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## Montogue

QUIZ GT504 Reviewed Solutions to Huang's Pavement Analysis and Design, $2^{\text {nd }}$ Ed. Lucas Montogue



## Chapter 2: Stresses and Strains in Flexible Pavements <br> PROBLEM 2.4

A 10,000-lb wheel load exerting contact pressure of 80 psi is applied on an elastic two-layer system, as shown in Figure P2.4. Layer 1 has an elastic modulus of 200,000 psi and thickness 8 in. Layer 2 has elastic modulus of 10,000 psi. Both layers are incompressible, with Poisson ratio equal to 0.5 . Assuming that the loaded area is a single circle, determine the maximum surface deflection, interface deflection, and interface stress.


## PROBLEM ᄅ. 6

Figure P2.6 shows a pavement structure composed of the following three layers: 5.75 in . HMA with elastic modulus 400,000 psi, 23 in . granular base with elastic modulus 20,000 psi, and a subgrade with elastic modulus 10,000 psi. All layers are assumed to have a Poisson ratio of 0.5 . Calculate the maximum horizontal tensile strain at the bottom of HMA and the maximum vertical compressive strain on the top of subgrade under a $40,000-\mathrm{lb}$ wheel load and 150 -psi contact pressure, assuming that the contact area is a circle.


## PROBLEM 2.7

In Problem 2-6, if the base and subgrade are combined as one layer, as shown in Figure P2.7a, what should be the equivalent elastic modulus of this combined layer so that the same tensile strain at the bottom of HMA can be obtained? If the HMA and base are combined as one layer with the same total thickness of 28.75 in., as shown in Figure $2.7 b$, what should be the equivalent elastic modulus of this combined layer so that the same compressive strain on the top of subgrade can be obtained?


## Chapter 6: Traffic Loading and Volume

 PROBLEM 6. 1A set of dual tires is spaced at 34 in . center to center and carries a total load of $45,000 \mathrm{lb}$ with a tire pressure of 100 psi . Assuming the pavement to be a homogeneous half-space, determine the equivalent single-wheel load (ESWL) for a pavement of 25 in . using
(a) The Boyd and Foster method;
(b) The Foster and Ahlvin method;
(c) Huang's chart based on equal contact radius.

## PROBLEM 6.2

A full-depth asphalt pavement is loaded by a set of dual wheels, each weighing 8000 lb and spaced at 20 in . on centers. The hot mix asphalt has a thickness of 10 in . and an elastic modulus of $250,000 \mathrm{psi}$; the subgrade has an elastic modulus of 10,000 psi. Both layers are incompressible with a Poisson ratio of 0.5 . If the dual wheels and the equivalent single wheel have the same contact radius of 6 in ., determine the equivalent single-wheel load (ESWL) based on (a) equal interface deflection and $(b)$ equal tensile strain at the bottom of asphalt layer.

## PROBLEM 6.3

A pavement is subjected to the single-axle loads shown in Table P6.3. Determine the ESAL for a design period of 20 years using (a) Al's equivalent axle load factors and $(b)$ the equivalent axle load factors from equation 6.23.

| Axle <br> load (kip) | Number <br> per day | Axle <br> load (kip) | Number <br> per day | Axle <br> load (kip) | Number <br> per day |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 12 | 200.0 | 20 | 47.2 | 28 | 2.9 |
| 14 | 117.4 | 22 | 21.4 | 30 | 1.2 |
| 16 | 84.5 | 24 | 12.9 | 32 | 0.7 |
| 18 | 61.4 | 26 | 6.1 | 34 | 0.3 |

## PROBLEM 6.6

Estimate the equivalent 18-kip single-axle load applications (ESAL) for a four-lane pavement (two lanes in each direction) of a rural interstate highway with a truck count of 1000 per day (including 2-axle, 4 -tire panel, and pickup trucks), an annual growth rate of $5 \%$, and a design life of 20 years.

## PROBLEM 6.7

Table P6.7 is abstracted from a W-4 table on tractor semitrailer combinations for a loadometer station from July 16 to August 8 . It is assumed that the traffic during the recorded period represents the average over the entire year. If the pavement to be constructed has a structural number SN of 5, estimate the ESAL of the tractor semitrailer combinations during the first year in two directions over all lanes based on a $p_{t}$ (terminal serviceability) of 2.5 .

| TABLE P6.7 |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Axle loads <br> (lb) | 3-axle | 4-axle | 5-axle | Tractor semitrailer combinations probable no. |
| Single axle under 3000 | - | - | - | - |
| 3000-6999 | 256 | 227 | 60 | 3188 |
| 7000-7999 | 148 | 243 | 85 | 2843 |
| 8000-11,999 | 345 | 939 | 363 | 9942 |
| 12,000-15,999 | 174 | 288 | 54 | 3111 |
| 16,000-17,999 | 67 | 225 | 12 | 1899 |
| 18,000-19,000 | 22 | 141 | 7 | 1078 |
| 19,001-19,999 | 8 | 54 | 5 | 423 |
| 20,000-21,999 | 5 | 80 | 9 | 598 |
| 22,000-23,999 | 1 | 29 |  | 144 |
| 24,000-25,999 | - | 1 | - | 6 |
| 26,000-29,999 | - | 1 | - | 6 |
| Total single axles weighed | 1026 | 2228 | 595 |  |
| Total single axles counted | 5541 | 14,526 | 3171 | 23,238 |
| Tandem axle under 6000 | - | 1 | - | 7 |
| 6000-11,999 | - | 237 | 173 | 2631 |
| 12,000-17,999 | - | 189 | 209 | 2541 |
| 18,000-23,999 | - | 218 | 150 | 2362 |
| 24,000-29,999 | - | 270 | 214 | 3103 |
| 30,000-31,500 | - | 62 | 52 | 703 |
| 31,501-31,999 | -- | 11 | 11 | 141 |
| 32,000-33,999 | - | 36 | 43 | 503 |
| 34,000-35,999 | - | 26 | 35 | 388 |
| 36,000-37,999 | - | 16 | 28 | 280 |
| 38,000-39,999 | - | 12 | 27 | 247 |
| 40,000-41,999 | - | 10 | 19 | 183 |
| 42,000-43,999 | - | 14 | 11 | 160 |
| 44,000-45,999 | - | 2 | 6 | 51 |
| 46,000-49,999 | - | 5 | 5 | 64 |
| 50,000-53,999 | - | 1 | - | 7 |
| Total tandem axles weighed |  | 1110 | 983 | - |
| Total tandem axles counted |  | 7263 | 6135 | 13,398 |
| Total vehicles counted | 1847 | 7263 | 3087 | 12,197 |

## PROBLEM 6.8

Same as Problem 6-7 but for a rigid pavement with a concrete thickness of 9 in.

## PROBLEM 6.9

Based on the axles weighted and the axles and vehicles counted, as shown in Table P6.7, determine the truck factor of all tractor semitrailer combinations for a flexible pavement with $S N=5$ and $p_{t}=2.5$.

## PROBLEM 6.10

Same as Problem 6-9 but for a rigid pavement with $D=9$.

## Chapter 7: Material Characterization

PROBLEM 7.5
An asphalt mixture has an asphalt content of $7 \%$ and a bulk specific gravity of 2.24 . The recovered asphalt has a specific gravity is 1.02 , a ring and ball softening point of $120^{\circ} \mathrm{F}$, and a penetration of 50 at $77^{\circ} \mathrm{F}$. The specific gravity of the aggregate is 2.61 . Determine the stiffness modulus of the mixture at a temperature of $74^{\circ} \mathrm{F}$ and a loading time of 0.02 s by the Shell nomographs shown in Figures 7.19 and 7.20. Check the result by equation 7.25 .

## PROBLEM 7.6

The asphalt mixture is the same as in Problem 7.5. The mineral aggregate has $5 \%$ passing through a No. 200 sieve, and the original asphalt has a penetration of 75 at $77^{\circ} \mathrm{F}$. Determine the dynamic modulus at a temperature of $74^{\circ} \mathrm{F}$ and a loading frequency of 8 Hz by the Al equations.

## PROBLEM 7.7

The asphalt mixture in Problem 7.5 is subjected to an initial tensile strain of 0.00015 . Determine the number of repetitions to failure by the nomograph shown in Figure 7.26, using constant stress and constant strain tests, respectively, and check the results by using equations 7.33 and 7.35 .

## PROBLEM 7.8

The asphalt mixture in Problem 7-6 is subjected to an initial strain of 0.00015 . Determine the number of repetitions to failure by equations (7.36) and (7.37).

## PROBLEM 7.9

The results of incremental static test on a HMA specimen are shown in Table P7.9. If the creep strain at a time duration of 0.03 s is $3.045 \times 10^{-5}$, determine the permanent deformation parameters $\alpha$ and $\mu$.

| Load <br> duration (s) | 0.1 | 1 | 10 | 100 | 1000 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Permanent <br> strain $\left(10^{-5}\right)$ | 1.158 | 3.474 | 11.533 | 31.172 | 62.483 |

## - Chapter 11: Flexible Pavement Design PROBLEM 11.3

The fatigue equation for an asphalt pavement is $N_{f}=5 \times 10^{-6}\left(\varepsilon_{t}\right)^{-3}$, where $\varepsilon_{t}$ is the tensile strain. If the pavement is subjected to 5000 thermal cycles with a maximum tensile strain of 0.0005 in ./in. and the coefficient of variation of $N_{f}, C\left[N_{f}\right]$, is 0.8 , estimate the percent area cracked that is due to thermal fatigue cracking.

## PROBLEM 11.4

Figure P11.4 shows an asphalt pavement with the thickness and the resilient moduli of the granular base and subgrade as indicated. The hot mix asphalt has a viscosity of $2.5 \times 10^{6}$ poise at $70^{\circ} \mathrm{F}$, a bitumen volume of $11 \%$, and a fine content of $5 \%$ passing No. 200, and is subjected to a loading frequency of 8 Hz . The air void content is $4 \%$ for the surface course and $7 \%$ for the binder course. The mean monthly air temperature is $68^{\circ} \mathrm{F}$. By the Asphalt Institute procedure, as used in the DAMA program, determine (a) temperatures for surface and binder courses using equation (3.27); (b) dynamic moduli of surface and binder courses using equation (7.27); and (c) the modulus of the untreated base using equations (3.28) and (3.29).


## PROBLEM 11.5

A six-lane (three in each direction) rural interstate highway has a truck count of 1885 per day (including two-axle, four-tire panel and pickup trucks) and an annual growth rate of $4 \%$. The HMA will be laid on an untreated granular base, 8 in. thick, which is placed on a subgrade with a resilient modulus of 10,000 psi. (a) Referring to Tables 6.9 and 6.10, estimate the equivalent single-axle load (ESAL) for a design period of 20 years by the Asphalt Institute method. (b) Determine the HMA thickness required by the Asphalt Institute method. (c) If part of the HMA is replaced by emulsified asphalt mix type I, determine the thickness of HMA and of emulsified asphalt required.

## PROBLEM 11.9

A 12-in. full-depth asphalt pavement is placed on a subgrade with an effective roadbed resilient modulus of 10,000 psi. Assuming a layer coefficient of 0.44 for the hot mix asphalt, a drop in PSI from 4.2 to 2.5 , an overall standard deviation of 0.5 , and a predicted ESAL of $3 \times 10^{7}$, determine the reliability of the design by the AASHTO equation, and check the result by the AASHTO design chart.

## PROBLEM 11.10

An interstate highway pavement composed of a HMA surface course, a cement treated base course, and a sand-gravel subbase is to be designed for an ESAL of $1.2 \times 10^{6}$. The quality of drainage is considered fair because water can be removed from the subbase within a week. However, there is a large amount of precipitation, so more than $25 \%$ of the time the pavement will be exposed to moisture levels approaching saturation. The material properties are as follows: effective roadbed soil resilient modulus $=5500$ psi, resilient modulus of subbase $=15,000$ psi, unconfined compressive strength of cement-treated at 7 days $=500$ psi (see Figure 7.15 c for correlation), and resilient modulus of $\mathrm{HMA}=4.3 \times 10^{5} \mathrm{psi}$. Assuming a minimum thickness for HMA, determine the thickness of the surface, base, and subbase courses required.

## Chapter 12: Rigid Pavement Design

PROBLEM 12.1
A 10-in. concrete pavement without concrete shoulders is placed on an untreated subbase having the $k$ value 150 pci. Estimate the allowable corner deflection by the PCA erosion criterion if the pavement is subjected to 2 million applications of a given axle load.

## PROBLEM12.2

Same as Problem 12-1, but with concrete shoulders. Estimate the allowable corner deflection.

## PROBLEM 12.3

Determine the thickness of a concrete pavement for a two-lane highway by the PCA method. The pavement has doweled joints and no concrete shoulders. The modulus of subgrade reaction is 200 pci and the concrete modulus of rupture is 650 psi. Assume a load safety factor of 1.1 and a design period of 20 years. The average daily traffic during the design period is 2500 , of which $35 \%$ are trucks. Truck weight distribution data for single $(S)$ and tandem ( $T$ ) loads are tabulated in Table P12.3.

| Axle loads <br> (kip) | No. axles per <br> 1000 trucks | Axle loads <br> (kip) | No. axles per <br> 1000 trucks |
| :---: | :---: | :---: | :---: |
| $16 S$ | 130.9 | $24 T$ | 80.2 |
| $18 S$ | 110.8 | $28 T$ | 34.4 |
| $20 S$ | 65.4 | $32 T$ | 24.0 |
| $22 S$ | 15.6 | $36 T$ | 17.2 |
| $24 S$ | 2.3 | $40 T$ | 16.8 |
| $26 S$ | 1.9 | $44 T$ | 10.5 |
| $28 S$ | 0.9 | $48 T$ | 9.6 |

## PROBLEM 12.4

A concrete pavement has doweled joints and tied concrete shoulders. Given a concrete modulus of rupture of 650 psi, a modulus of subgrade reaction of 150 pci, a load safety factor of 1.2 , a design period of 20 years, an average daily truck traffic of 3460 (excluding all two-axle, fourtire trucks) on the design lane during the design life, and an axle load distribution shown by category 3 in Table 12.13. Determine the slab thickness by the PCA method using the procedure similar to the worksheet shown in Figure 12.15. Check the result by the simplified procedure using Table 12.15.

## PROBLEM 12.5

Same as Problem 12-4, but with no concrete shoulders. Determine the slab thickness by the regular PCA procedure, and check the result with the simplified procedure.

## PROBLEM 12.7

Figure P12.7 shows a concrete pavement with the thicknesses and elastic moduli as indicated. Assume a modulus of rupture of 650 psi a load transfer coefficient of 3.2, a serviceability loss from 4.5 to 2.0 , and poor drainage with 5\% of time near saturation. Determine the performance traffic by equation (12.21) without the reliability term and check the result by the AASHTO design chart.


## PROBLEM 12.8

A $8.5-\mathrm{in}$. $(216-\mathrm{mm})$ concrete slab is placed directly on a subgrade. The relationship between the resilient modulus and $k$ value is indicated by equation (12.22). The monthly subgrade resilient moduli from January to December are tabulated below. Determine the effective modulus of subgrade reaction.

| Month | Subgrade <br> resilient <br> modulus | Month | Subgrade <br> resilient <br> modulus |
| :---: | :---: | :---: | :---: |
| January | 15,900 | July | 2340 |
| February | 27,300 | August | 3060 |
| March | 38,700 | September | 3780 |
| April | 50,000 | October | 4500 |
| May | 900 | November | 4500 |
| June | 1620 | December | 4500 |

## PROBLEM 12.9

Same as Problem 12-8, except that the relationship between the resilient modulus and $k$ is indicated by equation (5.7). Assuming that the concrete has an elastic modulus of 4,000,000 psi and a Poisson ratio of 0.15 and the subgrade has a Poisson ratio of 0.45 , determine the effective modulus of subgrade reaction.

## SOLUTIONS

## P.2.4 ■ Solution

We first determine the radius of the loaded area,

$$
\frac{10,000}{\pi a^{2}}=80 \rightarrow a=\sqrt{\frac{10,000}{\pi \times 80}}=6.31 \mathrm{in} .
$$

Now, the vertical surface deflection is given by

$$
w_{0}=\frac{1.5 q a}{E_{2}} F_{2}
$$

where $F_{2}$ is the deflection factor, which is a function of the ratio of Young moduli, $E_{1} / E_{2}$, and the dimension ratio, $h_{1} / a$, as shown in Fig. 2.17. In the case at hand,

$$
\frac{E_{1}}{E_{2}}=\frac{200,000}{10,000}=20 ; \frac{h_{1}}{a}=\frac{8}{6.31}=1.27
$$

so that $F_{2}$ can be read to be $\sim 0.34$. Substituting in the formula for $w_{0}$ brings to

$$
w_{0}=\frac{1.5 \times 80 \times 6.31}{10,000} \times 0.34=0.0257 \mathrm{in} .
$$

The vertical surface deflection is close to 0.026 inches
Next, to compute the vertical interface deflection, we write

$$
w=\frac{q a}{E_{2}} F
$$

Here, $F$ can be determined from Figure 2.19. Since there is no chart for a Young moduli ratio $E_{1} / E_{2}=20$, the best we can do is to interpolate the values of $F$ read from the chart for $E_{1} / E_{2}=25$ and the chart for $E_{1} / E_{2}=10$. First, we enter $h_{1} / a=1.27$ and $r / a=0$ into the chart for $E_{1} / E_{2}=10$ to read $F=0.59$. Then, we enter the same ratio into the chart for $E_{1} / E_{2}=25$ to read $F=0.43$. Interpolating between the two results, we obtain $F=0.483$.

| $E_{1} / E_{2}$ | $F$ |
| :---: | :---: |
| 10 | 0.59 |
| 25 | 0.43 |
| 20 | 0.483 (interpolated) |

We are now in position to compute the interface deflection,

$$
w=\frac{q a}{E_{2}} F=\frac{80 \times 6.31}{10,000} \times 0.483=0.0244 \mathrm{in} .
$$

The interface deflection is close to 0.024 inches.
Next, we compute the vertical interface stress $\sigma_{c}$. The ratio $\sigma_{c} / q$ can be read by entering $a / h_{1}=6.31 / 8=0.789$ and the Young moduli ratio $E_{1} / E_{2}$ into Figure 2.15. There is no curve for $E_{1} / E_{2}=20$, so the best we can do is interpolate between the curves for $E_{1} / E_{2}=10$ and $E_{1} / E_{2}=25$. The results are tabulated below.

| $E_{1} / E_{2}$ | $F$ |
| :---: | :---: |
| 10 | 0.22 |
| 25 | 0.13 |
| 20 | 0.160 (interpolated) |

Accordingly,

$$
\begin{aligned}
\frac{\sigma_{c}}{q} & =0.16 \rightarrow \sigma_{c}=0.16 q \\
\therefore \sigma_{c} & =0.16 \times 80=12.8 \mathrm{psi}
\end{aligned}
$$

## P.2.6 - Solution

First of all, $a$ is given by

$$
a=\sqrt{\frac{40,000}{\pi \times 150}}=9.21 \mathrm{in}
$$

We proceed to determine $k_{1}, k_{2}, A$, and $H$,

$$
\begin{gathered}
k_{1}=\frac{E_{1}}{E_{2}}=\frac{400,000}{20,000}=20 ; k_{2}=\frac{E_{2}}{E_{3}}=\frac{20,000}{10,000}=2 \\
A=\frac{a}{h_{2}}=\frac{9.21}{23}=0.400 ; H=\frac{h_{1}}{h_{2}}=\frac{5.75}{23}=0.25
\end{gathered}
$$

With $k_{1}=20$ and $k_{2}=2$, we refer to Figure 2.31(c) and read ( $R R 1-Z Z 1$ )/2 = 2.0. Using eq. (2.25), the radial strain is calculated as

$$
\varepsilon_{r}=-\frac{q}{E} \underbrace{\left(\frac{R R 1-Z Z 1}{2}\right)}_{=2.0}=\frac{150}{40,0000} \times 2.0=-7.5 \times 10^{-4}
$$

One could make use of Table 2.3 instead. Entering $k_{1}=20, k_{2}=2, H=$ 0.25 , and $A=0.4$, we obtain $(Z Z 1-R R 1)=3.86779$. It follows that

$$
\varepsilon_{r}=-\frac{150}{40,0000} \times\left(\frac{3.86779}{2}\right)=-7.25 \times 10^{-4}
$$

There is little difference relatively to the result obtained graphically.

| 0.04381 | 0.00530 | 0.63215 | 0.00962 | 0.00909 |
| :--- | :--- | :--- | :--- | :--- |
| 0.14282 | 0.02091 | 1.83766 | 0.03781 | 0.03269 |
| 0.37882 | 0.07933 | 3.86779 | 0.14159 | 0.10684 |
| 0.75904 | 0.26278 | 5.50796 | 0.44710 | 0.30477 |
| 0.98743 | 0.61673 | 4.24281 | 0.90115 | 0.66786 |
| 1.00064 | 0.91258 | 1.97494 | 0.93254 | 0.98447 |

Also from Table 2.3, we read $Z Z 2-R R 2=0.14159$, giving

$$
\sigma_{z 2}-\sigma_{r 2}=q(Z Z 2-R R 2)=150 \times 0.14159=21.2 \mathrm{psi}
$$

so that

$$
\sigma_{z 2}-\sigma_{r 2}^{\prime}=\frac{\sigma_{z 2}-\sigma_{r 2}}{k_{2}}=\frac{21.2}{2}=10.6 \mathrm{psi}
$$

The corresponding compressive strain is

$$
\varepsilon_{z}=\frac{\sigma_{z 2}-\sigma_{r 2}^{\prime}}{E_{3}}=\frac{10.6}{10,000}=1.06 \times 10^{-3}
$$

## P.2.7■ Solution

From the previous problem, $\varepsilon_{\mathrm{r}}=-7.25 \times 10^{-4}, \varepsilon_{z}=1.06 \times 10^{-3}$, and $a=$ 9.21 in . The base and subgrade are combined as one layer. We first solve equation (2.17) for strain factor $F_{e}$,

$$
\begin{gathered}
e=\left|\varepsilon_{r}\right|=\frac{q}{E_{1}} F_{e} \rightarrow F_{e}=\frac{\varepsilon_{r} E_{1}}{q} \\
\therefore F_{e}=\frac{\left|\varepsilon_{r}\right| E_{1}}{q}=\frac{\left(7.25 \times 10^{-4}\right) \times 400,000}{150}=1.93
\end{gathered}
$$

Further, $h_{1} / a=5.75 / 9.21=0.624$. From Figure $2.17, E_{1} / E_{2}=20$, so that

$$
E_{2}=\frac{E_{1}}{20}=\frac{400,000}{20}=20,000 \mathrm{psi}
$$

In the second combination of layers, trial-and-error can be used to yield $E_{1}=35,000$ psi. Indeed, when $E_{1}=35,000$ psi, and noting that $\varepsilon_{r}=0.5 \varepsilon_{z}=$ $0.5 \times 0.00106=0.00053$, we have

$$
F_{e}=\frac{0.00053 \times 35,000}{150}=0.124
$$

Further, $h_{1} / a=28.75 / 9.21=3.12$. Entering the two foregoing results into Figure 2.21 , we read $E_{1} / E_{2}=3.5$, giving

$$
E_{1}=3.5 E_{2}=3.5 \times 10,000=35,000 \mathrm{psi}
$$

## P.6.1 ■ Solution

In the Boyd and Foster method, the ESWL is computed with equation (6.1),

$$
\log (\mathrm{ESWL})=\log P_{d}+\frac{0.301 \log (2 z / d)}{\log \left(4 S_{d} / d\right)}
$$

In the present case, the load on one of the dual tires is $P_{d}=45,000 / 2=$ $22,500 \mathrm{lb}$; the pavement thickness is $z=25 \mathrm{in} . ;$ the center-to-center spacing between dual tires is $S_{d}=34 \mathrm{in}$.; and the clearance between dual tires is $d=S_{d}$ - $2 a$, where

$$
a=\sqrt{\frac{P_{d}}{\pi p}}=\sqrt{\frac{(45,000 / 2)}{\pi \times 100}}=8.46 \mathrm{in} .
$$

so that

$$
d=S_{d}-2 a=34-2 \times 8.46=17.1 \mathrm{in} .
$$

Accordingly,

$$
\log (\mathrm{ESWL})=\log (22,500)+\frac{0.301 \log (2 \times 25 / 17.1)}{\log (4 \times 34 / 17.1)}=4.51
$$

Resolving the logarithm,

$$
\begin{gathered}
\log (\mathrm{ESWL})=4.51 \rightarrow E S W L=10^{4.51} \\
\therefore E S W L=10^{4.51}=32,400 \mathrm{lb}
\end{gathered}
$$

| Point No. | Left wheel |  | Right wheel |  | Sum |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $r / a$ | $F^{*}$ | $r / a$ | $F^{*}$ |  |
| 1 | 0 | 0.48 | 4 | 0.22 | 0.70 |
| 2 | 1 | 0.42 | 3 | 0.27 | 0.69 |
| 3 | 2 | 0.35 | 2 | 0.35 | 0.70 |

Noting that $F_{d}=0.70$ (red cell in the table above) and $F_{s}=0.48$ (blue cell), we resort to equation (6.6) to obtain

$$
E S W L=\frac{F_{d}}{F_{s}} P_{d}=\frac{0.70}{0.48} \times 22,500=32,800 \mathrm{lb}
$$

We now turn to the solution by Huang's method. Since the chart in Figure 6.4 is designed for a tire spacing different from 48 in. and a contact radius different from 16 in., we must apply the alternative method recommended in Figure 6.4. We first compute the corrected values

$$
\begin{aligned}
& a^{\prime}=\frac{48}{S_{d}} a=\frac{48}{34} \times 8.46=11.9 \mathrm{in} \\
& h_{1}^{\prime}=\frac{48}{S_{d}} h_{1}=\frac{48}{34} \times 25=35.3 \mathrm{in}
\end{aligned}
$$

Further, $E_{1} / E_{2}=1$. Entering this ratio and $h_{1}^{\prime}$ into Figure 6.4, we read $L_{1}$ $=1.40$ (upper chart) and $L_{2}=1.34$ (lower chart). Substituting into equation (6.10) brings to

$$
L=L_{1}-\left(L_{1}-L_{2}\right) \frac{a^{\prime}-6}{10}=1.40-(1.40-1.34) \times \frac{11.9-6}{10}=1.36 \mathrm{in}
$$

so that, from equation (6.7b),

$$
E S W L=\frac{2 P_{d}}{L}=\frac{2 \times 22,500}{1.36}=33,100 \mathrm{lb}
$$

## P.6.2 $\quad$ Solution

Let us first analyze the problem per the equal interface deflection criterion. The upper layer, which is constituted of hot mix asphalt, has thickness $h_{1}=10 \mathrm{in}$, whence we have the ratio $h_{1} / a=10 / 6=1.67$. Further, $E_{1} / E_{2}$ $=250,000 / 10,000=25$. Refer to the illustration below.


At point 1, which is directly below the central axis of the left wheel, we have $r=0$ and $r / a=0$. Entering this ratio and $h_{1} / a=1.67$ into the chart of Figure 2.19 corresponding to $E_{1} / E_{2}=25$, we read $F=0.35$. Proceeding similarly with other points, we draw up the following table.

| Point No. | Left wheel |  | Right wheel |  | Sum |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $r / a$ | $F^{*}$ | $r / a$ | $F^{*}$ |  |
| 1 | $0 / 6=0$ | 0.35 | $20 / 6=3.33$ | 0.23 | 0.58 |
| 2 | $5 / 6=0.833$ | 0.32 | $15 / 6=2.5$ | 0.29 | 0.61 |
| 3 | $10 / 6=1.67$ | 0.3 | $10 / 6=1.67$ | 0.3 | 0.61 |

*Values read from Figure 2.19 with $h_{1} / a=1.67$ and $E_{1} / E_{2}=25$.
Accordingly, $F_{d}=0.61$ and $F_{s}=0.35$, so that, from equation (6.6),

$$
E S W L=\frac{F_{d}}{F_{s}} P_{d}=\frac{0.61}{0.35} \times 8000=13,900 \mathrm{lb}
$$

Next, we assess the ESWL with the equal tensile strain at the bottom of asphalt layer. The charts in Figure 2.23 are meant for center-to-center spacing between dual tires and contact radii different from those in the system at hand; we must use the extrapolation procedure presented under topic Dual Wheels of section 2.2.1. We first estimate the modified radius $a^{\prime}$ and the modified thickness $h_{1}^{\prime}$,

$$
a^{\prime}=\frac{24}{S_{d}} a=\frac{24}{20} \times 6=7.2 \mathrm{in} . ; h_{1}^{\prime}=\frac{24}{S_{d}} h_{1}=\frac{24}{20} \times 10=12 \mathrm{in}
$$

Then, we enter this modified thickness and the ratio $E_{1} / E_{2}=25$ into Figure 2.23 to read coefficients $C_{1}=1.27$ and $C_{2}=1.38$; interpolation is needed
because there are no curves for $E_{1} / E_{2}=25$. Substituting into equation (2.19) gives

$$
\begin{gathered}
C=C_{1}+0.2\left(a^{\prime}-3\right)\left(C_{2}-C_{1}\right) \\
\therefore C=1.27+0.2 \times(7.2-3) \times(1.38-1.27)=1.36
\end{gathered}
$$

Substituting into equation (6.14) brings to

$$
E S W L=C P_{d}=1.36 \times 8000=10,900 \mathrm{lb}
$$

## P.6.3 ■ Solution

The data for this traffic load analysis are processed in the following table. Columns [1] and [4] are reproduced from the given dataset. Column [2] contains the EALF's read from Table 6.4. Column [3] is obtained by substituting the load value $L_{x}$ into equation (6.23), namely

$$
E A L F=\left(\frac{L_{x}}{18}\right)^{4}
$$

Columns [5] and [6] list the products of the EALF's and the numbers-per-day of column [4]. The sums of the computed values are listed at the end of these two columns.

| $[1]$ | $[2]$ | $[3]$ | $[4]$ | $[5]$ | $[6]$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Axle load <br> (kip) | EALF <br> from <br> Table 6.4 | EALF <br> from <br> Eq. 6.23 | No. <br> per day | $[2] \times[4]$ | $[3] \times[4]$ |
| 12 | 0.189 | 0.198 | 200 | 37.80 | 39.51 |
| 14 | 0.36 | 0.366 | 117.4 | 42.26 | 42.96 |
| 16 | 0.623 | 0.624 | 84.5 | 52.64 | 52.75 |
| 18 | 1 | 1.000 | 61.4 | 61.40 | 61.40 |
| 20 | 1.51 | 1.524 | 47.2 | 71.27 | 71.94 |
| 22 | 2.18 | 2.232 | 21.4 | 46.65 | 47.75 |
| 24 | 3.03 | 3.160 | 12.9 | 39.09 | 40.77 |
| 26 | 4.09 | 4.353 | 6.1 | 24.95 | 26.55 |
| 28 | 5.39 | 5.855 | 2.9 | 15.63 | 16.98 |
| 30 | 6.97 | 7.716 | 1.2 | 8.36 | 9.26 |
| 32 | 8.88 | 9.989 | 0.7 | 6.22 | 6.99 |
| 34 | 11.18 | 12.730 | 0.3 | 3.35 | 3.82 |

Noting that 20 years amount to $20 \times 365$ days, the corresponding EAL's are, for the data extracted from Table 6.4,

$$
E A L=\sum F_{i} n_{i} \times 365 \times 20=409.63 \times 365 \times 20=2.99 \times 10^{6}
$$

while, for the data produced using equation (6.23),

$$
E A L=\sum F_{i} n_{i} \times 365 \times 20=420.69 \times 365 \times 20=3.07 \times 10^{6}
$$

## P.6.6 ■ Solution

The expression to use is equation (6.30), namely

$$
E S A L=(A D T)_{0}(T)\left(T_{f}\right)(G)(D)(L)(365)(Y)
$$

Since the truck count is 1000 per day, $(A D T)_{0}(T)=1000$. For the general truck mix in question, $T_{f}=0.52$ can be read from Table 6.10. For an annual growth rate $r=0.05$ and a design life of 20 years, equation (6.33) gives

$$
(G)(Y)=\frac{(1+r)^{Y}-1}{r}=\frac{(1+0.05)^{20}-1}{0.05}=33.1
$$

For a four-lane pavement, $(D)(L)=0.45$ is read from Table 6.15. Gleaning our data and substituting into the equation for $E S A L$, we find that

$$
E S A L=1000 \times 0.52 \times 33.1 \times 0.45 \times 365=2.83 \times 10^{6}
$$

## P.6.7 $\quad$ Solution

The data are processed below. Column [1] is simply the load-range data from Table P6.7; column [2] is the average load corresponding to each loading range in [1]; column [3] is EALF data read from Table 6.4 and may require interpolation for some of the average loads in [2]; column [4] is No. per day data read from the rightmost column of Table P6.7; column [5] is ESAL data obtained by multiplying [3] and [4].

| Single Axle |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| [1] | [2] | [3] | [4] | [5] |
| Axle load range (lb) | Average load (lb) | EALF <br> from <br> Table 6.4 | No. per day | ESAL |
| < 3000 | 1500 | 0.0001 | 0 | 0.000 |
| 3000-6999 | 5000 | 0.005 | 3188 | 15.940 |
| 7000-7999 | 7500 | 0.02695 | 2843 | 76.619 |
| 8000-11999 | 10000 | 0.0877 | 9942 | 871.913 |
| 12000-15999 | 14000 | 0.360 | 3111 | 1119.960 |
| 16000-17999 | 17000 | 0.796 | 1899 | 1511.604 |
| 18000-19000 | 18500 | 1.120 | 1078 | 1207.360 |
| 19001-19999 | 19500 | 1.375 | 423 | 581.625 |
| 20000-21999 | 21000 | 1.830 | 598 | 1094.340 |
| 22000-23999 | 23000 | 2.580 | 144 | 371.520 |
| 24000-25999 | 25000 | 3.530 | 6 | 21.180 |
| 26000-29999 | 28000 | 5.390 | 6 | 32.340 |
| Tandem Axle |  |  |  |  |
| < 6000 | 3000 | 0 | 7 | 0.000 |
| 6000-11999 | 9000 | 0 | 2631 | 0.000 |
| 12000-17999 | 15000 | 0.036 | 2541 | 91.476 |
| 18000-23999 | 21000 | 0.148 | 2362 | 349.576 |
| 24000-29999 | 27000 | 0.426 | 3103 | 1321.878 |
| 30000-31500 | 30750 | 0.72925 | 703 | 512.663 |
| 31501-31999 | 31750 | 0.831 | 141 | 117.171 |
| 32000-33999 | 33000 | 0.971 | 503 | 488.413 |
| 34000-35999 | 35000 | 1.23 | 388 | 477.240 |
| 36000-37999 | 37000 | 1.53 | 280 | 428.400 |
| 38000-39999 | 39000 | 1.89 | 247 | 466.830 |
| 40000-41999 | 41000 | 2.29 | 183 | 419.070 |
| 42000-43999 | 43000 | 2.76 | 160 | 441.600 |
| 44000-45999 | 45000 | 3.27 | 51 | 166.770 |
| 46000-49999 | 48000 | 4.17 | 64 | 266.880 |
| 50000-53999 | 52000 | 5.63 | 7 | 39.410 |
|  |  |  | Sum = | 12491.778 |

Noting that the period of July 16 to August 8 spans 24 days and that 1 year $=365.25$ days, the ESAL during the first year is calculated to be

$$
E S A L \text { during first year }=\frac{12,491.778 \times 365.25}{24}=190,109
$$

## P.6.8 ■ Solution

The only difference between the present problem and Problem 6.8 is that EALFs are to be read from Table 6.7 instead of Table 6.4. The calculations are tabulated below.

| Single Axle |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| [1] | [2] | [3] | [4] | [5] |
| Axle load range (lb) | Average load (lb) | EALF <br> from <br> Table 6.7 | No. per day | ESAL |
| < 3000 | 1500 | 0.0000 | 0 | 0.000 |
| 3000-6999 | 5000 | 0.006 | 3188 | 19.128 |
| 7000-7999 | 7500 | 0.02650 | 2843 | 75.340 |
| 8000-11999 | 10000 | 0.0820 | 9942 | 815.244 |
| 12000-15999 | 14000 | 0.341 | 3111 | 1060.851 |
| 16000-17999 | 17000 | 0.802 | 1899 | 1522.998 |
| 18000-19000 | 18500 | 1.143 | 1078 | 1231.615 |
| 19001-19999 | 19500 | 1.428 | 423 | 603.833 |
| 20000-21999 | 21000 | 1.955 | 598 | 1169.090 |
| 22000-23999 | 23000 | 2.850 | 144 | 410.400 |
| 24000-25999 | 25000 | 4.015 | 6 | 24.090 |
| 26000-29999 | 28000 | 6.290 | 6 | 37.740 |
| Tandem Axle |  |  |  |  |
| < 6000 | 3000 | 0.0003 | 7 | 0.002 |
| 6000-11999 | 9000 | 0.009 | 2631 | 23.679 |
| 12000-17999 | 15000 | 0.065 | 2541 | 165.165 |
| 18000-23999 | 21000 | 0.257 | 2362 | 607.034 |
| 24000-29999 | 27000 | 0.736 | 3103 | 2283.808 |
| 30000-31500 | 30750 | 1.271 | 703 | 893.513 |
| 31501-31999 | 31750 | 1.446 | 141 | 203.886 |
| 32000-33999 | 33000 | 1.705 | 503 | 857.615 |
| 34000-35999 | 35000 | 2.175 | 388 | 843.900 |
| 36000-37999 | 37000 | 2.73 | 280 | 764.400 |
| 38000-39999 | 39000 | 3.385 | 247 | 836.095 |
| 40000-41999 | 41000 | 4.145 | 183 | 758.535 |
| 42000-43999 | 43000 | 5.015 | 160 | 802.400 |
| 44000-45999 | 45000 | 6.005 | 51 | 306.255 |
| 46000-49999 | 48000 | 7.73 | 64 | 494.720 |
| 50000-53999 | 52000 | 10.6 | 7 | 74.200 |
|  |  |  | Sum = | 16885.535 |

Noting that the period of July 16 to August 8 spans 24 days and that 1 year $=365.25$ days, the ESAL during the first year is calculated to be

$$
E S A L \text { during first year }=\frac{16,885.535 \times 365.25}{24}=256,977
$$

## P.6.9 ■ Solution

Table P6.7 shows that 12,197 vehicles were counted over the course of 24 days. The equivalent total vehicle count is

Equivalent total vehicle count $=12,197 \times \frac{365}{24}=185,496$
Problem 6.7 gave us a total EAL of 190,109; it follows that the truck factor is

$$
\text { Truck factor }=\frac{190,109}{185,496}=1.025
$$

## P.6.10 ■ Solution

In Problem 6.9, the equivalent total vehicle count was calculated to be 185,496 . Problem 6.8 gave us a total EAL equal to 256,977 , so that

$$
\text { Truck factor }=\frac{256,977}{185,496}=1.385
$$

## P.7.5 ■ Solution

For convenience, we first convert the temperature values involved to Celsius: $74^{\circ} \mathrm{F}=23.3^{\circ} \mathrm{C} ; 77^{\circ} \mathrm{F}=25^{\circ} \mathrm{C}$; and $120^{\circ} \mathrm{F}=49^{\circ} \mathrm{C}$. Observing that the mixture has a penetration of 50 at $25^{\circ} \mathrm{C}\left(77^{\circ} \mathrm{F}\right)$ and that the ring and ball softening point corresponds to a temperature of $49^{\circ} \mathrm{C}\left(120^{\circ} \mathrm{F}\right)$, we substitute into (7.18) to determine the temperature susceptibility $A$,

$$
A=\frac{\log (\text { pen at } T)-\log 800}{T-T_{R \& B}}=\frac{\log 50-\log 800}{25-49}=0.0502
$$

Then, substituting $A$ into (7.16) gives the penetration index Pl ,

$$
P I=\frac{20-500 A}{1+50 A}=\frac{20-500 \times 0.0502}{1+50 \times 0.0502}=-1.45
$$

Entering a loading time of 0.02 s , a temperature difference $T_{\text {diff }}=49-$ $23.3=25.7^{\circ} \mathrm{C}$, and $P I=-1.45$ into Figure 2.19 , we read a stiffness modulus $\approx$ $2 \times 10^{7} \mathrm{~N} / \mathrm{m}^{2}$ for the asphalt binder.

Per the problem statement, asphalt content $P_{b}=0.07$, bulk specific gravity $G_{m}=2.24$, and aggregate specific gravity $G_{g}=2.61$. Substituting into (7.20) gives

$$
V_{g}=\frac{100\left(1-P_{b}\right) G_{m}}{G_{g}}=\frac{100 \times(1-0.07) \times 2.24}{2.61}=79.82 \%
$$

In turn, the percent volume of bitumen follows from equation (7.21),

$$
V_{b}=\frac{100 P_{b} G_{m}}{G_{b}}=\frac{100 \times 0.07 \times 2.24}{1.02}=15.37 \%
$$

Subtracting the two percent volume values above from 100, we obtain the percent volume of air void,

$$
V_{a}=100-V_{g}-V_{b}=100-79.82-15.37=4.81 \%
$$

Entering the binder stiffness modulus $2 \times 10^{7} \mathrm{~N} / \mathrm{m}^{2}$, the percent volume of binder $V_{b}=15.37 \%$, and the percent volume of aggregate $V_{a}=4.81 \%$ into Figure 7.20, the stiffness modulus of the asphalt mixture is found to be $\approx 2 \times 10^{9} \mathrm{~N} / \mathrm{m}^{2}$.

Let's see how this stiffness modulus compares to the result given by equation (7.25). We first substitute the pertaining quantities into equations (7.24a) through (7.24d), giving

$$
\begin{aligned}
& \beta_{1}=10.82-\frac{1.342\left(100-V_{g}\right)}{V_{g}+V_{b}}=10.82-\frac{1.342 \times(100-79.82)}{79.82+15.37}=10.54 \\
& \beta_{2}=8.0+0.00568 V_{g}+0.0002135 V_{g}^{2}=8.0+0.00568 \times 79.82+0.0002135 \times 79.82^{2} \\
& \therefore \beta_{2}=9.82 \\
& \beta_{3}=0.6 \log \left(\frac{1.37 V_{b}^{2}-1}{1.33 V_{b}-1}\right)=0.6 \times \log \left(\frac{1.37 \times 15.37^{2}-1}{1.33 \times 15.37-1}\right)=0.732 \\
& \beta_{4}=0.7582\left(\beta_{1}-\beta_{2}\right)=0.7582 \times(10.54-9.82)=0.546
\end{aligned}
$$

Now, since the stiffness modulus of the bitumen $S_{b}=2 \times 10^{7} \mathrm{~N} / \mathrm{m}^{2}$ falls into the interval $\left[5 \times 10^{6}, 10^{9}\right]$, the stiffness modulus $S_{m}$ of the mixture can be estimated with equation (7.25a),

$$
\begin{gathered}
\log S_{m}=\frac{\beta_{4}+\beta_{3}}{2}\left(\log S_{b}-8\right)+\frac{\beta_{4}-\beta_{3}}{2}\left|\log S_{b}-8\right|+\beta_{2} \\
\therefore \log S_{m}=\frac{0.546+0.732}{2}\left[\log \left(2 \times 10^{7}\right)-8\right]+\frac{0.546-0.732}{2}\left|\log \left(2 \times 10^{7}\right)-8\right|+9.82
\end{gathered}
$$

$$
\therefore \log S_{m}=9.31
$$

$$
\therefore S_{m}=10^{9.31}=2.04 \times 10^{9} \mathrm{~N} / \mathrm{m}^{2}
$$

The result obtained from equation (7.25) deviates very little from our graphical estimate of $2 \times 10^{9} \mathrm{~N} / \mathrm{m}^{2}$.

## P.7.6 ■ Solution

If the penetration at $77^{\circ} \mathrm{F}$ is equal to 75 , we can estimate the viscosity from equation (7.28),

$$
\lambda=29,508.2\left(P_{77^{\circ} \mathrm{F}}\right)^{-2.1939}=29,508.2 \times 75^{-2.1939}=2.27 \times 10^{6} \text { poise }
$$

Let us compute the $\beta_{i}$ coefficients, starting with $\beta_{5}$; the load frequency is $f=8 \mathrm{~Hz}$, the volume of bitumen is $V_{b}=15.37 \%$, the percent passing through a No. 200 sieve is $P_{200}=5 \%$, the air void volume is $V_{a}=4.81 \%$, and the operation temperature is $74^{\circ} \mathrm{F}$, giving

$$
\begin{gathered}
\beta_{5}=1.3+0.49825 \log f=1.3+0.49825 \log 8=1.75 \\
\beta_{4}=0.483 V_{b}=0.483 \times 15.37=7.42 \\
\beta_{3}=0.553833+0.028829\left(P_{200} f^{-0.1703}\right)-0.03476 V_{v} \\
+0.070377 \lambda+0.931757 f^{-0.02774}
\end{gathered}
$$

$$
\begin{aligned}
\therefore \beta_{3}= & 0.553833+0.028829 \times\left(5 \times 8^{-0.1703}\right)-0.03476 \times 4.81 \\
& +0.070377 \times 2.27+0.931757 \times 8^{-0.02774}=1.53
\end{aligned}
$$

$$
\beta_{2}=\beta_{4}^{0.5} T^{\beta_{5}}=7.42^{0.5} \times 74^{1.75}=5090
$$

$\beta_{1}=\beta_{3}+5 \times 10^{-5} \beta_{2}-0.00189 \beta_{2} f^{-1.1}=1.53+\left(5 \times 10^{-6}\right) \times 5090-0.00189 \times 5090 \times 8^{-1.1}=0.579$
Lastly, substituting $\beta_{1}$ into equation (7.27a) yields the dynamic modulus $\left|E^{*}\right|$,

$$
\left|E^{*}\right|=100,000 \times 10^{\beta_{1}}=100,000 \times 10^{0.579}=379,000 \mathrm{psi}
$$

## P.7.7 ■ Solution

To establish the number of cycles to failure, refer to Figure 7.26. Enter a penetration index $P I=-1.45$, a volume of bitumen $V_{b}=15.37 \%$, a stiffness modulus $S_{m}=2 \times 10^{9} \mathrm{~N} / \mathrm{m}^{2}=2.9 \times 10^{5} \mathrm{psi}$, and an initial tensile strain $\varepsilon_{r}=$ 0.00015 ; in doing so, we find a number of cycles to failure $N_{f} \approx 8.7 \times 10^{5}$ for constant stress test and $N_{f} \approx 9 \times 10^{7}$ for constant strain test. The number of repetitions to failure for constant stress can also be estimated from equation (7.33),

$$
\begin{aligned}
& N_{f}=\left[0.0252 P I-0.00126 P I\left(V_{b}\right)+0.00673 V_{b}-0.0167\right]^{5} \varepsilon_{t}^{-5} S_{m}^{-1.4} \\
\therefore & N_{f}=[0.0252 \times(-1.45)-0.00126 \times(-1.45) \times 15.37+0.00673 \times 15.37-0.0167]^{5} \times 0.00015^{-5} \times\left(2.9 \times 10^{5}\right)^{-1.4} \\
\therefore & N_{f}=8.72 \times 10^{5}
\end{aligned}
$$

which agrees fairly well with our graphical estimate. Likewise, the number of repetitions to failure for constant strain can be estimated from equation (7.35),

$$
\begin{gathered}
N_{f}=\left[0.17 P I-0.0085 P I\left(V_{b}\right)+0.0454 V_{b}-0.112\right]^{5} \varepsilon_{t}^{-5} S_{m}^{-1.8} \\
\therefore N_{f}=[0.17 \times(-1.45)-0.0085 \times(-1.45) \times 15.37+0.0454 \times 15.37-0.112]^{5} \times 0.00015^{-5} \times\left(2.9 \times 10^{5}\right)^{-1.8} \\
\therefore N_{f}=8.01 \times 10^{7}
\end{gathered}
$$

Again, this result agrees reasonably well with the graphical estimate.

## P.7.8 ■ Solution

Equation (7.36) gives the number of cycles to failure for a constant stress criterion,

$$
N_{f}=0.00432 C \varepsilon_{1}^{-3.291}\left|E^{*}\right|^{-0.854}
$$

Here, factor $C=10^{M}$, where $M$ is given by equation (7.37b),

$$
M=4.84\left(\frac{V_{b}}{V_{a}+V_{b}}-0.69\right)=4.84 \times\left(\frac{15.37}{4.81+15.37}-0.69\right)=0.347
$$

so that

$$
C=10^{0.347}=2.22
$$

and, noting that $\left|E^{*}\right|=379,000$ psi was determined in Problem 7-6,

$$
N_{f}=0.00432 \times 2.22 \times 0.00015^{-3.291} \times 379,000^{-0.854}=634,000
$$

## P.7.9 ■ Solution

Per the discussion in Section 7.4.1, a bilogarithmic plot of permanent strain versus the number of load repetitions should yield a straight line, as shown.


To find the coefficients that fit the linear curve above, one way to go is to apply Mathematica's FindFit command:

$$
\begin{aligned}
& \text { data }=\{\{0.1,1.158\},\{1,3.474\},\{10,11.533\},\{100,31.172\},\{1000,62.483\}\} \\
& \{(0.1,1.158\},\{1,3.474\},\{10,11.533\},(100,31.172\},\{1000,62.483\}\}
\end{aligned}
$$

## $\log$ Data $=\log 10[$ data $]$

$\{\{-1 ., 0.0637086\},\{0,0.54083\},\{1,1.06194\},\{2,1.49376\},\{3,1.79576\}\}$
line $=$ SetPrecision[FindFit[logData, $\{a * x+b\},\{a, b\}, x], 3]$
$\{a \rightarrow 0.442, b \rightarrow 0.549\}$
As the output of the last code indicates, we may write

$$
\log (\text { Strain })=0.442 \log (\text { Duration })+0.549
$$

It follows that, substituting the slope $S=0.442$ into equation (7.43),

$$
\alpha=1-S=1-0.442=0.558
$$

The intercept $I$, in turn, equals

$$
I=10^{0.442 \log (0.1)+0.549} \times 10^{-5}=1.28 \times 10^{-5}
$$

so that, using the reference strain $\varepsilon=3.045 \times 10^{-5}$ and substituting into equation (7.44),

$$
\mu=\frac{I S}{\varepsilon}=\frac{\left(1.28 \times 10^{-5}\right) \times 0.442}{3.045 \times 10^{-5}}=0.186
$$

## P.11.3 ■ Solution

Substituting the given tensile strain $\varepsilon_{t}=0.0005$ into the fatigue equation provided, we get

$$
N_{f}=5 \times 10^{-6} \varepsilon_{t}^{-3}=5 \times 10^{-6} \times 0.0005^{-3}=40,000
$$

With a coefficient of variation $C\left[N_{f}\right]=0.8$, the variance of $N_{f}$ is

$$
V\left[N_{f}\right]=(0.8 \times 40,000)^{2}=1.02 \times 10^{9}
$$

The cracking index $c$ is

$$
c=\frac{n}{N_{f}}=\frac{5000}{40,000}=0.125
$$

The variance of the cracking index is

$$
V[c]=\left(-\frac{n}{N_{f}^{2}}\right)^{2} V\left[N_{f}\right]=\left(-\frac{5000}{40,000^{2}}\right)^{2} \times\left(1.02 \times 10^{9}\right)=0.00996
$$

Using equation (10.58), the variance of $\log c$ can be determined from the variance of $c$,

$$
V[\log c]=\frac{0.1886}{c^{2}} V[c]=\frac{0.1886}{0.125^{2}} \times 0.00996=0.120
$$

Taking the square root of this result gives the standard deviation of $\log c$,

$$
S_{\log c}=\sqrt{V[\log c]}=\sqrt{0.120}=0.346
$$

The logarithm of $c$ itself is $\log c=\log 0.125=-0.903$, and the $Z$ score follows as

$$
Z=\frac{0-(-0.903)}{0.346}=2.61
$$

Referring to Table 10.1, we see that this Z-score corresponds to an area $\approx 0.495473$. The reliability is then $0.5+0.495473 \approx 0.995$, and the percent area cracked that is attributable to thermal fatigue cracking is $0.5 \%$.

## P.11.4 ■ Solution

The mean monthly air temperature is $68^{\circ} \mathrm{F}$. Noting that the temperature at the upper third point of each layer is to be used, we substitute $z=2 / 3=0.667$ for the surface course into equation (3.27), giving
$M_{p}=M_{a}\left(1+\frac{1}{z+4}\right)-\frac{34}{z+4}+6=68 \times\left(1+\frac{1}{0.667+4}\right)-\frac{34}{0.667+4}+6=81.3^{\circ} \mathrm{F}$
In turn, the depth used to compute the surface temperature of the binder course is $z=2+6 / 3=4 \mathrm{in}$., so that

$$
M_{p}=68 \times\left(1+\frac{1}{4.0+4}\right)-\frac{34}{4.0+4}+6=78.3^{\circ} \mathrm{F}
$$

To compute the dynamic modulus of the surface course, we substitute the pertaining data serially into equations (7.27f) through (7.27a) (see Problem 7.6),

$$
\begin{gathered}
\beta_{5}=1.3+0.49825 \log f=1.3+0.49825 \times \log 8=1.75 \\
\beta_{4}=0.483 V_{b}=0.483 \times 11=5.31 \\
\beta_{3}=0.553833+0.028829\left(P_{200} f^{-0.1703}\right)-0.03476 V_{v} \\
+0.070377 \lambda+0.931757 f^{-0.02774}
\end{gathered}
$$

$$
\begin{gathered}
\beta_{3}=0.553833+0.028829 \times 5 \times 8^{-0.1703}-0.03476 \times 4 \\
+0.070377 \times 2.5+0.931757 \times 8^{-0.02774}=1.57 \\
\beta_{2}=\beta_{4}^{0.5} T^{\beta_{5}}=5.31^{0.5} \times 81.3^{1.75}=5070 \\
\beta_{1}=\beta_{3}+5 \times 10^{-6} \beta_{2}-0.00189 \beta_{2} f^{-1.1}=1.57+5 \times 10^{-6} \times 5070-0.00189 \times 5070 \times 8^{-1.1}=0.622 \\
\left|E^{*}\right|=100,000 \times 10^{\beta_{1}}=100,000 \times 10^{0.622}=419,000 \mathrm{psi}
\end{gathered}
$$

Considering the binder, calculations are identical for $\beta_{5}$ and $\beta_{4}$; in calculating $\beta_{3}$, substitute $V_{a}=7 \%$ instead of $4 \%$ to obtain $\beta_{3}=1.47$. In calculating $\beta_{2}$, use $T=78.3^{\circ} \mathrm{F}$ instead of $81.3^{\circ} \mathrm{F}$ to obtain $\beta_{2}=4750$. The value of $\beta_{1}$ should be close to 0.582 . Finally,

$$
\left|E^{*}\right|=100,000 \times 10^{\beta_{1}}=100,000 \times 10^{0.582}=382,000 \mathrm{psi}
$$

Before using equation (3.28), we substitute the dynamic moduli obtained above into (3.29) to obtain the equivalent modulus

$$
E_{1}=\left[\frac{h_{1 a}\left(E_{1 a}\right)^{1 / 3}+h_{1 b}\left(E_{1 b}\right)^{1 / 3}}{h_{1 a}+h_{1 b}}\right]^{3}=\left(\frac{2 \times 4.19^{1 / 3}+6 \times 3.82^{1 / 3}}{2+6}\right)^{3}=3.91 \times 10^{5} \mathrm{psi}
$$

Lastly, the modulus of the untreated base is

$$
\begin{gathered}
E_{2}=10.447 h_{1}^{-0.471} h_{2}^{-0.041} E_{1}^{-0.139} E_{3}^{0.287} K_{1}^{0.868} \\
E_{2}=10.447 \times 8^{-0.471} \times 8^{-0.041} \times\left(3.91 \times 10^{5}\right)^{-0.139} \times(10,000)^{0.287} \times 8000^{0.868}=20,700 \mathrm{psi}
\end{gathered}
$$

## P.11.5 ■ Solution

The ESAL is to be determined with equation (6.30),

$$
E S A L=(A D T)_{0}(T)\left(T_{f}\right)(G)(D)(L)(365)(Y)
$$

From the problem statement, $(A D T)_{0}(T)=1885$. This is an interstate highway with two-axle, four-tire panel and pickup trucks, so a truck factor $T_{f}=$ 0.52 can be read from Table 6.10. Further, this is a six-lane highway, so a percentage of trucks $(D)(L)=40 \%$ can be read from Table 6.15. For a growth rate $r=0.04$ and a reference period $Y=20$ years, equation (6.33) gives

$$
(G)(Y)=\frac{(1+r)^{Y}-1}{r}=\frac{(1+0.04)^{20}-1}{0.04}=29.8
$$

so that

$$
E S A L=1885 \times 29.8 \times 0.4 \times 0.52 \times 365=4.27 \times 10^{6}
$$

Entering the ESAL value computed just now and the subgrade resilient modulus $M_{R}=10,000$ psi into Figure 11.17, the appropriate HMA thickness is found to equal 10 in .

Entering $E S A L=4.27 \times 10^{6}$ and $M_{R}=10,000$ psi into Figure 11.11 gives a thickness of full-depth HMA equal to 11.5 in . Entering the same data into Figure 11.12, we read a combined thickness of HMA and emulsified asphalt mix of 12 in . Section 11.2.5 recommends assuming a 2 -in. surface course, so we may compute the substitution ratio

$$
\text { Substitution ratio }=\frac{12-2}{11.5-2}=1.053
$$

If we use 2 in. of HMA, the thickness of the emulsified asphalt base shall be $(10-2) \times 1.053=8.42 \mathrm{in}$. If we use 3.5 in . of HMA, the thickness of the emulsified asphalt base shall be $(10-3.5) \times 1.053=6.85 \mathrm{in}$.

## P.11.9 ■ Solution

For a layer coefficient $a_{1}=0.44$ for the HMA, the structural number $S N$ is found as $S N=a_{1} D_{1}=0.44 \times 12=5.28$. To compute the reliability of the design, we need the normal deviate $Z_{R}$ as given by equation (11.36),

$$
Z_{R}=\frac{\log W_{18}-\log W_{t 18}}{S_{0}}(\mathrm{I})
$$

We were given $W_{18}=3 \times 10^{7}$ and $S_{0}=0.5 ; W_{t 18}$ can be determined from equation (11.34),

$$
\begin{aligned}
& \log W_{t 18}=9.36 \log (S N+1)-0.20+\frac{\log \left[\left(4.2-p_{t}\right) / 2.7\right]}{0.4+1094 /(S N+1)^{5.19}}+2.32 \log M_{R}-8.07 \\
& \therefore \log W_{t 18}=9.36 \log (5.28+1)-0.20+\frac{\log [(4.2-2.5) / 2.7]}{0.4+1094 /(5.28+1)^{5.19}}+2.32 \log 10,000-8.07=8.059 \\
& \therefore W_{t 18}=10^{8.059}=1.15 \times 10^{8}
\end{aligned}
$$

so that, substituting in (I),

$$
Z_{R}=\frac{\log \left(3 \times 10^{7}\right)-\log \left(1.15 \times 10^{8}\right)}{0.5}=-1.167
$$

Entering this normal deviate into Table 11.15 shows that the reliability must be between 85 and $90 \%$, as shown to the left. Referring to Table 10.1, we find that the shaded area in the Gaussian curve drawn to the right equals 0.377 , so the reliability must be $0.5+0.377=0.877 \approx 88 \%$.


## P.11.10 ■ Solution

Entering a HMA resilient modulus of $4.3 \times 10^{5}$ psi into Figure 11.27, we read a structural coefficient $a_{1} \approx 0.44$. Taking a resilient modulus of $6.5 \times 10^{5}$ psi for cement-treated base along with the given 7-day UCS of 500 psi, we refer to the nomograph in Figure $7.15 c$ and read a structural coefficient $a_{2}=0.17$ (see illustration to the side). For the granular subbase course, note that layer coefficient $a_{3}$ is related to resilient modulus $E_{3}=15,000$ psi by equation (11.46),


$$
a_{3}=0.227\left(\log E_{3}\right)-0.839=0.227(\log 15,000)-0.839=0.109
$$

Since the resilient modulus of the HMA is well above the maximum assumed in the AASHTO method, we size the HMA layer with the minimum thickness suggested in Table 11.21. For $E S A L=1,200,000$, which is between 500,000 and 2,000,000, we read a minimum thickness of 3.0 in.

We then proceed to size the CTB course. Entering an $E S A L=1.2 \times 10^{6}$, a resilient modulus $M_{R}=15,000$ psi, and a $\triangle P S I=2$ into Figure 11.25, we read a structural number $S N_{2}=2.3$. Now, $S N_{2}$ is related to the thicknesses of the two uppermost layers by an expression of the form (equation 11.48)

$$
S N_{2}=a_{1} D_{1}+a_{2} D_{2} m_{2}
$$

If the CTB is unsusceptible to moisture change, $m_{2}=1$. Substituting and solving for $D_{2}$,

$$
\begin{gathered}
2.3=3 \times 0.44+0.17 \times D_{2} \times 1 \\
\therefore D_{2}=5.76 \mathrm{in} .
\end{gathered}
$$

For convenience, take $D_{2}=6$ in. Next, we turn to the subbase. Entering an $E S A L=1.2 \times 10^{6}$, a resilient modulus $M_{R}=5500 \mathrm{psi}$, and a $\triangle P S I=2$ into Figure 11.25 gives $S N_{3}=3.1$. For fair drainage quality and $25 \%$ percent of time for moisture approaching saturation, we read $m_{3}=0.80$ from Table 11.20. The depth of the subbase can be established from equation (11.49),

$$
D_{3} \geq \frac{S N_{3}-a_{1} D_{1}-a_{2} D_{2} m_{2}}{a_{3} m_{3}}=\frac{3.1-0.44 \times 3-0.17 \times 6 \times 1}{0.109 \times 0.8}=8.72 \mathrm{in} .
$$

For convenience, take $D_{3}=9$ in.

## P.12.1 $\quad$ Solution

The allowable number $N$ of load repetitions is given by $N=C_{2} n$, where $C_{2}=0.06$ for pavements with concrete shoulders, giving

$$
N=C_{2} n=0.06 \times\left(2 \times 10^{6}\right)=1.2 \times 10^{5}
$$

We then appeal to equation (12.7),

$$
\begin{gathered}
\log N=14.524-6.777\left(C_{1} P-9.0\right)^{0.103} \\
\therefore \log \left(1.2 \times 10^{5}\right)=14.524-6.777(1.0 \times P-9.0)^{0.103}
\end{gathered}
$$

$$
\therefore P=34.1
$$

The rate of work $P$ is defined by equation (12.8),

$$
P=268.7 \frac{p^{2}}{h k^{0.73}}
$$

We have $P$, the thickness $h=10 \mathrm{in}$., and $k=150$ pci. Solving for pressure and substituting,

$$
\begin{aligned}
& P=268.7 \frac{p^{2}}{h k^{0.73}} \rightarrow p=\sqrt{\frac{P h k^{0.73}}{268.7}} \\
& \therefore p=\sqrt{\frac{34.1 \times 10 \times 150^{0.73}}{268.7}}=7.02 \mathrm{psi}
\end{aligned}
$$

Finally, we divide this pressure value by $k$ to obtain the allowable corner deflection $w$,

$$
w=\frac{p}{k}=\frac{7.02}{150}=0.0468 \mathrm{in} .
$$

## P.12.2 $\quad$ Solution

For a pavement with concrete shoulders, $C_{2}$ changes from 0.06 to 0.94 , giving $N=0.94 \times\left(2 \times 10^{6}\right)=1.88 \times 10^{6}$. Substituting into (12.7) and solving for $P$,

$$
\begin{gathered}
\log \left(1.88 \times 10^{6}\right)=14.524-6.777(1.0 \times P-9.0)^{0.103} \\
\therefore \log \left(1.88 \times 10^{6}\right)=14.524-6.777(1.0 \times P-9.0)^{0.103} \\
\therefore P=15.8
\end{gathered}
$$

Substituting into (12.8),

$$
p=\sqrt{\frac{P h k^{0.73}}{268.7}}=\sqrt{\frac{15.8 \times 10 \times 150^{0.73}}{268.7}}=4.77 \mathrm{psi}
$$

Lastly,

$$
w=\frac{p}{k}=\frac{4.77}{150}=0.0318 \mathrm{in} .
$$

## P. 12.3 ■ Solution

The total number of trucks during the design period in the design lane is given by the product

Total No. of trucks $=2500 \times 365 \times 20 \times 0.35 \times 0.5=3.19 \times 10^{6}$
To compute the expected repetitions for a given weight category, multiply the No. of axles per 1000 trucks given in Table P12.3 by the total No. of trucks; for an axle-load value of 28 kip, for example,
Expected repetitions of 16-kip axle loads $=\frac{0.9}{1000} \times\left(3.19 \times 10^{6}\right)=2870$
Entering an assumed thickness of 8 in . and a modulus of subgrade reaction of 200 pci into Table 12.6, we read an equivalent stress of 242 psi. Dividing this quantity by the concrete modulus of rupture gives the stress ratio factor $242 / 650=0.372$. Equipped with the stress ratio factor, we can enter its value and the axle loads multiplied by LSF into Figure 12.12 to read the corresponding allowable repetitions. For example, entering into the nomograph an axle load of 28 kips , or $1.1 \times 28=30.8 \mathrm{kips}$, and $S R F=$ 0.372 , we read an allowable number of repetitions of about 20,000, as indicated by the red line to the side. Dividing the expected number of 28 -kip repetitions, which is 2870 , by the allowable No. of repetitions, which is 20,000 , we obtain a fatigue damage of $14.4 \%$.

To determine the erosion factor, enter a slab thickness of 8 in . and a modulus of subgrade-subbase of 200 pci into Table 12.8 (doweled joints, no concrete shoulders); the EF is found to be 2.80 . Equipped with the erosion factor, we can enter its value and the axle loads multiplied by LSF into Figure 12.13 to read the pertaining allowable repetitions. For instance, entering into the nomograph an axle load of 28 kips, or $1.1 \times 28=30.8$ kips, and $E F=2.80$, we read an allowable number of repetitions of about 900,000 , as indicated by the blue line to the side.

Proceed similarly with the tandem-axle portion of the data. In this case, use 208 psi as the equivalent stress (Table 12.6), 208/650 $=0.32$ as the stress ratio factor, and 2.93 as the erosion factor (Table 12.8). Maximum allowable repetitions for fatigue and erosion are read from Figs. 12.12 and 13 , respectively, as before. The calculations are tabulated below.


| Single Axle |  |  | Fatigue analysis |  | Erosion analysis |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $E S=242 \mathrm{psi}, S R F=0.372, E F=2.80$ |  |  |  |  |  |  |
| Load (kips) | Multiplied by LSF | Expected repetitions | Allowable repetitions | Fatigue damage (\%) | Allowable repetitions | Erosion damage (\%) |
| 28 | 30.8 | 2870 | 20000 | 14.4 | 900000 | 0.3 |
| 26 | 28.6 | 6070 | 45000 | 13.5 | 1400000 | 0.4 |
| 24 | 26.4 | 7400 | 160000 | 4.6 | 2100000 | 0.4 |
| 22 | 24.2 | 50000 | 800000 | 6.3 | 3500000 | 1.4 |
| 20 | 22 | 200000 | - | - | 7200000 | 2.8 |
| 18 | 19.8 | 360000 | - | - | 16000000 | 2.3 |
| 16 | 17.6 | 420000 | - | - | 40000000 | 1.1 |
| Tandem Axles |  |  | Fatigue analysis |  | Erosion analysis |  |
| $E S=208 \mathrm{psi}, S R F=0.320, E F=2.93$ |  |  |  |  |  |  |
| Load (kips) | Multiplied by LSF | Expected repetitions | Allowable repetitions | Fatigue damage (\%) | Allowable repetitions | Erosion damage (\%) |
| 48 | 52.8 | 30700 | 10000000 | 0.31 | 950000 | 3.2 |
| 44 | 48.4 | 33500 | - | - | 1600000 | 2.1 |
| 40 | 44 | 53700 | - | - | 2600000 | 2.1 |
| 36 | 39.6 | 55000 | - | - | 5600000 | 1.0 |
| 32 | 35.2 | 76700 | - | - | 12000000 | 0.6 |
| 28 | 30.8 | 110000 | - | - | 40000000 | 0.3 |
| 24 | 26.4 | 256000 | - | - | - | - |
|  |  |  | Total $=$ | 39.0 | Total $=$ | 17.9 |

Note that with the thickness at hand the cumulative damage and erosion criteria are well below $100 \%$. Accordingly, we design the pavement with a thickness of 8.0 in .

## P.12.4 ■ Solution

The total No. of trucks on the design lane during the design period is $3460 \times 365 \times 20=2.53 \times 10^{7}$. Initially, a trial thickness of 8.5 in . is selected. For a modulus of subgrade reaction of 150 pci and the given trial thickness of 8.5 in., we read equivalent stresses of 191 and 166 from Table 12.7. The corresponding stress ratio factors are 191/650 $=0.294$ and 166/650 $=0.255$. For a pavement with doweled joints and concrete shoulders, the erosion factor can be read from Table 12.10; since there is no entry for a $k$ value of 150 pci, we interpolate using the data points for $k=100$ and $k=200 \mathrm{pci}$, yielding $2.32 / 2.45$. The pertaining data are processed below.

| Single Axle |  |  |  | Fatigue analysis |  | Erosion analysis |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $E S=191 \mathrm{psi}, S R F=0.294, E F=2.32$ |  |  |  |  |  |  |  |
| Load (kips) | Multiplied by LSF | Category 3 axle load dist. (Table 12.13) | Expected repetitions | Allowable repetitions | Fatigue damage (\%) | Allowable repetitions | Erosion damage (\%) |
| 30 | 36 | 0.45 | 11385 | 110000 | 10.4 | 680000 | 1.7 |
| 28 | 33.6 | 0.85 | 21505 | 300000 | 7.2 | 1200000 | 1.8 |
| 26 | 31.2 | 1.78 | 45034 | 1750000 | 2.6 | 2000000 | 2.3 |
| 24 | 28.8 | 5.21 | 131813 | - | - | 5000000 | 2.6 |
| 22 | 26.4 | 7.85 | 198605 | - | - | 15000000 | 1.3 |
| 20 | 24 | 16.33 | 413149 | - | - | - | - |
| 18 | 21.6 | 25.15 | 636295 | - | - | - | - |
| 16 | 19.2 | 31.82 | 805046 | - | - | - | - |
| 14 | 16.8 | 47.73 | 1207569 | - | - | - | - |
| 12 | 14.4 | 182.02 | 4605106 | - | - | - | - |
| Tandem Axles |  |  |  | Fatigue analysis |  | Erosion analysis |  |
| $E S=166 \mathrm{psi}, S R F=0.255, E F=2.45$ |  |  |  |  |  |  |  |
| Load (kips) | Multiplied by LSF | Category 3 axle load dist. (Table 12.13) | Expected repetitions | Allowable repetitions | Fatigue damage (\%) | Allowable repetitions | Erosion damage (\%) |
| 52 | 62.4 | 1.19 | 30107 | 10000000 | 0.30 | 680000 | 4.4 |
| 48 | 57.6 | 2.91 | 73623 | - | - | 1300000 | 5.7 |
| 44 | 52.8 | 8.01 | 202653 | - | - | 3000000 | 6.8 |
| 40 | 48 | 21.31 | 539143 | - | - | 8000000 | 6.7 |
| 36 | 43.2 | 56.25 | 1423125 | - | - | - | - |
| 32 | 38.4 | 103.63 | 2621839 | - | - | - | - |
| 28 | 33.6 | 121.22 | 3066866 | - | - | - | - |
| 24 | 28.8 | 72.54 | 1835262 | - | - | - | - |
| 20 | 24 | 85.94 | 2174282 | - | - | - | - |
| 16 | 19.2 | 99.34 | 2513302 | - | - | - | - |
|  |  |  |  | Total $=$ | 20.4 | Total $=$ | 33.3 |

As can be seen, a pavement with a trial thickness of 8.5 in . exceeds neither the fatigue damage nor the erosion damage criterion; accordingly, the design is sound. Try performing the calculations for a trial thickness of 8 in . and you'll find that the ensuing fatigue damage will be greater than 100\%; it follows that a thickness of 8.5 in . is close to the minimum we can propose for the situation at hand.

In the simplified approach, we can enter the given $M R$ of 650 psi, a concrete-shoulder endowed road, and a subgrade-subbase support assumed to be medium in nature into Table 12.15; for a slab thickness of 8 in., the allowable ADTT is found to be 5700, which is less than the given ADTT of 3460 . Thus, a 8-in. thickness is taken as a sound design per the simplified method.

| Concrete shoulder or curb |  |  |  |  |
| :--- | ---: | ---: | ---: | ---: |
| Slab <br> thickness <br> (in.) | Subgrade-subbase support |  |  |  |
|  | Low | Medium | High | Very high |
|  |  |  |  |  |
| 6.5 |  |  | 83 | 320 |
| 7 | 52 | 220 | 550 | 1900 |
| 7.5 | 320 | 1200 | 2900 | 9800 |
| 8 | 1600 | 5700 | 13,800 |  |
| 8.5 | 6900 | $23,700^{\text {b }}$ |  |  |
|  |  |  |  |  |

## P.12.5 ■ Solution

Use a trial thickness of 9.5 in . For a modulus of subgrade reaction of 150 pci and the given trial thickness of 8.5 in ., we read equivalent stresses of 200 and 183 from Table 12.6 (not Table 12.7, as the road has no concrete shoulders). The corresponding stress ratio factors are 200/650 $=0.308$ and $183 / 650=0.282$. For a pavement with doweled joints and no concrete shoulders, the erosion factor can be read from Table 12.8; since there is no entry for a $k$ value of 150 pci, we interpolate using the data points for $k=100$ and $k=200$ pci, yielding 2.59/2.78. The pertaining data are processed below.

| Single Axle |  |  |  | Fatigue analysis |  | Erosion analysis |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $E S=200 \mathrm{psi}, S R F=0.308, E F=2.59$ |  |  |  |  |  |  |  |
| Load (kips) | Multiplied by LSF | Category 3 axle load dist. (Table 12.13) | Expected repetitions | Allowable repetitions | Fatigue damage (\%) | Allowable repetitions | Erosion damage (\%) |
| 30 | 36 | 0.45 | 11385 | 40000 | 28.5 | 1400000 | 0.8 |
| 28 | 33.6 | 0.85 | 21505 | 120000 | 17.9 | 2000000 | 1.1 |
| 26 | 31.2 | 1.78 | 45034 | 400000 | 11.3 | 3400000 | 1.3 |
| 24 | 28.8 | 5.21 | 131813 | 3000000 | 4.4 | 5800000 | 2.3 |
| 22 | 26.4 | 7.85 | 198605 | - | - | 10000000 | 2.0 |
| 20 | 24 | 16.33 | 413149 | - | - | 21000000 | 2.0 |
| 18 | 21.6 | 25.15 | 636295 | - | - | 60000000 | 1.1 |
| 16 | 19.2 | 31.82 | 805046 | - | - | - | - |
| 14 | 16.8 | 47.73 | 1207569 | - | - | - | - |
| 12 | 14.4 | 182.02 | 4605106 | - | - | - | - |
| Tandem Axles |  |  |  | Fatigue analysis |  | Erosion analysis |  |
| $E S=183 \mathrm{psi}, S R F=0.282, E F=2.78$ |  |  |  |  |  |  |  |
| Load (kips) | Multiplied by LSF | Category 3 axle load dist. (Table 12.13) | Expected repetitions | Allowable repetitions | Fatigue damage (\%) | Allowable repetitions | Erosion damage (\%) |
| 52 | 62.4 | 1.19 | 30107 | 4000000 | 0.75 | 900000 | 3.3 |
| 48 | 57.6 | 2.91 | 73623 | - | - | 1400000 | 5.3 |
| 44 | 52.8 | 8.01 | 202653 | - | - | 2100000 | 9.7 |
| 40 | 48 | 21.31 | 539143 | - | - | 4000000 | 13.5 |
| 36 | 43.2 | 56.25 | 1423125 | - | - | 8000000 | 17.8 |
| 32 | 38.4 | 103.63 | 2621839 | - | - | 20000000 | 13.1 |
| 28 | 33.6 | 121.22 | 3066866 | - | - | 80000000 | 3.8 |
| 24 | 28.8 | 72.54 | 1835262 | - | - | - | - |
| 20 | 24 | 85.94 | 2174282 | - | - | - | - |
| 16 | 19.2 | 99.34 | 2513302 | - | - | - | - |
|  |  |  |  | Total $=$ | 62.8 | Total $=$ | 77.0 |

The fatigue damage and erosion damage criteria are closer to 100 than in the analysis of the design with concrete shoulders, but remain below this threshold. The design is adequate. From Table 12-15, a thickness of 9.5 in . is prescribed to withstand a ADTT of up to 10,800; a thickness of 9 in . is not adequate because the daily truck traffic of 3460 exceeds the 2700 truck traffic limit for this thickness level.

| No concrete shoulder or curb |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Slab thickness (in.) | Subgrade-subbase support |  |  |  |
|  | Low | Medium | High | Very high |
| $\mathrm{MR}=650 \mathrm{psi}$ |  |  |  |  |
| 7.5 |  |  |  | 250 |
| 8 |  | 130 | 350 | 1300 |
| 8.5 | 160 | 640 | 1600 | 6200 |
| 9 | 700 | 2700 | 7000 | $11,500^{\text {b }}$ |
| 9.5 | 2700 | 10,800 |  |  |
| 10 | 9900 |  |  |  |

## P.12.7 ■ Solution

From Table 11.20, the drainage coefficient for poor drainage with 5\% of time at saturation is taken as 0.90 . From Figure 12.18 , we can read a composite modulus of subgrade reaction $k \approx 350$ pci. To determine $W_{18}$, we appeal to equation (12.21) without the reliability term, giving

$$
\begin{gathered}
\log W_{18}=7.35 \log (D+1)-0.06+\frac{\log [\Delta P S I /(4.5-1.5)]}{1+1.624 \times 10^{7} /(D+1)^{8.46}} \\
+\left(4.22-0.32 p_{t}\right) \log \left\{\frac{S_{c} C_{d}\left(D^{0.75}-1.132\right)}{215.63 J\left[D^{0.75}-18.42 /\left(E_{c} / k\right)^{0.25}\right]}\right\} \\
\therefore \log W_{18}=7.35 \log (8.0+1)-0.06+\frac{\log [2.5 /(4.5-1.5)]}{1+1.624 \times 10^{7} /(8.0+1)^{8.46}} \\
+(4.22-0.32 \times 2.0) \log \left(\frac{650 \times 0.9 \times\left(8.0^{0.75}-1.132\right)}{215.63 \times 3.2 \times\left\{8.0^{0.75}-18.42 /\left[\left(4.0 \times 10^{6}\right) / 100\right]^{0.25}\right\}}\right\} \\
\therefore \log W_{18}=6.93 \\
\therefore W_{18}=10^{6.93}=8.51 \times 10^{6}
\end{gathered}
$$

Entering our data into the AASHTO chart, we read $W_{18} \approx 8.5 \times 10^{6}$, which is close to our result.

## P.12.8 ■ Solution

Per the discussion in section 12.3.2, if a slab is placed directly on the subgrade without a subbase the modulus of subgrade reaction can be related to the resilient modulus by the simple equation

$$
k=\frac{M_{R}}{19.4}(\mathrm{I})
$$

with $k$ in pci and $M_{R}$ in psi. The relative damage to rigid pavements, $u_{r}$, is given by equation (12.29),

$$
\begin{equation*}
u_{r}=\left(D^{0.75}-0.39 k^{0.25}\right)^{3.42} \tag{II}
\end{equation*}
$$

This equation can be used to compute the effective modulus of subgrade reaction $k$ in terms of the seasonal moduli $k_{\text {}}$. The data are processed below.

| Month | $M_{R}$ (psi) | $k$ (pci) <br> (Eq. I) | $u_{r}$ <br> (Eq. II) |
| :---: | :---: | :---: | :---: |
| JAN | 15900 | 819.6 | 37.8 |
| FEB | 27300 | 1407.2 | 25.9 |
| MAR | 38700 | 1994.8 | 19.2 |
| APR | 50000 | 2577.3 | 14.8 |
| MAY | 900 | 46.4 | 110.7 |
| JUNE | 1620 | 83.5 | 96.1 |
| JULY | 2340 | 120.6 | 86.6 |
| AUGUST | 3060 | 157.7 | 79.6 |
| SEPTEMBER | 3780 | 194.8 | 74.1 |
| OCTOBER | 4500 | 232.0 | 69.5 |
| NOVEMBER | 4500 | 232.0 | 69.5 |
| DECEMBER | 4500 | 232.0 | 69.5 |
|  |  | Total $=$ | 753.2 |

Dividing the cumulative $u_{r}$ by 12 gives 62.8. Substituting into (II),

$$
\begin{aligned}
& \bar{u}_{r}=\left(D^{0.75}-0.39 k_{\mathrm{eff}}^{0.25}\right)^{3.42}=62.8 \\
& \therefore\left(8.5^{0.75}-0.39 k_{\mathrm{eff}}^{0.25}\right)^{3.42}=62.8
\end{aligned}
$$

The equation above can be easily solved for $k_{\text {eff }} u s i n g$ logarithms. For convenience, we simply employ the Mathematica code

$$
\begin{aligned}
\ln [118]== & \text { Solve }\left[\left(\mathbf{8 . 5} \mathbf{5}^{0.75}-\boldsymbol{0 . 3 9 * \mathbf { k } _ { \text { eff } }}{ }^{\mathbf{0 . 2 5}}\right)^{\left.\mathbf{3 . 4 2}=\mathbf{6 2 . 8}, \mathbf{k}_{\text {eff }}\right]}\right. \\
& \ldots \text { Solve: Inverse functions are being used by Solve, so some solu } \\
& \quad \text { Reduce for complete solution information. }
\end{aligned}
$$

That is, $k_{\text {eff }} \approx 300$ pci. This result is different from the 305 figure that the textbook indicates as the correct answer; my explanation for the disparity is the following. Computing the relative damage contributions given by (II) with an exponent of 3.24 instead of 3.42 yields a cumulative damage of 601.8, which in turn leads to a 12-month average of 50.15 . Substituting this $\bar{u}_{r}$ into (II) and solving for $k_{\text {eff, }}$ again using an exponent of 3.24 , we get

… Solve: Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information.

Out[120] $=\left\{\left\{k_{\text {eff }} \rightarrow 305.275\right\}\right\}$
that is, $k_{\text {eff }}=305$ pci in this case. In short, my guess is that whoever prepared the "official" solutions used an exponent of 3.24 instead of 3.42 in equation (12.30): I stand by my reviewed solution, which gives $k_{\text {eff }} \approx 300 \mathrm{pci}$.

## P.12.9 ■ Solution

The equation in question is (5.7), namely

$$
k=0.95\left(\frac{E_{f}}{E}\right)^{\frac{1}{3}} \frac{E_{f}}{\left(1-v_{f}^{2}\right) h}
$$

Here, $E_{f}=M_{R}, E=4 \times 10^{6} \mathrm{psi}, v_{f}=0.45$, and $h=D=8.5 \mathrm{in} .$, giving

$$
\begin{equation*}
k=0.95\left(\frac{M_{R}}{4 \times 10^{6}}\right)^{\frac{1}{3}} \frac{M_{R}}{\left(1-0.45^{2}\right) \times 8.5}=8.83 \times 10^{-4} M_{R}^{4 / 3} \tag{I}
\end{equation*}
$$

The expression for cumulative damage is no different from the one used in the previous problem,

$$
\begin{equation*}
u_{r}=\left(D^{0.75}-0.39 k^{0.25}\right)^{3.42} \tag{II}
\end{equation*}
$$

The data are processed below.

| Month | $M_{R}$ (psi) | $k$ (pci) <br> (Eq. I) | $u_{r}$ <br> (Eq. II) |
| :---: | :---: | :---: | :---: |
| JAN | 15900 | 353.0 | 58.6 |
| FEB | 27300 | 725.8 | 40.6 |
| MAR | 38700 | 1155.9 | 30.0 |
| APR | 50000 | 1626.5 | 23.0 |
| MAY | 900 | 7.7 | 150.1 |
| JUNE | 1620 | 16.8 | 134.1 |
| JULY | 2340 | 27.4 | 123.2 |
| AUGUST | 3060 | 39.2 | 114.8 |
| SEPTEMBER | 3780 | 52.0 | 107.9 |
| OCTOBER | 4500 | 65.6 | 102.2 |
| NOVEMBER | 4500 | 65.6 | 102.2 |
| DECEMBER | 4500 | 65.6 | 102.2 |
|  |  | Total $=$ | 1088.8 |

Dividing the cumulative $u_{r}$ by 12 gives 90.7 . Substituting into (II),

$$
\begin{aligned}
& \bar{u}_{r}=\left(D^{0.75}-0.39 k_{\mathrm{eff}}^{0.25}\right)^{3.42}=90.7 \\
& \therefore\left(8.5^{0.75}-0.39 k_{\mathrm{eff}}^{0.25}\right)^{3.42}=90.7
\end{aligned}
$$

As before, we apply the Mathematica code
$\ln [128]=$ Solve $\left[\left(8.5^{0.75}-0.39 * k_{\text {eff }}{ }^{0.25}\right)^{3.42}=\mathbf{9 0 . 7}, \mathrm{k}_{\text {eff }}\right]$
... Solve: Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information.
Out [128] $=\left\{\left\{\mathbf{k}_{\text {eff }} \rightarrow \mathbf{1 0 2 . 9 1 1}\right\}\right\}$
Clearly, $k_{\text {eff }} \approx 103$ pci.

## - REFERENCE

- HUANG, Y.H. (2004). Pavement Analysis and Design. 2nd edition. Upper Saddle River: Pearson.

Got any questions related to this quiz? We can help!
Send a message to contact@montogue.com and we'll answer your question as soon as possible.

