the start-up and clearance lost times in each signal cycle. The capacity of this movement may then be computed as

$$
c=\frac{1488}{3.0}=496 \mathrm{veh} / \mathrm{h} / \ln
$$

As expected, the result is the same as that of the first approach.
$\Rightarrow$ The correct answer is $\mathbf{B}$.

## P. 2 ■ Solution

Part A: The maximum sum of critical-lane flows is
$V_{c}=\frac{1}{h}\left[3600-N t_{L}\left(\frac{3600}{C}\right)\right]=\frac{1}{2.05} \times\left[3600-2 \times 4.3 \times\left(\frac{3600}{60}\right)\right]=1504 \mathrm{veh} / \mathrm{h}$
The required number of lanes is now determined. Suppose we had a single lane in each direction, as illustrated below.


The sum of critical-lane volumes would then be $800+1400=2200 \mathrm{veh} / \mathrm{h}$. Since this is greater than the maximum volume of $1504 \mathrm{veh} / \mathrm{h}$, this situation is not acceptable. Further lane divisions are in order. Suppose now that the EB approach is divided into two lanes, as shown.


In this case, the sum of critical-lane volumes becomes $700+800=1500$ veh/h. Since this is less than the calculated $V_{c}$ of 1504 , the situation is acceptable. Recalling that critical volumes reverse in the peak hour, the prescribed number of lanes will be 2 for the NB movement and 4 for the EB movement, with $2+4=6$ lanes in total.

- The correct answer is B.

Part B: The maximum sum of critical lane volumes, $V_{c}$, is determined as

$$
V_{c}=\frac{1}{2.30} \times\left[3600-4 \times 4.5 \times\left(\frac{3600}{90}\right)\right]=1252 \mathrm{veh} / \mathrm{h}
$$

Suppose we divided the 1000 veh/h SB flow into two lanes, as indicated by the red arrows.


In this case, the sum of critical-lane volumes becomes

$$
V_{c}=700+200+500+300=1700 \mathrm{veh} / \mathrm{h}
$$

which is greater than the capacity of $1250 \mathrm{veh} / \mathrm{h}$. Two other possible segmentations are illustrated below.


Referring to the figure to the left, the value of $V_{c}$ is

$$
V_{c}=350+200+500+300=1350 \mathrm{veh} / \mathrm{h}
$$

which is still greater than the analytical value of $V_{c}$. As for the figure to the right, we have

$$
V_{c}=350+200+500+150=1200 \mathrm{veh} / \mathrm{h}
$$

which is acceptable. Noting that the number of lanes must be doubled for opposing movement, we prescribe $2 \times 3=6$ lanes for the E-W artery and $2 \times 4=8$ lanes for the N-S artery, with a total of $6+8=14$ lanes.

- The correct answer is D.


## P. 3 ■ Solution

The change interval is given by

$$
y=t+\frac{1.47 S_{85}}{2 a+64.4 \times 0.01 G}
$$

where $t=1.0 \mathrm{~s}$ is the reaction time, $S_{85}=S+5=35+5=40 \mathrm{mi} / \mathrm{h}$ is the 85 th percentile speed, $a=10 \mathrm{ft} / \mathrm{s}^{2}$ is the deceleration rate, and $G=-2 \%$ is the grade of the approach. Accordingly,

$$
y=1.0+\frac{1.47 \times 40}{2 \times 10+64.4 \times 0.01 \times(-2)}=4.1 \mathrm{~s}
$$

The clearance interval for a case in which significant pedestrian traffic exists is expressed as

$$
a r=\frac{P+L}{1.47 S_{15}}
$$

where $P=50+10=60 \mathrm{ft}$ is the distance from the stop line to the far side of the farthest conflicting crosswalk, $L=20 \mathrm{ft}$ is the length of a standard vehicle, and $S_{15}=$ $35-5=30 \mathrm{mi} / \mathrm{h}$ is the 15 th percentile speed. Therefore,

$$
a r=\frac{P+L}{1.47 S_{15}}=\frac{60+20}{1.47 \times 30}=1.8 \mathrm{~s}
$$

- The correct answer is $\mathbf{A}$.


## P. 4 ■ Solution

Part A: The pedestrians will be safely accommodated depending on the inequality $G_{p} \geq G+Y$. Computing it for phases $A$ and $B$ gives, respectively, we have

$$
\begin{gathered}
G_{p, A} \geq G_{A}+Y_{A} \rightarrow 30 \geq 18+4.5=22.5 \mathrm{~s}(\mathrm{NG}) \\
G_{p, B} \geq G_{B}+Y_{B} \rightarrow 15 \geq 60+4=64 \mathrm{~s}(\mathrm{OK})
\end{gathered}
$$

The correct answer is $\mathbf{C}$.

Part B: The effective green times for phases A and B are calculated as

$$
\begin{gathered}
g=G+Y-t_{L} \\
g_{A}=18+4.5-4.0=18.5 \mathrm{~s} \\
g_{B}=60+4-4.0=60 \mathrm{~s}
\end{gathered}
$$

The signal must be re-timed to result in a $G+Y$ for phase A of at least 30.0 seconds while maintaining the relative balance of effective green time needed by vehicles in both phases (i.e., a ratio of 18.5 to 60). For phase A to have a $G+Y$ of 30.0 seconds, the effective green time would have to be increased to

$$
g_{A}^{\prime}=30.0-4.0=26.0 \mathrm{~s}
$$

To maintain the original ratio of vehicular green time, the effective green time for phase B must also be increased,

$$
\begin{gathered}
\frac{g_{B}^{\prime}}{g_{A}^{\prime}}=\frac{g_{B}}{g_{A}} \rightarrow \frac{g_{B}^{\prime}}{26.0}=\frac{60}{18.5} \\
\therefore g_{B}^{\prime}=84.3 \mathrm{~s}
\end{gathered}
$$

The actual green times would become

$$
\begin{aligned}
& G_{A}=26.0-4.5+4.0=25.5 \mathrm{~s} \\
& G_{B}=84.3-4.0+4.0=84.3 \mathrm{~s}
\end{aligned}
$$

The cycle length follows as

$$
L=(25.5+4.5)+(84.3+4.0)=118 \mathrm{~s}
$$

If this intersection were under pretimed control, a 120 -second cycle would be imposed.
$\Rightarrow$ The correct answer is $\mathbf{B}$

## P. 5 ■ Solution

Part A: The arrival rate is $\lambda=200 / 3600=0.0556$ veh/s and the departure rate is $\mu=1400 / 3600=0.389 \mathrm{veh} / \mathrm{s}$. If the queue clears at the end of each cycle, we can write

$$
\lambda \times C=\mu \times g
$$

where $C=120 \mathrm{~s}$ is the cycle length and $g$ is the effective green time. Inserting our data and solving for $g$, it follows that

$$
0.0556 \times 120=0.389 \times g \rightarrow g=17.2 \mathrm{~s}
$$

Recall that $g$ can be expressed as

$$
g=G+A R+Y-t_{L}
$$

Substituting and solving for the displayed green time $G$, we get

$$
\begin{gathered}
g=G+A R+Y-t_{L} \rightarrow 17.2=G+2+4-3 \\
\therefore G=14 \mathrm{~s}
\end{gathered}
$$

$\Rightarrow$ The correct answer is A.
Part B: Before proceeding, we require the effective red time $r$, which is given by

$$
r=C-g=120-17.2=103 \mathrm{~s}
$$

We also need the volume/capacity ratio $v / c$, which is calculated as

$$
\frac{v}{c}=\frac{v}{s(g / C)}=\frac{0.0556}{0.389 \times(17.2 / 120)}=1.0
$$

Now, the total vehicle delay for each cycle is estimated as

$$
D_{t}=\frac{v r^{2}}{2\left(1-\frac{v}{s}\right)}=\frac{0.0556 \times 103^{2}}{2 \times\left(1-\frac{0.0556}{0.389}\right)}=344 \mathrm{veh}-\mathrm{sec}
$$

As for the average vehicle delay, we find that

$$
\bar{d}=\frac{0.5 C\left(1-\frac{g}{C}\right)^{2}}{1-\left(\frac{v}{c} \times \frac{g}{C}\right)}=\frac{0.5 \times 120 \times\left(1-\frac{17.2}{120}\right)^{2}}{1-\left(1.0 \times \frac{17.2}{120}\right)}=51.4 \mathrm{~s}
$$

- The correct answer is $\mathbf{C}$.


## P. 6 ■ Solution

The effective red time is

$$
r=R+t_{L}=58+4=62 \mathrm{~s}
$$

Given the effective green time $g=28 \mathrm{~s}$, the cycle length is found as

$$
C=g+r=28+62=90 \mathrm{~s}
$$

The volume/capacity ratio is then

$$
\left(\frac{v}{c}\right)=\frac{v}{s \times \frac{g}{C}}=\frac{550}{1900 \times \frac{28}{90}}=0.930
$$

The average uniform delay per vehicle is determined next,

$$
\bar{d}=\frac{0.5 C\left(1-\frac{g}{C}\right)^{2}}{1-\left(\frac{v}{c} \times \frac{g}{C}\right)}=\frac{0.5 \times 90 \times\left(1-\frac{28}{90}\right)^{2}}{1-\left(0.930 \times \frac{28}{90}\right)}=30.1 \mathrm{~s}
$$

- The correct answer is $\mathbf{C}$.


## P. 7 ■ Solution

The arrival rate is $v=800 / 3600=0.222 \mathrm{veh} / \mathrm{h}$ and the saturation flow rate is $s=1500 / 3600=0.417 \mathrm{veh} / \mathrm{h}$. Recall that the effective red time is given by the difference between cycle length and effective green time,

$$
r=C-g
$$

Since approach queues are to dissipate 10 s before the end of effective green, the queue clearance time is given by $t_{c}=g-10$. However, we know that $t_{c}$ can be calculated as

$$
t_{c}=\frac{v r}{s-v}
$$

Substituting $r=60-g$ for the effective red and other pertaining variables, we obtain

$$
\begin{gathered}
t_{c}=\frac{v r}{s-v} \rightarrow g-10=\frac{0.222 \times(60-g)}{0.417-0.222} \\
\therefore g=36.6 \mathrm{~s}
\end{gathered}
$$

Lastly, substituting in the first equation, the maximum effective red is determined to be

$$
r=C-g=60-36.6=23.4 \approx 23 \mathrm{~s}
$$

$\downarrow$ The correct answer is $\mathbf{B}$.

## P. 8 ■ Solution

Given the effective red time $r=40 \mathrm{~s}$ and the number of vehicles in queue $Q=8$ veh, the corresponding flow rate is

$$
\begin{gathered}
Q=v r \rightarrow v=\frac{Q}{r} \\
\therefore v=\frac{8}{40}=0.2 \mathrm{veh} / \mathrm{s}
\end{gathered}
$$

The saturation flow rate is converted as $s=1440 / 3600=0.4$ veh/s. Lastly, the time required to clear this queue is determined to be

$$
t_{c}=\frac{v r}{s-v}=\frac{0.2 \times 40}{0.4-0.2}=40 \mathrm{~s}
$$

- The correct answer is $\mathbf{D}$.


## P. 9 ■ Solution

The saturation flow rate is converted as $s=3600 / 3600=1.0$ veh/s. Given the number of vehicles $Q_{\max }=13$ in the queue, the effective red can be expressed in terms of the arrival rate as

$$
\begin{gathered}
Q=v r \rightarrow r=\frac{Q}{v} \\
\therefore r=\frac{13}{v}
\end{gathered}
$$

However, we know that the arrival rate and the saturation flow rate are related by

$$
v(C-t)=s(C-t-r)
$$

Substituting our data, the equation becomes

$$
\begin{gathered}
v(60-8)=1.0 \times\left(60-8-\frac{13}{v}\right) \\
\therefore 52 v=60-8-\frac{13}{v} \\
\therefore 52 v=\frac{60 v-8 v-13}{v} \\
\therefore 52 v^{2}=60 v-8 v-13 \\
\therefore 52 v^{2}-52 v+13=0
\end{gathered}
$$

The positive solution to this quadratic equation is $v=0.5 \mathrm{veh} / \mathrm{s}$ or, equivalently, 1800 veh/h.

- The correct answer is $\mathbf{C}$.


## P. 10 ■ Solution

The converted arrival rate is $\lambda=1064 / 3600=0.296 \mathrm{veh} / \mathrm{s}$, while the departure rate is $\mu=2640 / 3600=0.733 \mathrm{veh} / \mathrm{s}$. We were told that there are 3 vehicles in the queue at the beginning of effective red. The signal for the approach is timed such that the effective green starts 8 s after the vehicle queue reaches 10 vehicles. In mathematical terms, we have

$$
3+\lambda t=10
$$

which, substituting $\lambda$ and solving for $t$, yields

$$
3+0.296 t=10 \rightarrow t=23.6 \mathrm{~s}
$$

The maximum queue length occurs 8 s after the start of effective green, which is to say that the red time exceeds time $t$ by 8 seconds, or

$$
r=t+8 \rightarrow r=23.6+8=31.6 \mathrm{~s}
$$

Given the effective green time $g=15 \mathrm{~s}$, the cycle length amounts to

$$
C=r+g=31.6+15=46.6 \mathrm{~s}
$$

The arrival and departure lines are plotted below.


Line $A B$ represents the number of vehicles at the beginning of the effective red (= $=3$ ), line $B C$ represents the arrivals, and line DE represents the departures. The total delay is given by the shaded area and can be determined as

$$
\begin{gathered}
D_{t}=A_{A B C F}-A_{D E F}=\frac{1}{2} \times(A B+C F) \times A F-\frac{1}{2} \times E F \times D F \\
\therefore D_{t}=0.5 \times[3+(3+\lambda C)] \times C-0.5 \times g \times \mu g \\
\therefore D_{t}=0.5 \times(3+3+0.296 \times 46.6) \times 46.6-0.5 \times 15 \times 0.733 \times 15=379 \mathrm{veh}-\mathrm{sec} \\
\triangleright \text { The correct answer is } \mathbf{B} .
\end{gathered}
$$

## P. 11 ■ Solution

The vehicle delay is converted as $D_{t}=4.0 \times 60=240$ veh-sec. The departure rate, in turn, is $\mu=1500 / 3600=0.417 \mathrm{veh} / \mathrm{s}$. At the beginning of effective green, there are 9 vehicles in the queue. 4 of the 9 vehicles are left over from the previous cycle. Thus, given the arrival rate $\lambda$ and the effective green $g$, the number of vehicles at the end of the cycle length is $(9-4)+\lambda g$. Since there are 2 vehicles left in the queue at the end of the cycle, the number of departing vehicles at the end of the cycle is $(4-2)+\lambda C$. The arrival and departure lines are plotted below.


Line $A B$ represents the number of vehicles at the beginning of the effective red (=4), line BC represents the arrivals, and line DE represents the departures. The total delay is given by the shaded area and follows as

$$
\begin{gathered}
D_{t}=A_{A B C F}-A_{D E F}=\frac{1}{2} \times(A B+C F) \times A F-\frac{1}{2} \times E F \times D F \\
\therefore D_{t}=0.5 \times[4+(5+\lambda g)] \times C-0.5 \times g \times(2+\lambda C)
\end{gathered}
$$

Substituting $D_{t}=240$ veh-sec and $C=60 \mathrm{~s}$ brings to

$$
\begin{aligned}
& 240=0.5 \times[4+(5+\lambda \times g)] \times C-0.5 \times g \times(2+\lambda C) \\
& \therefore 240=0.5 \times(9+\lambda g) \times C-g-0.5 g \lambda C \\
& \therefore 240=4.5 C+0.5 g 2 C-g-0.5 g \lambda C \\
& \therefore g=4.5 \times 60-240=30 \mathrm{~s}
\end{aligned}
$$

Finally, the arrival rate is calculated as

$$
\begin{gathered}
\lambda r=5 \rightarrow \lambda \times(C-g)=5 \\
\therefore \lambda=\frac{5}{C-g} \\
\therefore \lambda=\frac{5}{60-30}=0.167 \mathrm{veh} / \mathrm{s} \\
\therefore \lambda=601 \mathrm{veh} / \mathrm{h}
\end{gathered}
$$

The correct answer is $\mathbf{A}$.

## P. 12 ■ Solution

The converted saturation rate is $1400 / 3600=0.389 \mathrm{veh} / \mathrm{s}$. The effective green time is

$$
g=G+A R+Y-t_{L}=20+3+4-4=23 \mathrm{~s}
$$

The effective red, in turn, is $r=60-23=37 \mathrm{~s}$. In order to graph the arrivals and departures, consider the following data points. For 30 seconds starting from the beginning of effective red, vehicles arrive at three-quarters of the saturation rate, i.e., at a rate of $0.75 \times 0.389=0.292 \mathrm{veh} / \mathrm{s}$, amounting to $30 \times 0.292=8.76 \approx$ 9 veh. Accordingly, one of the data points is $B=(30,9)$. After this period, there is no flow for 10 seconds. Thus, a second pertaining data point is $C=(40,9)$. Then, from 40 s until the end of the cycle, vehicles arrive at one-half the saturation rate, i.e., at a rate of $0.5 \times 0.389=0.195 \mathrm{veh} / \mathrm{s}$, totaling $20 \times 0.195=3.9 \approx 4$ veh. At the end of the cycle, $9+4=13$ vehicles will have arrived. Thus, a third pertaining data point is $D=(60,13)$. Departures start at the onset of effective green, which is at 37 s. Thus, a fourth point of interest is $E=(37,0)$. The number of vehicles that will have departed the approach in the green period is $0.389 \times 23=8.95 \approx 9$ veh. Consequently, a fifth point of interest is $F=(60,9)$. We are now ready to outline the data points in the plane of No. of vehicles versus time, as illustrated below.


The total vehicle delay is the shaded area in the figure above; that is,
$D_{t}=\frac{1}{2} \times 30 \times 9+(60-30) \times 9+\frac{1}{2} \times(60-40) \times(13-9)-\frac{1}{2} \times(60-37) \times 9=342 \mathrm{veh}-\mathrm{sec}$

- The correct answer is $\mathbf{B}$.


## P. 13 ■ Solution

The converted initial arrival rate is $\lambda=800 / 3600=0.222 \mathrm{veh} / \mathrm{s}$. The number of vehicles that will arrive in the first 40 seconds is then $0.222 \times 40=8.88$ $\approx 9$ veh. After the first 40 s elapse, the arrival rate becomes $\lambda=500 / 3600=0.139$ veh/s. The departure rate, in turn, is $\mu=1200 / 3600=0.333 \mathrm{veh} / \mathrm{s}$, and the number of vehicles that will have left the approach during the effective green is $0.333 \times 20$ $=6.66 \approx 7$ veh. With these quantities, we can outline the following data points. At the initial 40 s of the first cycle, 9 vehicles will have arrived, whence we verify that $B=(40,9)$ is a point of interest. In the second cycle, the rate of arrivals will have reduced to $0.139 \mathrm{veh} / \mathrm{s}$, and the corresponding number of vehicles will then be $0.222 \times 40+0.139 \times 40=14.4 \approx 14$ veh. Thus, $C=(80,14)$ is another point of interest. Departures start at the beginning of effective green time. Since the effective green time is 20 s and the cycle length is 40 s , departures start in the 20th second and amount to 7 veh as calculated just now. Accordingly, $D=(40,7)$ is also a point of interest. Another 7 vehicles will depart the approach in the interval that goes from 40 s to 60 s because of the effective red time of the second cycle and, consequently, $E=(60,7)$ is also part of the plot. Finally, the number of departures at the end of the second cycle is $0.333 \times 20+0.333 \times 20=13.3 \approx 13$ veh. Thus, $F=$ $(80,13)$ is also part of the departure line. We are now ready to plot the data points in the time-No. of vehicles plane, as shown.


The total vehicle delay is the shaded area in the figure above; that is,
$D_{t}=\frac{1}{2} \times 40 \times 9-\frac{1}{2} \times 20 \times 7+40 \times(9-7)+\frac{1}{2} \times 40 \times(14-9)-\frac{1}{2} \times 20 \times(13-7)=230$ veh-sec

- The correct answer is $\mathbf{D}$.
- ANSWER SUMMARY

| Problem 1 |  | B |
| :---: | :---: | :---: |
| Problem 2 | 2A | B |
|  | 2B | D |
| Problem 3 |  | A |
| Problem 4 | 4A | C |
|  | 4B | B |
| Problem 5 | 5A | A |
|  | 5B | C |
| Problem 6 |  | C |
| Problem 7 |  | B |
| Problem 8 |  | D |
| Problem 9 |  | C |
| Problem 10 |  | B |
| Problem 11 |  | A |
| Problem 12 |  | B |
| Problem 13 |  | D |

## REFERENCES

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