Quiz HD101

# Specific Energy & Momentum

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## Problems

### Problem 1

Which of the following statements is *false*?

**A)** For a given specific energy $E$, the unit discharge, $q$, is maximum at the critical depth, $y_c$.

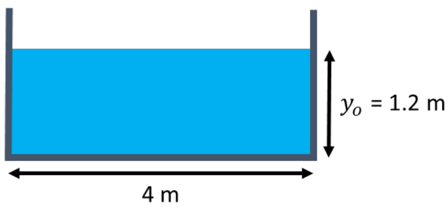
**B)** Simple explicit, non-empirical formulas for critical depth are available for rectangular and trapezoidal channels, but not for triangular channels.

**C)** An open channel flow is said to be *subcritical* if its Froude number is less than unity and *supercritical* if it is greater than unity.

**D)** For a rectangular channel, it can be shown that $y_c = (3/2)E$, that is, the critical depth is three halves of the specific energy.

### Problem 2

A rectangular channel is 4-m wide and carries a discharge of 12 m$^3$/s at a normal depth of 1.2 m. Calculate its critical depth and determine whether the flow is subcritical or supercritical.




**A)** $y_c$ = 0.66 m and the flow is subcritical.

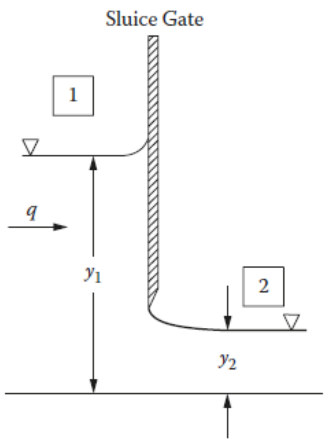
**B)** $y_c$ = 0.97 m and the flow is subcritical.

**C)** $y_c$ = 1.25 m and the flow is supercritical.

**D)** $y_c$ = 1.51 m and the flow is supercritical.

### Problem 3 (Moglen, 2015, w/ permission)

The system shown in the figure below has a unit discharge of 3 m$^3$/s. The depth $y_1$ upstream of the sluice gate is 2 m. Regarding this system, which of the following statements is *false*?



**A)** The upstream Froude number equals 0.34.

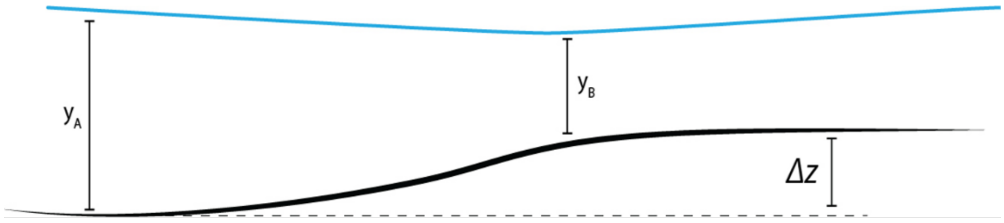
**B)** The specific energy upstream of the sluice gate is 2.11 m.

**C)** The depth of flow downstream of the sluice gate is 1.08 m.

**D)** The downstream Froude number equals 2.41.

## Problem 4 (Akan, 2006)

A rectangular channel has a bottom width $b$ = 6 m and carries a discharge $Q$ = 8 m$^3$/s. Suppose the channel is nearly horizontal, except that there is a smooth step rise in the channel bottom, as shown below, such that $\Delta z$ = 0.30 m. The energy loss due to this step is negligible. At section A, the depth of flow is $y_A$ = 1.5 m. What is the depth over the step, at section B?




**A)** $y_B$ = 0.31 m

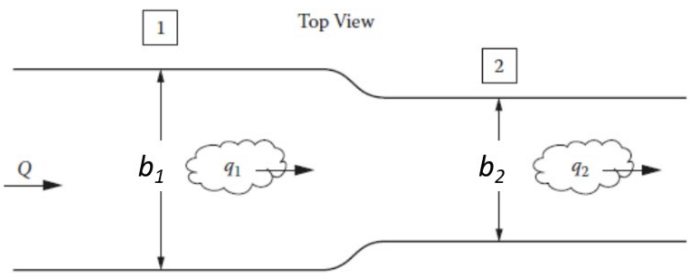
**B)** $y_B$ = 0.60 m

**C)** $y_B$ = 0.89 m

**D)** $y_B$ = 1.18 m

## Problem 5 (Moglen, 2015, w/ permission)

Consider a channel flow system with a discharge of 9 m$^3$/s. The channel is rectangular. The width at location 1 is $b_1$ = 4.5 m. A constriction is encountered at location 2 downstream such that the width $b_2$ = 3.0 m. Regarding this system, which of the following statements is *false*?




**A)** The Froude number upstream of the constriction is 0.22.

**B)** The Froude number downstream of the constriction is 0.34.

**C)** The constriction causes the flow depth to decrease 0.12 m.

**D)** Assuming the constriction is energy-conserving, the specific energy downstream of the constriction is 2.11 m.

## Problem 6 (Chaudhry, 2008)

A bridge is planned on 50-m wide rectangular channel carrying a flow of 200 m$^3$/s at a flow depth of 4.0 m. For reducing the length of the bridge, what is the minimum channel width such that the upstream water level is not influenced for this discharge?

**A)** $b$ = 14.4 m

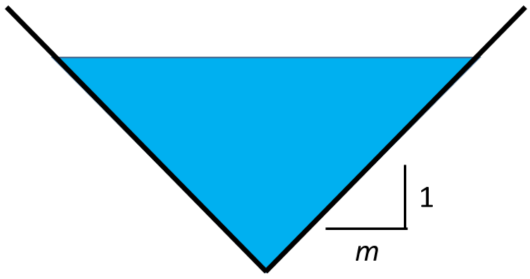
**B)** $b$ = 19.5 m

**C)** $b$ = 24.1 m

**D)** $b$ = 29.6 m

## Problem 7

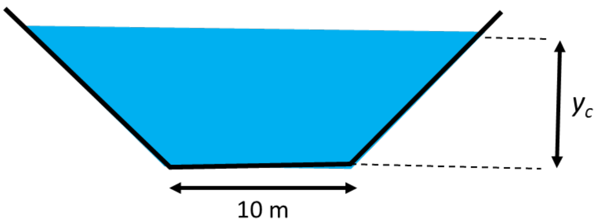
Consider a channel with triangular cross-section of which the slope is $m$. The channel passes a discharge $Q$. What is the critical depth of this channel? Assume that the longitudinal slope is small and that $\alpha$ = 1.0.




**A)** $y_c = (Q^2/gm^2)^{1/5}$
**B)** $y_c = (2Q^2/gm^2)^{1/5}$
**C)** $y_c = (Q^2/gm^2)^{2/5}$
**D)** $y_c = (2Q^2/gm^2)^{2/5}$

## Problem 8

Compute the critical depth of a trapezoidal channel with the cross-section illustrated below. The discharge is 45 $m^3$/s, the channel bottom width is 10.0 m, and the side slopes are 2H:1V. Neglect the bottom slope and consider the velocity head coefficient $\alpha$ = 1.0.



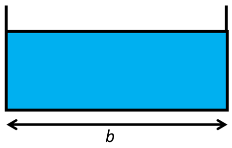
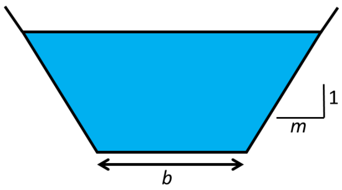
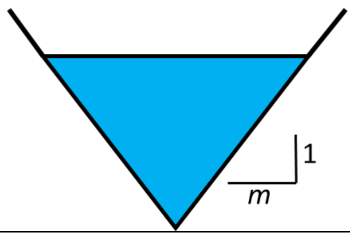

**A)** $y_c$ = 0.51 m
**B)** $y_c$ = 1.08 m
**C)** $y_c$ = 1.50 m
**D)** $y_c$ = 2.04 m

## Problem 9

In the 1980s, Straub developed a series of empirical equations for the estimation of the critical depth. Use of these formulas requires computation of factor $\psi$, given by

$$\psi = \frac{\alpha Q^2}{g}$$

where $\alpha$ is the kinetic energy coefficient, $Q$ is the discharge, and g is the acceleration due to gravity. Refer to the table below.

| Channel Type | Equation for $y_c$ | Notes |
|---|---|---|
| Rectangle   | $y_c = \left(\frac{\psi}{b^2}\right)^{\frac{1}{3}}$ | |
| Trapezoid   | $y_c = 0.81\left(\frac{\psi}{m^{0.75}b^{1.25}}\right)^{0.27} - \frac{b}{30m}$ | Range of applicability: $0.1 < \frac{Q}{b^{2.5}} < 4.0$ If $Q/b^{2.5} < 0.1$, use equation for rectangular channel |
| Triangle   | $y_c = \left(\frac{2\psi}{m^2}\right)^{0.20}$ | |

Using Straub's formulas, find the critical depth for a trapezoidal channel with 5 m width and side slope $m$ = 1 carrying a discharge of 8 m$^3$/s. Assume $\alpha$ = 1.0.

**A)** $y_c$ = 0.54 m

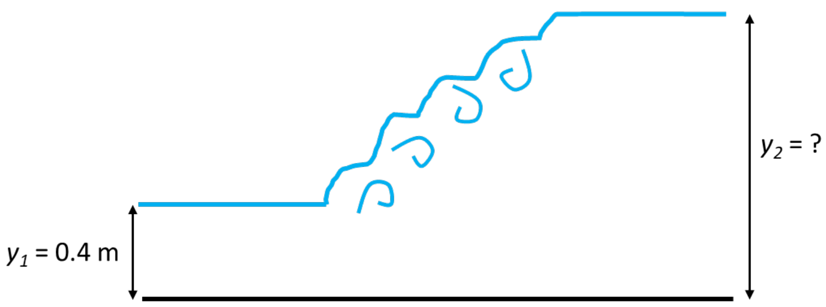
**B)** $y_c$ = 1.12 m

**C)** $y_c$ = 1.65 m

**D)** $y_c$ = 2.13 m

## Problem 10

A rectangular channel has a bottom width $b$ = 6 m and carries a discharge $Q$ = 15 m$^3$/s. A hydraulic jump occurs at a point in the course of the channel. The flow depth just before the jump is $y_1$ = 0.4 m. Find the depth after the jump and the head loss.




**A)** $y_2$ = 1.20 m and $h_L$ = 0.43 m

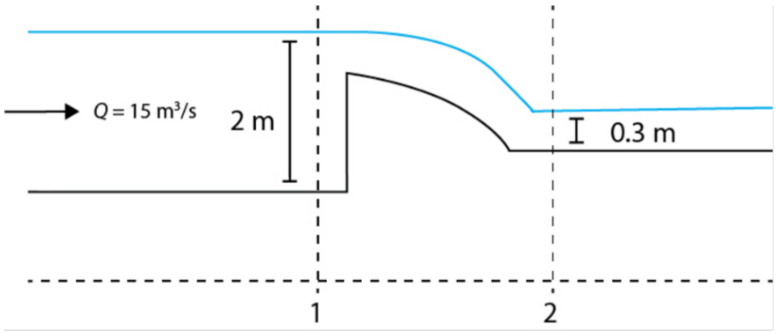
**B)** $y_2$ = 1.20 m and $h_L$ = 0.68 m

**C)** $y_2$ = 1.60 m and $h_L$ = 0.43 m

**D)** $y_2$ = 1.60 m and $h_L$ = 0.68 m

## Problem 11 (Akan, 2006)

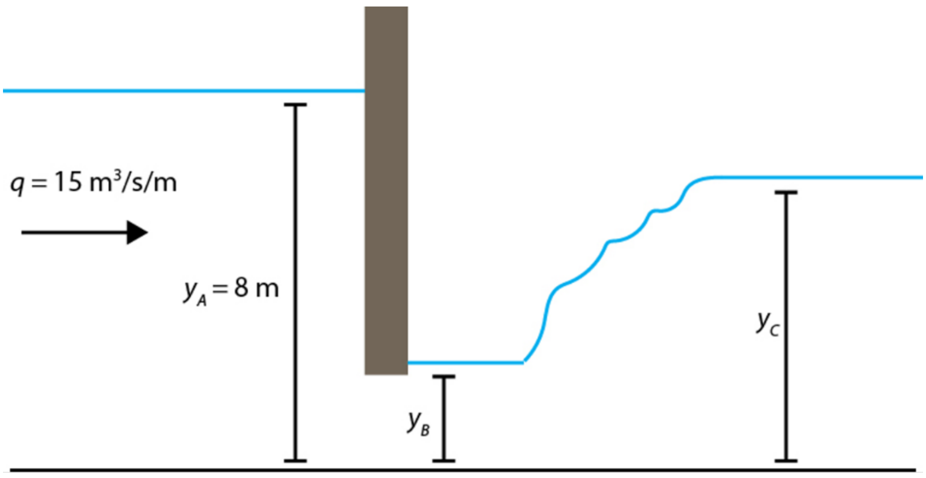
The channel shown in the figure below has a rectangular cross-section and is 6-m wide. Suppose that the friction forces and the weight component in the flow direction are negligible. Determine the magnitude of the force exerted by the flow onto the spillway.




**A)** $F_e = 68$ kN
**B)** $F_e = 94$ kN
**C)** $F_e = 115$ kN
**D)** $F_e = 141$ kN

## Problem 12 (Akan, 2006)

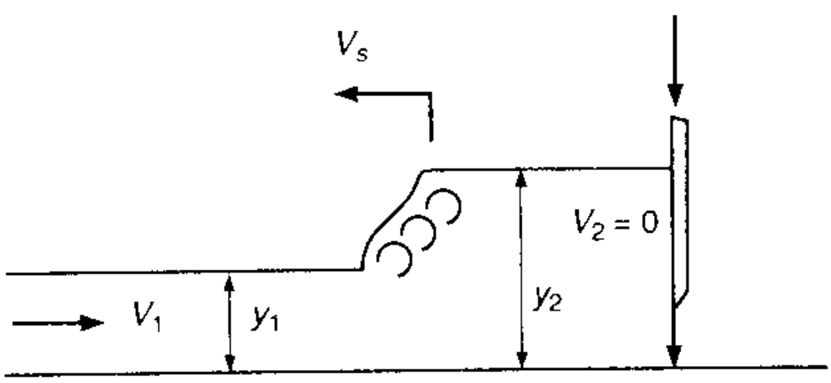
The rectangular channel shown in the figure below is nearly horizontal and carries 15 $m^3/s/m$. The flow depth upstream of the sluice gate is 8 m. A hydraulic jump occurs on the downstream side of the sluice gate. Find the flow depth in section C and the head loss due to the hydraulic jump.




**A)** $y_C = 1.29$ m and $h_L = 2.44$ m
**B)** $y_C = 1.29$ m and $h_L = 7.32$ m
**C)** $y_C = 5.36$ m and $h_L = 2.44$ m
**D)** $y_C = 5.36$ m and $h_L = 7.32$ m

## Problem 13 (Sturm, 2009)

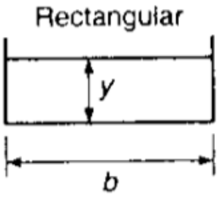
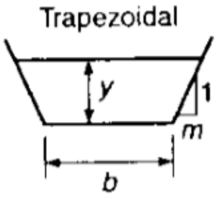
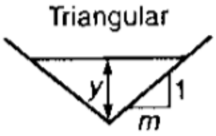
A steady flow occurs in a rectangular channel upstream of a sluice gate. The flow velocity is 2 m/s and the depth of flow is 3.5 m just upstream of the gate. If the sluice gate is suddenly slammed shut, what are the height and speed of the upstream surge?



**A)** $y_2$ = 4.8 m and $V_S$ = 5.5 m/s
**B)** $y_2$ = 4.8 m and $V_S$ = 7.2 m/s
**C)** $y_2$ = 6.1 m and $V_S$ = 5.5 m/s
**D)** $y_2$ = 6.1 m and $V_S$ = 7.2 m/s

# Additional Information

**Table 1** Geometric elements for channels of different shape ($y$ = flow depth)

| Section | Area, $A$ | Wetted Perimeter, $P$ | Top Width, $B$ |
|---|---|---|---|
| Rectangular  | $by$ | $b + 2y$ | $b$ |
| Trapezoidal  | $y(b + my)$ | $b + 2y(1 + m^2)^{1/2}$ | $b + 2my$ |
| Triangular  | $my^2$ | $2y(1 + m^2)^{1/2}$ | $2my$ |

# Solutions

## P.1 ■ Solution

Statement A is correct, because one of the properties of critical flow is that it provides the maximum unit discharge for a given specific energy. Statement C is also true, since the Froude number provides the threshold between subcritical flow ($Fr < 1$) and supercritical flow ($Fr > 1$). Statement D is also true. Finally, statement B is false because there are simple explicit formulas for rectangular, triangular, and parabolic channels, but not for trapezoidal channels, although in recent years authors have devised empirical (e.g., Problem 9) and analytical relations for this purpose.

★ The false statement is **B**.

## P.2 ■ Solution

The critical depth for a rectangular channel is given by the expression

$$y_c = \sqrt[3]{\frac{q^2}{g}}$$

Substituting $q = Q/b$ = 12/4 = 3 m²/s and $g$ = 9.81 m/s², we obtain

$$y_c = \sqrt[3]{\frac{3^2}{9.81}} = \boxed{0.97\text{ m}}$$

Since the depth $y_o$ = 1.2 > $y_c$, we conclude that the flow is subcritical. This can be confirmed if we calculate the Froude number,

$$\text{Fr} = \frac{V}{\sqrt{gy_0}}$$

With $V = 12/(4 \times 1.2)$ = 2.5 m/s and $y_0$ = 1.2 m, it follows that

$$\mathrm{Fr} = \frac{2.5}{\sqrt{9.81 \times 1.2}} = 0.73 < 1$$

Because *Fr* < 1, the flow is indeed subcritical.

★ The correct answer is **B**.

## P.3 ■ Solution

The upstream Froude number can be obtained with the usual formula

$$\mathrm{Fr}_1 = \frac{V}{\sqrt{gy_1}} = \frac{q}{\sqrt{gy_1^3}} = \frac{3.0}{\sqrt{9.81 \times 2.0^3}} = 0.34$$

The specific energy upstream of the sluice gate is

$$E_1 = \frac{q^2}{2gy_1^2} + y_1 = \frac{3.0^2}{2 \times 9.81 \times 2.0^2} + 2.0 = 2.11 \text{ m}$$

The alternate depth can be obtained with the formula

$$y_2 = \frac{2y_1}{\sqrt{1 + \frac{8}{\mathrm{Fr}_1^2}} - 1} = \frac{2 \times 2.0}{\sqrt{1 + \frac{8}{0.34^2}} - 1} = 0.54 \text{ m}$$

The downstream Froude number, in turn, is

$$\mathrm{Fr}_2 = \frac{q}{\sqrt{gy_2^3}} = \frac{3.0}{\sqrt{9.81 \times 0.54^3}} = 2.41$$

Statement C is false because $y_2$ = 0.54 m ≠ 1.08 m.

★ The false statement is **C**.

## P.4 ■ Solution

The specific energy at A is

$$E_A = y_A + \frac{q^2}{2gy_A^2} = 1.5 + \frac{(8/6)^2}{2 \times 9.81 \times (1.5)^2} = 1.54 \text{ m}$$

Because the head loss in the bottom increase is negligible, we can state that the specific energy is going to be reduced by an amount of 0.3 m; that is,

$$E_B = E_A - 0.3 = 1.24 \text{ m}$$

We must then solve the specific energy equation for $y_B$,

$$1.24 = y_B + \frac{1.33^2}{2 \times 9.81 \times y_B^2}$$

which yields $y_B$ = 0.31 m and $y_B$ = 1.18 m. The former is the supercritical solution, whereas the latter is the subcritical solution. Using the specific energy diagram, it can be shown that the flow, which was subcritical before the step, will remain as such *after* the step (see, e.g., Example 2.9 in Akan's textbook). This implies that $y_B$ = 1.18 m is the flow depth after the rise in channel bottom.

★ The correct answer is **D**.

## P.5 ■ Solution

The specific discharges are $q_1$ = 9.0/4.5 = 2 m²/s and $q_2$ = 9.0/3.0 = 3 m²/s. The downstream Froude number is

$$\mathrm{Fr}_2 = \frac{q}{\sqrt{gy_2^3}} = \frac{3.0}{\sqrt{9.81 \times 2.0^3}} = 0.34$$

Assuming that the constriction is energy-conserving, we can state that $E_1 = E_2$ and, accordingly,

$$E_2 = E_1 = \frac{3.0^2}{2 \times 9.81 \times 2.0^2} + 2.0 = 2.11 \text{ m}$$

The flow depth upstream, $y_1$, must satisfy the specific energy equation for the energy calculated above,

$$E_1 = y_1 + \frac{2.0^2}{2 \times 9.81 \times y_1^2} = 2.11$$

which can be solved to give $y_1$ = 0.34 m and $y_2$ = 2.06 m. These two depths are an alternate depth pair in which both depths produce a specific energy of 2.11 m. It can be reasoned that the larger depth, $y_2$ = 2.06 m, will prevail, and subcritical flow will occur. We can use this depth to compute the upstream Froude number, $Fr_1$,

$$\mathrm{Fr}_1 = \frac{q_1}{\sqrt{gy_1^3}} = \frac{2.0}{\sqrt{9.81 \times 2.06^3}} = 0.22$$

Thus, the constriction, much like the upward step in Problem 4, serves to increase the Froude number, in this case from a value of 0.22 to a value of 0.34. In other words, the constriction drives the flow toward critical conditions. Finally, we must determine the absolute change in water depth from location 1 to location 2. This quantity, $\Delta h$, is given by the difference

$$\Delta h = y_2 - y_1 = 2.00 - 2.06 = -0.06 \text{ m} = -6 \text{ cm}$$

Therefore, the flow depth has decreased 6 cm within the constriction. Because $\Delta h = 0.06 \neq 0.12$ m, we conclude that statement C is false.

★ The false statement is **C**.

## P.6 ■ Solution

The flow velocity and specific energy are, respectively,

$$V = \frac{200}{50 \times 4} = 1.0 \text{ m/s}$$

$$E = 4 + \frac{1.0^2}{2 \times 9.81} = 4.05 \text{ m}$$

For the discharge to be maximum at the bridge site for a given upstream specific energy of 4.05 m, the flow should be critical. Hence, the critical depth should be

$$y_c = \frac{2}{3}H = 0.667 \times 4.05 = 2.70 \text{ m}$$

The unit discharge corresponding to this critical depth can be computed with the relation

$$q = \sqrt{gy_c^3} = 13.9 \text{ m}^3\text{/s/m}$$

The width needed for this unit discharge is

$$q = \frac{Q}{b} \rightarrow b = \frac{Q}{q} = \frac{200}{13.9} = \boxed{14.4 \text{ m}}$$

Therefore, the channel width may be reduced from 50 to 14.4 m without affecting the upstream level for a flow of 200 $m^3$/s.

★ The correct answer is **A**.

## P.7 ■ Solution

We begin by finding the hydraulic depth $D$ of the channel, which is equal to the ratio of critical depth to top width,

$$D = \frac{A}{B} = \frac{my_c^2}{2my_c} = \frac{y_c}{2}$$

We then take the root of $D$ and multiply it by the area $A$, giving

$$A\sqrt{D} = my_c^2 \times \sqrt{\frac{y_c}{2}} = \frac{my_c^{2.5}}{\sqrt{2}}$$

The section factor $A\sqrt{D}$ must be equal to the ratio $Q/\sqrt{g}$. Therefore,

$$\frac{m y_c^{\frac{5}{2}}}{\sqrt{2}} = \frac{Q}{\sqrt{g}}$$

$$\therefore \frac{m^2 y_c^5}{2} = \frac{Q^2}{g}$$

$$\therefore y_c^5 = \frac{2Q^2}{gm^2}$$

$$\therefore y_c = \boxed{\left(\frac{2Q^2}{gm^2}\right)^{\frac{1}{5}}}$$

★ The correct answer is **B**.

## P.8 ■ Solution

The critical depth for a specified discharge may be computed from the equation *Fr* = 1, which becomes

$$\frac{V}{\sqrt{gD\cos\theta/\alpha}} = 1$$

Since $Q = VA$, we have

$$\frac{Q/A}{\sqrt{gD\cos\theta/\alpha}} = 1$$

which, given the definition of section factor $Z = A\sqrt{D}$, becomes

$$Z = A\sqrt{D} = \frac{Q/\sqrt{\cos\theta}}{\sqrt{g/\alpha}}$$

For the present channel, we have $Q$ = 45 m³/s. In addition, $g$ = 9.81 m/s², $\cos\theta \approx 1$, and $\alpha \approx 1$. Substituting these data into the right-hand side of the equation, we obtain

$$Z = A\sqrt{D} = \frac{45}{\sqrt{9.81}} = 14.37$$

We can then substitute the data for the geometry of the channel into the left-hand side. Denoting the critical depth as $y_c$, the cross-sectional area $A$ and top width $B$ are (Table 1)

$$A = (10.0 + 2.0y_c)y_c$$

and

$$B = 10.0 + 4.0y_c$$

The hydraulic depth is then

$$D = \frac{A}{B} = \frac{(10.0 + 2.0y_c)y_c}{10.0 + 4.0y_c}$$

Substituting this result into the expression for the section factor $Z$, we get

$$\mathrm{A}\sqrt{\mathrm{D}} = (10.0 + 2.0\mathrm{y}_c)\mathrm{y}_c\sqrt{\frac{(10.0 + 2.0\mathrm{y}_c)\mathrm{y}_c}{10.0 + 4.0\mathrm{y}_c}} = 14.37$$

This intricate equation requires use of trial-and-error to be solved. We can apply the *Solve* function in Mathematica,

$$\text{Solve}\left[(10.+2.*y_c)\sqrt{\frac{(10.+2.y_c)*y_c}{10.+4.y_c}} == 14.37, y_c\right]$$

which returns $y_c$ = 1.50 m.

★ The correct answer is **C**.

## P.9 ■ Solution

Before anything else, we calculate factor $\psi$,

$$\psi = \frac{\alpha Q^2}{g} = \frac{1 \times 20^2}{9.81} = 6.52$$

To verify the validity of the formula for trapezoidal channels, we need to check the value of $Q/b^{2.5}$,

$$\frac{Q}{b^{2.5}} = \frac{20}{5^{2.5}} = 0.143$$

which is greater than 0.1 but less than 4.0, and hence implies that the empirical expression for a trapezoidal cross-section is valid. Applying the relation in question gives

$$y_c = 0.81\left(\frac{6.52}{1 \times 5^{1.25}}\right) - \frac{5}{30 \times 1} = \boxed{0.54\text{ m}}$$

The result is reasonably close to the value obtained via numerical methods, which would yield about $y_c$ = 0.61 m.

★ The correct answer is **A**.

## P.10 ■ Solution

One way to solve this is by postulating that the momentum before the jump must equal the momentum after the jump; that is,

$$M_1 = M_2 \rightarrow \frac{by_1^2}{2} + \frac{Q^2}{g \times by_1} = \frac{6 \times 0.4^2}{2} + \frac{15^2}{9.81 \times 6(0.4)} = 10.04\text{ m}^3$$

$M_1$ must equal the momentum after the jump. Thus,

$$10.04 = \frac{6y_2^2}{2} + \frac{15^2}{9.81 \times 6y_2}$$

which is a cubic equation in $y_2$. Solving it via the *Solve* command in Mathematica, we obtain $y_2$ = –1.996 m, $y_2$ = 0.40 m, and $y_2$ = 1.60 m. The first solution is meaningless, which leaves us with the sequent depths $y_1$ = 0.40 m and $y_2$ = 1.60 m. $y_2$ is the depth after the jump. Another way to find the depth after the jump is to apply the formula

$$y_2 = \frac{y_1}{2}\left(\sqrt{1 + 8\text{Fr}_1^2} - 1\right)$$

where the Froude number $Fr_1$ is determined as

$$\text{Fr}_1 = \frac{q}{\sqrt{gy_1^3}} = \frac{(15/6)}{\sqrt{9.81 \times 0.4^3}} = 3.16$$

Thus,

$$y_2 = \frac{0.4}{2}\left(\sqrt{1 + 8 \times 3.16^2} - 1\right) = \boxed{1.60\text{ m}}$$

This confirms the result obtained via the momentum equation. Then, the energy loss can be calculated with the expression

$$h_L = \frac{(y_2 - y_1)^3}{4y_1y_2} = \frac{(1.60 - 0.40)^3}{4(1.60 \times 0.40)} = \boxed{0.68\text{ m}}$$

★ The correct answer is **D**.

## P.11 ■ Solution

To compute the force that acts on a system, such as the force imparted by the flow on the spillway wall, all we need to do is add a term $F_e/\gamma$ to the momentum conservation equation,

$$\left(\frac{Q^2}{gA_1} + \bar{z}A_1\right) - \frac{F_e}{\gamma} = \left(\frac{Q^2}{gA_2} + \bar{z}A_2\right)$$

Substituting the data we were given, it follows that

$$\left[\frac{15^2}{9.81 \times (2 \times 6)} + 1 \times (2 \times 6)\right] - \frac{F_e}{9.81} = \left[\frac{15^2}{9.81 \times (2 \times 6)} + 0.15 \times (0.3 \times 6)\right]$$

$$\therefore \boxed{F_e = 115 \text{ kN}}$$

The positive value indicates that the assumed direction is correct, that is, the force imparted by the flow onto the spillway points to the right.

★ The correct answer is **C**.

## P.12 ■ Solution

First, we need to calculate the flow depth at section B. The sluice gate applies a force on the flow, causing its momentum to change. We cannot apply the momentum equation, because the equation would have two unknowns – namely, $y_B$ and the force $F_e$. However, we can neglect the energy loss due to the sluice gate and apply the principle of conservation of energy. At section A, the specific energy is given by

$$y_A + \frac{q^2}{2gy_A^2} = 8 + \frac{15^2}{2 \times 9.81 \times 8^2} = 8.18 \text{ m}$$

This value must equal the energy equation at section B, namely

$$8.18 = y_B + \frac{15^2}{2 \times 9.81 \times y_B^2}$$

Solving this equation, we find three possible values of depth: $y_B$ = 8.0 m, $y_B$ = 1.29 m, and $y_B$ = −1.11 m. The latter solution is obviously impossible, leaving us with $y_B$ = 8.0 m, which is the subcritical depth, and $y_B$ = 1.29 m, which is the supercritical depth. Flow after the gate is bound to be supercritical, which implies that $y_B$ = 1.29 m. To see that the flow after the gate is indeed supercritical, we compute the Froude number at section B,

$$\text{Fr}_\text{B} = \frac{15}{\sqrt{9.81 \times 1.29^3}} = 3.27$$

We may then use the relation between sequent depths to obtain the flow depth after the jump,

$$y_C = \frac{y_B}{2}\left(\sqrt{1 + 8\text{Fr}_1^2} - 1\right) = \frac{1.29}{2}\left(\sqrt{1 + 8 \times 3.27^2} - 1\right) = \boxed{5.36 \text{ m}}$$

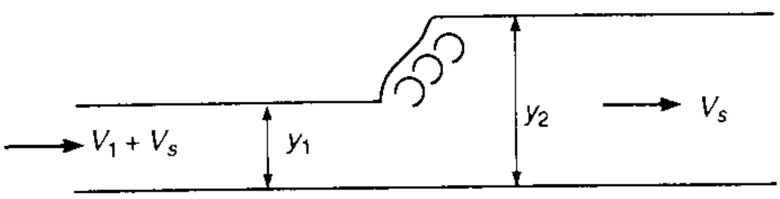
Hence, the flow depth after the jump is $y_C$ = 5.36 m. To obtain the head loss, we appeal to the usual relation

$$h_L = \frac{(y_2 - y_1)^3}{4y_1y_2} = \frac{(5.36 - 1.29)^3}{4 \times 1.29 \times 5.36} = \boxed{2.44 \text{ m}}$$

★ The correct answer is **C**.

## P.13 ■ Solution

Although the discussion of a surge problem belongs in the domain of unsteady flow, this type of system can be analyzed with principles of steady flow by making it stationary, as illustrated below.




In this transformation, a surge velocity $V_S$ is superimposed to the right so that the surge becomes stationary. From this viewpoint, which is that of an observer moving at the speed of the surge, the problem becomes simply the steady flow formation of a hydraulic jump. By making the surge stationary, the steady-flow form of the continuity and momentum equations can be applied to the previous figure. The continuity equation for a rectangular channel of unit width is

$$(V_1 + V_s)y_1 = (V_2 + V_s)y_2$$

which can be rearranged to give

$$V_s = \frac{V_1 y_1 - V_2 y_2}{y_2 - y_1} \quad \text{(I)}$$

In this form, the continuity equation states that the net flow rate through the surge is given by the rate of volume increase effected by the surge movement. The momentum equation written for the stationary surge becomes

$$\frac{(V_1 + V_s)^2}{g y_1} = \frac{1}{2}\frac{y_2}{y_1}\left(1 + \frac{y_2}{y_1}\right) \quad \text{(II)}$$

which is of the same form as the hydraulic jump equation except that the velocity of flow, $V_1$, has been replaced by ($V_1$ + $V_s$). In the present case, we have $y_1$ = 3.5 m, $V_1$ = 2.0 m/s, and $V_2$ = 0. Substituting these data into Equations (I) and (II), we have

$$V_s = \frac{3.5 \times 2}{y_2 - 3.5}$$

and

$$\frac{(2 + V_s)^2}{9.81 \times 3.5} = \frac{1}{2}\frac{y_2}{3.5}\left(1 + \frac{y_2}{3.5}\right)$$

These equations can be solved simultaneously for $V_s$ and $y_2$, for example, by using the Mathematica code

$$\text{Solve}\left[\left\{V_s == \frac{3.5 * 2}{y_2 - 3.5}, \frac{(1 + V_s)^2}{9.81 * 3.5} == \frac{1}{2} * \frac{y_2}{3.5} * \left(1 + \frac{y_2}{3.5}\right)\right\}, \{V_s, y_2\}\right]$$

which yields $y_2 = 4.8$ m and $V_s$ = 5.5 m/s. The latter is the speed that would be seen by a stationary observer.

★ The correct answer is **A**.

## Answer Summary

| | |
|---|---|
| Problem 1 | B |
| Problem 2 | B |
| Problem 3 | C |
| Problem 4 | D |
| Problem 5 | C |
| Problem 6 | A |
| Problem 7 | B |
| Problem 8 | C |
| Problem 9 | A |
| Problem 10 | D |
| Problem 11 | C |
| Problem 12 | C |
| Problem 13 | A |

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