Quiz EG2
Solar Energy and Photovoltaics

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Problems

Problem 1

Regarding the theory of solar power, are the following statements true or false?

1.( ) In the visible spectrum (λ = 600 nm), crystalline silicon has a light absorption coefficient of approximately 4000 cm^{-1}. Accordingly, the corresponding penetration depth for this material is greater than 3 μm.

2.( ) A certain semiconductor cell of absorption coefficient 40 cm^{-1} is irradiated with infrared light (wavelength $\lambda \approx$ 1000 nm). Taking R = 0 as the reflection factor, the minimum thickness required for this cell to achieve an absorption efficiency of 95% is greater than 700 μm.

3.( ) A certain solar cell has open-circuit voltage V_{OC} = 0.3 V and short-circuit current I_{SC} = −5.2 mA. The maximum operating voltage and current are 0.18 V and −3.6 mA, respectively. The fill factor of this cell is greater than 0.43.

4.( ) A solar energy collector uses a single glass cover with thickness of 6 mm. In the visible solar range, the refraction index of the glass material is 1.53 and the extinction coefficient K is 36 m^{-1}. The transmissivity of the glass sheet at an angle of incidence of 50° is greater than 0.74.

5.( ) A certain solar cell is irradiated with sunlight at 1200 W/m^2. The electrical efficiency of the cell is 10%, the fill factor is 0.550, and the aperture area is 4 cm^2, The power output associated with this cell can be calculated to be greater than 30 mW.

6.( ) The thermal efficiency η_T of a flat-plate solar collector can be estimated with the empirical formula

$$\eta_T = 0.78 - \frac{7.7\left(T_c - T_a\right)}{G}$$

where T_a is water inflow temperature, T_c is outflow temperature, and G is the intensity of incident irradiation. With reference to the formula above, consider a flat-plate solar collector receiving water at 20°C and returning water at 50°C while being irradiated with sunlight at 900 W/m^2. The thermal efficiency of this system is greater than 50%.

7.( ) A flat plate solar collector with an aperture area of 4 m^2 is irradiated by sunlight at an average rate of 830 W/m^2 for a period of one hour. The average transmittance-absorptance product is 0.86. The average rate of heat loss from the collector to the ambient has been estimated to be 430 W/m^2. The efficiency of this collector can be calculated to be greater than 35%.

8.( ) A solar water heater is mounted on a south-facing roof and feeds a storage tank of 150 liters. The circulating pump operates at a rate of 50×10^{-6} m^3/s. The temperature rise of the collector water is 18°C and the heating process lasts for 6 hours. If the temperature of the water in the storage tank is increased by 14°C, the thermal efficiency of this system can be calculated to be greater than 20%. In your analysis of this statement, take 4.18 J/g·K as the specific heat capacity of water.

9.( ) Five collector panels are connected in series. Each collector is 3 m^2 in area and the product of heat removal factor and heat loss coefficient is $F_R U_c$ = 5 W/m^2·°C at a flow rate of water (c_p = 4.18 kJ/kg·K) equal to 0.02 kg/s. The correction factor for this arrangement of panels is greater than 0.74.

A crystalline silicon solar cell at 298 K generates a photocurrent density J_{ph} = 400 A/m^2. The wafer is doped with 10^{23} acceptor atoms per cubic meter and the emitter layer with a uniform concentration of 10^{25} donors per cubic meter. The minority-carrier diffusion length in the *p*-type region and *n*-type region are 520×10^{-6} and 16×10^{-6} m, respectively. Further, the intrinsic carrier concentration in silicon at 300 K is 1.5×10^{16} m^{-3}, the mobility of electrons in the *p*-type region is μ_n = 980 cm^2 V^{-1} s^{-1} and holes in the *n*-type region is μ_p = 120 cm^2 V^{-1} s^{-1}. When analyzing the following two statements, assume that the solar cell behaves as an ideal diode.
10.( ) The open circuit voltage of the cell described above is greater than 0.55 V.
11.( ) The conversion efficiency of the cell when irradiated at 1000 W/m^2 is greater than 30%.

Problem **2**

The average direct beam radiation intensity at a location is 825 W/m^2 and the average ambient temperature is 30°C. A compound parabolic collector with the specifications listed below is used to operate a low-temperature power plant:

→ Concentration ratio = 3.2
→ Transmittance-absorptance product = 0.8
→ Reflectivity = 0.89
→ Collector heat loss coefficient = 2.5 W/m^2·K

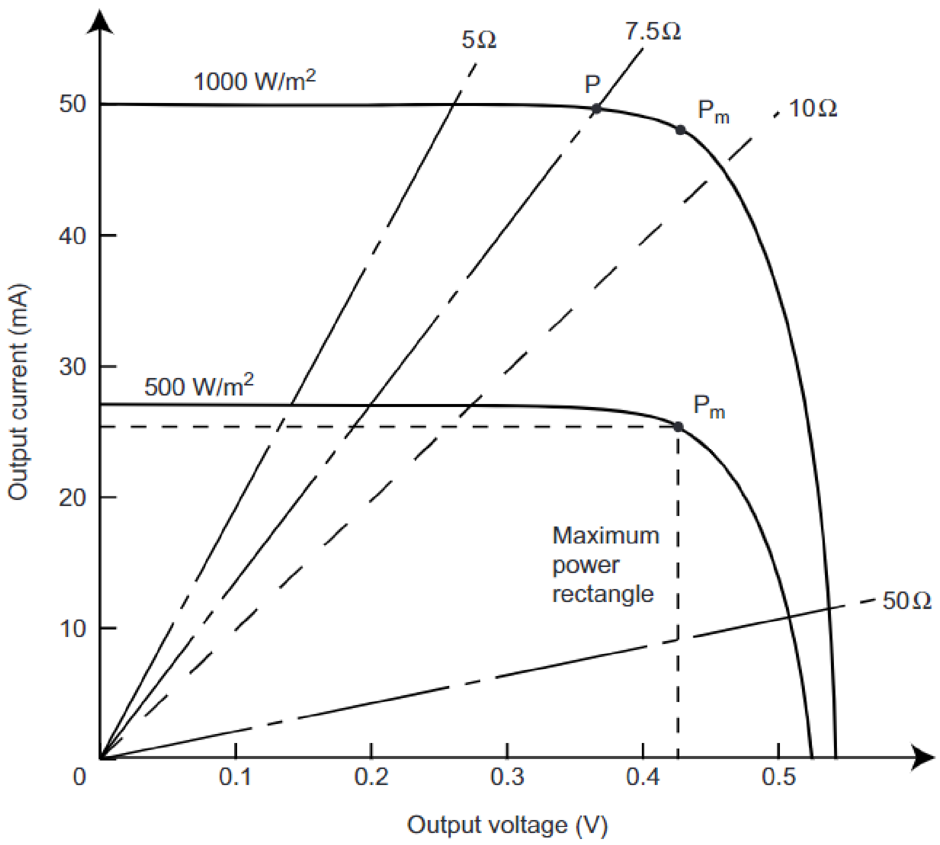
Assuming that the power plant operates in an ideal Carnot cycle, evaluate the following statements.
1.( ) The temperature at which the receiver in question yields the maximum overall efficiency is greater than 260°C.
2.( ) The maximum overall efficiency for the receiver in question is greater than 30%. Note that, for the receiver in question, overall efficiency is the product of collector efficiency and power cycle efficiency.

Problem **3.1** (Modified from Shepherd and Shepherd, 2008)

Twenty-five solar cells of the type characterized by the chart on the next page are connected in parallel to a 7.5 Ω load. The cells are being irradiated at 500 W/m^2. Are the following statements true or false?
1.( ) The voltage for this arrangement of cells is greater than 0.3 V.
2.( ) The load current for this arrangement of cells is greater than 500 mA.
3.( ) The power dissipated by this arrangement of cells is greater than 100 mW.

Problem **3.2** (Modified from Shepherd and Shepherd, 2008)

Suppose that 40 solar cells described by the same chart used in the previous problem are connected in series. The radiation intensity is 1000 W/m^2 and the load resistance is 10 Ω. Are the following statements true or false?
1.( ) The load current for this arrangement of cells is greater than 40 mA.
2.( ) The load voltage for this arrangement of cells is greater than 22 V.
3.( ) The power dissipated by this arrangement of cells is greater than 1 W.



Problem 4

The band gap of silicon (Si) is 1.11 eV. Light of intensity 4 mW and wavelength 0.549 μm is incident on a Si solar cell. Calculate:

Part 1: The wavelength of radiation whose photons have energies equal to the band gap of silicon.

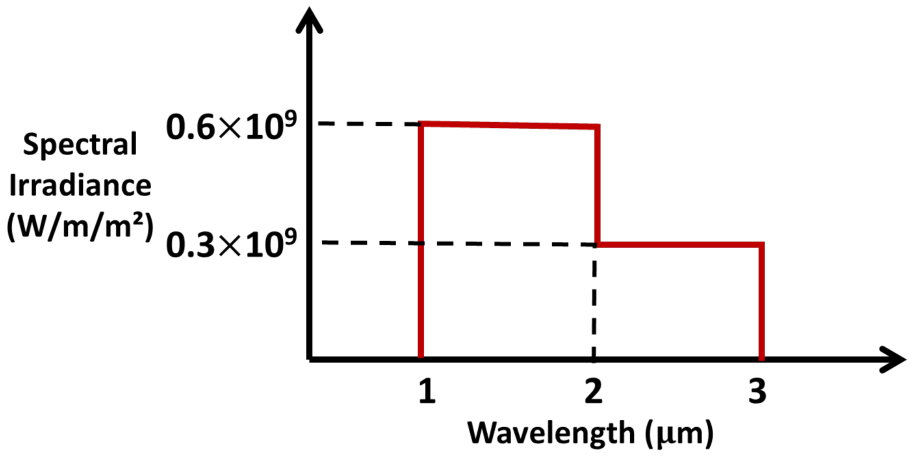
Part 2: The number of photons incident per second.

Part 3: The maximum power available.

Part 4: The maximum percentage of incident power that can be converted to electricity.

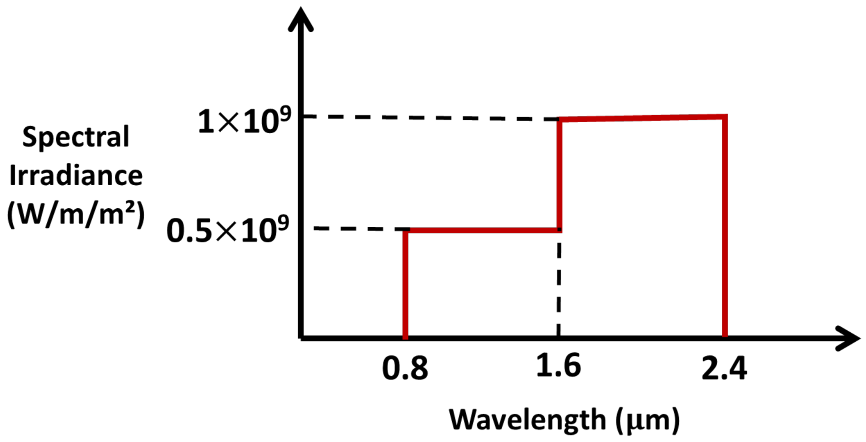
Problem 5

A certain solar spectrum is approximated by the rectangular form graph shown next. Find **(1)** the total radiation and **(2)** the total photon flux of the spectrum.



Problem 6

A certain solar spectrum is approximated by the following distribution. Radiation with this spectrum is incident on a solar cell with a band gap of 1.11 eV. Assuming a quantum efficiency of 1, find **(1)** the total irradiation; **(2)** the total photon flux; **(3)** the amount of incident radiation lost as heat in the cell.



Problem 7

A silicon solar cell of band gap equal to 1.11 eV is uniformly illuminated by monochromatic light of intensity equal to 500 W/m^2 and wavelength 760 nm. The external quantum efficiency at this wavelength is 0.75. Calculate **(1)** the photon flux; and **(2)** the light current of the cell if the area of the cell is 8 cm^2.

Problem 8

An abrupt silicon *p-n* junction solar cell at room temperature is irradiated with sunlight. Assume that the sunlight is uniformly intense throughout the silicon, yielding an optical generation rate of 6×10^{19} cm^{-3}s^{-1}. The solar cell has a junction area of 120 cm^2, a depletion region width of 4 μm, and reverse saturation current density of 1.25×10^{-11} A cm^{-2}. The silicon has a carrier lifetime of 2×10^{-6} s. Find **(1)** the optically generated current that is generated inside the depletion region; **(2)** the total optically generated current; **(3)** the short-circuit current; **(4)** the open-circuit voltage. Finally, if the solar cell fill factor is 0.77, find **(5)** the maximum power available. Use D_n = 35 cm^{-2}s^{-1} and D_P = 13 cm^{-2}s^{-1} as the diffusivity of electrons and holes, respectively.

Problem 9

Monochromatic light of wavelength 736 nm and intensity 330 W/m^2 is incident on a silicon solar cell with a band gap of 1.11 eV and area 5 cm^2. The dark saturation current density is 1.4×10^{-12} A. The cell temperature is 32°C. The quantum efficiency and the ideality factor are both equal to 1. Find **(1)** the short-circuit current; **(2)** the open circuit voltage; **(3)** the maximum power; **(4)** the fill factor; and **(5)** the conversion efficiency.

Problem 10

A photovoltaic module has dimensions equal to 0.5 × 0.5 m. The I-V characteristic of the module has been measured under specified conditions (temperature T = 298 K), yielding an open-circuit voltage V_{OV} = 0.7 V and a short-circuit current I_{SC} = 36 A. The ideality factor is 1.5. Find **(1)** the reverse saturation current; **(2)** the characteristic equation of the module; **(3)** the maximum power; **(4)** the fill factor; and **(5)** the energy conversion efficiency for standard incidence radiation at 1000 W/m^2.

Problem 11.1

For a plate collector having the following characteristics and ignoring the bond resistance, calculate the fin efficiency and the collector efficiency factor.

→ Overall heat loss coefficient, U_c = 7.5 W/m^2·°C
→ Tube spacing = 160 mm
→ Tube outside diameter = 20 mm
→ Tube inside diameter = 18 mm
→ Plate thickness = 0.5 mm
→ Plate material: copper (thermal conductivity = 390 W/m· °C)
→ Heat transfer coefficient inside the tubes = 400 W/m^2·°C

Problem 11.2

For the collector described in the previous part, compute the useful energy and the collector efficiency if the collector area is 5 m^2, the flow rate is 0.08 kg/s, the transmittance-absorptance product is $(\tau\alpha)$ = 0.81, the global solar radiation for one hour is 2.9 MJ/m^2, and the collector operates at a temperature difference of 10°C. Use 4.18 kJ/kg·K as the specific heat capacity of water.

Problem 12

Calculate the averaged hourly and daily efficiency of a water solar collector on February 1st in a location at 40°N. The collector is tilted at an angle of 60° and has an overall conductance of 10 W/m^2·K on the upper surface. The collector is made of copper (thermal conductivity k = 390 W/m·K) tubes with 1 cm internal diameter and 0.05 cm thickness, which are connected by a 0.06-cm-thick plate at a center-to-center distance of 18 cm. The heat transfer coefficient for the water in the tubes is 1600 W/m^2·K, the cover transmittance is 0.92, and the solar absorptance of the copper surface is 0.82. The collector is 1 m wide and 2 m long, the water inlet temperature is 310 K, and the water flow rate is 0.03 kg/s. The horizontal insolation (total) and the environmental temperature are tabulated below. Assume that diffuse radiation accounts for 30% of the total insolation.

Time (h)	I_h (W/m^2)	T_{amb} (K)
7 – 8	14	274
8 – 9	82	280
9 – 10	176	283
10 – 11	340	286
11 – 12	480	291
12 – 13	464	291
13 – 14	401	288
14 – 15	299	282
15 – 16	176	280
16 – 17	39	277

Problem **13**

A compound parabolic collector has an aperture area of 5 m^2 and a concentration ratio of 2.0. Estimate the collector efficiency given the following:

→ Circulating fluid: water (spec. heat cap. = 4180 J/kg·°C)
→ Total radiation = 920 W/m^2
→ Diffuse to total radiation ratio = 0.15
→ Receiver absorptivity = 0.86
→ Receiver emissivity = 0.14
→ Mirror reflectivity = 0.93
→ Glass cover transmissivity = 0.90
→ Collector heat loss coefficient = 3 W/m^2·K
→ Entering fluid temperature = 75°C
→ Fluid flow rate = 0.02 kg/s
→ Ambient temperature = 20°C
→ Collector efficiency factor = 0.93

Problem **14**

Calculate the collector-plate efficiency factor F' and heat-removal factor F_R of a smooth, 2-m-wide, 4-m-long air collector with the following design. The flow rate per unit collector area is 0.76 m^3/min·m^2. The air duct height is 1.6 cm, the air density is 1.05 kg/m^3, the specific heat is 1 kJ/kg·K, and the viscosity is 1.8×10^{-5} Pa·s. The collector heat-loss coefficient U_c is 21 kJ/h·m^2·K. Take 0.72 as the Prandtl number of air.

Problem **15**

The following are steps involved in the conversion of sunlight to electricity in organic photovoltaics. Which of the following alternatives ranks the four steps in correct order?

P. Exciton diffusion to the donor-acceptor interface.
Q. Charge transport to, and collection at, the electrodes.
R. Absorption of sunlight and formation of an exciton.
S. Exciton dissociation into free charges (electrons and holes).

(A) S → P → Q → R
(B) R → P → S → Q
(C) R → S → P → Q
(D) P → S → R → Q

Problem **16**

Regarding the theory of third-generation solar cells, are the following statements true or false?

1.( ) The performance of an organic photocell can be improved by constructing a bulk heterojunction (BHJ) constituted of a nanoscale photoactive blend of acceptor and donor materials. In the common approach to BHJ construction, a fullerene derivative is used as the acceptor and a conjugated polymer is used as the donor.

Recommended book: Zdyb (2023).

Recently, the ternary blend photoactive layer strategy has emerged as a means to improve the device performance of organic solar cells. Indeed, the ternary blend strategy is simple to implement and has greater potential for use in large-scale fabrication than, say, the tandem solar cell strategy.

2.( ) Duan *et al.* (2019) showed that a ternary organic solar cell incorporating the nonfullerene acceptor Y6 can achieve a 29% efficiency improvement over an equivalent binary cell. Duan's group went on to explore the mechanism behind this improvement; they posited the existence of a tradeoff between exciton dissociation and the carrier recombination process. The addition of Y6 was found to induce a more efficient exciton dissociation process, but accelerates the carrier recombination process. ■ **(A black square indicates the end of a multi-paragraph statement.)**

Recommended research: Duan *et al.* (2019).

The conventional structure of a dye-sensitized solar cell (DSSC) includes a photoanode made of glass covered by TCO (transparent conductive oxide) and a layer of titanium dioxide nanoparticles sensitized with the dye.
3.( ) The glass used as substrate of the photoanode is generally fabricated with indium tin oxide, which has supplanted fluorine tin oxide due to its low cost and superior performance at annealing temperatures. ■

In 1961, Shockley and Queisser developed a theoretical framework for determining the limiting efficiency of a single junction solar cell based on the principle of detailed balance, equating the incoming and outgoing fluxes of photons for a device at open-circuit conditions. Work with recent devices, however, has indicated that the Shockley-Queisser (SQ) limit can be exceeded; for instance, Krogstrup *et al.* (2013) proposed this for nanowire solar cells.
4.( ) More recently, Xu *et al.* (2015) argued that any nanostructured solar cell, irrespective of its geometry, has limiting efficiency identical to a planar solar cell with concentrating optics. ■

Recommended research: Krogstrup (2013); Xu *et al.* (2015).

Solar cells made from perovskites have shown promise as a third-generation technology. Organic-inorganic hybrid perovskite cells exhibit outstanding optoelectronic features such as a high absorption coefficient, long carrier lifetimes, and large diffusion length. On the other hand, it is well-reported that hybrid perovskites are somewhat unstable and can switch phases when subjected to even mild temperature gradients. Recently, researchers have tried to overcome these issues by replacing hybrid perovskite-based cells with all-inorganic solutions that do away with any volatile organic constituents. The first all-inorganic perovskite cell, made from cesium lead iodide ($CsPbI_3$), was introduced by Eperon *et al.* (2015).
5.( ) However, the efficiencies reported in research work with $CsPbI_3$ solar cells are very low. This discourages further research on this type of material as a potential solar technology for commercial use. Indeed, as of early 2023 there are no reports of all-inorganic perovskite cells with power conversion efficiency greater than 10%. ■

Recommended research: Eperon *et al.* (2015); Wang *et al.* (2019).

Although inorganic perovskites are generally stabler than their organic counterparts, they are still somewhat unreliable. $CsPbI_3$ does not have a stable perovskite phase at ambient temperature due to the relatively small ionic radius of Cs to hold the PbI_8 octahedra together. Eperon *et al.* (2015) reported that $CsPbI_3$ could be stabilized by adding HI acid in the perovskite precursor solution. More recently, Lau *et al.* (2018) used calcium to partially substitute Pb in the $CsPbI_3$ precursor to produce $CsPbI_3$ films.
6.( ) Lau's group noted that the Ca addition led to improvements in performance. Stability also improved: if we were to take a sample of their calcium-modified perovskite cell 2 months after preparation, we'd find that most of the power conversion efficiency would have been retained. ■

Recommended research: Eperon *et al.* (2015); Lau *et al.* (2018).

7.( ) One common issue that workers in the photovoltaics community have faced when working with lead halide perovskite-based solar cells is the fact that these cells' current-voltage scans – determined so as to yield a cell's efficiency – yield different results depending on the scan direction, and also sometimes on rate and range. This phenomenon, called thixotropy, continues to be a hurdle for the development of commercial perovskite cells.

Recommended research: Habisreutinger (2018).

Increases in solar cell area are usually accompanied by substantial efficiency loss because of increased series resistance, decreased shunt resistance, unavoidable dead area at interconnection regions, and film inhomogeneity. The efficiency loss per increased area is a critical parameter to evaluate the upscalability of solar cell technology.

8.( ) Commercial solar cells such as silicon and CIGS follow an apparent inverse scaling law, with an absolute power conversion efficiency loss of 0.8% when the device area is increased by one order of magnitude (i.e., 0.8% PCE loss/(10 × area)). In turn, perovskite solar cells have been shown to have a much higher PCE loss rate with increasing area. ■

Recommended research: Kothandaraman *et al.* (2020).

9.( ) Quantum dots can be exploited in solar cell designs. Usually, QDs are implemented to replace dyes as sensitizers. Researchers have also experimented with inclusion of dopants in quantum dots, which have been successfully used to broaden the cell's bandgap, suppress carrier recombination, and enhance electron conduction.

Recommended research: Ganguly and Nath (2020).

10.( ) The performance of quantum dots in solar cells can be improved by use of a core-shell structure, which occurs as either a 'type I' or 'type II' formulation. The potential benefits of the core-shell approach to quantum dots have been exemplified by Yang *et al.* (2015), who described a type-II CdSeTe/CdS core-shell QD cell that achieved an efficiency of nearly 9.5% and hence ranks as one of the most efficient designs of its kind.

Recommended research: Yang *et al.* (2015).

11.( ) One important limitation of early quantum dot solar cells is that most prospective designs were based on toxic cadmium and lead chalcogenides. This has led researchers to propose 'greener' designs that eschew toxic materials. For example, Pan *et al.* (2014) described a CIS-Z quantum dot cell that contains no Cd or Pb; however, despite its compositional advantages, the cell reported by Yan's group had a lackluster power conversion efficiency – much less, in fact, than the PCE achieved by equivalent toxic metal-based QD cells available at the time.

Recommended research: Pan *et al.* (2014).

Additional Information

Figure 1. Average number of reflections for full and truncated compound parabolic concentrators (CPCs). θ_c denotes acceptance half angle; C denotes concentration ratio.

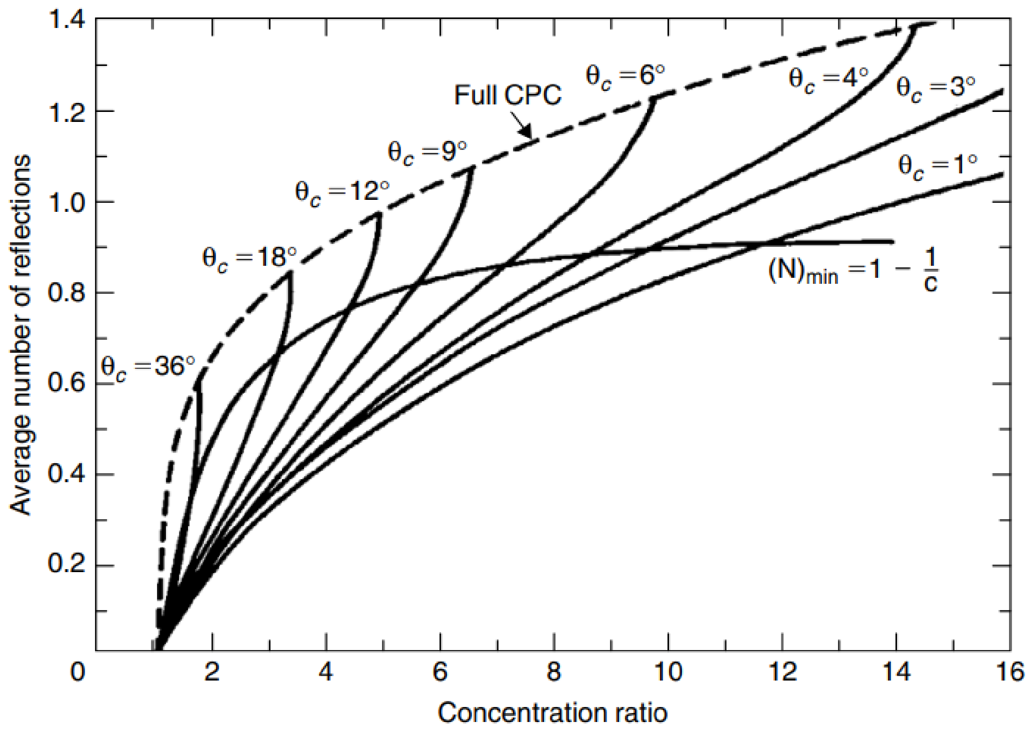


Figure 2. Ratio of height to aperture for full and truncated compound parabolic concentrators (CPCs).

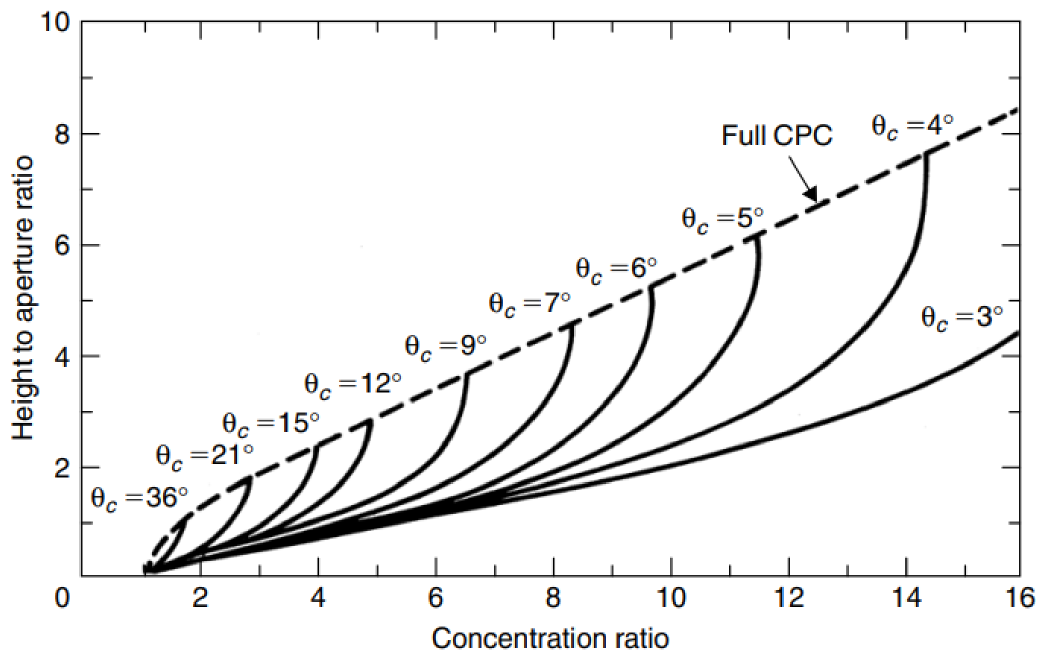
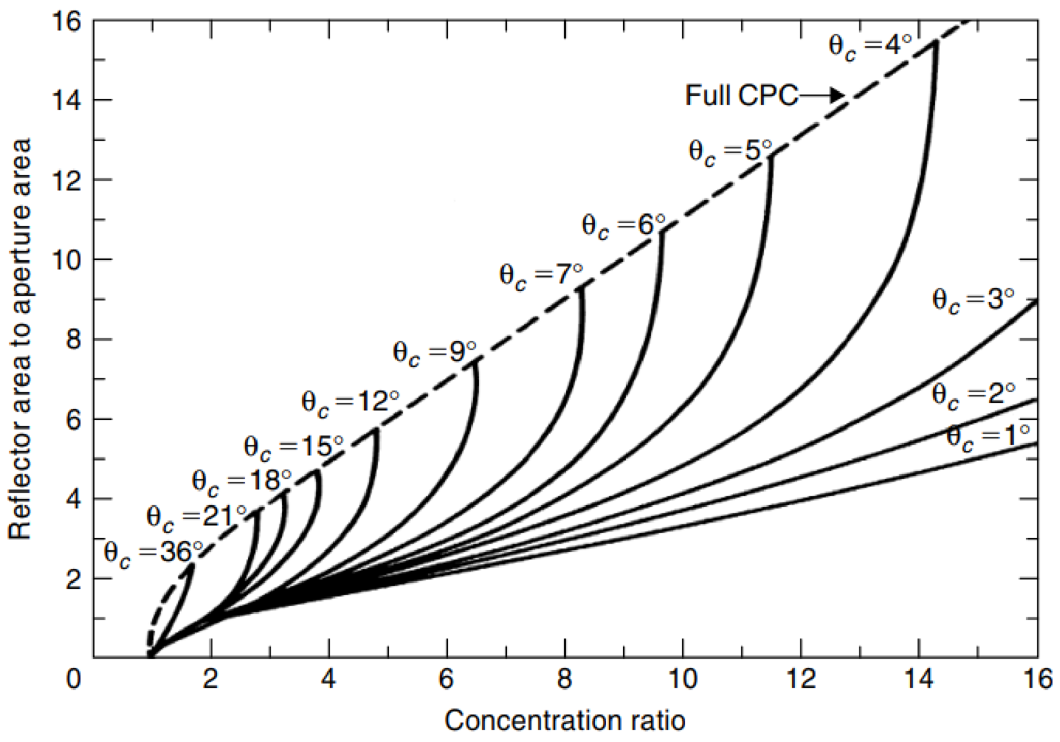


Figure 3. Ratio of reflector to aperture area for full and truncated CPCs.



SOLUTIONS

■ Problem 1

1. False. In the case at hand, the penetration depth x_p is the reciprocal of absorption coefficient α:

$$x_p = \frac{1}{\alpha} = \frac{1}{4000} = 2.5\times10^{-4}\ \text{cm} = \boxed{2.5\ \mu\text{m}}$$

2. True. The absorption efficiency η_A may be expressed as

$$\eta_A = (1-R)\left(1-e^{-\alpha d}\right)$$

where R = 0 is the reflection factor, α = 40 cm^{-1} is the absorption coefficient, and d is thickness. Setting η_A = 0.95 and solving for d, we get

$$\eta_A = (1-R)\left(1-e^{-\alpha d}\right) \rightarrow 0.95 = (1-0)\times\left(1-e^{-40d}\right)$$

$$\therefore 0.95 = 1-e^{-40d}$$

$$\therefore e^{-40d} = 0.05$$

$$\therefore -40d = \ln(0.05)$$

$$\therefore d = \frac{\ln(0.05)}{-40} = 0.0749\ \text{cm} = \boxed{749\ \mu\text{m}}$$

3. False. Substituting the appropriate values into the definition of fill factor, we obtain

$$F = \frac{V_{\max} \times |I_{\max}|}{V_{OC} \times |I_{SC}|} = \frac{0.18 \times 3.6}{0.3 \times 5.2} = \boxed{0.415}$$

4. False. We first compute the refraction angle θ_2 using Snell's law:

$$\theta_2 = \sin^{-1}\left(\frac{\sin\theta_1}{n}\right) = \sin^{-1}\left(\frac{\sin 50^\circ}{1.53}\right) = 30.0^\circ$$

The transmittance τ_α is then

$$\tau_\alpha = e^{-KL/\cos\theta_2} = \exp\left(-\frac{36 \times 0.006}{\cos 30.0^\circ}\right) = 0.779$$

The perpendicular and parallel components of unpolarized radiation are

$$r_\perp = \frac{\sin^2(\theta_2 - \theta_1)}{\sin^2(\theta_2 + \theta_1)} = \frac{\sin^2(30.0^\circ - 50.0^\circ)}{\sin^2(30.0^\circ + 50.0^\circ)} = 0.121$$

$$r_\parallel = \frac{\tan^2(\theta_2 - \theta_1)}{\tan^2(\theta_2 + \theta_1)} = \frac{\tan^2(30.0^\circ - 50.0^\circ)}{\tan^2(30.0^\circ + 50.0^\circ)} = 0.00412$$

The transmissivity τ follows as

$$\tau = \frac{\tau_\alpha}{2}\left\{\frac{1-r_\perp}{1+r_\perp}\left[\frac{1-r_\perp^2}{1-(r_\perp\tau_\alpha)^2}\right] + \frac{1-r_\parallel}{1+r_\parallel}\left[\frac{1-r_\parallel^2}{1-(r_\parallel\tau_\alpha)^2}\right]\right\}$$

$$\therefore \tau = \frac{0.779}{2} \times \left\{\frac{1-0.121}{1+0.121} \times \left[\frac{1-0.121^2}{1-(0.121 \times 0.779)^2}\right] + \frac{1-0.00412}{1+0.00412} \times \left[\frac{1-0.00412^2}{1-(0.00412 \times 0.779)^2}\right]\right\}$$

$$\therefore \boxed{\tau = 0.690}$$

5. False. The power output is given by

$$\Pi = I_S \times \eta \times F \times A = 1200 \times 0.1 \times 0.550 \times \left(4.0 \times 10^{-4}\right) = 0.0264 \text{ W}$$

$$\therefore \boxed{\Pi = 26.4 \text{ mW}}$$

6. True. All we have to do is substitute the pertaining values into the empirical formula:

$$\eta_T = 0.78 - \frac{7.7 \times (50 - 20)}{900} = 0.523 = \boxed{52.3\%}$$

7. True. The average rate of absorption of solar radiation is

$$Q_s = (\tau\alpha)_{av} A_c I_S = 0.86 \times 4 \times 830 = 2860 \text{ W}$$

The average rate of heat loss is, in turn,

$$Q_L = A_c q_L = 4 \times 430 = 1720 \text{ W}$$

The rate of useful energy collection follows as

$$Q_u = Q_S - Q_L = 2860 - 1720 = 1140 \text{ W}$$

Finally, the average hourly efficiency is

$$\eta = \frac{Q_u}{Q_S} = \frac{1140}{2860} = 0.399 = \boxed{39.9\%}$$

8. False. The mass of water transferred over the course of the six-hour period is

$$m = \left(50 \times 10^{-6}\right) \times (6.0 \times 3600) \times 10^6 \text{ g}$$

$$\therefore m = 1.08 \times 10^6 \text{ g}$$

The amount of heat transferred then becomes

$$Q = mc\Delta T = \left(1.08\times 10^6\right)\times 4.18\times 18 = 8.13\times 10^7 \text{ J}$$

The amount of heat directly involved to raise the temperature of water within the tank is

$$Q = 150{,}000\times 4.18\times 14 = 8.78\times 10^6 \text{ J}$$

The efficiency of the thermal system follows as

$$\eta_T = \frac{8.78\times 10^6}{8.13\times 10^7} = 0.108 = \boxed{10.8\%}$$

9. False. The correction factor we aim for is given by

$$f = \frac{1-(1-K)^N}{NK}$$

where N = 5 and

$$K = \frac{A_c F_R U_c}{\dot{m}c_p} = \frac{3\times 5}{0.02\times 4180} = 0.179$$

so that

$$f = \frac{1-(1-0.179)^5}{5\times 0.179} = \boxed{0.701}$$

10. True. We begin by computing the diffusion coefficients:

$$D_N = \left(\frac{k_B T}{q}\right)\mu_n = \frac{\left(1.38\times 10^{-23}\right)\times 298}{1.60\times 10^{-19}}\times\left(980\times 10^{-4}\right) = 2.52\times 10^{-3} \text{ m}^2\text{s}^{-1}$$

$$D_P = \left(\frac{k_B T}{q}\right)\mu_P = \frac{\left(1.38\times 10^{-23}\right)\times 298}{1.60\times 10^{-19}}\times\left(120\times 10^{-4}\right) = 3.08\times 10^{-4} \text{ m}^2\text{s}^{-1}$$

The saturation current density is determined as

$$J_0 = qn_i^2\left(\frac{D_N}{L_N N_A}+\frac{D_P}{L_P N_D}\right)$$

$$\therefore J_0 = \left(1.6\times 10^{-9}\right)\times\left(1.5\times 10^{16}\right)^2\times\left[\frac{2.52\times 10^{-3}}{\left(520\times 10^{-6}\right)\times 10^{23}}+\frac{3.08\times 10^{-4}}{\left(16\times 10^{-6}\right)\times 10^{25}}\right]$$

$$\therefore J_0 = 1.81\times 10^{-9} \text{ A/m}^2$$

Then, the open circuit voltage is found as

$$V_{OC} = \frac{k_B T}{q}\ln\left(\frac{J_{\text{SC}}}{J_o}+1\right) = \underbrace{\frac{\left(1.38\times 10^{-23}\right)\times 298}{1.6\times 10^{-19}}}_{=0.0257\text{ V}}\times\ln\left(\frac{400}{1.81\times 10^{-9}}+1\right) = \boxed{0.671 \text{ V}}$$

11. False. We proceed to normalize V_{OC} with respect to thermal voltage:

$$v_{oc} = \frac{V_{OC}}{k_B T/q} = \frac{0.671}{0.0257} = 26.1$$

We can determine the fill factor with the empirical formula:

$$FF = \frac{v_{oc}-\ln\left(v_{oc}+0.72\right)}{v_{oc}+1} = \frac{26.1-\ln\left(26.1+0.72\right)}{26.1+1} = 0.842$$

Finally, the conversion efficiency is

$$\eta = \frac{FF\times J_{SC}V_{OC}}{I} = \frac{0.842\times 400\times 0.671}{1000} = 0.226 = \boxed{22.6\%}$$

■ Problem 2

1. True. For solar radiation transfer from aperture to receiver, we may write

$$\rho I_s A_a = A_r q_{sr} \quad \text{(I)}$$

where ρ is the effective reflectivity of the concentrator. Subscripts a and r refer to aperture and receiver, respectively. In the equation above, the left-hand side describes radiation incident on the aperture, and the right-hand side is the radiation reaching the receiver. For an overall energy balance of the receiver, we may write

$$Q_r = (\tau\alpha)_r A_r q_{sr} - U_r A_r (T_r - T_a)$$

where $(\tau\alpha)_r$ is the transmittance-absorptance product of the receiver. Note that this is similar to the energy balance equation for a flat plate collector. The efficiency of the collector may be written as

$$\eta_c = \frac{Q_r}{A_a I_s} = \frac{(\tau\alpha)_r A_r q_{sr} - U_r A_r (T_r - T_a)}{A_a I_s}$$

Using (I) and manipulating, we have

$$\eta_c = \frac{\rho(\tau\alpha)_r I_s A_a - U_r A_r (T_r - T_a)}{A_a I_s}$$

$$\therefore \eta_c = \rho(\tau\alpha)_r \left[1 - \frac{U_r (T_r - T_a)}{C\rho(\tau\alpha)_r I_s}\right] \text{ (II)}$$

where $C = A_a/A_r$ is the concentration ratio. Now, the efficiency of a Carnot engine is given by

$$\eta_{\text{carnot}} = 1 - \frac{T_a}{T_r} \text{ (III)}$$

where we have taken the receiver temperature of the solar collector as the Carnot-cycle heat source temperature, and the ambient temperature as the Carnot-cycle heat sink temperature. The overall efficiency of the power system is given by the product

$$\eta_o = \eta_c \eta_{\text{carnot}}$$

where η_c is given by (II) and η_{carnot} is given by (III), that is,

$$\eta_o = \rho(\tau\alpha)_r \left[1 - \frac{U_r (T_r - T_a)}{C\rho(\tau\alpha)_r I_s}\right]\left(1 - \frac{T_a}{T_r}\right) \text{ (IV)}$$

To obtain the temperature at which maximum efficiency is achieved, we differentiate the relationship above with respect to T_r and set the result to zero. Upon doing so, we find that the optimum receiver temperature is

$$T_{r,\text{opt}} = \left[\left(T_a + \frac{C\rho(\tau\alpha)_r I_s}{U_r}\right) T_a\right]^{\frac{1}{2}}$$

Substituting the pertaining variables, we obtain

$$T_{r,\text{opt}} = \left[\left(303 + \frac{3.2 \times 0.89 \times 0.8 \times 825}{2.5}\right) \times 303\right]^{\frac{1}{2}} = 565 \text{ K} = \boxed{292^\circ\text{C}}$$

2. False. Substituting $T_{r,opt}$ = 565 K and other known quantities into equation (IV), we obtain

$$\eta_c = 0.89 \times 0.8 \times \left[1 - \frac{2.5 \times (565 - 303)}{3.2 \times 0.89 \times 0.8 \times 825}\right] \times \left(1 - \frac{303}{565}\right) = 0.215 = \boxed{21.5\%}$$

■ Problem 3.1

1. False. The characteristic curve associated with 500-W/m^2 radiation intensity and the load line for a resistance of 7.5 Ω intercept at I = 27 mA and V = 0.2 V. The voltage for an arrangement of cells in parallel is the same for all cells, hence $V_{25} = V$ = 0.2 V.

2. True. To find the load current, we multiply I = 27 mA by the number of cells:

$$I_{25} = 25 \times 0.027 = \boxed{0.675 \text{ A}}$$

3. True. The power dissipated by the cells is such that

$$\Pi = V_{25}I_{25} = 0.2\times 0.675 = \boxed{0.135\ \text{W}}$$

■ Problem 3.2

1. True. At a radiation level of 1000 W/m^2 and a load resistance of 10 Ω, the chart can be used to read $V = 0.46$ V and $I = 45$ mA. The current for an arrangement of cells in series is the same for all cells, hence $I_{40} = I = 45$ mA.

2. False. To find the load voltage, we multiply $V = 0.46$ V by the number of cells:

$$V_{40} = 40\times 0.46 = \boxed{18.4\ \text{V}}$$

3. False. The power dissipated by the cells is such that

$$\Pi = V_{40}I_{40} = 18.4\times 0.045 = \boxed{0.828\ \text{W}}$$

■ Problem 4

Part 1: The wavelength of radiation corresponding to the band gap of silicon is determined as

$$\lambda_{\text{BG}} = \frac{hc}{B} = \frac{\left(6.63\times 10^{-34}\right)\times\left(3\times 10^{8}\right)}{1.11\times\left(1.6\times 10^{-19}\right)} = 1.13\times 10^{-6}\ \text{m}$$

$$\therefore \boxed{\lambda_{\text{BG}} = 1.13\ \mu\text{m}}$$

or, equivalently, 1130 nm.

Part 2: The photon flux n_{ph} can be established from the intensity of the light, which was given as 4 mW:

$$I = n_{\text{ph}}\left(\frac{hc}{\lambda_i}\right) \rightarrow n_{\text{ph}} = \frac{I\times\lambda_i}{hc}$$

$$\therefore n_{\text{ph}} = \frac{\left(4\times 10^{-3}\right)\times\left(0.549\times 10^{-6}\right)}{\left(6.63\times 10^{-34}\right)\times\left(3\times 10^{8}\right)} = \boxed{1.10\times 10^{16}\ \text{photons/s}}$$

Part 3: The maximum available power from the solar cell is given by the product of photon flux and band-gap energy:

$$\Pi_{\max} = n_{\text{ph}}B = \left(1.10\times 10^{16}\right)\times\left[1.11\times\left(1.6\times 10^{-19}\right)\right] = 0.00195\ \text{W}$$

$$\therefore \boxed{\Pi_{\max} = 1.95\ \text{mW}}$$

The maximum power is 1.95 milliwatts.

Part 4: If the light source has power of 4 mW and the maximum available power is 1.95 mW, the percentage we're looking for becomes

$$\eta = \frac{1.95}{4}\times 100\% = \boxed{48.8\%}$$

Approximately 49% of the incident power can be converted to energy.

■ Problem 5

The total radiation intensity of the spectrum is obtained by integrating the spectrum over the wavelength intervals. Since the graph consists of rectangular shapes, integration is straightforward:

$$I_e = \int_{1\times 10^{-6}}^{2\times 10^{-6}} 0.6\times 10^{9}\,d\lambda + \int_{2\times 10^{-6}}^{3\times 10^{-6}} 0.3\times 10^{9}\,d\lambda$$

$$\therefore I_e = 0.6\times 10^{9}\times\left[(2-1)\times 10^{-6}\right] + 0.3\times 10^{9}\times\left[(3-2)\times 10^{-6}\right] = 600+300$$

$$\therefore \boxed{I_e = 900\ \text{W/m}^2}$$

The spectral photon flux E_λ is given by the differential relation

$$\delta E_\lambda = I_{e\lambda}d\lambda = n_{ph}\left(\frac{hc}{\lambda}\right)d\lambda$$

Solving for n_{ph} and integrating,

$$N_{ph} = \int n_{ph}d\lambda = \int\left(\frac{\lambda}{hc}\right)I_{e\lambda}d\lambda$$

$$\therefore N_{ph} = \left(\frac{1}{hc}\right)\left[\int_{1\times10^{-6}}^{2\times10^{-6}} 0.6\times10^{9}\lambda d\lambda + \int_{2\times10^{-6}}^{3\times10^{-6}} 0.3\times10^{9}\lambda d\lambda\right]$$

$$\therefore N_{ph} = \left(\frac{1}{hc}\right)\left[0.6\times10^{9}\times\frac{\left(2^2-1^2\right)\times10^{-12}}{2} + 0.3\times10^{9}\times\frac{\left(3^2-2^2\right)\times10^{-12}}{2}\right]$$

$$\therefore N_{ph} = \left[\frac{1}{\left(6.63\times10^{-34}\right)\times\left(3\times10^{8}\right)}\right]\times0.00165$$

$$\therefore \boxed{N_{ph} = 8.30\times10^{21}\ \text{m}^{-2}\ \text{s}^{-1}}$$

■ Problem 6

The total solar radiation intensity is given by the area of the spectrum:

$$I = 0.5\times10^{9}\times\left[\left(1.6-0.8\right)\times10^{-6}\right] + 1\times10^{9}\times\left[\left(2.4-1.6\right)\times10^{-6}\right]$$

$$\therefore I = 400+800 = \boxed{1200\ \text{W/m}^2}$$

The total photon flux is given by

$$N_{ph} = \int I_{e\lambda}\left(\frac{\lambda}{hc}\right)d\lambda = \frac{1}{hc}\left[\int_{0.8\times10^{-6}}^{1.6\times10^{-6}} 0.5\times10^{9}\lambda d\lambda + \int_{1.6\times10^{-6}}^{2.4\times10^{-6}} 1.0\times10^{9}\lambda d\lambda\right]$$

$$\therefore N_{ph} = \frac{1}{hc}\left[0.5\times10^{9}\times\left(1.6^2-0.8^2\right)\times10^{-12} + 1.0\times10^{9}\times\left(2.4^2-1.6^2\right)\times10^{-12}\right]$$

$$\therefore N_{ph} = \frac{1}{\left(6.63\times10^{-34}\right)\times\left(3.0\times10^{8}\right)}\times0.00416 = \boxed{2.09\times10^{22}\ \text{m}^{-2}\text{s}^{-1}}$$

Only photons with wavelength less than or equal to the band-gap wavelength will produce electron-hole pairs in the cell. The band-gap wavelength for silicon was calculated in Part 1 of Problem 4 and is such that λ_{bg} = 1.13 μm. The energy of the electrons in question is calculated as

$$Q_{ec} = B\times\int_{\lambda_0}^{\lambda_{bg}} \phi_{ph} d\lambda = \left(\frac{hc}{\lambda_{bg}}\right)\int_{\lambda_0}^{\lambda_{bg}} I_{e\lambda}\left(\frac{\lambda}{hc}\right)d\lambda$$

$$\therefore Q_{ec} = \left(\frac{hc}{\lambda_{bg}}\right)\int_{\lambda_0}^{\lambda_{bg}} I_{e\lambda}\left(\frac{\lambda}{hc}\right)d\lambda = \frac{1}{1.13\times10^{-6}}\times\int_{0.8\times10^{-6}}^{1.13\times10^{-6}}\left(0.5\times10^{9}\right)\lambda d\lambda$$

$$\therefore Q_{ec} = \frac{0.5\times10^{9}}{1.13\times10^{-6}}\times\frac{\left[\left(1.13^2-0.8^2\right)\times10^{-12}\right]}{2} = 141\ \text{W}$$

The energy lost as heat is the difference between the incident energy determined in Part 1 and the output of the cell:

$$Q_{\text{loss}} = 1200-141 = \boxed{1059\ \text{W}}$$

■ Problem 7

The photon flux is given by

$$\phi_{ph,a} = I_s\left(\frac{\lambda}{hc}\right) = 500\times\left[\frac{760\times10^{-9}}{\left(6.63\times10^{-34}\right)\times\left(3\times10^{8}\right)}\right] = \boxed{1.91\times10^{21}\ \text{m}^{-2}\text{s}^{-1}}$$

For a quantum efficiency *EQE* = 0.75, the electron generation rate becomes

$$\psi_{el} = EQE\times\phi_{ph,a} = 0.75\times\left(1.91\times10^{21}\right) = 1.43\times10^{21}$$

The light current follows as

$$I_L = A\times q\times\psi_{el} = \left(8\times10^{-4}\right)\times\left(1.6\times10^{-19}\right)\times\left(1.43\times10^{21}\right)$$

$$\therefore \boxed{I_L = 0.183\ \text{A}}$$

■ Problem 8

The depletion current is given by the product of elementary charge q, optical generation rate G, depletion region width W, and junction area A:

$$I_d = qGWA = \left(1.60\times10^{-19}\right)\times\left(6\times10^{19}\right)\times\left(4\times10^{-4}\right)\times120 = \boxed{0.461\ \text{A}}$$

Then, given the appropriate diffusivities and the carrier lifetime $\tau_n = \tau_p = 2\times10^{-6}$ s, we compute the diffusion lengths:

$$L_n = \sqrt{D_n\tau_n} = \sqrt{35\times\left(2\times10^{-6}\right)} = 8.37\times10^{-3}\ \text{cm}$$

$$L_p = \sqrt{D_p\tau_p} = \sqrt{13\times\left(2\times10^{-6}\right)} = 5.10\times10^{-3}\ \text{cm}$$

The total optically generated current follows as

$$I_L = qAG\left(L_n + W + L_p\right) = \left[\begin{array}{c}\left(1.60\times10^{-19}\right)\times120\times\left(6\times10^{19}\right)\\ \times\left(8.37\times10^{-3} + 4\times10^{-4} + 5.10\times10^{-3}\right)\end{array}\right] = \boxed{16.0\ \text{A}}$$

The short-circuit current I_{SC} is the same as I_L, that is,

$$I_{SC} = I_L = \boxed{16.0\ \text{A}}$$

To find the open-circuit voltage, we first compute current component I_0,

$$I_0 = J_0A = \left(1.25\times10^{-11}\right)\times120 = 1.5\times10^{-9}\ \text{A}$$

so that

$$V_{OC} = \frac{kT}{q}\ln\left(\frac{I_L}{I_0}+1\right) = \frac{\left(1.38\times10^{-23}\right)\times298}{1.6\times10^{-19}}\times\ln\left(\frac{16.0}{1.5\times10^{-9}}+1\right) = \boxed{0.593\ \text{V}}$$

It remains to compute the maximum output power P_{max}:

$$P_{\max} = FF\times I_{SC}V_{OC} = 0.77\times16.0\times0.593 = \boxed{7.31\ \text{W}}$$

■ Problem 9

The band-gap wavelength of silicon has been determined in other problems and equals 1.13 μm. Since the wavelength of the given monochromatic radiation is less than the band gap wavelength and the quantum efficiency is one, all incident photons will produce electron-hole pairs. The photon flux is then

$$n_{ph} = \frac{330}{hc/\lambda} = \frac{330}{\left(6.63\times10^{-34}\right)\times\left(3\times10^{8}\right)/\left(736\times10^{-9}\right)} = 1.22\times10^{21}\ \text{m}^{-2}\text{s}^{-1}$$

The corresponding light current density is

$$J_L = qn_{ph} = \left(1.6\times10^{-19}\right)\times\left(1.22\times10^{21}\right) = 195\ \text{A m}^{-2}$$

The characteristic equation of the ideal solar cell is

$$I = I_L - I_o\left[\exp\left(\frac{qV}{k_BT}\right)-1\right]$$

Here, the short-circuit current – that is, the current observed when $V = 0$ – is such that

$$I_{sc} = I_L = J_LA = 195\times\left(5\times10^{-4}\right) = \boxed{0.0975\ \text{A}}$$

The open-circuit voltage is given by

$$V_{oc} = \frac{k_BT}{q}\ln\left(1+\frac{I_L}{I_o}\right) = \frac{\left(1.38\times10^{-23}\right)\times305}{1.60\times10^{-19}}\times\ln\left(1+\frac{0.0975}{1.4\times10^{-12}}\right) = \boxed{0.657\ \text{V}}$$

The thermal voltage V_T is, in turn,

$$V_T = \frac{k_BT}{q} = \frac{\left(1.38\times10^{-23}\right)\times305}{1.60\times10^{-19}} = 0.0263\ \text{V}$$

so that V_{oc} can be nondimensionalized as

$$v_{oc} = \frac{V_{oc}}{V_T} = \frac{0.657}{0.0263} = 25.0$$

The dimensionless voltage v_m at the maximum power is obtained by solving the transcendental equation

$$v_{oc} - v_m = \ln(1 + v_m)$$
$$\therefore 25.0 - v_m = \ln(1 + v_m)$$

This equation can be solved with MATLAB's *fsolve* function:

```
>> voltage = @(vm) 25 - vm - log(1+vm);
v0 = 5;
fsolve(voltage, v0)

ans =
   21.8702
```

That is, v_m = 21.87. Turning this into a dimensional voltage, we get

$$V_m = 21.87 \times 0.0263 = 0.575 \text{ V}$$

Then, the current at maximum power is determined as

$$I_m = (I_L + I_o) - I_o \exp(v_m) = \left(0.0975 + 1.4 \times 10^{-12}\right) - 1.4 \times 10^{-12} \exp(21.87)$$
$$\therefore I_m = 0.0931 \text{ A}$$

so that the maximum power becomes

$$P_{\max} = V_m I_m = 0.575 \times 0.0931 = 0.0535 \text{ W} = \boxed{53.5 \text{ mW}}$$

The fill factor is

$$FF = \frac{P_{\max}}{V_{oc} I_{sc}} = \frac{53.5 \times 10^{-3}}{0.657 \times 0.0975} = \boxed{0.835}$$

Finally, the energy conversion efficiency η is

$$\eta = \frac{0.0535}{330 \times \left(5 \times 10^{-4}\right)} = 0.324 = \boxed{32.4\%}$$

■ Problem 10

The characteristic equation of a non-ideal solar cell is

$$I = I_L - I_o \left[\exp\left(\frac{qV}{nk_B T}\right) - 1\right]$$

where n = 1.5 is the ideality factor. We have the short-circuit; current I_{SC}:

$$I_{SC} = I_L = 36 \text{ A}$$

The open-circuit voltage is given by

$$V_{OC} = \left(\frac{nk_B T}{q}\right) \ln\left(1 + \frac{I_L}{I_o}\right) = \left[\frac{1.5 \times \left(1.38 \times 10^{-23}\right) \times 298}{1.60 \times 10^{-19}}\right] \times \ln\left(1 + \frac{36}{I_o}\right)$$
$$\therefore V_{OC} = 0.0386 \ln\left(1 + \frac{36}{I_o}\right)$$

But we were given V_{OC} = 0.7 V, that is,

$$0.0386 \times \ln\left(1 + \frac{36}{I_o}\right) = 0.7$$

This logarithmic equation can be solved with the following MATLAB code:

```
>> fun = @(i_0) 0.0386*log(1 + 36/i_0) - 0.7;
x0 = 0.1;
fsolve(fun, x0)
ans =
   4.7918e-07 - 2.3306e-14i
```

Ignoring the tiny imaginary component, the reverse saturation current is found to be

$$\boxed{I_o = 4.79\times10^{-7}\ \text{A}}$$

The characteristic equation that describes the module is

$$I = 36 - 4.79\times10^{-7}\left[\exp\left(\frac{1}{0.0386}\times T\right)-1\right]$$

$$\therefore I = 36 - 4.79\times10^{-7}\left[\exp(25.91T)-1\right]$$

The dimensionless form of V_{oc} is

$$v_{oc} = 25.91\times0.7 = 18.1$$

The dimensionless voltage v_m at the maximum power point is the solution to

$$v_{oc} - v_m = \ln(1+v_m)$$

$$\therefore 18.1 - v_m = \ln(1+v_m)$$

This equation can be solved with the following MATLAB code:

```
>> voltage = @(vm) 18.1 - vm - log(1+vm);
v0 = 5;
fsolve(voltage, v0)
 ans =
    15.3083
```

That is, v_m = 15.3. The voltage for maximum power then becomes

$$V_m = \frac{15.3}{25.91} = 0.591\ \text{V}$$

The current associated with maximum power is

$$I_m = (I_L + I_o) - I_o\exp(v_m) = \left(36 + 4.79\times10^{-7}\right) - 4.79\times10^{-7}\exp(15.3)$$

$$\therefore I_m = 33.9\ \text{A}$$

The maximum power follows as

$$P_{\max} = V_m I_m = 0.591\times33.9 = \boxed{20.0\ \text{W}}$$

The fill factor is determined next,

$$FF = \frac{P_{\max}}{V_{OC}I_{SC}} = \frac{20.0}{0.7\times36} = \boxed{0.794}$$

The energy conversion efficiency under irradiation at 1000 W/m^2 is

$$\eta = \frac{20.0}{1000\times(0.5\times0.5)} = 0.08 = \boxed{8.0\%}$$

■ Problem 11.1

We first compute parameter m:

$$m = \sqrt{\frac{U_c}{kt}} = \sqrt{\frac{7.5}{390\times0.0005}} = 6.20\ \text{m}^{-1}$$

Then, the fin efficiency becomes

$$\eta_f = \frac{\tanh\left[m(\ell-D)/2\right]}{m(\ell-D)/2} = \frac{\tanh\left[6.20\times(0.16-0.02)/2\right]}{6.20\times(0.16-0.02)/2} = \boxed{0.942}$$

The collector efficiency factor is, in turn,

$$F' = \frac{1/U_c}{\ell\left[\dfrac{1}{U_c\left(D+(\ell-D)\eta_f\right)} + \dfrac{1}{h_{c,i}\pi D}\right]}$$

$$\therefore F' = \frac{1/7.5}{0.16\times\left[\dfrac{1}{7.5\times\left(0.02+(0.16-0.02)\times0.942\right)} + \dfrac{1}{400\times\pi\times0.02}\right]} = \boxed{0.908}$$

■ Problem 11.2

Noting that $F' = 0.908$ was determined in the previous part, the dimensionless collector capacitance rate becomes

$$\frac{\dot{m}c_p}{A_c U_c F'} = \frac{0.08 \times 4180}{5 \times 7.5 \times 0.908} = 9.82$$

This can be used to determine the flow factor F'':

$$F'' = \frac{\dot{m}c_p}{A_c U_c F'}\left[1 - \exp\left(-\frac{U_c F' A_c}{\dot{m}c_p}\right)\right] = 9.82 \times \left[1 - \exp\left(-\frac{1}{9.82}\right)\right] = 0.951$$

The heat removal factor F_R follows as

$$F_R = F' \times F'' = 0.908 \times 0.951 = 0.864$$

The amount of useful heat, Q_u, is (note that factor 3.6, or 3600/1000, was included because I_t is given in MJ/m^2, or 1000 kJ/m^2, and the time span considered is one hour, or 3600 sec):

$$Q_u = A_c F_R \left[I_t(\tau\alpha) - U_c(T_i - T_a) \times 3.6\right]$$

$$Q_u = 5 \times 0.864 \times (2900 \times 0.81 - 7.5 \times 10 \times 3.6) = 8980 \text{ kJ} = 8.98 \text{ MJ}$$

Finally, the collector efficiency is

$$\eta_c = \frac{Q_u}{A_c I_t} = \frac{8.98}{5 \times 2.90} = 0.619 = \boxed{61.9\%}$$

■ Problem 12

Neglecting the ground reflected radiation, the total radiation received by the collector is given by

$$I_c = I_{d,c} + I_{b,c} = 0.3 \times I_b \times \cos^2\left(\frac{\beta}{2}\right) + (1 - 0.3) \times I_h \times R_b \;\; \text{(Equation (1))}$$

where $I_{d,c}$ denotes the diffuse component of radiation and $I_{b,c}$ denotes the beam component of radiation; further, inclination β is set at 60° and the tilt factor R_b is given by

$$R_b = \frac{\sin(L-\beta)\sin\delta_a + \cos(L-\beta)\cos\delta_a\cos\omega}{\sin L \sin\delta_a + \cos L\cos\delta_a\cos\omega}$$

Here, L = 40°N is latitude, ω is the hour angle (and equals 15° for each hour away from noon) and δ_a is the declination angle, which, for February 1st (n = 32), equals

$$\delta = -23.44^\circ \times \cos\left[\frac{360}{365} \times (n + 10)\right] = -23.44^\circ \times \cos\left[\frac{360}{365} \times (32 + 10)\right] = -17.58^\circ$$

The fin efficiency is given by

$$\eta_f = \frac{\tanh\left[m(\ell - D)/2\right]}{m(\ell - D)/2}$$

where

$$m = \sqrt{\frac{U_c}{kt}} = \sqrt{\frac{10}{390 \times (0.06 \times 10^{-2})}} = 6.54$$

so that

$$\eta_f = \frac{\tanh\left[6.54 \times (0.18 - 0.01)/2\right]}{6.54 \times (0.18 - 0.01)/2} = 0.908$$

We proceed to compute the collector efficiency factor F:

$$F' = \frac{1/U_c}{\ell\left[\dfrac{1}{U_c\left(D + (\ell - D)\eta_f\right)} + \dfrac{1}{h_{c,i}\pi D}\right]}$$

$$\therefore F' = \frac{1/10}{0.18\times\left[\frac{1}{8.0\times\left(0.01+\left(0.18-0.01\right)\times 0.908\right)}+\frac{1}{1600\times\pi\times 0.01}\right]} = 0.712$$

The flow rate per unit surface area of collector $G = \dot{m}/A_c$ is

$$G = \frac{\dot{m}}{A_c} = \frac{0.03}{1\times 2} = 0.015\ \frac{\text{kg}}{\text{s}\cdot\text{m}^2}$$

Then, we obtain the heat-removal factor F_R:

$$F_R = \frac{Gc_p}{U_c}\left[1-\exp\left(-\frac{U_c F}{Gc_p}\right)\right] = \frac{0.015\times 4184}{10}\times\left[1-\exp\left(-\frac{10\times 0.712}{0.015\times 4184}\right)\right] = 0.673$$

$$q_u = A_c F_R\left[\tau_s\alpha_s I_c - U_c\left(T_{f,in}-T_a\right)\right]$$

$$\therefore q_u = 2\times 0.673\times\left[0.92\times 0.82\times I_c - 10\times\left(330-T_a\right)\right]\ \text{(Equation (2))}$$

The efficiency of the collector in each hour is given by the ratio

$$\eta_{c,\text{hour}} = \frac{q_u}{A_c I_c}\ \text{(Equation (3))}$$

where A_c = 1 × 2 = 2 m^2. The calculations are tabulated below. You can download the Excel spreadsheet in this Google Drive folder.

Time (h)	I_h (W/m²)	ω (deg)	R_b	$I_{d,c}$ (W/m²)	$I_{b,c}$ (W/m²)	I_c (W/m²)	T_{amb} (K)	q_u (W)	η_c
						Eq. (1)		Eq. (2)	Eq. (3)
7 to 8	14	60	3.224	3.15	31.59	34.74	274	0.00	0.000
8 to 9	82	45	2.286	18.45	131.24	149.69	280	0.00	0.000
9 to 10	176	30	2.006	39.60	247.11	286.71	283	0.00	0.000
10 to 11	340	15	1.895	76.50	450.91	527.41	286	212.51	0.201
11 to 12	480	0	1.864	108.00	626.16	734.16	291	489.75	0.334
12 to 13	464	15	1.895	104.40	615.36	719.76	291	475.12	0.330
13 to 14	401	30	2.006	90.23	563.02	653.24	288	367.20	0.281
14 to 15	299	45	2.286	67.28	478.53	545.81	282	177.35	0.162
15 to 16	176	60	3.224	39.60	397.15	436.75	280	39.69	0.045
16 to 17	39	0	0.000	8.78	0.00	8.78	277	0.00	0.000
					ΣI_c (W/m²)	**4097.05**	Σq_u (W)	**1761.61**	

It remains to calculate the daily average efficiency:

$$\eta_{c,\text{day}} = \frac{\sum q_u}{A_c\sum I_c} = \frac{1761.61}{\left(1\times 2\right)\times 4097.05} = 0.215$$

$$\therefore \boxed{\eta_{c,\text{day}} = 21.5\%}$$

■ Problem 13

The diffuse radiation correction factor, γ, is estimated as

$$\gamma = 1-\left(1-\frac{1}{C}\right)\frac{G_D}{G_t} = 1-\left(1-\frac{1}{2}\right)\times 0.15 = 0.925$$

Entering C = 2.0 into Figure 1, we see that the average number of reflections for a full CPC is 0.65; accordingly, the effective transmissivity τ_{CPC} becomes

$$\tau_{CPC} = \rho^n = 0.93^{0.65} = 0.954$$

The absorber radiation is

$$S = G_t\tau_{\text{cover}}\tau_{CPC}\alpha_t\gamma = 920\times 0.90\times 0.954\times 0.86\times 0.925 = 628\ \text{W/m}^2$$

Next, we compute the heat removal factor:

$$F_R = \frac{\dot{m}c_p}{A_c U_c}\left[1-\exp\left(-\frac{U_c F' A_c}{\dot{m}c_p}\right)\right] = \frac{0.02\times 4180}{5\times 3.0}\times\left[1-\exp\left(-\frac{3.0\times 0.93\times 5}{0.02\times 4180}\right)\right] = 0.857$$

The receiver area is

$$A_r = \frac{A_a}{C} = \frac{5}{2.0} = 2.5\ \text{m}^2$$

The useful energy gain is

$$Q_u = F_R\left[SA_a - A_r U_c\left(T_i - T_a\right)\right] = 0.857\times\begin{bmatrix} 628\times 5.0 \\ -2.5\times 3.0\times\left(75-20\right)\end{bmatrix} = 2340\ \text{W}$$

Lastly, the collector efficiency is

$$\eta = \frac{Q_u}{A_a G_t} = \frac{2340}{5\times 920} = 0.509 = \boxed{50.9\%}$$

■ Problem 14

The first step is to determine the duct heat transfer coefficient, h_c. The average velocity $\bar{V}$ is the volume flow rate divided by flow area:

$$\bar{V} = \frac{0.76\times 2\times 4}{2\times 0.016} = 190\ \text{m/min} = 3.17\ \text{m/s}$$

The hydraulic diameter D_H is the ratio of cross-sectional area to perimeter:

$$D_H = \frac{4\times 2\times 0.016}{2+2+0.016+0.016} = 0.0317\ \text{m}$$

The Reynolds number follows as

$$\text{Re} = \frac{\rho\bar{V}D_H}{\mu} = \frac{1.05\times 3.17\times 0.0317}{1.8\times 10^{-5}} = 5860$$

The Nusselt number for a smooth air-heating collector is expressed as

$$\text{Nu}_{sm} = \frac{0.0192\,\text{Re}^{3/4}\,\text{Pr}}{1+1.22\,\text{Re}^{-1/8}\left(\text{Pr}-2\right)}$$

The Prandtl number for air is 0.72 and the Reynolds number was calculated above; accordingly,

$$\text{Nu}_{sm} = \frac{0.0192\times 5860^{3/4}\times 0.72}{1+1.22\times 5860^{-1/8}\times\left(0.72-2\right)} = 19.6$$

Then, h_c can be determined from the definition of Nu:

$$\text{Nu} = \frac{h_c\times D_H}{k}\ \rightarrow\ h_c = \frac{\text{Nu}\times k}{D_H}$$

$$\therefore h_c = \frac{\text{Nu}\times\mu c_p}{\text{Pr}\times D_H}$$

In the final passage, we've replaced the thermal conductivity k with the definition of Prandtl number, $Pr = \mu c_p/k$. It follows that

$$h_c = \frac{19.6\times\left(1.8\times 10^{-5}\right)\times 1.0}{0.72\times 0.0317} = 0.0155\ \text{kW/m}^2\cdot\text{K}$$

$$\therefore h_c = 55.8\ \text{kJ/m}^2\cdot\text{h}\cdot^{\circ}\text{C}$$

The latter unit conversion is achieved by multiplying the result by 3600. We can proceed to determine the efficiency factor F:

$$F = \frac{h_c}{h_c + U_c} = \frac{55.8}{55.8+21} = \boxed{0.727}$$

To compute the heat-removal factor, we write

$$F_R = \frac{\dot{m}_a c_{p,a}}{U_c A_c}\left[1-\exp\left(-\frac{F U_c A_c}{\dot{m}_a c_{p,a}}\right)\right]$$

The only missing quantity is the air flow rate $\dot{m}_a$,

$$\frac{\dot{m}_a}{A_c} = \rho\left(q/A_c\right) = 1.05\times\frac{0.76}{60} = 0.0133\ \frac{\text{kg}}{\text{s}\cdot\text{m}^2}$$

Finally,

$$F_R = \frac{0.0133 \times 1.0}{(21/3600)} \times \left[1 - \exp\left(-\frac{0.727 \times (21/3600)}{0.0133 \times 1.0}\right)\right] = \boxed{0.622}$$

■ Problem 15

In the commercially available silicon solar cells, when a photon from the solar spectrum is absorbed, the exciton binding energy is so small that the electron and hole can separate, and thus a current can be produced. However, in an organic material, the absorption of a photon creates a strongly bound exciton, which is basically a neutral electron-hole pair. This means that absorbed photons produce a neutral excitation, not free carriers, and thus a dissociation interface is required. Within most organic solar cells, for example in the case of the well-known P3HT/PCBM heterojunction, the necessary steps required to convert sunlight into electricity are:

1. Absorption of sunlight and formation of the exciton;
2. Exciton diffusion to the donor-acceptor interface;
3. Exciton dissociation into free charges (electrons and holes);
4. Charge transport to and collection at the electrodes;

Thus, option B is correct.

■ Problem 16

1. False. On the contrary, a bulk heterojunction is generally constructed with a conjugated donor as the donor and a fullerene derivative as the acceptor. Fullerene derivatives such as $PC_{60}BM$ (phenyl-C61-butyric-acid-methyl-ester) and $PC_{70}BM$ (phenyl-C71-butyric-acid-methyl-ester) are commonly used in BHJ schemes. Of note, the numerous disadvantages of fullerene acceptors, such as limited absorption of visible light, problems with functionalization, degradation and high costs have motivated the search for non-fullerene (NF) acceptors.

Reference: Zdyb (2023).

2. True. The final part of the statement is taken verbatim from section 3 of Duan *et al.* (2019). The nearly-30% efficiency improvement achieved by Duan's group showcases the promising gains that even a simple, laboratory-scale ternary blend scheme can achieve.

Reference: Duan *et al.* (2019) (verbatim from section 3).

3. False. According to Zdyb (2023), fluorine tin oxide works best as a glass substrate due to its constant, low resistivity up to 600°C. Maintenance of good conductivity at high temperatures is crucial for DSSCs, since the sintering of TiO_2 deposited on the glass electrode requires annealing at 450°C. Accordingly, FTO outperforms indium tin oxide (ITO), whose resistance is more sensitive to increases in temperature. Another important advantage of FTO is its lower cost when compared to ITO.

Reference: Zdyb (2023).

4. True. Xu *et al.* (2015) demonstrated, on theoretical grounds, that any nanostructured solar cell (e.g., composed from wires, cones, pyramids, etc.) has efficiency identical to a planar solar cell with concentrating optics; any additional improvement arises solely from improvements in open-circuit voltage.

References: Krogstrup (2013); Xu *et al.* (2015).

5. False. Wang *et al.* (2019) reported an all-inorganic perovskite solar cell with power conversion efficiency exceeding 18%.

References: Eperon *et al.* (2015); Wang *et al.* (2019).

6. True. Indeed, Lau *et al.* (2018) noted that 85% of the cell's initial efficiency had been retained over 2 months with encapsulation.

Reference: Lau *et al.* (2018).

7. False. Of course, the dependence of a variable on its past history is called hysteresis. Perovskite solar cells exhibit hysteretic current-voltage relationships when subjected to forward and reverse bias sweeps. As the cause of this phenomenon remains widely debated, researchers meanwhile have tried to make their results more consistent by correcting current-voltage data with a so-called "hysteresis index," which, as noted by Habisreutinger (2018), is both scientifically and practically meaningless.

Reference: Habisreutinger (2018).

8. True. As noted by Kothandaraman *et al.* (2020), the PCE loss rate with area of perovskite solar cells is substantially greater than that of other photovoltaic technologies, which makes their upscaling particularly problematic.

Reference: Kothandaraman *et al.* (2020).

9. True. CdS dots can be applied as sensitizers in the ZnO wide-bandgap electron transport layer of quantum dot-sensitized solar cells. The CdS dots can be doped with transition metal ions, leading to effects such as suppression of recombination and enhancement of electron conduction. For example, Ganguly and Nath (2020) reported that doping of CdS quantum dots with Mn^{2+} increased power conversion efficiency from 1% to 2.09%.

Reference: Ganguly and Nath (2020).

10. False. Yang *et al.* (2015) actually worked with a type-I cell. It is true, however, that at the time Yang's cell ranked as one of the most efficient core-shell QD cells ever certified.

Reference: Yang *et al.* (2015).

11. False. Not only was the cell described by Pan *et al.* (2014) built from nontoxic materials, it also had an efficiency value (~7%) comparable to that of QD cells constructed with toxic metal-based materials. Pan's results offer a wide range of possibilities for future research on green quantum dot photovoltaics.

Reference: Pan *et al.* (2014).

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