Quiz SM209

STATICALLY INDETERMINATE BEAMS

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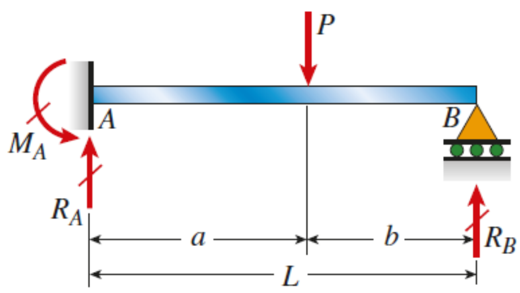
PROBLEMS

Problem 1 (Gere & Goodno, 2009, w/ permission)

A propped cantilever beam *AB* of length *L* carries a concentrated load *P* acting at the position shown in the figure. Solve for all reactions. What is the value of M_A?



A) $M_A = \frac{Pab}{2L^2}(L+a)$

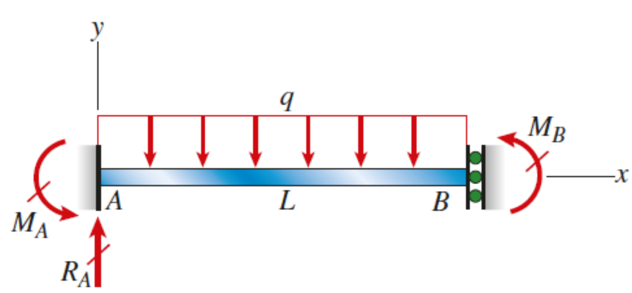
B) $M_A = \frac{Pab}{2L^2}(L+b)$

C) $M_A = \frac{Pab}{L^2}(L+a)$

D) $M_A = \frac{Pab}{L^2}(L+b)$

Problem 2 (Gere & Goodno, 2009, w/ permission)

A beam with a guided support at *B* is loaded by a uniformly distributed load with intensity *q*. Solve for all reactions. What is the value of moment reaction M_A?



A) $M_A = \frac{qL^2}{12}$

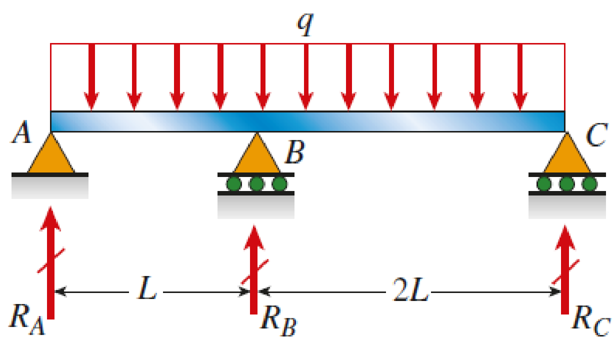
B) $M_A = \frac{qL^2}{6}$

C) $M_A = \frac{qL^2}{3}$

D) $M_A = \frac{qL^2}{2}$

Problem 3 (Gere & Goodno, 2009, w/ permission)

A continuous beam *ABC* with two unequal spans, one of length *L* and one of length 2*L*, supports a uniform load of intensity *q* (see figure). Solve for all reactions. What is the value of R_C?



A) $R_C = \dfrac{13qL}{16}$

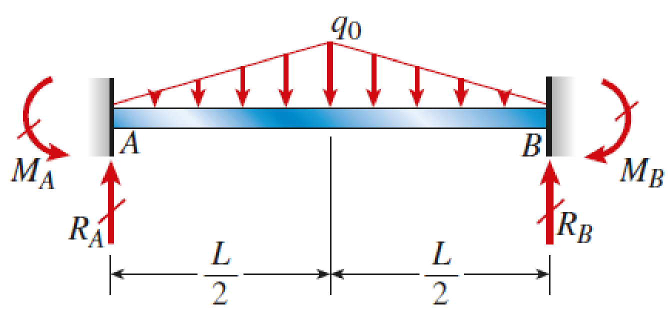
B) $R_C = \dfrac{19qL}{16}$

C) $R_C = \dfrac{23qL}{16}$

D) $R_C = \dfrac{33qL}{16}$

Problem 4 (Gere & Goodno, 2009, w/ permission)

The beam shown supports a triangular load of maximum intensity q_0. Solve for all reactions. What is the value of M_B?



A) $M_B = \dfrac{q_0L^2}{96}$

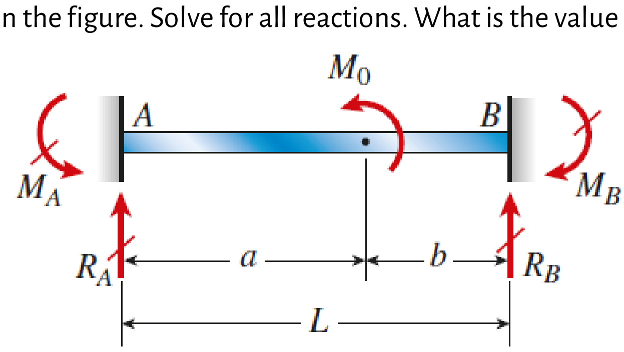
B) $M_B = \dfrac{q_0L^2}{32}$

C) $M_B = \dfrac{5q_0L^2}{96}$

D) $M_B = \dfrac{7q_0L^2}{96}$

Problem 5 (Gere & Goodno, 2009, w/ permission)

A fixed-end beam *AB* of length *L* is subjected to a moment M_0 acting at the position shown in the figure. Solve for all reactions. What is the value of M_A?



A) $M_A = \dfrac{M_0a}{2L^2}(3b - L)$

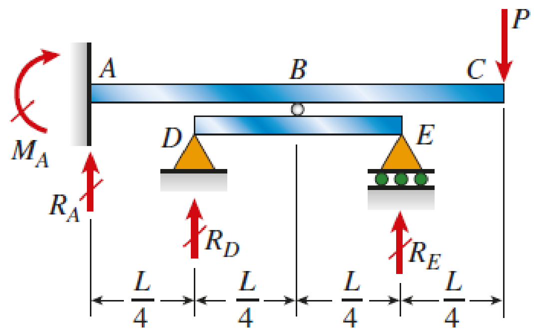
B) $M_A = \dfrac{M_0b}{2L^2}(3a - L)$

C) $M_A = \dfrac{M_0a}{L^2}(3b - L)$

D) $M_A = \dfrac{M_0b}{L^2}(3a - L)$

Problem 6 (Gere & Goodno, 2009, w/ permission)

A beam ABC is fixed at end *A* and supported by beam *DE* at point *B*. Both beams have the same cross-section and are made of the same material. Solve for all reactions. What is the value of M_A?



A) $M_A = \frac{3PL}{17}$

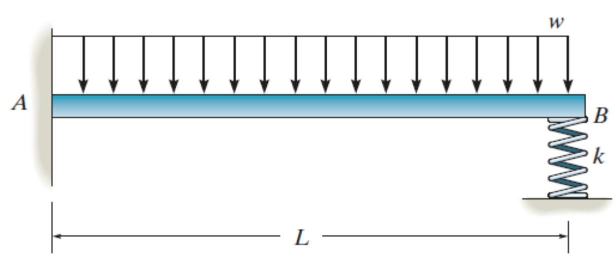
B) $M_A = \frac{5PL}{17}$

C) $M_A = \frac{7PL}{17}$

D) $M_A = \frac{9PL}{17}$

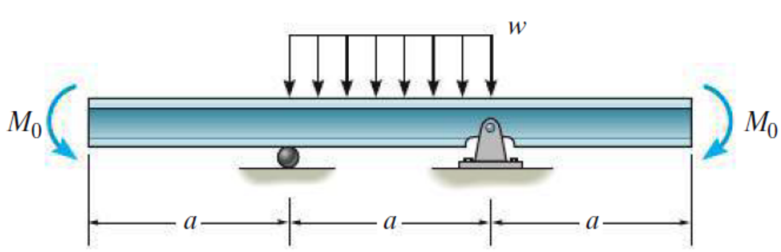
Problem 7 (Hibbeler, 2014, w/ permission)

Determine the force in the spring. *EI* is constant.



Problem 8 (Hibbeler, 2014, w/ permission)

Determine the magnitude of M_0 in terms of the distributed load *w* and dimension *a* so that the deflection at the center of the beam is zero. *EI* is constant.



A) $M_0 = \frac{wa^2}{16}$

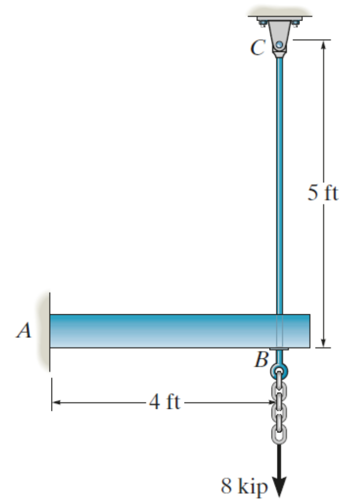
B) $M_0 = \frac{5wa^2}{48}$

C) $M_0 = \frac{7wa^2}{48}$

D) $M_0 = \frac{3wa^2}{16}$

Problem 9 (Hibbeler, 2014, w/ permission)

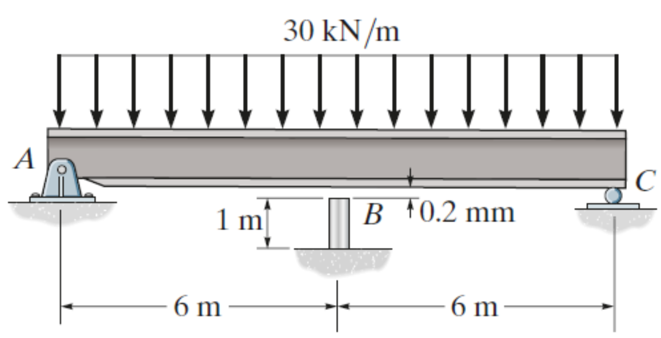
The steel beam and rod ($E = 29$ ksi) are used to support the load of 8 kip. If it is required that the allowable normal stress for the steel is $\sigma_{allow} = 18$ ksi, and the maximum deflection should not exceed 0.05 in., determine the smallest diameter rod that should be used. The beam is rectangular, having a height of 5 in. and a thickness of 3 in.



A) $d = 0.254$ in.
B) $d = 0.707$ in.
C) $d = 1.21$ in.
D) $d = 1.72$ in.

Problem 10 (Hibbeler, 2014, w/ permission)

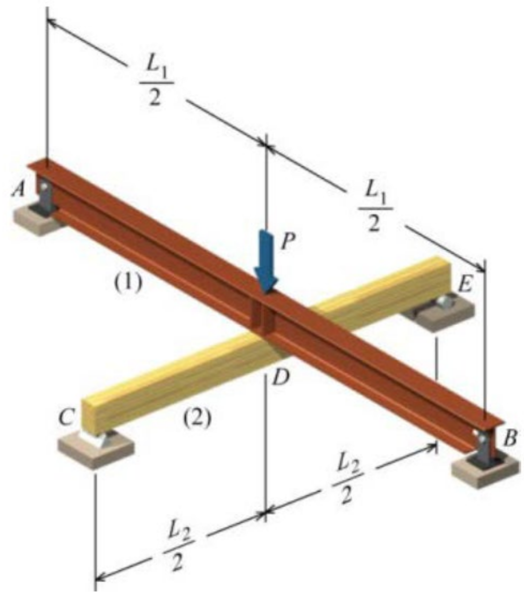
Before the uniform load is applied on the beam, there is a small gap of 0.2 mm between the beam and the post at *B*. The post at *B* has a diameter of 40 mm, and the moment of inertia of the beam is $I = 875(10^6)$ mm^4. The post and the beam are made of material having a modulus of elasticity of $E = 200$ GPa. Determine the support reactions at *A*, *B*, and *C*. What is the value of the vertical reaction at *A*?



A) $A_y = 70$ kN
B) $A_y = 100$ kN
C) $A_y = 130$ kN
D) $A_y = 160$ kN

Problem 11 (Philpot, 2013, w/ permission)

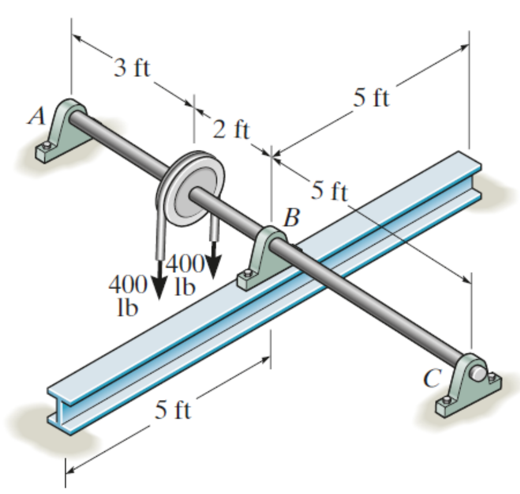
Steel beam (1) carries a concentrated load of $P = 13$ kips that is applied at midspan, as shown in the next figure. The steel beam is supported at ends *A* and *B* by nondeflecting supports and at its middle by simply supported timber beam (2). In the unloaded condition, steel beam (1) touches but exerts no force on timber beam (2). The length of the steel beam is $L_1 = 30$ ft, and its flexural rigidity is $EI_1 = 7.2\times10^6$ kip-in.^{2}. The length and the flexural rigidity of the timber beam are $L_2 = 2.0$ ft and $EI_2 = 1.0\times10^6$ kip-in.^{2}, respectively. Determine the vertical reaction force that acts on the steel beam at *A* and on the timber beam at *C*.



A) $|A_y| = 1.79$ kips and $|C_y| = 2.08$ kips
B) $|A_y| = 1.79$ kips and $|C_y| = 5.22$ kips
C) $|A_y| = 4.43$ kips and $|C_y| = 2.08$ kips
D) $|A_y| = 4.43$ kips and $|C_y| = 5.22$ kips

Problem 12 (Hibbeler, 2014, w/ permission)

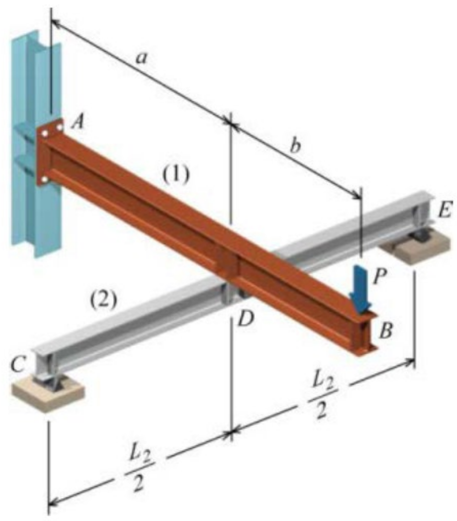
The 1-in. diameter steel shaft is supported by bearings at *A* and *C*. The bearing at *B* rests on a simply supported steel wide flange beam having a moment of inertia of $I = 500$ in.^{4}. If the belt loads on the pulley are 400 lb each, determine the vertical reactions at *A*, *B* and *C*. What are the values of vertical reactions A_y and C_y?



A) $|A_y| = 243$ lb and $|C_y| = 78$ lb
B) $|A_y| = 243$ lb and $|C_y| = 188$ lb
C) $|A_y| = 391$ lb and $|C_y| = 78$ lb
D) $|A_y| = 391$ lb and $|C_y| = 188$ lb

Problem 13 (Philpot, 2013, w/ permission)

Two steel beams support a concentrated load of P =45 kN, as shown. Beam (1) is supported by a fixed support at A and by a simply supported beam (2) at D. In the unloaded condition, beam (1) touches but exerts no force on beam (2). The beam lengths are a = 4.0 m, b = 1.5 m, and L_2 = 6 m. The flexural rigidities of the beams are E_1I_1 = 40,000 kN·m² and E_2I_2 = 14,000 kN·m². Determine the deflections of beam (1) at D and at B.



A) $|\delta_D|$= 7.0 mm and $|\delta_B|$ = 25.9 mm
B) $|\delta_D|$ = 7.0 mm and $|\delta_B|$ = 40.2 mm
C) $|\delta_D|$ = 14.1 mm and $|\delta_B|$ = 25.9 mm
D) $|\delta_D|$ = 14.1 mm and $|\delta_B|$ = 40.2 mm

ADDITIONAL INFORMATION

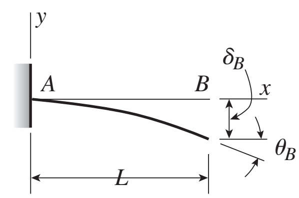
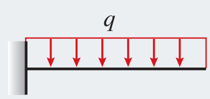
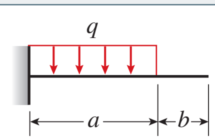
The following pages are taken from Gere and Goodno's *Mechanics of Materials*, 8th edition, reproduced with permission of Cengage Learning, 200 First Stamford Place, Suite 400, Stamford, CT 06902, USA).

APPENDIX G

Deflections and Slopes of Beams

Table G-1

Deflections and Slopes of Cantilever Beams

		v = deflection in the y direction (positive upward) $v' = dv/dx$ = slope of the deflection curve $\delta_B = -v(L)$ = deflection at end B of the beam (positive downward) $\theta_B = -v'(L)$ = angle of rotation at end B of the beam (positive clockwise) EI = constant
1		$v = -\dfrac{qx^2}{24EI}(6L^2 - 4Lx + x^2) \qquad v' = -\dfrac{qx}{6EI}(3L^2 - 3Lx + x^2)$ $\delta_B = \dfrac{qL^4}{8EI} \qquad \theta_B = \dfrac{qL^3}{6EI}$
2		$v = -\dfrac{qx^2}{24EI}(6a^2 - 4ax + x^2) \qquad (0 \le x \le a)$ $v' = -\dfrac{qx}{6EI}(3a^2 - 3ax + x^2) \qquad (0 \le x \le a)$ $v = -\dfrac{qa^3}{24EI}(4x - a) \qquad v' = -\dfrac{qa^3}{6EI} \qquad (a \le x \le L)$ $\text{At } x = a\text{:} \; v = -\dfrac{qa^4}{8EI} \qquad v' = -\dfrac{qa^3}{6EI}$ $\delta_B = \dfrac{qa^3}{24EI}(4L - a) \qquad \theta_B = \dfrac{qa^3}{6EI}$

(*Continued*)

Table G-1 (Continued)

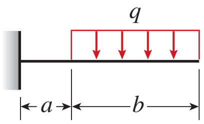
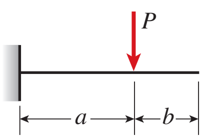
3		$v = -\frac{qbx^2}{12EI}(3L + 3a - 2x) \quad (0 \le x \le a)$ $v' = -\frac{qbx}{2EI}(L + a - x) \quad (0 \le x \le a)$ $v = -\frac{q}{24EI}(x^4 - 4Lx^3 + 6L^2x^2 - 4a^3x + a^4) \quad (a \le x \le L)$ $v' = -\frac{q}{6EI}(x^3 - 3Lx^2 + 3L^2x - a^3) \quad (a \le x \le L)$ At $x = a$: $v = -\frac{qa^2b}{12EI}(3L + a) \quad v' = -\frac{qabL}{2EI}$ $\delta_B = \frac{q}{24EI}(3L^4 - 4a^3L + a^4) \quad \theta_B = \frac{q}{6EI}(L^3 - a^3)$
4		$v = -\frac{Px^2}{6EI}(3L - x) \quad v' = -\frac{Px}{2EI}(2L - x)$ $\delta_B = \frac{PL^3}{3EI} \quad \theta_B = \frac{PL^2}{2EI}$
5		$v = -\frac{Px^2}{6EI}(3a - x) \quad v' = -\frac{Px}{2EI}(2a - x) \quad (0 \le x \le a)$ $v = -\frac{Pa^2}{6EI}(3x - a) \quad v' = -\frac{Pa^2}{2EI} \quad (a \le x \le L)$ At $x = a$: $v = -\frac{Pa^3}{3EI} \quad v' = -\frac{Pa^2}{2EI}$ $\delta_B = \frac{Pa^2}{6EI}(3L - a) \quad \theta_B = \frac{Pa^2}{2EI}$
6		$v = -\frac{M_0x^2}{2EI} \quad v' = -\frac{M_0x}{EI}$ $\delta_B = \frac{M_0L^2}{2EI} \quad \theta_B = \frac{M_0L}{EI}$

Table G-1 (Continued)

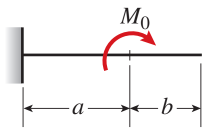
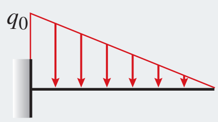
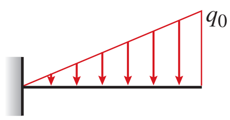
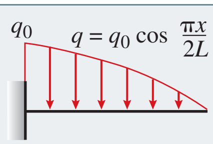
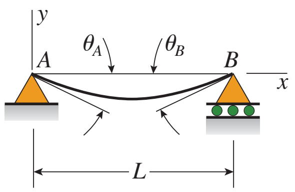
7	 	$v = -\frac{M_0x^2}{2EI} \qquad v' = -\frac{M_0x}{EI} \qquad (0 \le x \le a)$ $v = -\frac{M_0a}{2EI}(2x - a) \qquad v' = -\frac{M_0a}{EI} \qquad (a \le x \le L)$ At $x = a$: $\quad v = -\frac{M_0a^2}{2EI} \qquad v' = -\frac{M_0a}{EI}$ $\delta_B = \frac{M_0a}{2EI}(2L - a) \qquad \theta_B = \frac{M_0a}{EI}$
8	 	$v = -\frac{q_0x^2}{120LEI}(10L^3 - 10L^2x + 5Lx^2 - x^3)$ $v' = -\frac{q_0x}{24LEI}(4L^3 - 6L^2x + 4Lx^2 - x^3)$ $\delta_B = \frac{q_0L^4}{30EI} \qquad \theta_B = \frac{q_0L^3}{24EI}$
9	 	$v = -\frac{q_0x^2}{120LEI}(20L^3 - 10L^2x + x^3)$ $v' = -\frac{q_0x}{24LEI}(8L^3 - 6L^2x + x^3)$ $\delta_B = \frac{11q_0L^4}{120EI} \qquad \theta_B = \frac{q_0L^3}{8EI}$
10	 	$v = -\frac{q_0L}{3\pi^4EI}\left(48L^3 \cos \frac{\pi x}{2L} - 48L^3 + 3\pi^3Lx^2 - \pi^3x^3\right)$ $v' = -\frac{q_0L}{\pi^3EI}\left(2\pi^2Lx - \pi^2x^2 - 8L^2 \sin \frac{\pi x}{2L}\right)$ $\delta_B = \frac{2q_0L^4}{3\pi^4EI}(\pi^3 - 24) \qquad \theta_B = \frac{q_0L^3}{\pi^3EI}(\pi^2 - 8)$

Table G-2

Deflections and Slopes of Simple Beams



$v =$ deflection in the y direction (positive upward)

$v' = dv/dx =$ slope of the deflection curve

$\delta_C = -v(L/2) =$ deflection at midpoint C of the beam (positive downward)

$x_1 =$ distance from support A to point of maximum deflection

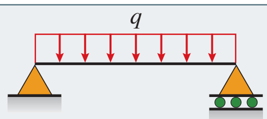
$\delta_{\text{max}} = -v_{\text{max}} =$ maximum deflection (positive downward)

$\theta_A = -v'(0) =$ angle of rotation at left-hand end of the beam (positive clockwise)

$\theta_B = v'(L) =$ angle of rotation at right-hand end of the beam (positive counterclockwise)

$EI =$ *constant*

1

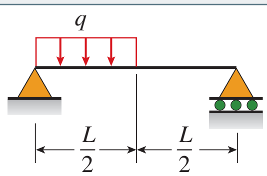


$$v = -\frac{qx}{24EI}(L^3 - 2Lx^2 + x^3)$$

$$v' = -\frac{q}{24EI}(L^3 - 6Lx^2 + 4x^3)$$

$$\delta_C = \delta_{\text{max}} = \frac{5qL^4}{384EI} \qquad \theta_A = \theta_B = \frac{qL^3}{24EI}$$

2



$$v = -\frac{qx}{384EI}(9L^3 - 24Lx^2 + 16x^3) \qquad \left(0 \le x \le \frac{L}{2}\right)$$

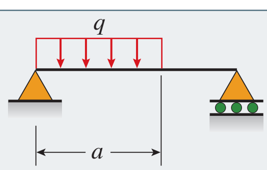
$$v' = -\frac{q}{384EI}(9L^3 - 72Lx^2 + 64x^3) \qquad \left(0 \le x \le \frac{L}{2}\right)$$

$$v = -\frac{qL}{384EI}(8x^3 - 24Lx^2 + 17L^2x - L^3) \qquad \left(\frac{L}{2} \le x \le L\right)$$

$$v' = -\frac{qL}{384EI}(24x^2 - 48Lx + 17L^2) \qquad \left(\frac{L}{2} \le x \le L\right)$$

$$\delta_C = \frac{5qL^4}{768EI} \qquad \theta_A = \frac{3qL^3}{128EI} \qquad \theta_B = \frac{7qL^3}{384EI}$$

3



$$v = -\frac{qx}{24LEI}(a^4 - 4a^3L + 4a^2L^2 + 2a^2x^2 - 4aLx^2 + Lx^3) \qquad (0 \le x \le a)$$

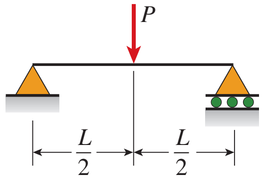
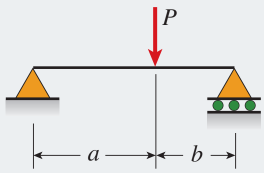
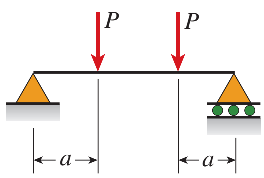
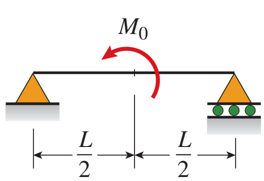
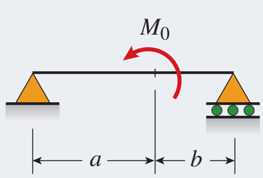
$$v' = -\frac{q}{24LEI}(a^4 - 4a^3L + 4a^2L^2 + 6a^2x^2 - 12aLx^2 + 4Lx^3) \qquad (0 \le x \le a)$$

$$v = -\frac{qa^2}{24LEI}(-a^2L + 4L^2x + a^2x - 6Lx^2 + 2x^3) \qquad (a \le x \le L)$$

$$v' = -\frac{qa^2}{24LEI}(4L^2 + a^2 - 12Lx + 6x^2) \qquad (a \le x \le L)$$

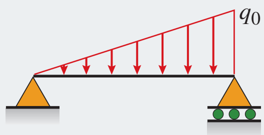
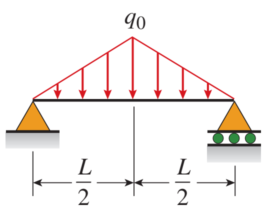
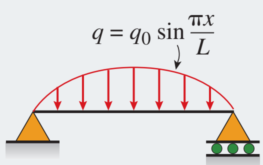
$$\theta_A = \frac{qa^2}{24LEI}(2L - a)^2 \qquad \theta_B = \frac{qa^2}{24LEI}(2L^2 - a^2)$$

Table G-2 (Continued)

4		$v = -\frac{Px}{48EI}(3L^2 - 4x^2) \quad v' = -\frac{P}{16EI}(L^2 - 4x^2) \quad \left(0 \le x \le \frac{L}{2}\right)$ $\delta_C = \delta_{max} = \frac{PL^3}{48EI} \quad \theta_A = \theta_B = \frac{PL^2}{16EI}$
5		$v = -\frac{Pbx}{6LEI}(L^2 - b^2 - x^2) \quad v' = -\frac{Pb}{6LEI}(L^2 - b^2 - 3x^2) \quad (0 \le x \le a)$ $\theta_A = \frac{Pab(L + b)}{6LEI} \quad \theta_B = \frac{Pab(L + a)}{6LEI}$ If $a \ge b,\ \delta_C = \frac{Pb(3L^2 - 4b^2)}{48EI}$ If $a \le b,\ \delta_C = \frac{Pa(3L^2 - 4a^2)}{48EI}$ If $a \ge b,\ x_1 = \sqrt{\frac{L^2 - b^2}{3}}$ and $\delta_{max} = \frac{Pb(L^2 - b^2)^{3/2}}{9\sqrt{3}LEI}$
6		$v = -\frac{Px}{6EI}(3aL - 3a^2 - x^2) \quad v' = -\frac{P}{2EI}(aL - a^2 - x^2) \quad (0 \le x \le a)$ $v = -\frac{Pa}{6EI}(3Lx - 3x^2 - a^2) \quad v' = -\frac{Pa}{2EI}(L - 2x) \quad (a \le x \le L - a)$ $\delta_C = \delta_{max} = \frac{Pa}{24EI}(3L^2 - 4a^2) \quad \theta_A = \theta_B = \frac{Pa(L - a)}{2EI}$
7		$v = -\frac{M_0x}{6LEI}(2L^2 - 3Lx + x^2) \quad v' = -\frac{M_0}{6LEI}(2L^2 - 6Lx + 3x^2)$ $\delta_C = \frac{M_0L^2}{16EI} \quad \theta_A = \frac{M_0L}{3EI} \quad \theta_B = \frac{M_0L}{6EI}$ $x_1 = L\left(1 - \frac{\sqrt{3}}{3}\right)$ and $\delta_{max} = \frac{M_0L^2}{9\sqrt{3}EI}$
8		$v = -\frac{M_0x}{24LEI}(L^2 - 4x^2) \quad v' = -\frac{M_0}{24LEI}(L^2 - 12x^2) \quad \left(0 \le x \le \frac{L}{2}\right)$ $\delta_C = 0 \quad \theta_A = \frac{M_0L}{24EI} \quad \theta_B = -\frac{M_0L}{24EI}$
9		$v = -\frac{M_0x}{6LEI}(6aL - 3a^2 - 2L^2 - x^2) \quad (0 \le x \le a)$ $v' = -\frac{M_0}{6LEI}(6aL - 3a^2 - 2L^2 - 3x^2) \quad (0 \le x \le a)$ At $x = a$: $v = \frac{M_0ab}{3LEI}(2a - L) \quad v' = -\frac{M_0}{3LEI}(3aL - 3a^2 - L^2)$ $\theta_A = \frac{M_0}{6LEI}(6aL - 3a^2 - 2L^2) \quad \theta_B = \frac{M_0}{6LEI}(3a^2 - L^2)$

(Continued)

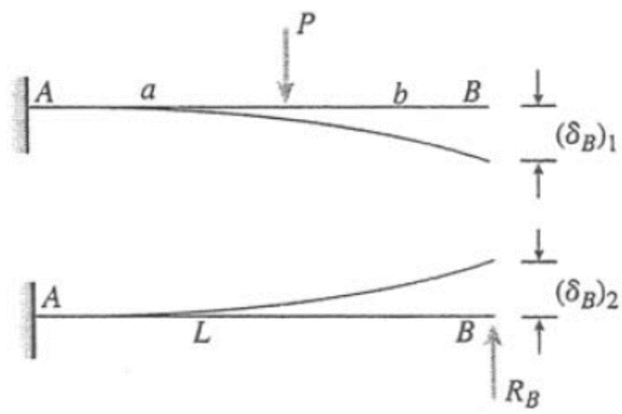
Table G-2 (Continued)

10		$v = -\frac{M_0 x}{2EI}(L - x) \quad v' = -\frac{M_0}{2EI}(L - 2x)$ $\delta_C = \delta_{max} = \frac{M_0 L^2}{8EI} \quad \theta_A = \theta_B = \frac{M_0 L}{2EI}$
11		$v = -\frac{q_0 x}{360LEI}(7L^4 - 10L^2x^2 + 3x^4)$ $v' = -\frac{q_0}{360LEI}(7L^4 - 30L^2x^2 + 15x^4)$ $\delta_C = \frac{5q_0L^4}{768EI} \quad \theta_A = \frac{7q_0L^3}{360EI} \quad \theta_B = \frac{q_0L^3}{45EI}$ $x_1 = 0.5193L \quad \delta_{max} = 0.00652\frac{q_0L^4}{EI}$
12		$v = -\frac{q_0 x}{960LEI}(5L^2 - 4x^2)^2 \quad \left(0 \le x \le \frac{L}{2}\right)$ $v' = -\frac{q_0}{192LEI}(5L^2 - 4x^2)(L^2 - 4x^2) \quad \left(0 \le x \le \frac{L}{2}\right)$ $\delta_C = \delta_{max} = \frac{q_0L^4}{120EI} \quad \theta_A = \theta_B = \frac{5q_0L^3}{192EI}$
13		$v = -\frac{q_0L^4}{\pi^4EI}\sin\frac{\pi x}{L} \quad v' = -\frac{q_0L^3}{\pi^3EI}\cos\frac{\pi x}{L}$ $\delta_C = \delta_{max} = \frac{q_0L^4}{\pi^4EI} \quad \theta_A = \theta_B = \frac{q_0L^3}{\pi^3EI}$

SOLUTIONS

P.1 ➡ *Solution*

The system has three reactions, but we have only two equations of equilibrium. This means that the system is statically indeterminate with a degree of static indeterminacy of one. Suppose R_B is the redundant. The released structure is a cantilever beam, as illustrated below. $(\delta_B)_1$ is the deflection of point *B* due to concentrated load *P*, while $(\delta_B)_2$ is the deflection due to the redundant reaction R_B.



Summing forces in the *y*-direction, we have

$$\Sigma F_y = 0 \rightarrow R_A + R_B = P$$

$$\therefore R_A = P - R_B \quad \text{(I)}$$

Taking moments about fixed end *A*, we see that

$$\Sigma M_A = 0 \rightarrow M_A + R_B L - Pa = 0$$

$$\therefore M_A = Pa - R_B L \quad \text{(II)}$$

Deflection $(\delta_B)_1$ is taken from case 5 of Table G-1,

$$v = -\frac{Pa^2}{6EI}(3x - a) \rightarrow (\delta_B)_1 = \frac{Pa^2}{6EI}(3L - a)$$

Deflection $(\delta_B)_2$ can be obtained from case 4 of Table G-1,

$$v = -\frac{Px^2}{6EI}(3L - x) \rightarrow (\delta_B)_2 = \frac{R_B L^2}{6EI}(3L - L) = \frac{R_B L^3}{3EI}$$

Since point *B* is fixed, the deflection therein must equal zero, or, in mathematical terms,

$$\delta_B = 0 \rightarrow -(\delta_B)_1 + (\delta_B)_2 = 0$$

$$\therefore -\frac{Pa^2}{6\cancel{EI}}(3L - a) + \frac{R_B L^3}{3\cancel{EI}} = 0$$

$$\therefore -\frac{Pa^2(3L - a)}{6} + \frac{R_B L^3}{3} = 0$$

$$\therefore R_B = \frac{Pa^2}{2L^3}(3L - a)$$

Once the redundant reaction has been determined, the remaining forces easily follow. Substituting R_B in equation (I) and manipulating, we have

$$R_A = P - \frac{Pa^2}{2L^3}(3L-a)$$

$$\therefore R_A = \frac{P}{2L^3}\left[2L^3 - a^2(3L-a)\right]$$

$$\therefore R_A = \frac{P}{2L^3}\left[3L^3 - 3a^2L + a^3\right]$$

$$\therefore R_A = \frac{P}{2L^3}\left[3L\left(L^2-a^2\right) - \left(L^3-a^3\right)\right]$$

$$\therefore R_A = \frac{P}{2L^3}\left[3L(L-a)(L+a) - (L-a)\left(L^2+La+a^2\right)\right]$$

However, $L - a = b$,

$$R_A = \frac{Pb}{2L^3}\left[3L(L+a) - \left(L^2+La+a^2\right)\right]$$

$$\therefore R_A = \frac{Pb}{2L^3}\left(3L^2+3La-L^2-La-a^2\right)$$

$$\therefore R_A = \frac{Pb}{2L^3}\left[2L^2+La+a(L-a)\right]$$

$$\therefore R_A = \frac{Pb}{2L^3}\left[2L^2+(a+b)a+ab\right]$$

$$\therefore R_A = \frac{Pb}{2L^3}\left[2L^2+(a+b)^2-b^2\right]$$

$$\therefore R_A = \frac{Pb}{2L^3}\left(3L^2-b^2\right)$$

Likewise, we can substitute R_B in equation (II) to determine moment M_A,

$$M_A = Pa - \frac{Pa^2}{2L^2}(3L-a)$$

$$\therefore M_A = \frac{Pa}{2L^2}\left(2L^2-3aL+a^2\right)$$

$$\therefore M_A = \frac{Pa}{2L^2}\left[3L(L-a)-\left(L^2-a^2\right)\right]$$

$$\therefore M_A = \frac{Pa}{2L^2}\left[3L-(L+a)\right](L-a)$$

Noting that $L - a = b$ as before, we get

$$M_A = \frac{Pab}{2L^2}\left[3L-(L+a)\right]$$

$$\therefore M_A = \frac{Pab}{2L^2}(2L+a)$$

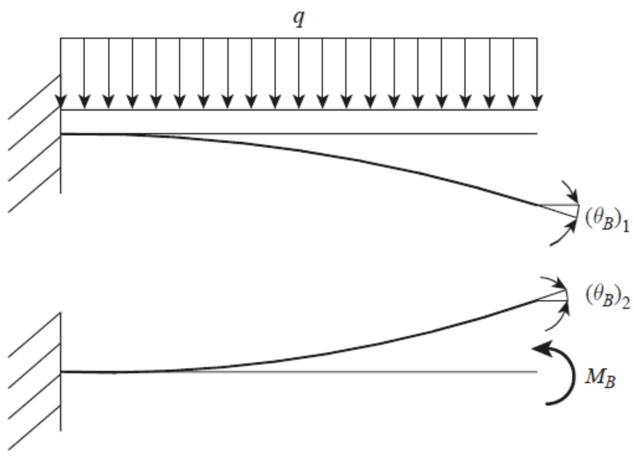
$$\therefore M_A = \frac{Pab}{2L^2}\left[L+(a+b)-a\right]$$

$$\therefore \boxed{M_A = \frac{Pab}{2L^2}(L+b)}$$

The correct answer is **B**.

P.2 ➡ *Solution*

The system has three reactions, but we have only two equations of equilibrium. As a result, the system is statically indeterminate and has a degree of static indeterminacy of one. Let us choose moment M_B as the redundant load. The released structure is a cantilever beam, as shown. $(\theta_B)_1$ is the rotation of point B due to uniform load q, while $(\theta_B)_2$ is the rotation due to the redundant moment M_B.



Summing forces in the *y*-direction, we obtain

$$\Sigma F_y = 0 \rightarrow R_A - qL = 0$$

$$\therefore R_A = qL \ \ \text{(I)}$$

Taking moments about point *B*, in turn, we have

$$\Sigma M_A = 0 \rightarrow M_A - \frac{qL^2}{2} + M_B = 0$$

$$\therefore M_A = \frac{qL^2}{2} - M_B \ \ \text{(II)}$$

The rotation due to the uniform load can be obtained from case 1 of Table G-1,

$$(\theta_B)_1 = \frac{qL^3}{6EI}$$

The rotation due to the redundant moment can be obtained from case 6 of Table G-1,

$$(\theta_B)_2 = \frac{M_B L}{EI}$$

Since point *B* is fixed with a roller support, the angle of rotation therein must equal zero, which leads to the compatibility equation

$$\theta_A = 0 \rightarrow (\theta_A)_1 = (\theta_A)_2$$

Substituting the relations for the angles obtained just now and solving for M_B, we get

$$(\theta_A)_1 = (\theta_A)_2 \rightarrow \frac{qL^3}{6\cancel{EI}} = \frac{M_B L}{\cancel{EI}}$$

$$\therefore M_B = \frac{qL^2}{6}$$

Having determined the redundant moment, we can easily compute the remaining forces. Reaction R_A has already been established to be equal to qL. Substituting M_B in equation (II), in turn, we get

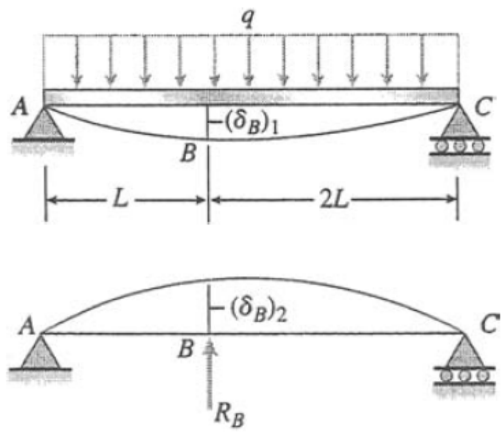
$$M_A = \frac{qL^2}{2} - \frac{qL^2}{6} = \boxed{\frac{qL^2}{3}}$$

Note that the moment reaction at support *A* is twice as large as the moment in support *B*.

The correct answer is **C**.

P.3 ➡ *Solution*

The system has three reactions, but we have only two equations of equilibrium. Consequently, the system is statically indeterminate with a degree of static indeterminacy of one. Let R_B be the redundant load. The released structure is a simple beam, as shown. $(\delta_B)_1$ is the deflection of point *B* due to uniform load q, while $(\delta_B)_2$ is the deflection due to the redundant reaction R_B.



Taking moments about point *C*, we have

$$\Sigma M_C = 0 \rightarrow R_A \times 3L = \frac{q \times (3L)^2}{2} - R_B \times 2L$$

$$\therefore R_A = \frac{3qL}{2} - \frac{2R_B}{3} \quad \text{(I)}$$

Taking moments about point *A*, in turn, we have

$$\Sigma M_A = 0 \rightarrow R_C \times 3L + R_B \times L - \frac{q \times (3L)^2}{2} = 0$$

$$\therefore R_C = \frac{3qL}{2} - \frac{R_B}{3} \quad \text{(II)}$$

The deflection due to the uniform load can be obtained from case 1 of Table G-2.

$$v = -\frac{qx}{24EI}\left[(3L)^3 - 2(3L)x^2 + x^3\right] = -\frac{qx}{24EI}\left(27L^3 - 6Lx^2 + x^3\right)$$

Substituting $x = L$, it follows that

$$(\delta_B)_1 = \frac{q \times L}{24EI} \times \left(27L^3 - 6L \times L^2 + L^3\right) = \frac{11qL^4}{12EI}$$

The deflection due to the redundant load can be obtained from case 5 of Table G-2,

$$v = \frac{P(3L - x)x}{6 \times 3L \times EI} \times \left[(3L)^2 - x^2 - (3L - x)^2\right]$$

With $x = L$ and $P = R_B$, it follows that

$$(\delta_B)_2 = \frac{R_B \times (3L - L) \times L}{18LEI} \times \left[(3L)^2 - L^2 - (3L - L)^2\right]$$

$$\therefore (\delta_B)_2 = \frac{R_B L}{9EI} \times \left(9L^2 - L^2 - 4L^2\right) = \frac{4R_B L^3}{9EI}$$

Since point *B* is fixed, the deflection therein must equal zero, or, in mathematical terms,

$$\delta_B = 0 \rightarrow -(\delta_B)_1 + (\delta_B)_2 = 0$$

$$\therefore -\frac{11qL^4}{12EI} + \frac{4R_BL^3}{9EI} = 0$$

$$\therefore \frac{11qL^4}{12\cancel{EI}} = \frac{4R_BL^3}{9\cancel{EI}}$$

$$\therefore R_B = \frac{33qL}{16}$$

Having determined the redundant reaction, we can easily establish the remaining forces. Substituting R_B in equations (I) and (II) gives

$$R_A = \frac{3qL}{2} - \frac{2}{3} \times \left(\frac{33qL}{16}\right) = \frac{qL}{8}$$

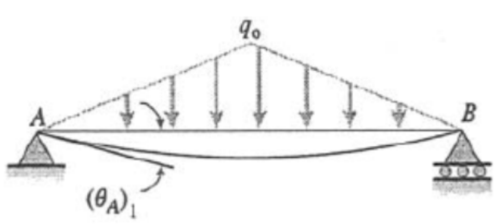
$$R_C = \frac{3qL}{2} - \frac{1}{3} \times \left(\frac{33qL}{16}\right) = \boxed{\frac{13qL}{16}}$$

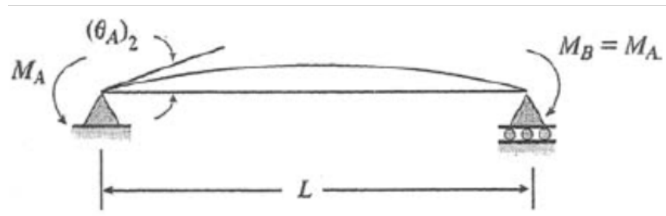
Reaction R_C is sensibly greater than R_A, but less than R_B.

The correct answer is **A**.

P.4 ➡ *Solution*

The system has four reactions, but there are only two equations of equilibrium. As a result, the structure is statically indeterminate with a degree of static indeterminacy of two. We shall take moments M_A and M_B as the redundant loadings, so that the released structure turns out to be a simple beam, as shown in continuation. $(\theta_A)_1$ is the angle of rotation of point *A* due to the triangular distributed load, while $(\theta_A)_2$ is the angle of rotation due to the action of reaction R_B.





Summing forces in the *y*-direction, we find that

$$\Sigma F_y = 0 \rightarrow R_A + R_C - \frac{q_0L}{2} = 0$$

$$\therefore R_A + R_C = \frac{q_0L}{2}$$

Due to symmetry, we surmise that both reactions must have the same intensity; that is,

$$R_A = R_C = \frac{q_0L}{4}$$

Likewise, moments M_A and M_B should have the same intensity as well. Consider the displacement of point A in the released structure. Angle of rotation $(\theta_A)_1$ due to the action of the triangular load is taken from case 12 of Table G-2,

$$(\theta_A)_1 = \frac{5q_0L^3}{192EI}$$

Angle $(\theta_A)_2$ due to action of two equal moments follows from case 10 of Table G-2,

$$(\theta_A)_2 = \frac{M_A L}{2EI}$$

Since point *A* is guarded against rotation, the following compatibility equation is cast and solved for M_A.

$$(\theta_A)_1 = (\theta_A)_2 \rightarrow \frac{5q_0L^3}{192EI} = \frac{M_A L}{2EI}$$

$$\therefore M_A = \frac{5q_0L^2}{96}$$

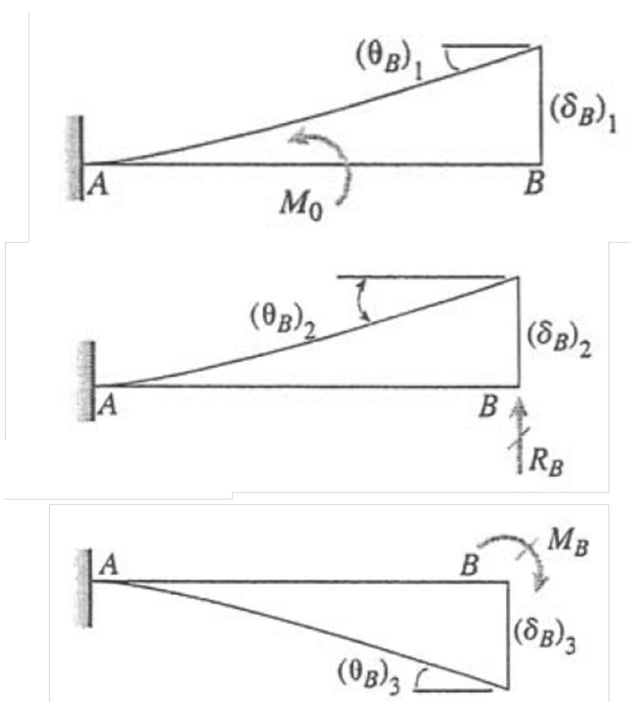
Given the symmetry of the system, we have

$$M_B = M_A = \boxed{\frac{5q_0L^2}{96}}$$

The correct answer is **C**.

P.5 ➡ *Solution*

The system has four reactions, but statics provides only two equilibrium equations. Consequently, the system is statically indeterminate and has a degree of static indeterminacy of two. Let us choose moment M_B and force reaction R_B as the redundant loads. The released structure is a cantilever beam, as shown. $(\theta_B)_1$ and $(\delta_B)_1$ represent the angle of rotation and displacement in point *B* due to moment M_0, respectively; $(\theta_B)_2$ and $(\delta_B)_2$ represent the angle of rotation and displacement at the same point due to force reaction R_B; finally, $(\theta_B)_3$ and $(\delta_B)_3$ represent the angle of rotation and displacement due moment reaction M_B.



Summing forces in the *y*-direction, we have

$$\Sigma F_y = 0 \rightarrow R_A + R_B = 0$$

$$\therefore R_A = -R_B \quad \text{(I)}$$

Taking moments about point *A*, in turn, we have

$$\Sigma M_A = 0 \rightarrow M_A - M_B + R_B L + M_0 = 0$$

$$\therefore M_A = M_B - R_B L - M_0 \quad \text{(II)}$$

The deflection and rotation due to moment M_0 can be obtained from case 7 of Table G-1,

$$(\delta_B)_1 = \frac{M_0 a}{2EI} \times (2L - a) = \frac{M_0 a}{2EI}\left[2(a+b) - a\right]$$

$$\therefore (\delta_B)_1 = \frac{M_0 a}{2EI}(a + 2b) \; ; \; (\theta_B)_1 = \frac{M_0 a}{EI}$$

The deflection and rotation due to reaction R_B can be obtained from case 4 of Table G-1,

$$(\delta_B)_2 = \frac{R_B L^3}{3EI} \; ; \; (\theta_B)_2 = \frac{R_B L^2}{2EI}$$

The deflection and rotation due to moment M_B can be obtained from case 6 of Table G-1,

$$(\delta_B)_3 = \frac{M_B L^2}{2EI} \; ; \; (\theta_B)_3 = \frac{M_B L}{EI}$$

We now turn to the compatibility equations. Since point *B* is fixed against translation, the angle of displacement therein must equal zero,

$$\delta_B = -(\delta_B)_1 - (\delta_B)_2 + (\delta_B)_3 = 0$$

$$\therefore -\frac{M_0 a}{2EI}(a+2b) - \frac{R_B L^3}{3EI} + \frac{M_B L^2}{2EI} = 0$$

$$\therefore -3M_0 a(a+2b) - 2R_B L^3 + 3M_B L^2 = 0 \;\; \text{(III)}$$

Point *B* is also guarded against rotation, which leads to the relation

$$\theta_B = (\theta_B)_1 + (\theta_B)_2 - (\theta_B)_3 = 0$$

$$\therefore \frac{M_0 a}{EI} + \frac{R_B L^2}{2EI} - \frac{M_B L}{EI} = 0$$

$$\therefore 2M_0 a + R_B L^2 - 2M_B L = 0 \;\; \text{(IV)}$$

Equations (III) and (IV) constitute a system of linear equations with R_B and M_B as the unknowns,

$$\begin{cases} -2R_B L^3 + 3M_B L^2 = 3M_0 a(a+2b) \\ R_B L^2 - 2M_B L = -2M_0 a \end{cases}$$

The solutions are

$$R_B = \frac{6M_0 ab}{L^3} \; ; \; M_B = -\frac{M_0 a}{L^2}(3b - L)$$

Substituting R_B into equation (I), we effortlessly obtain

$$R_A = -R_B = -\frac{6M_0 ab}{L^3}$$

Substituting M_B into equation (II), it follows that

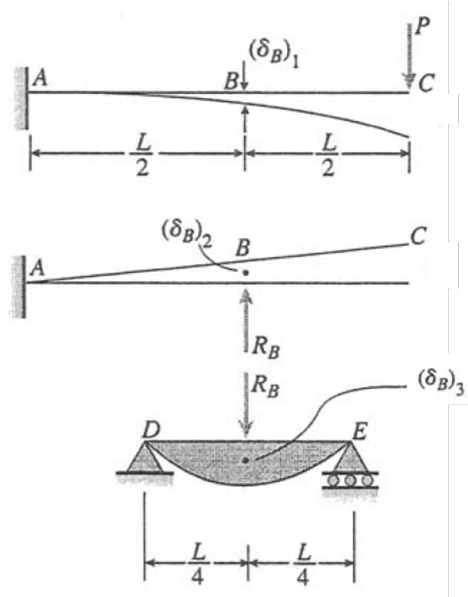
$$M_A = M_B - R_B L - M_0 = -\frac{M_0 a}{L^2}(3b - L) - \frac{6M_0 ab}{L^3} \times L - M_0$$

$$\therefore \boxed{M_A = \frac{M_0 b}{L^2}(3a - L)}$$

The correct answer is **D**.

P.6 ➡ *Solution*

Let reaction R_B on point *B* be the redundant load. The released structures are a cantilever beam *ABC* and a simple beam *DE*, as shown. $(\delta_B)_1$ represents the deflection at point *B* due to concentrated load *P*; $(\delta_B)_2$ is the deflection at point *B* of beam ABC due to reaction R_B; and $(\delta_B)_3$ is the deflection at point *B* of beam *DE* due to the same reaction.



In view of the lower beam's symmetry, it is easy to see that reactions R_D and R_E are equal in intensity and such that

$$R_D = R_E = \frac{R_B}{2} \text{ (I)}$$

Summing vertical forces acting on beam ABC, we obtain

$$\Sigma F_y = 0 \rightarrow R_A + R_B - P = 0$$

$$\therefore R_A = P - R_B \text{ (II)}$$

Taking moments about point *A*, we obtain

$$M_A - R_B \frac{L}{2} - PL = 0$$

$$\therefore M_A = PL + \frac{R_B L}{2} \text{ (III)}$$

The deflection due to concentrated load *P* can be taken from case 4 of Table G-1,

$$v = -\frac{Px^2}{6EI}(3L - x) \rightarrow (\delta_B)_1 = \frac{P}{6EI} \times \left(\frac{L}{2}\right)^2 \times \left(3L - \frac{L}{2}\right)$$

$$\therefore (\delta_B)_1 = \frac{P}{6EI} \times \frac{L^2}{4} \times \frac{5L}{2}$$

$$\therefore (\delta_B)_1 = \frac{5PL^3}{48EI}$$

The deflection due to upward reaction R_B can be taken from case 5 of Table G-1,

$$v = -\frac{Px^2}{6EI}(3a - x) \rightarrow (\delta_B)_2 = \frac{R_B}{6EI} \times \left(\frac{L}{2}\right)^2 \times \left(3 \times \frac{L}{2} - \frac{L}{2}\right)$$

$$\therefore (\delta_B)_2 = \frac{R_B L^2}{24EI} \times \left(3 \times \frac{L}{2} - \frac{L}{2}\right)$$

$$\therefore (\delta_B)_2 = \frac{R_B L^3}{24EI}$$

The deflection due to downward reaction R_B can be taken from case 5 of Table G-2,

$$v=-\frac{Pbx}{6LEI}\left(L^2-b^2-x^2\right)\rightarrow\left(\delta_B\right)_3=\frac{R_B}{6\times\frac{L}{2}\times EI}\times\frac{L}{4}\times\frac{L}{4}\times\left[\left(\frac{L}{2}\right)^2-\left(\frac{L}{4}\right)^2-\left(\frac{L}{4}\right)^2\right]$$

$$\therefore\left(\delta_B\right)_3=\frac{R_BL^3}{384EI}$$

Since point B is fixed against translation, the deflection therein must equal zero; that is,

$$\delta_B=0\rightarrow\left(\delta_B\right)_1-\left(\delta_B\right)_2=\left(\delta_B\right)_3$$

Substituting the appropriate expressions and solving for R_B, we find that

$$\left(\delta_B\right)_1-\left(\delta_B\right)_2=\left(\delta_B\right)_3\rightarrow\frac{5PL^3}{48\cancel{EI}}-\frac{R_BL^3}{24\cancel{EI}}=\frac{R_BL^3}{384\cancel{EI}}$$

$$\therefore 40P\cancel{L^3}-16R_B\cancel{L^3}=R_B\cancel{L^3}$$

$$\therefore R_B=\frac{40P}{17}$$

Substituting this result into equation (I), we can easily determine the reactions on the lower beam,

$$R_D=R_E=\frac{1}{2}\times\frac{40P}{17}=\frac{20P}{17}$$

Likewise, we can substitute the available quantities into equation (II) and determine reaction R_A,

$$R_A=P-R_B=P-\frac{40P}{17}=-\frac{23P}{17}$$

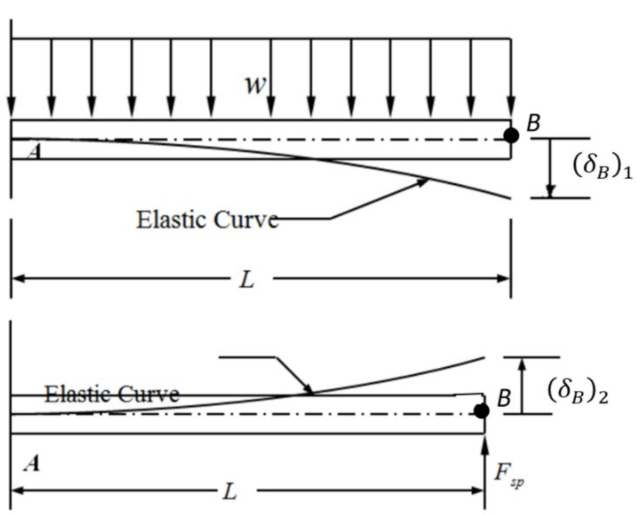
The negative sign indicates that the force points downward. Finally, we can establish the value of moment M_A from equation (III),

$$M_A=R_B\times\frac{L}{2}-PL=\frac{40PL}{34}-PL=\boxed{\frac{3PL}{17}}$$

The correct answer is **A**.

P.7 ➡ *Solution*

The deflections due to the uniform load and the elastic force F_{sp} are illustrated below.



The deflection at B due to the uniform load can be taken from case 1 of Table G-1,

$$\left(\delta_B\right)_1=\frac{wL^4}{8EI}$$

The deflection at B due to the elastic force can be taken from case 4 of Table G-1,

$$(\delta_B)_2 = \frac{F_{sp}L^3}{3EI}$$

From Hooke's law, the overall deflection at point B is determined as

$$F_{sp} = k\delta_B \rightarrow \delta_B = \frac{F_{sp}}{k}$$

We can then propose the equation of compatibility

$$-\delta_B = -(\delta_B)_1 + (\delta_B)_2$$

Substituting each variable and solving for F_{sp}, we obtain

$$-\delta_B = -(\delta_B)_1 + (\delta_B)_2 \rightarrow -\frac{F_{sp}}{k} = -\frac{wL^4}{8EI} + \frac{F_{sp}L^3}{3EI}$$

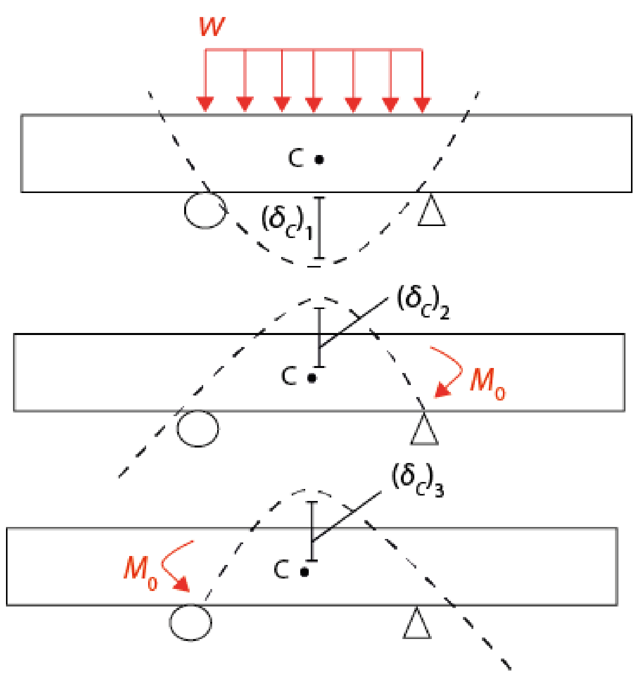
$$\therefore F_{sp}\left(\frac{1}{k} + \frac{L^3}{3EI}\right) = \frac{wL^4}{8EI}$$

$$\therefore F_{sp}\left(\frac{3EI + kL^3}{3\cancel{EI}k}\right) = \frac{wL^4}{8\cancel{EI}}$$

$$\therefore \boxed{F_{sp} = \frac{3kwL^4}{24EI + 8kL^3}}$$

P.8 ➡ *Solution*

The deflections due to each load are illustrated below.



The deflection at center C due to uniform load w is taken from case 1 of Table G-2,

$$(\delta_C)_1 = \frac{5wa^4}{384EI}$$

The deflection due to clockwise moment M_0 is taken from case 7 of Table G-2,

$$(\delta_C)_2 = \frac{M_0a^2}{16EI}$$

The counter-clockwise moment M_0 produces the same deflection,

$$(\delta_C)_3 = \frac{M_0a^2}{16EI}$$

The deflection at center C must equal zero. Mathematically,

$$\delta_C = 0 \rightarrow -(\delta_C)_1 + (\delta_C)_2 + (\delta_C)_3 = 0$$

Substituting and solving for moment M_0, we have

$$-(\delta_C)_1 + (\delta_C)_2 + (\delta_C)_3 = 0 \rightarrow -\frac{5wa^4}{384EI} + \frac{M_0 a^2}{16EI} + \frac{M_0 a^2}{16EI} = 0$$

$$\therefore -\frac{5wa^4}{384\cancel{EI}} + \frac{M_0 a^2}{8\cancel{EI}} = 0$$

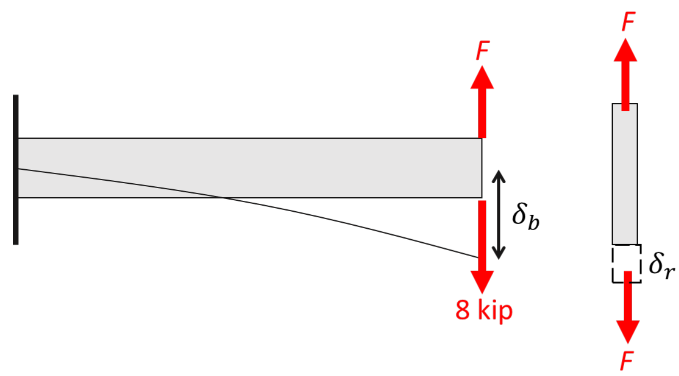
$$\therefore \frac{M_0 a^2}{8} = \frac{5wa^4}{384}$$

$$\therefore \boxed{M_0 = \frac{5wa^2}{48}}$$

The correct answer is **B**.

P.9 ➡ *Solution*

Let F be the force of interaction between the rod and the beam, as shown.



The deflection δ_b of the beam at point B can be taken from case 4 of Table G-1,

$$\delta_b = \frac{(8-F)L_b^3}{3EI} = \frac{(8-F)\times(4\times12)^3}{3E\times\left(\frac{1}{12}\times3\times5^3\right)} = \frac{1180(8-F)}{E}$$

The elongation δ_r of the rod, in turn, is determined with the axial load formula

$$\delta_r = \frac{FL_r}{AE} = \frac{F\times(5\times12)}{AE} = \frac{60F}{AE}$$

Suppose the rod reaches its maximum stress of 18 ksi, that is, $\sigma = F/A = 18$ ksi. Substituting in the preceding formula, we obtain

$$\delta_r = \frac{60}{E}\times\frac{F}{A} = \frac{60}{E}\times18 = \frac{1080}{E}$$

However, compatibility requires that $\delta_b = \delta_r$, so that

$$\delta_b = \delta_r \rightarrow \frac{1180(8-F)}{\cancel{E}} = \frac{1080}{\cancel{E}}$$

$$\therefore F = 7.08 \text{ kip}$$

We should also verify that the maximum stress in the beam does not exceed the maximum value,

$$\sigma = \frac{Mc}{I} = \frac{(8-7.08)\times\left(\frac{5}{2}\times4\times12\right)}{\left(\frac{1}{12}\times3\times5^3\right)} = 3.53 \text{ ksi} < 18 \text{ ksi}$$

Furthermore, the maximum deflection should not exceed 0.05 in. Substituting in the deflection formula, we obtain

$$\delta = \frac{PL^3}{3EI} = \frac{(8-7.08)\times(4\times12)^3}{3\times(29\times10^3)\times\left(\frac{1}{12}\times3\times5^3\right)} = 0.0374\text{ in.} < 0.05\text{ in.}$$

Thus, force *F* is acceptable and the smallest feasible diameter for the rod is such that

$$\sigma_{\text{allow}} = \frac{F}{A} \rightarrow A = \frac{F}{\sigma_{\text{allow}}} = \frac{7.08}{18} = 0.393\text{ in.}^2$$

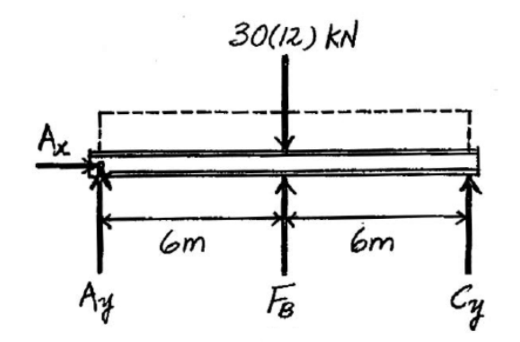
$$\therefore \frac{\pi}{4}\times d^2 = 0.393$$

$$\therefore \boxed{d = 0.707\text{ in.}}$$

The correct answer is **B**.

P.10 ➡ *Solution*

The free body diagram of the beam is shown in continuation.



Taking moments about point *A* gives

$$\Sigma M_A = 0 \rightarrow F_B\times6 + C_y\times12 - 30\times12\times6 = 0$$

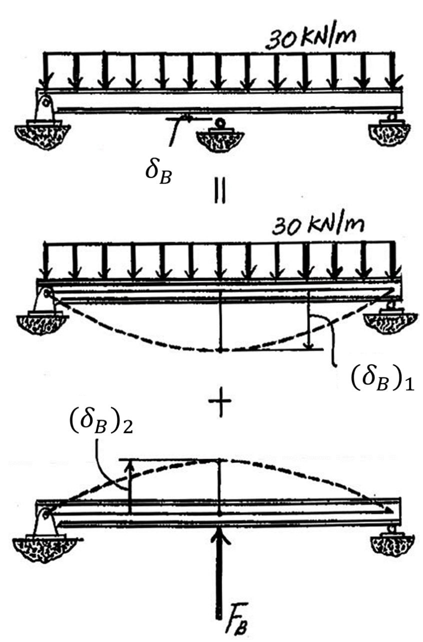
$$\therefore 6F_B + 12C_y = 2160 \quad \text{(I)}$$

Summing forces in the *x*-direction, it is easy to see that $A_x = 0$. Summing forces in the *y*-direction, in turn, we have

$$\Sigma F_y = 0 \rightarrow A_y + F_B + C_y - 30\times12 = 0$$

$$\therefore A_y + F_B + C_y = 360 \quad \text{(II)}$$

The deflection of point *B* consists of the superposition of two loading cases: a downward deflection $(\delta_B)_1$ due to a uniform load of 30 kN/m intensity and an upward deflection due to a concentrated reaction F_B whose intensity is not yet established.



Deflection $(\delta_B)_1$ is obtained from case 1 of Table G-2,

$$(\delta_B)_1 = \frac{5wL^4}{384EI} = \frac{5\times\left(30\times10^3\right)\times12^4}{384\times\left(200\times10^9\right)\times\left(875\times10^{-6}\right)} = 0.0463 \text{ m}$$

Deflection $(\delta_B)_2$ is obtained from case 4 of Table G-2,

$$(\delta_B)_2 = \frac{PL^3}{48EI} = \frac{F_B\times12^3}{48\times\left(200\times10^9\right)\times\left(875\times10^{-6}\right)} = 2.057\times10^{-7}F_B$$

We know that the overall deflection of point *B* is

$$\delta_B = 0.2\times10^{-3} + \frac{F_BL_B}{AE} = 0.2\times10^{-3} + \frac{1\times F_B}{\left(\frac{\pi}{4}\times0.04^2\right)\times\left(200\times10^9\right)} = 0.2\times10^{-3} + 4\times10^{-9}F_B$$

However, compatibility at support *B* requires that

$$-\delta_B = -(\delta_B)_1 + (\delta_B)_2$$

Accordingly,

$$-0.2\times10^{-3} - 4\times10^{-9}F_B = -0.0463 + 2.057\times10^{-7}F_B$$

$$\therefore F_B = 220 \text{ kN}$$

Substituting F_B in equation (I) gives

$$6\times220 + 12\times C_y = 2160 \rightarrow C_y = 70 \text{ kN}$$

Lastly, substituting F_B and C_y in equation (II), we obtain

$$A_y + 220 + 70 = 360 \rightarrow A_y = C_y = \boxed{70 \text{ kN}}$$

The correct answer is **A**.

P.11 ➡ *Solution*

Removing wood beam (2), steel beam (1) becomes a simply supported beam subjected to a concentrated load of *P* = 13 kips at midspan. The deflection at midpan can be taken from case 4 of Table G-2, and is given by

$$(\delta_D)_1 = -\frac{PL_1^3}{48E_1I_1}$$

Timber beam (2) imparts an upward reaction force on the steel beam at *D*. Consider steel beam (1) subjected to this upward reaction force D_y. The pertaining deflection formula is the same as the one that defines $(\delta_D)_2$; that is,

$$(\delta_D)_2 = \frac{D_yL_1^3}{48E_1I_1}$$

Timber beam (2) exerts an upward force to the steel beam at *D*. Conversely, steel beam (1) exerts an equal magnitude force on the timber beam, causing it to deflect downward. The downward deflection of beam (2) that is produced by reaction force D_y follows from the same expression as in the two preceding cases; that is,

$$(\delta_D)_3 = \frac{D_yL_2^3}{48E_2I_2}$$

The sum of the downward deflection of the steel beam due to the concentrated load, $(\delta_D)_1$. and the upward deflection produced by the reaction force exerted by the timber beam, $(\delta_D)_2$, must equal the downward deflection of the timber beam, $(\delta_D)_3$. Consequently, we can propose the compatibility equation

$$-(\delta_D)_1 + (\delta_D)_2 = -(\delta_D)_3$$

Substituting and solving for D_y, we obtain

$$-(\delta_D)_1+(\delta_D)_2=-(\delta_D)_3 \rightarrow -\frac{PL_1^3}{\cancel{48}E_1I_1}+\frac{D_yL_1^3}{\cancel{48}E_1I_1}=-\frac{D_yL_2^2}{\cancel{48}E_2I_2}$$

$$\therefore D_y\left(\frac{L_1^3}{E_1I_1}+\frac{L_2^3}{E_2I_2}\right)=\frac{PL_1^3}{E_1I_1}$$

$$\therefore D_y=\frac{PL_1^3}{E_1I_1\left(\frac{L_1^3}{E_1I_1}+\frac{L_2^3}{E_2I_2}\right)}$$

Substituting the numerical data we were given, it follows that

$$D_y=\frac{13\times(30\times12)^3}{(7.2\times10^6)\times\left[\frac{(30\times12)^3}{(7.2\times10^6)}+\frac{(20\times12)^3}{(1.0\times10^6)}\right]}=4.15\text{ kips}$$

The reaction at *A* can be determined by summing moments with respect to point *B*,

$$\Sigma M_B=0\rightarrow A_yL_1-P\times\frac{L_1}{2}+D_y\times\frac{L_1}{2}=0$$

$$\therefore A_y\times30-13\times\frac{30}{2}+4.15\times\frac{30}{2}=0$$

$$\therefore \boxed{A_y=4.43\text{ kips}}$$

The reaction at *C* on the timber beam can be obtained by taking moments about point *E*,

$$\Sigma M_E=0\rightarrow C_yL_2-D_y\times\frac{L_2}{2}=0$$

$$\therefore C_y=\frac{D_y}{2}$$

$$\therefore \boxed{C_y=2.08\text{ kips}}$$

The correct answer is **C**.

P.12 ➡ *Solution*

The moment of inertia of the shaft is

$$I_S=\frac{\pi}{4}r^4=\frac{\pi}{4}\times0.5^4=0.0491\text{ in.}^4$$

Taking moments about point *C*, we have

$$\Sigma M_C=0\rightarrow A_y\times10=800\times7-B_y\times5$$

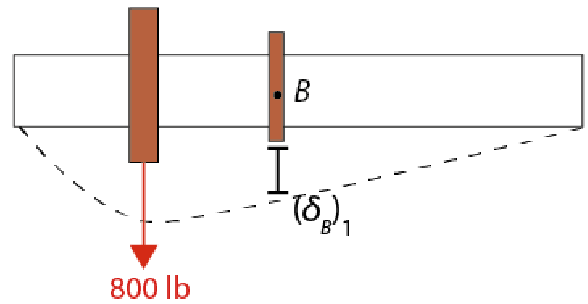
$$\therefore 10A_y+5B_y=5600\ \ \text{(I)}$$

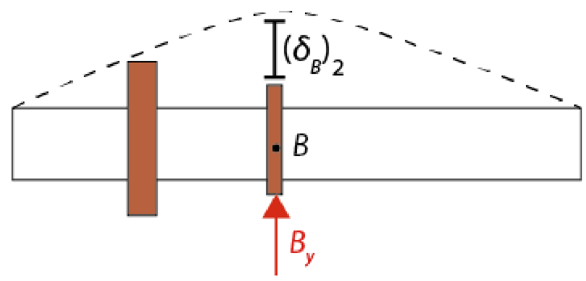
Summing forces in the *y*-direction, we find that

$$\Sigma F_y=0\rightarrow A_y+B_y+C_y-400-400=0$$

$$\therefore A_y+B_y+C_y=800\ \text{(II)}$$

Consider the displacement of point *B* of the structure.





The deflection due to the 800 lb load at point *B* can be determined from case 5 of Table G-2,

$$v=-\frac{Pbx}{6LEI}\left(L^2-b^2-x^2\right)\rightarrow(\delta_B)_1=\frac{800\times3\times5}{6\times10\times EI_S}\times\left(10^2-3^2-5^2\right)=\frac{13,200}{EI_S}$$

The deflection due to reaction B_y can be determined from case 4 of Table G-2,

$$v=\frac{PL^3}{48EI_x}\rightarrow(\delta_B)_2=\frac{B_y\times1000}{48\times EI_S}=\frac{20.8B_y}{EI_S}$$

Turning our attention to the beam that underlies the shaft, it is easy to see that reaction R_B causes the beam to deflect as in case 4 of Table G-2; that is,

$$\delta_B=\frac{PL^3}{48EI}\rightarrow\delta_B=\frac{B_y\times10^3}{48\times EI_B}=\frac{20.8B_y}{EI_B}$$

Since the translation of support *B* must equal zero, we can propose the compatibility equation

$$(\delta_B)_1-(\delta_B)_2=\delta_B$$

Substituting and solving for B_y, we obtain

$$\delta_B=(\delta_B)_1-(\delta_B)_2\rightarrow\frac{20.8B_y}{\cancel{E}I_B}=\frac{13,200}{\cancel{E}I_S}-\frac{20.8B_y}{\cancel{E}I_S}$$

$$\therefore\frac{20.8B_y}{500}=\frac{13,200}{0.0491}-\frac{20.8B_y}{0.0491}$$

$$\therefore B_y=635\text{ lb}$$

Backsubstituting into equation (I), the reaction at *A* is determined to be

$$10A_y+5\times635=5600\rightarrow\boxed{A_y=243\text{ lb}}$$

Lastly, substituting into equation (II), the reaction at *C* is calculated as

$$243+635+C_y=800\rightarrow\boxed{C_y=-78\text{ lb}}$$

The negative sign indicates that this reaction points downward.

The correct answer is **A.**

P.13 ➡ *Solution*

To begin, suppose we removed beam (2) and considered only cantilever beam (1), which is subjected to a concentrated load of $P = 45$ kN at its tip. The deflection at point D due to this load can be extracted from case 4 of Table G-1,

$$v = -\frac{Px^2}{6EI}(3L - x) \rightarrow (\delta_D)_1 = \frac{Pa^2}{6EI}\left[3(a+b) - a\right]$$

$$\therefore (\delta_D)_1 = \frac{Pa^2}{6EI}(2a + 3b)$$

Beam (2) exerts an upward reaction force on beam (1) at D. Consider beam (1) subjected to this upward reaction force D_y. The deflection of the cantilever beam at the point of application of the upward reaction force D_y is given by case 5 of Table G-1,

$$v = -\frac{Px^2}{6EI}(3a - x) \rightarrow (\delta_D)_2 = \frac{D_y a^2}{6EI}(3a - a)$$

$$\therefore (\delta_D)_2 = \frac{D_y a^3}{3EI}$$

Beam (2) imparts an upward force to beam (1) at D. Conversely, beam (1) exerts an equal magnitude force on beam (2), causing it to deflect downward. The corresponding deflection follows from case 4 of Table G-1,

$$v = -\frac{Px}{48EI}(3L^2 - 4x^2) \rightarrow (\delta_D)_3 = \frac{D_y}{48EI} \times \frac{L}{2} \times \left[3L^2 - 4 \times \left(\frac{L}{2}\right)^2\right]$$

$$\therefore (\delta_D)_3 = \frac{D_y L_2^3}{48E_2 I_2}$$

The sum of the downward deflection of beam (1) due to the concentrated load, $(\delta_D)_1$, and the upward deflection produced by the reaction force supported by beam (2), $(\delta_D)_2$, must equal the downward deflection of beam (2), $(\delta_D)_3$. This leads to the following compatibility equation,

$$-(\delta_D)_1 + (\delta_D)_2 = -(\delta_D)_3$$

Substituting and solving for D_y, we obtain

$$-(\delta_D)_1 + (\delta_D)_2 = -(\delta_D)_3 \rightarrow -\frac{Pa^2}{6E_1 I_1}(2a + 3b) + \frac{D_y a^3}{3E_1 I_1} = -\frac{D_y L_2^3}{48E_2 I_2}$$

$$\therefore D_y\left(\frac{a^3}{3E_1 I_1} + \frac{L_2^3}{48E_2 I_2}\right) = \frac{Pa^2}{6E_1 I_1}(2a + 3b)$$

$$\therefore D_y = \frac{Pa^2(2a + 3b)}{6E_1 I_1\left(\frac{a^3}{3E_1 I_1} + \frac{L_2^3}{48E_2 I_2}\right)}$$

Substituting the numerical data we were given, it follows that

$$D_y = \frac{45 \times 4.0^2 \times (2 \times 4.0 + 3 \times 1.5)}{6 \times 40{,}000 \times \left(\frac{4.0^3}{3 \times 40{,}000} + \frac{6.0^3}{48 \times 14{,}000}\right)} = 43.8 \text{ kN}$$

The deflection of beam (1) at D is equal to the deflection of beam (2) at that same point. Mathematically,

$$\delta_D = \frac{D_y L_2^3}{48E_2 I_2} = \frac{43.8 \times 6.0^3}{48 \times 14{,}000} = 0.0141 \text{ m} = \boxed{14.1 \text{ mm}}$$

The deflection of beam (1) at *B* consists of two contributions: the downward deflection due to concentrated load *P* and the upward deflection due to reaction D_y. The former, denoted as $(\delta_B)_1$, is easily obtained from case 4 of table G-1,

$$(\delta_B)_1 = \frac{P(a+b)^3}{3E_1I_1} = \frac{45\times(4.0+1.5)^3}{3\times 40,000} = 0.0624 \text{ m} = 62.4 \text{ mm}$$

The deflection of the cantilever beam at *B* due to reaction D_y, in turn, is given by the geometric relation

$$(\delta_B)_2 = v_D + \theta_D b$$

Here, v_D is the deflection of point *D* of beam segment *AD*, which is regarded as a cantilever beam with a concentrated load at the tip, and can be calculated from case 4 of Table G-1,

$$\delta_B = \frac{D_y a^3}{3E_1I_1} = \frac{43.8\times 4.0^3}{3\times 40,000} = 0.0234 \text{ m} = 23.4 \text{ mm}$$

Then, θ_D is the corresponding angle of rotation, taken from the same aforementioned case,

$$\theta_D = \frac{D_y a^2}{2E_1I_1} = \frac{43.8\times 4.0^2}{2\times 40,000} = 0.00876 \text{ rad}$$

With these data and *b* = 1.5 m, we ultimately obtain

$$(\delta_B)_2 = v_D + \theta_D b = 23.4 + 0.00876\times 1500 = 36.5 \text{ mm}$$

Thus, the deflection at *B* is determined to be

$$\delta_B = -(\delta_B)_1 + (\delta_B)_2 = -62.4 + 36.5 = \boxed{-25.9 \text{ mm}}$$

The negative sign indicates that the deflection points downward.

The correct answer is **C**.

ANSWER SUMMARY

Problem 1	**B**
Problem 2	**C**
Problem 3	**A**
Problem 4	**C**
Problem 5	**D**
Problem 6	**A**
Problem 7	Open-ended pb.
Problem 8	**B**
Problem 9	**B**
Problem 10	**A**
Problem 11	**C**
Problem 12	**A**
Problem 13	**C**

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