Montogue

Quiz EL106

Symmetrical Components and Unbalanced Fault

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PROBLEMS

Problem 1

Determine the symmetrical components of the following line currents.

$I_a = 6\angle 90°$ A
$I_b = 6\angle 320°$ A
$I_c = 6\angle 220°$ A

Problem 2

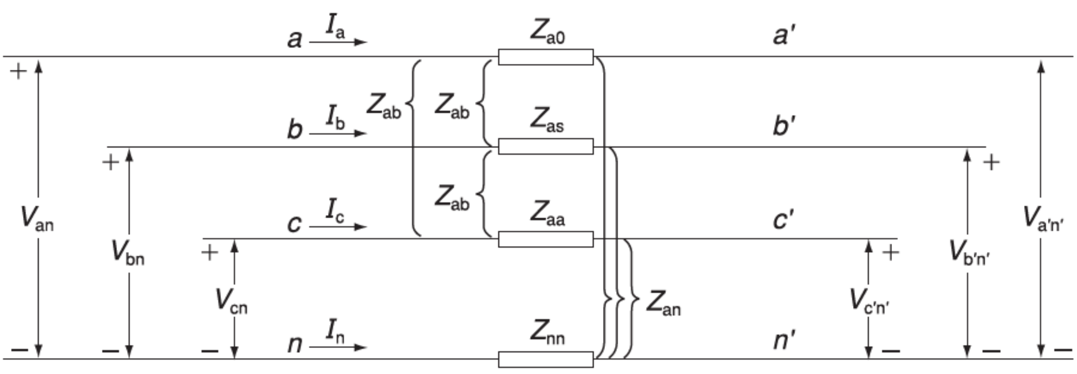
Problem 2.1: Given the line-to-ground voltages $V_{ag} = 280\angle 0°$, $V_{bg} = 250\angle -110°$, and $V_{cg} = 290\angle 130°$ volts, calculate the sequence components of the line-to-ground voltages, denoted $V_{L,g0}$, $V_{L,g1}$, and $V_{L,g2}$.
Problem 2.2: Determine the line-to-line voltages $V_{LL,0}$, $V_{LL,1}$, and $V_{LL,2}$.
Problem 2.3: Determine the sequence components of the line-to-line voltages $V_{LL,0} = 0$, $V_{LL,1}$, and $V_{LL,2}$.
Problem 2.4: The voltages given in the previous problem are applied to a balanced-Y load consisting of $(12 + j16)$ ohms per phase. The load neutral is solidly grounded. Draw the sequence networks and calculate I_0, I_1, and I_2, the sequence components of the line currents. Then calculate the line currents I_a, I_b, and I_c.
Problem 2.5: Repeat Problem 2.4 with the load neutral open.
Problem 2.6: Repeat Problem 2.4 with a balanced-Δ load consisting of $(12 + j16)$ ohms per phase.
Problem 2.7: Repeat the previous problem for the load described in Example 8.4 of Glover *et al.*'s book. The text is reproduced below.

A balanced-Y load is in parallel with a balanced-Δ-connected capacitor bank. The Y load has an impedance $Z_Y = (3 + j4)$ Ω per phase, and its neutral is grounded through an inductive reactance $X_n = 2$ Ω. The capacitor bank has a reactance $X_c = 30$ Ω per phase.

Problem 2.8: Repeat Problem 2.4 but include balanced three-phase impedances of $(3 + j4)$ ohms per phase between the source and load.

Problem 3 (Glover *et al.*, 2017, w/ permission)

Problem 3.1: Consider the flow of unbalanced currents in the symmetrical three-phase line section with neutral conductor illustrated below. Express the voltage drops across the line conductors given by V_{aa}, V_{bb}, and V_{cc} in terms of line currents, self-impedances defined by $Z_S = Z_{aa} + Z_{nn} - 2\,Z_{an}$, and mutual impedances defined by $Z_m = Z_{ab} + Z_{nn} - 2Z_{an}$.



Problem 3.2: Show that the sequence components of the voltage drops between the ends of the line section can be written as $V_{aa'0} = Z_0 I_{a0}$, $V_{aa'1} = Z_1 I_{a1}$, and $V_{aa'2} = Z_2 I_{a2}$, where

$$Z_0 = Z_S + 2Z_M = Z_{aa} + 2Z_{ab} + 3Z_{nn} - 6Z_{an}$$
$$Z_1 = Z_2 = Z_S = Z_M = Z_{aa} - Z_{ab}$$

Problem 4 (Glover *et al.*, 2017, w/ permission)

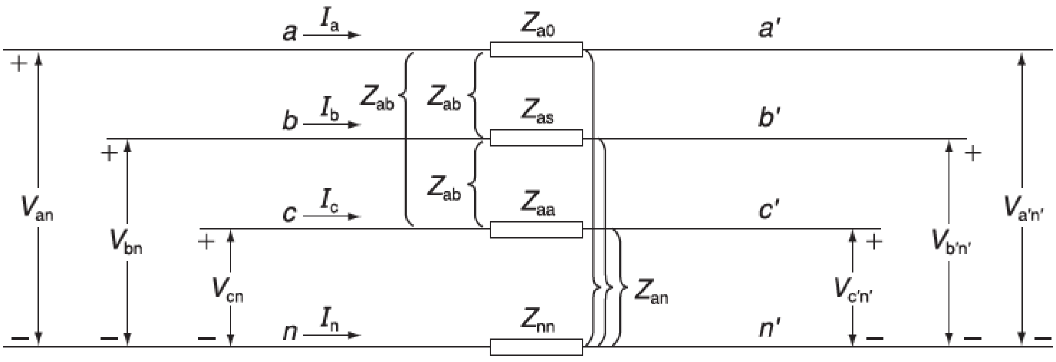
Problem 4.1: Let the terminal voltages at the two ends of the line section shown below be given by

$$V_{an} = (182 + j70)\,\text{kV} \;;\; V_{an'} = (154 + j28)\,\text{kV}$$
$$V_{bn} = (72.24 - j32.62)\,\text{kV} \;;\; V_{bn'} = (44.24 - j74.62)\,\text{kV}$$
$$V_{cn=} = (-170.24 + j88.62)\,\text{kV} \;;\; V_{cn'} = (-198.24 + j46.62)\,\text{kV}$$

The line impedances are $Z_{aa} = j60\ \Omega$, $Z_{ab} = j20\ \Omega$, $Z_{nn} = j80\ \Omega$, and $Z_{an} = j30\ \Omega$. Compute the line currents using symmetrical components.

Hint: Use the results developed in Problem 3.

Problem 4.2: Compute the line currents without using symmetrical components.



Problem 5

Using Saadat's (1999) MATLAB toolbox, obtain the symmetrical components for the following set of unbalanced voltages.

$V_a = 300\angle -120°$
$V_b = 200\angle 90°$
$V_c = 100\angle -30°$

Problem 6

The symmetrical components of a set of unbalanced three-phase currents are $I_a^0 = 3\angle -30°$, $I_a^1 = 5\angle 90°$, and $I_a^2 = 4\angle 30°$. Obtain the original unbalanced phasors.

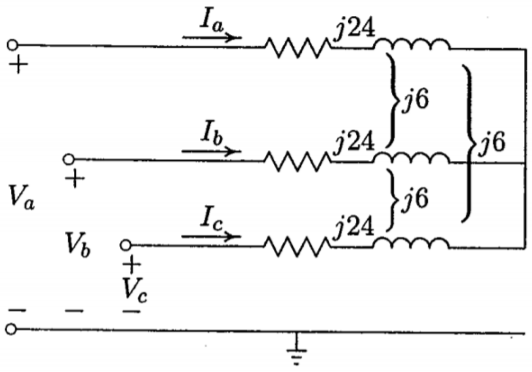
Problem 7

The line-to-line voltages in an unbalanced three-phase supply are $V_{ab} = 1000\angle 0°$, $V_{bc} = 866.0254\angle -150°$, and $V_{ca} = 500\angle 120°$. Determine the symmetrical components for line and phase voltages, then find the phase voltages V_{an}, V_{bn}, and V_{cn}.

Problem 8 (Saadat, 1999, w/ permission)

Problem 8.1: A balanced three-phase voltage of 360-V line-to-neutral is applied to a balanced Y-connected load with ungrounded neutral, as shown below. The three-phase load consists of three mutually-coupled reactances. Each phase has a series reactance of $Z_S = J24\ \Omega$, and the mutual coupling between phases is $Z_M = j6.0\ \Omega$. Determine the line currents by mesh analysis without using symmetrical components.

Problem 8.2: Determine the line currents using symmetrical components.



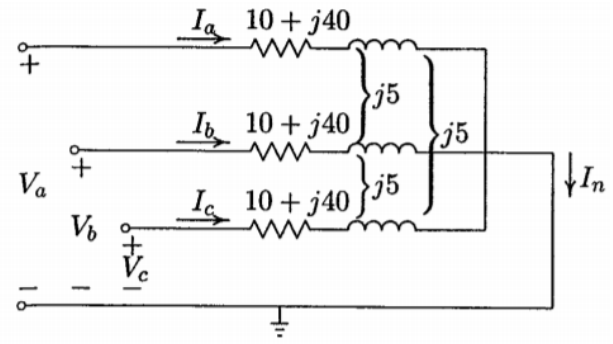
Problem 9 (Saadat, 1999, w/ permission)

A three-phase unbalanced source with the following phase-to-neutral voltages

$$\mathbf{V}^{abc} = \begin{bmatrix} 300\angle -120^\circ \\ 200\angle 90^\circ \\ 100\angle -30^\circ \end{bmatrix}$$

is applied to the circuit illustrated below. The load series impedance per phase is $Z_S = 10 + j40$ and the mutual impedance between phases is $Z_M = j5$. Determine:

Problem 9.1: The load sequence impedance matrix, $\mathbf{Z}^{012} = \mathbf{A}^{-1}\mathbf{Z}^{abc}\mathbf{A}$.
Problem 9.2: The symmetrical components of voltage.
Problem 9.3: The symmetrical components of current.
Problem 9.4: The load phase currents.
Problem 9.5: The complex power delivered to the load in terms of symmetrical components, $S_{3\phi} = 3(V_a^0 I_a^{0*} + V_a^1 I_a^{1*} + V_a^2 I_a^{2*})$.
Problem 9.6: The complex power delivered to the load by summing up the power in each phase, $S_{3\phi} = V_a I_a^* + V_b I_b^* + V_c I_c^*$.



Problem 10

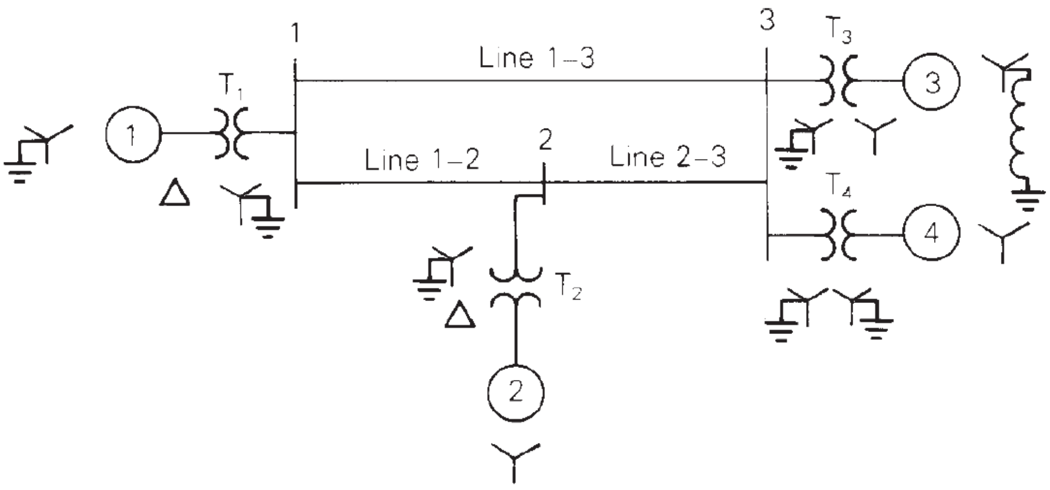
The line-to-line voltages in an unbalanced three-phase supply are $V_{ab} = 600\angle -36.87^\circ$, $V_{bc} = 800\angle 126.87^\circ$, $V_{ca} = 1000\angle -90^\circ$. A Y-connected load with a resistance of 37 Ω per phase is connected to the supply. Determine

Problem 10.1: The symmetrical components of voltage.
Problem 10.2: The phase voltages.
Problem 10.3: The line currents.

Problem 11 (Glover *et al.*, 2017, w/ permission)

The single-line diagram of a three-phase power system is shown below. Equipment ratings are given as follows.

Synchronous generators	
G1 1000 MVA, 15 kV	$X_d'' = X_2 = 0.18$, $X_0 = 0.07$ per unit
G2 1000 MVA, 15 kV	$X_d'' = X_2 = 0.20$, $X_0 = 0.10$ per unit
G3 500 MVA, 13.8 kV	$X_d'' = X_2 = 0.15$, $X_0 = 0.05$ per unit
G4 750 MVA, 13.8 kV	$X_d'' = 0.30$, $X_2 = 0.40$, $X_0 = 0.10$ per unit
Transformers	
T1 1000 MVA, 15 kV Δ/765 kV Y	$X = 0.10$ per unit
T2 1000 MVA, 15 kV Δ/765 kV Y	$X = 0.10$ per unit
T3 500 MVA, 15 kV Δ/765 kV Y	$X = 0.12$ per unit
T4 750 MVA, 15 kV Δ/765 kV Y	$X = 0.11$ per unit
Transmission lines	
1-2 765 kV	$X_1 = 50\ \Omega$, $X_0 = 150\ \Omega$
1-3 765 kV	$X_1 = 40\ \Omega$, $X_0 = 100\ \Omega$
2-3 765 kV	$X_1 = 40\ \Omega$, $X_0 = 100\ \Omega$

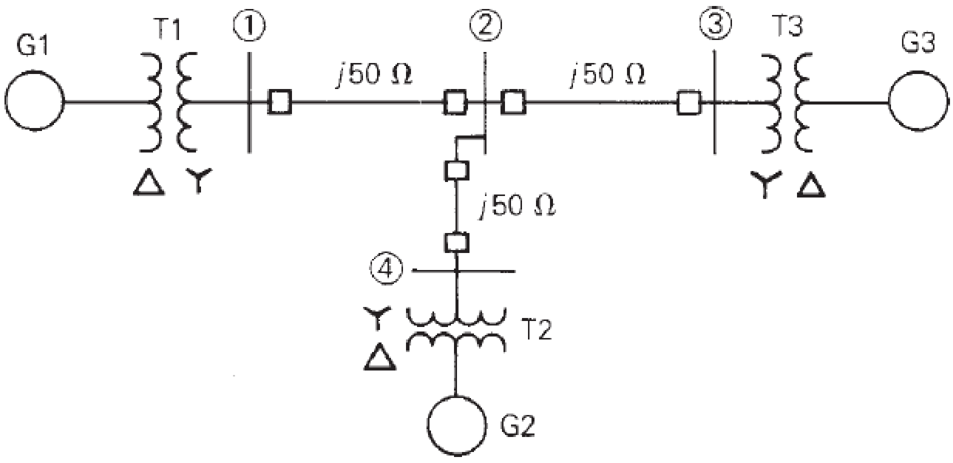


Problem 11.1: The inductor connected to generator 3 neutral has a reactance of 0.05 per unit using generator 3 ratings as a base. Draw the zero-, positive-, and negative-sequence reactance diagrams using a 1000-MVA, 765-kV base in the zone of line 1-2. Neglect the Δ-Y transformers phase shifts.
Problem 11.2: Faults at bus n of Problem 11.1 are of interest (the instructor selects n = 1, 2, or 3). Determine the Thévenin equivalent of each sequence network as viewed from the fault bus. Prefault voltage is 1.0 per unit. Prefault load currents and Δ-Y transformer phase shifts are neglected.
Problem 11.3: Determine the subtransient fault current in per-unit and in kA during a bolted three-phase fault at the fault bus selected in Problem 11.2.
Problem 11.4: Reconsidering the system introduced in Problem 11.1, determine the subtransient fault current in per-unit and in kA, as well as the per-unit line-to-ground voltages at the fault bus for a bolted single line-to-ground fault at the fault bus selected in Problem 11.2.
Problem 11.5: Repeat the previous problem for a single line-to-ground arcing fault with arc impedance $Z_F = 15 + j0\ \Omega$.
Problem 11.6: Repeat Problem 11.4 for a bolted line-to-line fault.
Problem 11.7: Repeat Problem 11.4 for a bolted double line-to-ground fault.

Problem 12 (Glover *et al.*, 2017, w/ permission)

Equipment ratings for the four-bus power system illustrated below are given as follows.

Generators
Generator **G1**: 500 MVA, 13.8 kV, $X''_d = X_2 = 0.20$, $X_0 = 0.10$ per unit
Generator **G2**: 750 MVA, 18 kV, $X''_d = X_2 = 0.18$, $X_0 = 0.09$ per unit
Generator **G3**: 1000 MVA, 20 kV, $X''_d = 0.17$, $X_2 = 0.20$, $X_0 = 0.09$ per unit
Transformers
Transformer **T1**: 500 MVA, 13.8 kV Δ/500 kV Y, $X = 0.12$ per unit
Transformer **T2**: 750 MVA, 18 kV Δ/500 kV Y, $X = 0.10$ per unit
Transformer **T3**: 1000 MVA, 20 kV Δ/500 kV Y, $X = 0.10$ per unit
Each line: $X_1 = 50\ \Omega$, $X_0 = 150\ \Omega$

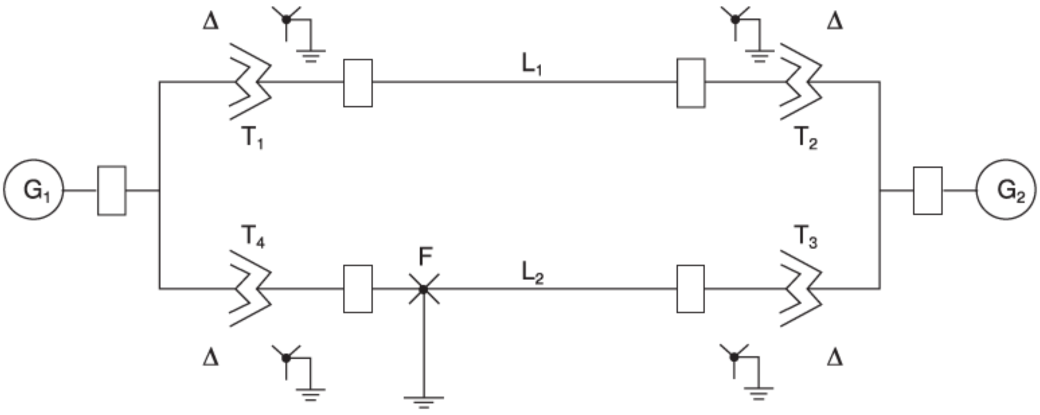


Problem 12.1: The inductor connected to generator G3 neutral has a reactance of 0.028 Ω. Draw the zero-, positive, and negative-sequence reactance diagrams using a 1000-MVA, 20-kV base in the zone of generator G3. Neglect Δ-Y transformer phase shifts.
Problem 12.2: Faults at bus n in Problem 12.1 are of interest (the instructor selects n = 1, 2, 3, or 4). Determine the Thévenin equivalent of each sequence network as viewed from the fault bus. Prefault voltage is 1.0 per unit. Prefault load currents and Δ-Y phase shifts are neglected.
Problem 12.3: Determine the subtransient fault current in per-unit and in kA during a bolted three-phase fault at the fault bus selected in Problem 12.2.
Problem 12.4: Determine the subtransient fault current in per unit and in kA, as well as contributions to the fault current from each line and transformer connected to the fault bus for a bolted single line-to-ground fault at the fault bus selected in Problem 12.2.
Problem 12.5: Repeat the previous problem for a bolted line-to-line fault.
Problem 12.6: Repeat Problem 12.4 for a bolted double line-to-ground fault.

Problem 13 (Glover *et al.*, 2017, w/ permission)

Problem 13.1: Consider the system shown below. As viewed from the fault at F, determine the Thévenin equivalent of each sequence network. Neglect Δ–Y phase shifts. Compute the fault currents for a balanced three-phase fault at fault point F through three fault impedances $Z_{FA} = Z_{FB} = Z_{FC} = j0.5$ per unit. Equipment data in per-unit on the same base are given as follows:

Synchronous generators
Generator **G1**: $X_1 = 0.2$, $X_2 = 0.12$, $X_0 = 0.06$ per unit
Generator **G2**: $X_1 = 0.33$, $X_2 = 0.22$, $X_0 = 0.066$ per unit
Transformers
Transformer **T1**: $X_1 = X_2 = X_0 = 0.2$
Transformer **T2**: $X_1 = X_2 = X_0 = 0.225$
Transformer **T3**: $X_1 = X_2 = X_0 = 0.27$
Transformer **T4**: $X_1 = X_2 = X_0 = 0.16$
Transmission lines
Line **L1**: $X_1 = X_2 = 0.14$, $X_0 = 0.3$
Line **L2**: $X_1 = X_2 = 0.35$, $X_0 = 0.6$

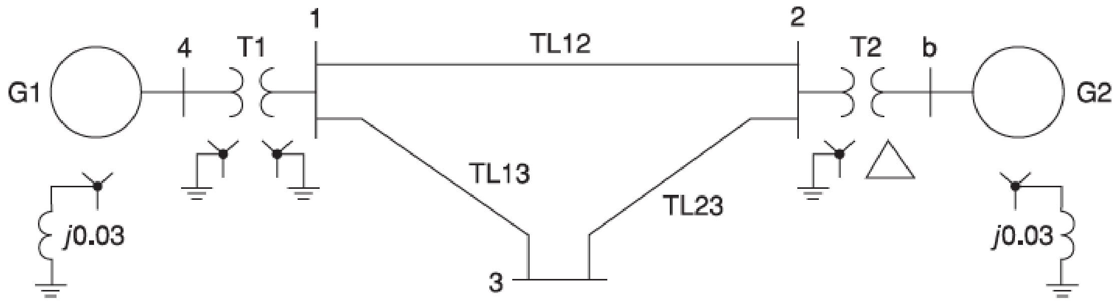


Problem 13.2: For the system of Problem 13.1, compute the fault current and voltages at the fault for the following faults at point F: (*a*) a bolted single line-to-ground fault; (*b*) a line-to-line fault through a fault impedance $Z_F = j0.05$ per unit; (*c*) a double line-to-ground fault from phase B to C to ground, where phase B has a fault impedance $Z_F = j0.05$ per unit, phase C also has a fault impedance $Z_F = j0.05$ per unit, and the common line-to-ground fault impedance is $Z_G = j0.033$ per unit.

Problem 14 (Glover *et al.*, 2017, w/ permission)

Equipment ratings and per-unit reactances for the system illustrated below are given as follows:

Synchronous generators
Generator **G1**: 100 MVA, 25 kV, $X_1 = X_2 = 0.2$, $X_0 = 0.05$
Generator **G2**: 100 MVA, 13.8 kV, $X_1 = X_2 = 0.2$, $X_0 = 0.05$
Transformers
Transformer **T1**: 100 MVA, 25/230 kV, $X_1 = X_2 = X_0 = 0.05$
Transformer **T2**: 100 MVA, 13.8/230 kV, $X_1 = X_2 = X_0 = 0.05$
Transmission lines
Transmission line **TL12**: 100 MVA, 230 kV, $X_1 = X_2 = 0.1$, $X_0 = 0.3$
Transmission line **TL13**: 100 MVA, 230 kV, $X_1 = X_2 = 0.1$, $X_0 = 0.3$
Transmission line **TL23**: 100 MVA, 230 kV, $X_1 = X_2 = 0.1$, $X_0 = 0.3$



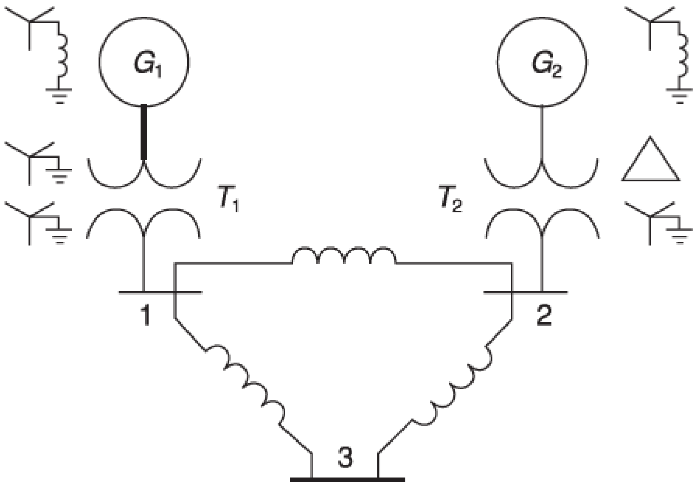
Problem 14.1: Using a 100-MVA, 230-kV base for the transmission lines, draw the per-unit sequence networks and reduce them to their Thévenin equivalents, "looking in" at bus 3. Neglect Δ-Y phase shifts. Compute the fault currents for a bolted three-phase fault at bus 3.

Problem 14.2: Reconsidering the system of Problem 14.1, compute the fault current and voltages at the fault for the following faults at bus 3: (*a*) a bolted single line-to-ground fault; (*b*) a bolted line-to-line fault; (*c*) a bolted double line-to-ground fault. Also, for the single line-to-ground fault at bus 3, determine the currents and voltages at the terminals of generators G1 and G2.

Problem 15 (Glover *et al.*, 2017, w/ permission)

Consider the oneline diagram of a simple power system shown below. System data in per-unit on a 100-MVA base are given as follows:

Synchronous generators
Generator **G1**: 100 MVA, 20 kV, $X_1 = X_2 = 0.15$, $X_0 = 0.05$
Generator **G2**: 100 MVA, 20 kV, $X_1 = X_2 = 0.15$, $X_0 = 0.05$
Transformers
Transformer **T1**: 100 MVA, 20/220 kV, $X_1 = X_2 = X_0 = 0.1$
Transformer **T2**: 100 MVA, 20/220 kV, $X_1 = X_2 = X_0 = 0.1$
Transmission lines
Transmission line **L12**: 100 MVA, 220 kV, $X_1 = X_2 = 0.125$, $X_0 = 0.3$
Transmission line **L13**: 100 MVA, 220 kV, $X_1 = X_2 = 0.15$, $X_0 = 0.35$
Transmission line **L23**: 100 MVA, 220 kV, $X_1 = X_2 = 0.25$, $X_0 = 0.7125$



Problem 15.1: The neutral of each generator is grounded through a current-limiting reactor of 0.08333 per unit on a 100-MVA base. All transformer neutrals are solidly grounded. The generators are operating no-load at their rated voltages and rated frequency with their EMFs in phase. Determine the fault current for a balanced three-phase fault at bus 3 through a fault impedance Z_F = 0.1 per unit on a 100-MVA base. Neglect Δ-Y phase shifts.

Problem 15.2: For the system of Problem 15.1, compute the fault current for the following faults at bus 3: (*a*) a single line-to-ground fault through a fault impedance $Z_F = j0.1$ per unit; (*b*) a line-to-line fault through a fault impedance $Z_F = j0.1$ per unit; (*c*) a double line-to-ground fault through a common fault impedance to ground $Z_F = j0.1$ per unit.

SOLUTIONS

P.1 ➡ *Solution*

Let I_0, I_1, and I_2 denote the symmetrical components of I_a, I_b, and I_c, respectively. Noting that $a = 1\angle 120°$, we may write

$$\begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} 6\angle 90° \\ 6\angle 320° \\ 6\angle 220° \end{bmatrix}$$

$$\therefore \begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix} = \frac{6}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} 1\angle 90° \\ 1\angle 320° \\ 1\angle 220° \end{bmatrix}$$

$$\therefore \begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix} = 2\begin{bmatrix} 1\times 1\angle 90° + 1\times 1\angle 320° + 1\times 1\angle 220° \\ 1\times 1\angle 90° + (1\angle 120°)\times 1\angle 320° + (1\angle 120°)^2\times 1\angle 220° \\ 1\times 1\angle 90° + (1\angle 120°)^2\times 1\angle 320° + (1\angle 120°)\times 1\angle 220° \end{bmatrix}$$

$$\therefore \begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix} = 2\begin{bmatrix} -j0.286 \\ j2.97 \\ j0.316 \end{bmatrix} = \begin{bmatrix} -j0.572 \\ j5.94 \\ j0.632 \end{bmatrix}$$

$$\therefore \boxed{I_0 = 0.572\angle -90^\circ \;;\; I_1 = 5.94\angle 90^\circ \;;\; I_2 = 0.632\angle 90^\circ}$$

P.2 ➡ Solution

Problem 2.1: To calculate $V_{L,g0}$, $V_{L,g1}$ and $V_{L,g2}$, we write

$$\begin{bmatrix} V_{L,g0} \\ V_{L,g1} \\ V_{L,g2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} 280\angle 0^\circ \\ 250\angle -110^\circ \\ 290\angle 130^\circ \end{bmatrix}$$

$$\therefore \begin{bmatrix} V_{L,g0} \\ V_{L,g1} \\ V_{L,g2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 280\angle 0^\circ + 250\angle -110^\circ + 290\angle 130^\circ \\ 280\angle 0^\circ + a\times 250\angle -110^\circ + a^2\times 290\angle 130^\circ \\ 290\angle 130^\circ + a^2\times 250\angle -110^\circ + a\times 290\angle 130^\circ \end{bmatrix}$$

$$\therefore \begin{bmatrix} V_{L,g0} \\ V_{L,g1} \\ V_{L,g2} \end{bmatrix} = \begin{bmatrix} 2.70 - j4.26 \\ 271 + j31.3 \\ 6.71 - j27.0 \end{bmatrix}$$

$$\therefore \begin{bmatrix} V_{L,g0} \\ V_{L,g1} \\ V_{L,g2} \end{bmatrix} = \begin{bmatrix} 5.04\angle -57.6^\circ \\ 273\angle 6.59^\circ \\ 27.8\angle -76.0^\circ \end{bmatrix} \text{V}$$

Problem 2.2: To determine the line-to-line voltages, we first establish the values of V_{ab}, V_{bc}, and V_{ca},

$$\begin{bmatrix} V_{ab} \\ V_{bc} \\ V_{ca} \end{bmatrix} = \begin{bmatrix} V_{ag} - V_{bg} \\ V_{bg} - V_{cg} \\ V_{cg} - V_{ag} \end{bmatrix} = \begin{bmatrix} 280\angle 0^\circ - 250\angle -110^\circ \\ 250\angle -110^\circ - 290\angle 130^\circ \\ 290\angle 130^\circ - 280\angle 0^\circ \end{bmatrix} = \begin{bmatrix} 366 + j235 \\ 101 - j457 \\ -466 + j222 \end{bmatrix}$$

$$\therefore \begin{bmatrix} V_{ab} \\ V_{bc} \\ V_{ca} \end{bmatrix} = \begin{bmatrix} 435\angle 32.7^\circ \\ 468\angle -77.5^\circ \\ 516\angle 155^\circ \end{bmatrix} \text{V}$$

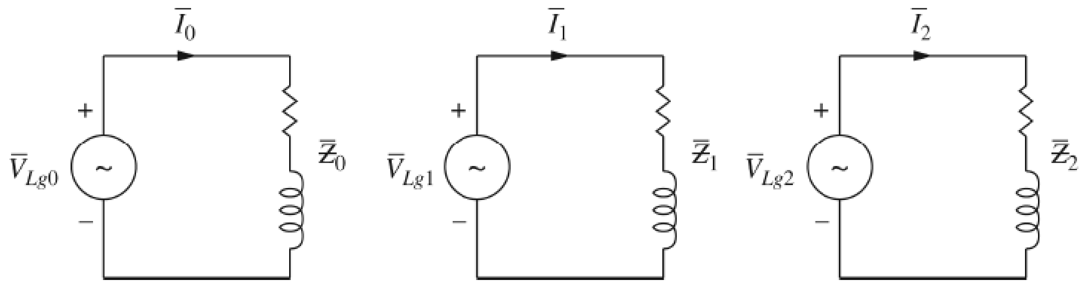
Problem 2.3: The values of the V_{LL}'s are determined next,

$$\begin{bmatrix} V_{LL,0} \\ V_{LL,1} \\ V_{LL,2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} V_{ab} \\ V_{bc} \\ V_{ca} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} 435\angle 32.7^\circ \\ 468\angle -77.5^\circ \\ 516\angle 155^\circ \end{bmatrix}$$

$$\therefore \begin{bmatrix} V_{L,g0} \\ V_{L,g1} \\ V_{L,g2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 435\angle 32.7^\circ + 468\angle -77.5^\circ + 516\angle 155^\circ \\ 435\angle 32.7^\circ + a\times 468\angle -77.5^\circ + a^2\times 516\angle 155^\circ \\ 435\angle 32.7^\circ + a^2\times 468\angle -77.5^\circ + a\times 516\angle 155^\circ \end{bmatrix} = \begin{bmatrix} -0.101 - j1.28 \\ 378 + j282 \\ -11.8 - j46.1 \end{bmatrix}$$

$$\therefore \begin{bmatrix} V_{LL,0} \\ V_{LL,1} \\ V_{LL,2} \end{bmatrix} = \begin{bmatrix} 1.28\angle -94.5^\circ \\ 472\angle 36.7^\circ \\ 47.6\angle -104^\circ \end{bmatrix} \text{V}$$

Problem 2.4: A load of $12 + j16\ \Omega$ can be written in exponential form as $20\angle 53.1^\circ$. The sequence networks are sketched below.



Given $V_{L,g0}$ = 5.04∠ −57.6°, $V_{L,g1}$ = 273∠6.59°, and $V_{L,g2}$ = 27.8∠ −76.0°, we compute the sequence components of the line currents as follows,

$$I_0 = \frac{V_{L,g0}}{Z_0} = \frac{5.04\angle -57.6^\circ}{20\angle 53.1^\circ} = \frac{5.04}{20}\angle\left(-57.6^\circ - 53.1^\circ\right) = 0.252\angle -111^\circ\,\text{A}$$

$$I_1 = \frac{V_{L,g1}}{Z_1} = \frac{273\angle 6.59^\circ}{20\angle 53.1^\circ} = 13.7\angle -46.5^\circ\,\text{A}$$

$$I_2 = \frac{V_{L,g2}}{Z_2} = \frac{27.8\angle -76.0^\circ}{20\angle 53.1^\circ} = 1.39\angle -129^\circ\,\text{A}$$

The next step is to calculate I_a, I_b, and I_c,

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0.252\angle -111^\circ \\ 13.7\angle -46.5^\circ \\ 1.39\angle -129^\circ \end{bmatrix} = \begin{bmatrix} 14.1\angle -53.0^\circ \\ 12.6\angle -163^\circ \\ 14.6\angle 76.9^\circ \end{bmatrix}\text{A}$$

Observe that the source and load neutrals are connected with a zero-ohm wire. It follows that a quicker, simpler way to compute the same currents is to write

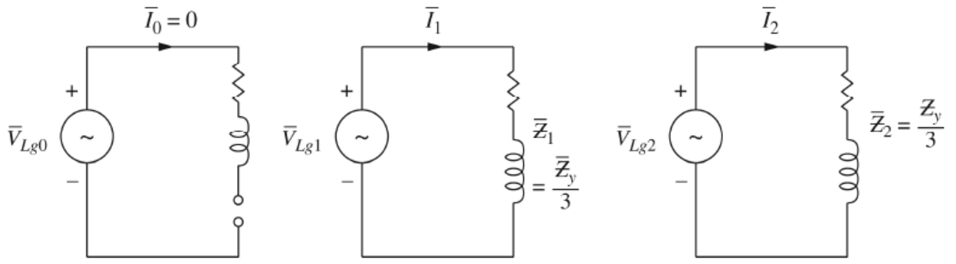
$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} V_{ag}/z_Y \\ V_{bg}/z_Y \\ V_{cg}/z_Y \end{bmatrix} = \begin{bmatrix} \dfrac{280\angle 0^\circ}{20\angle 53.1^\circ} \\ \dfrac{250\angle -110^\circ}{20\angle 53.1^\circ} \\ \dfrac{290\angle 130^\circ}{20\angle 53.1^\circ} \end{bmatrix} = \begin{bmatrix} 14\angle -53.1^\circ \\ 12.5\angle -153^\circ \\ 14.5\angle 76.9^\circ \end{bmatrix}\ \text{A}$$

which agrees with the above result; the slight difference is due to round-off.

Problem 2.5: With the load neutral open, I_0 = 0; The remaining currents continue to be I_1 = 13.7∠ −46.5° and I_2 = 1.39∠ −129° A. We proceed to determine I_a, I_b, and I_c.

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ 13.7\angle -46.5^\circ \\ 1.39\angle -129^\circ \end{bmatrix} = \begin{bmatrix} 8.56 - j11.1 \\ -11.9 - j3.42 \\ 3.39 + j14.4 \end{bmatrix} = \begin{bmatrix} 13.9\angle -52.2^\circ \\ 12.4\angle -164^\circ \\ 14.8\angle 76.8^\circ \end{bmatrix}\text{A}$$

Problem 2.6: The sequence networks are redrawn as follows.



Currents I_0, I_1, and I_2 are restated as

$$I_0 = 0\ ;\ I_1 = \frac{273\angle 6.59^\circ}{\left(\frac{20}{3}\right)\angle 53.1^\circ} = 41.0\angle -46.5^\circ\,\text{A}\ ;\ I_2 = \frac{27.8\angle -76.0^\circ}{\left(\frac{20}{3}\right)\angle 53.1^\circ} = 4.17\angle -129^\circ\,\text{A}$$

so that

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ 41.0\angle -46.5^\circ \\ 4.17\angle -129^\circ \end{bmatrix} = \begin{bmatrix} 25.6 - j33.0 \\ -35.7 - j10.2 \\ 10.2 + j43.2 \end{bmatrix} = \begin{bmatrix} 41.7\angle -52.2^\circ \\ 37.2\angle -164^\circ \\ 44.4\angle 76.8^\circ \end{bmatrix}\text{A}$$

Note that these currents are approximately 3 times the currents obtained in Problem 2.5, as they should be.

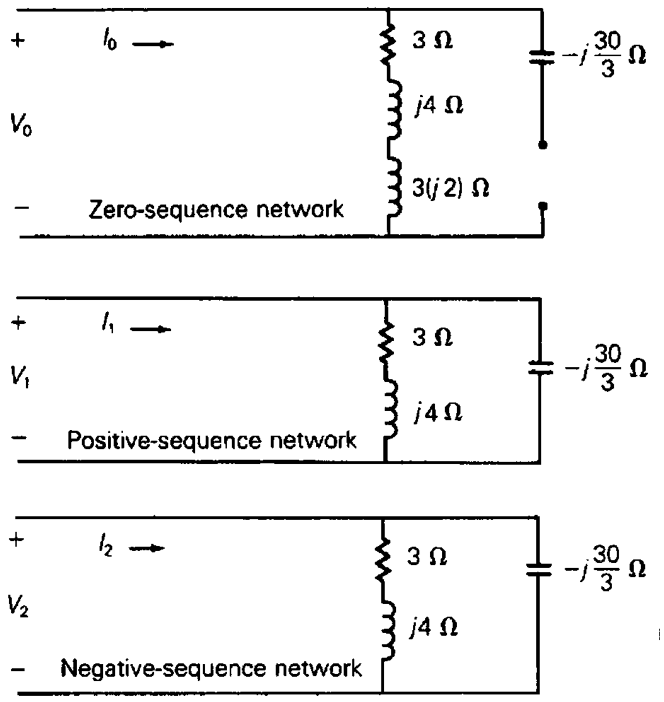
Problem 2.7: The sequence networks are illustrated below. As shown, the Y-load impedance in the zero-sequence network is in series with three times the neutral impedance. Also, the Δ-load branch in the zero-sequence network is open, since no zero-sequence current flows into the Δ-load. In the positive- and negative-sequence circuits, the Δ-load impedance is divided by

3 and placed in parallel with the Δ-load impedance. The equivalent sequence impedances are

$$Z_0 = Z_Y + 3Z_n = (3+j4) + 3\times j2 = 3 + j10\,\Omega$$

$$Z_1 = Z_Y // (Z_\Delta/3) = \frac{(3+j4)\times(-j30/3)}{(3+j4) - j30/3} = 6.67 + j3.33 = 7.45\angle 26.6^\circ\,\Omega$$

$$Z_2 = Z_1 = 7.45\angle 26.6^\circ\,\Omega$$



Having determined the pertaining impedances, we proceed to compute currents I_0, I_1, and I_2,

$$I_0 = \frac{V_{L,g0}}{Z_0} = \frac{5.04\angle -57.6^\circ}{3+j10} = \frac{5.04\angle -57.6^\circ}{10.4\angle 73.3^\circ} = 0.485\angle -131^\circ\,\text{A}$$

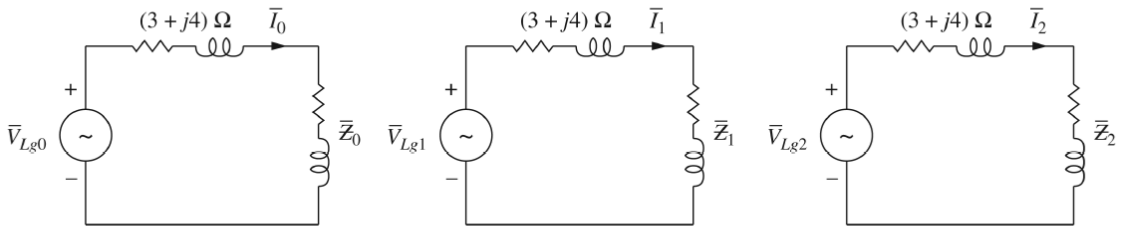
$$I_1 = \frac{V_{L,g1}}{Z_1} = \frac{273\angle 6.59^\circ}{7.45\angle 26.6^\circ} = 36.6\angle -20.0^\circ\,\text{A}$$

$$I_2 = \frac{V_{L,g2}}{Z_2} = \frac{27.8\angle -76.0^\circ}{7.45\angle 26.6^\circ} = 3.73\angle -103^\circ\,\text{A}$$

Lastly, we make use of the usual matrix equation to determine I_a, I_b, and I_c,

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0.485\angle -131^\circ \\ 36.6\angle -20.0^\circ \\ 3.73\angle -103^\circ \end{bmatrix} = \begin{bmatrix} 33.2 - j16.5 \\ -24.8 - j22.8 \\ -9.40 + j38.2 \end{bmatrix} = \begin{bmatrix} 37.1\angle -26.4^\circ \\ 33.7\angle -137^\circ \\ 39.4\angle 104^\circ \end{bmatrix} \text{A}$$

Problem 2.8: The only change relatively to Problem 2.4 is the addition of a series $(3+j4)$-Ω impedance to each sequence network, as shown.



First determine I_0, I_1, and I_2,

$$I_0 = \frac{V_{L,g0}}{(3+j4)+Z_0} = \frac{5.04\angle -57.6^\circ}{(3+j4)+20\angle 53.1^\circ} = \frac{5.04\angle -57.6^\circ}{5\angle 53.1^\circ + 20\angle 53.1^\circ} = \frac{5.04\angle -57.6^\circ}{25\angle 53.1^\circ}$$

$$\therefore I_0 = 0.202\angle -111^\circ\,\text{A}$$

$$I_1 = \frac{V_{L,g1}}{(3+j4)+Z_1} = \frac{273\angle 6.59^\circ}{(3+j4)+20\angle 53.1^\circ} = 10.9\angle -46.5^\circ\,\text{A}$$

$$I_2 = \frac{V_{L,g2}}{(3+j4)+Z_2} = \frac{27.8\angle -76.0^\circ}{(3+j4)+20\angle 53.1^\circ} = 1.11\angle -129^\circ \text{ A}$$

It follows that

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0.202\angle -111^\circ \\ 10.9\angle -46.5^\circ \\ 1.11\angle -129^\circ \end{bmatrix} = \begin{bmatrix} 6.73 - j8.96 \\ -9.57 - j2.91 \\ 2.63 + j11.3 \end{bmatrix} = \begin{bmatrix} 11.2\angle -53.1^\circ \\ 10.0\angle -163^\circ \\ 11.6\angle 76.9^\circ \end{bmatrix} \text{A}$$

Since the source and load neutrals are connected with a zero-ohm neutral wire, another way to express the currents above is, using the voltages given in 2.1,

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} V_{ag}/(3+j4+Z_Y) \\ V_{bg}/(3+j4+Z_Y) \\ V_{bg}/(3+j4+Z_Y) \end{bmatrix} = \begin{bmatrix} 280\angle 0^\circ/(3+j4+20\angle 53.1^\circ) \\ 250\angle -110^\circ/(3+j4+20\angle 53.1^\circ) \\ 290\angle 130^\circ/(3+j4+20\angle 53.1^\circ) \end{bmatrix} = \begin{bmatrix} 6.72 - j8.96 \\ -9.57 - j2.91 \\ 2.63 + j11.3 \end{bmatrix} = \begin{bmatrix} 11.2\angle -53.1^\circ \\ 10.0\angle -163^\circ \\ 11.6\angle 76.9^\circ \end{bmatrix} \text{A}$$

which confirms the results obtained above.

P.3 ➡ *Solution*

Problem 3.1: Applying Kirchhoff's volage law brings to

$$\begin{aligned} V_{an} &= Z_{aa}I_a + Z_{ab}I_b + Z_{ab}I_c + Z_{an}I_n + V_{a'n'} \\ &\quad -(Z_{nn}I_n + Z_{an}I_c + Z_{an}I_b + Z_{an}I_a) \end{aligned}$$

The voltage drop across the line section is given by

$$\begin{aligned} V_{an} - V_{a'n'} &= Z_{aa}I_a + Z_{ab}I_b + Z_{ab}I_c + Z_{an}I_n \\ &\quad -Z_{nn}I_n - Z_{an}I_c - Z_{an}I_b - Z_{an}I_a \\ \therefore V_{an} - V_{a'n'} &= (Z_{aa} - Z_{an})I_a + (Z_{ab} - Z_{an})(I_b + I_c) + (Z_{an} + Z_{nn})I_n \end{aligned}$$

Similarly for phases *b* and *c*,

$$\begin{aligned} V_{bn} - V_{b'n'} &= (Z_{aa} - Z_{an})I_b + (Z_{ab} - Z_{an})(I_a + I_c) + (Z_{an} + Z_{nn})I_n \\ V_{cn} - V_{c'n'} &= (Z_{aa} - Z_{an})I_c + (Z_{ab} - Z_{an})(I_a + I_b) + (Z_{an} + Z_{nn})I_n \end{aligned}$$

From Kirchoff's circuit law,

$$I_n = -(I_a + I_b + I_c)$$

Upon substitution,

$$\begin{aligned} V_{an} - V_{a'n'} &= (Z_{aa} + Z_{nn} - 2Z_{an})I_a + (Z_{ab} + Z_{nn} - 2Z_{an})I_b + (Z_{ab} + Z_{nn} - 2Z_{an})I_c \\ V_{bn} - V_{b'n'} &= (Z_{ab} + Z_{nn} - 2Z_{an})I_a + (Z_{aa} + Z_{nn} - 2Z_{an})I_b + (Z_{ab} + Z_{nn} - 2Z_{an})I_c \\ V_{cn} - V_{c'n'} &= (Z_{ab} + Z_{nn} - 2Z_{an})I_a + (Z_{ab} + Z_{nn} - 2Z_{an})I_b + (Z_{aa} + Z_{nn} - 2Z_{an})I_c \end{aligned}$$

We were given the definitions

$$\begin{aligned} Z_S &\equiv Z_{aa} + Z_{nn} - 2Z_{an} \\ Z_M &\equiv Z_{ab} + Z_{nn} - 2Z_{an} \end{aligned}$$

so that

$$\begin{aligned} V_{an} - V_{a'n'} &= Z_S I_a + Z_M I_b + Z_M I_c \\ V_{bn} - V_{b'n'} &= Z_M I_a + Z_S I_b + Z_M I_c \\ V_{cn} - V_{c'n'} &= Z_M I_a + Z_M I_b + Z_S I_c \end{aligned}$$

or, in matrix form,

$$\begin{bmatrix} V_{aa'} \\ V_{bb'} \\ V_{cc'} \end{bmatrix} = \begin{bmatrix} V_{an} - V_{a'n'} \\ V_{bn} - V_{b'n'} \\ V_{cn} - V_{c'n'} \end{bmatrix} = \begin{bmatrix} Z_S & Z_M & Z_M \\ Z_M & Z_S & Z_M \\ Z_M & Z_M & Z_S \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

where the voltage drops across the phase conductors are denoted by V_{aa}, V_{bb}, and V_{cc}.

Problem 3.2: The a-b-c voltage drops and currents of the line section can be written in terms of their symmetrical components according to $\boldsymbol{V_P}$ =

$\boldsymbol{AV_S}$, where $\boldsymbol{V_P}$ is the column vector of phase voltages, $\boldsymbol{V_S}$ is the column vector of sequence voltages, and $\boldsymbol{A}$ is a 3×3 transformation matrix. With phase a as the reference phase, we may write

$$A\begin{bmatrix} V_{aa'0} \\ V_{aa'1} \\ V_{aa'2} \end{bmatrix} = \left\{\begin{bmatrix} Z_S - Z_M & \bullet & \bullet \\ \bullet & Z_S - Z_M & \bullet \\ \bullet & \bullet & Z_S - Z_M \end{bmatrix} + \begin{bmatrix} Z_M & Z_M & Z_M \\ Z_M & Z_M & Z_M \\ Z_M & Z_M & Z_M \end{bmatrix}\right\} A\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}$$

Multiplying across by A^{-1},

$$\begin{bmatrix} V_{aa'0} \\ V_{aa'1} \\ V_{aa'2} \end{bmatrix} = A^{-1}\left\{(Z_S - Z_M)\begin{bmatrix} 1 & \bullet & \bullet \\ \bullet & 1 & \bullet \\ \bullet & \bullet & 1 \end{bmatrix} + Z_M\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}\right\} A\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}$$

or

$$\begin{bmatrix} V_{aa'0} \\ V_{aa'1} \\ V_{aa'2} \end{bmatrix} = \begin{bmatrix} Z_S - 2Z_M & \bullet & \bullet \\ \bullet & Z_S - Z_M & \bullet \\ \bullet & \bullet & Z_S - Z_M \end{bmatrix}\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}$$

Now define zero-, positive-, and negative-sequence impedances in terms of Z_S and Z_M as

$$Z_0 = Z_S + 2Z_M = Z_{aa} + 2Z_{ab} + 3Z_{nn} - 6Z_{an}$$
$$Z_1 = Z_S - Z_M = Z_{aa} - Z_{ab}$$
$$Z_2 = Z_S - Z_M = Z_{aa} - Z_{ab}$$

With reference to the last matrix equation, the sequence components of the voltage drops between the two ends of the line section can be written as three uncoupled equations, namely

$$V_{aa'0} = V_{an0} - V_{a'n'0} = Z_0 I_{a0}$$
$$V_{aa'1} = V_{an1} - V_{a'n'1} = Z_1 I_{a1}$$
$$V_{aa'2} = V_{an2} - V_{a'n'2} = Z_2 I_{a2}$$

P.4 ➡ *Solution*

Problem 4.1: Using the results from the previous problem, the sequence impedances are calculated as

$$Z_0 = Z_{aa} + 2Z_{ab} + 3Z_{nn} - 6Z_{an} = j60 + 2\times j20 + 3\times j80 - 6\times j30 = j160\,\Omega$$
$$Z_1 = Z_2 = j60 - j20 = j40\,\Omega$$

The sequence components of the voltage drops in the line are

$$\begin{bmatrix} V_{aa'0} \\ V_{aa'1} \\ V_{aa'2} \end{bmatrix} = A^{-1}\begin{bmatrix} V_{an} - V_{a'n'} \\ V_{bn} - V_{b'n'} \\ V_{cn} - V_{c'n'} \end{bmatrix} = A^{-1}\begin{bmatrix} (182 + j70) - (154 + j28) \\ (72.24 - j32.62) - (44.24 - j74.62) \\ (-170.24 + j88.62) - (-198.24 + j46.62) \end{bmatrix} = A^{-1}\begin{bmatrix} 28.0 + j42.0 \\ 28.0 + j42.0 \\ 28.0 + j42.0 \end{bmatrix}$$

$$\therefore \begin{bmatrix} V_{aa'0} \\ V_{aa'1} \\ V_{aa'2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}\begin{bmatrix} 28.0 + j42.0 \\ 28.0 + j42.0 \\ 28.0 + j42.0 \end{bmatrix} = \begin{bmatrix} 28.0 + j42.0 \\ 0 \\ 0 \end{bmatrix}\ \text{kV}$$

Now, the solution to Problem 3 tells us that

$$V_{aa'0} = V_{an0} - V_{a'n'0} = Z_0 I_{a0}$$
$$V_{aa'1} = V_{an1} - V_{a'n'1} = Z_1 I_{a1}$$
$$V_{aa'2} = V_{an2} - V_{a'n'2} = Z_2 I_{a2}$$

Substituting Z_n's and the results from the previous matrix equation, we find that

$$V_{aa'0} = 28{,}000 + j42{,}000 = j160 I_{a0}$$
$$\therefore I_{a0} = 262.5 - j175\ \text{A}$$

$$V_{aa'1} = 0 = j40 I_{a1}$$
$$\therefore I_{a1} = 0$$
$$V_{aa'2} = 0 = j40 I_{a2}$$
$$\therefore I_{a2} = 0$$

The line currents are then

$$\boxed{I_a = I_b = I_c = 262.5 - j175 \text{ A}}$$

Problem 4.2: The self- and mutual impedances are

$$Z_S = Z_{aa} + Z_{nn} - 2Z_{an} = j60 + j80 - 2 \times j30 = j80\,\Omega$$
$$Z_M = Z_{ab} + Z_{nn} - 2Z_{an} = j20 + j80 - 2 \times j30 = j40\,\Omega$$

Accordingly, line currents can be calculated as

$$\begin{bmatrix} V_{aa'} \\ V_{bb'} \\ V_{cc'} \end{bmatrix} = \begin{bmatrix} 28.0 + j42.0 \\ 28.0 + j42.0 \\ 28.0 + j42.0 \end{bmatrix} \times 10^3 = \begin{bmatrix} j80 & j40 & j40 \\ j40 & j80 & j40 \\ j40 & j40 & j80 \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

$$\therefore \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} j80 & j40 & j40 \\ j40 & j80 & j40 \\ j40 & j40 & j80 \end{bmatrix}^{-1} \begin{bmatrix} 28.0 + j42.0 \\ 28.0 + j42.0 \\ 28.0 + j42.0 \end{bmatrix} \times 10^3$$

Using MATLAB,

```
Iabc =

   1.0e+02 *

   2.6250 - 1.7500i
   2.6250 - 1.7500i
   2.6250 - 1.7500i
```

that is,

$$\boxed{\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 262.5 - j175 \\ 262.5 - j175 \\ 262.5 - j175 \end{bmatrix} \text{A}}$$

P.5 ➡ *Solution*

A link to Saadat's MATLAB toolbox can be found in our website. The command to use is `abc2sc`, which converts a set of three phasors to its symmetrical components. Since the output of `abc2sc` is in rectangular form, finish by applying `rec2pol`.

```
>> Vabc = [300,-120; 200,90; 100,-30]

Vabc =

   300  -120
   200    90
   100   -30

>> V012 = abc2sc(Vabc);
>> V012p = rec2pol(V012)

V012p =

   42.2650 -120.0000
  193.1852 -135.0000
   86.9473  -84.8961
```

The symmetrical components are V_0 = 42.27∠ −120°, V_1 = 193.2∠ −135°, and V_2 = 86.95∠ −84.9°.

P.6 ➡ *Solution*

To convert a set of symmetrical components back to the original unbalanced phasors, use `sc2abc`.

```
>> I012 = [3,-30; 5,90; 4,30];
>> Iabc = sc2abc(I012);
>> Iabcp = rec2pol(Iabc)

Iabcp =

    8.1854   42.2163
    4.0000  -30.0000
    8.1854 -102.2163
```

The unbalanced phasors are I_a = 8.185∠42.22°, I_b = 4∠ −30°, and I_c = 8.185∠ −102.2°.

P.7 ➡ Solution

First find the symmetrical components of line voltages, then find the symmetrical components of phase voltages. Use the inverse symmetrical components transformation to obtain the phase voltages. The commands involved are `abc2sc`, `rec2pol`, and `sc2abc`.

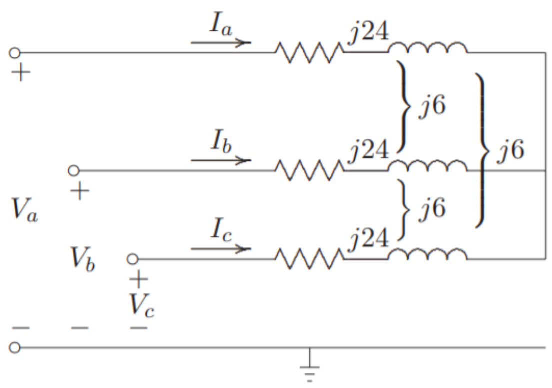
```
a = -0.5 + 1i*sqrt(3)/2; %a operator
Vabbcca = [1000,0; 866.0254,-150; 500,120];
VL012 = abc2sc(Vabbcca); %Finds the sym. comp. of line volt., rect.
VL012p = rec2pol(VL012) %Converts the sym. comp. of line volt. to polar
Va012 = [0; VL012(2)/(sqrt(3)*(0.866+1i*0.5));
VL012(3)/(sqrt(3)*(0.866-1i*0.5))]; %Sym. comp. of phase voltages, rect.
Va012p = rec2pol(Va012) %Converts the sym. comp. of phase volt. to polar
Vabc = sc2abc(Va012); %Unbalanced phase volt., rect.
Vabcp = rec2pol(Vabc) %Converts unbalanced phase volt. to polar
```

The output of the above code is shown below. Matrix `VL012p` contains the symmetrical components of the line voltages in polar form; matrix `Va012p` contains the symmetrical components of phase voltages in polar form; finally, matrix `Vabcp` contains the unbalanced phase voltages in polar form.

```
VL012p =

     0.0000    30.0000
   763.7626   -10.8934
   288.6751    30.0000

Va012p =

          0          0
   440.9683   -40.8941
   166.6703    60.0007

Vabcp =

   440.9641   -19.1072
   600.9394  -166.1024
   333.3443    59.9989
```

P.8 ➡ Solution

Problem 8.1:



Applying Kirchhoff's voltage law to the two independent mesh equations, and writing one node equation, results in

$$\begin{bmatrix} (Z_S - Z_M) & -(Z_S - Z_M) & 0 \\ 0 & (Z_S - Z_M) & -(Z_S - Z_M) \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} |V_L|\angle \pi/6 \\ |V_L|\angle -\pi/2 \\ 0 \end{bmatrix}$$

In matrix notation,

$$\mathbf{Z}_{\text{mesh}} \mathbf{I}^{\text{abc}} = \mathbf{V}_{\text{mesh}}$$

or, equivalently,

$$\mathbf{I}^{\text{abc}} = \mathbf{Z}_{\text{mesh}}^{-1} \mathbf{V}_{\text{mesh}}$$

Following Saadat, we write a short piece of code to determine the currents.

```
disp('(a) Solution by mesh analysis')
VL = 360*sqrt(3); %Line voltage magnitude
Zs = 1i*24; Zm = 1i*6; %Series and mutual impedances
Z = [(Zs-Zm), -(Zs-Zm), 0;
     0, (Zs-Zm), -(Zs-Zm);
            1, 1, 1]; %Matrix from 2 mesh eqs. and one node
eq.
V = [VL*cos(pi/6)+1i*VL*sin(pi/6); VL*cos(-pi/2)+1i*VL*sin(-
pi/2); 0];
Iabc = Z\V; %Line currents, rect.
Iabcp = [abs(Iabc), angle(Iabc)*180/pi] %Converts line
currents to polar
```

The output is

```
Iabcp =

20.0000  -90.0000

20.0000  150.0000

20.0000   30.0000
```

That is, $I_a = 20\angle -90^\circ$, $I_b = 20\angle 150^\circ$, and $I_c = 20\angle 30^\circ$.

Problem 8.2: Using the symmetrical components method, we have

$$\mathbf{V}^{012} = \mathbf{Z}^{012}\mathbf{I}^{012}$$

where

$$\mathbf{V}^{012} = \begin{bmatrix} 0 \\ V_a \\ 0 \end{bmatrix} ; \ \mathbf{Z}^{012} = \begin{bmatrix} Z_S + 2Z_M & 0 & 0 \\ 0 & Z_S - Z_M & 0 \\ 0 & 0 & Z_S - Z_M \end{bmatrix}$$

so that

$$\mathbf{I}^{012} = \left[\mathbf{Z}^{012}\right]^{-1} \mathbf{V}^{012}$$

A second piece of code is provided below.

```
disp('(b) Solution by symmetrical components')
Z012 = [Zs+2*Zm, 0, 0; 0, Zs-Zm, 0; 0, 0, Zs-Zm]; %Sym.
comp. matrix
V012 = [0; VL/sqrt(3); 0]; %Sym. comp. of phase voltages
I012 = inv(Z012)*V012 %Sym. comp. of line currents
a = cos(2*pi/3)+1i*sin(2*pi/3);
A = [1, 1, 1; 1, a^2, a; 1, a, a^2]; %Transformation matrix
Iabc = A*I012; %Line currents, rect.
Iabcp = [abs(Iabc), angle(Iabc)*180/pi] %Converts line curr.
to polar
```

The outputs are

```
I012 =
   0.0000 + 0.0000i
   0.0000 -20.0000i
   0.0000 + 0.0000i

Iabcp =
   20.0000  -90.0000
   20.0000  150.0000
   20.0000   30.0000
```

As before, $I_a = 20\angle -90^\circ$, $I_b = 20\angle 150^\circ$, and $I_c = 20\angle 30^\circ$.

P.9 ➡ *Solution*

Problems 9.1 to 9.6: All tasks can be accomplished with the following MATLAB code.

```
Vabc = [300,-120; 200,90; 100,-30]; %Phase-to-neutral
voltages
Zabc = [10+1i*40, 1i*5, 1i*5; 1i*5, 10+1i*40, 1i*5; 1i*5,
1i*5, 10+1i*40]; %Self and mutual impedances matrix
Z012 = zabc2sc(Zabc) %Sym. components of impedance
V012 = abc2sc(Vabc); %Sym. components of voltage, rect.
V012p = rec2pol(V012) %Converts sym. components of volt. to
polar
I012 = inv(Z012)*V012; %Sym. components of current, rect.
I012p = rec2pol(I012) %Converts sym. components of curr. to
polar
Iabc = sc2abc(I012); %Phase currents, rect.
Iabcp = rec2pol(Iabc) %Converts phase currents to polar
S3ph = 3*(V012.')*conj(I012) %Power using symmetrical
components
Vabcr = Vabc(:,1).*(cos(pi/180*Vabc(:,2)) +
1i*sin(pi/180*Vabc(:,2)));
S3ph = (Vabcr.')*conj(Iabc) %Power using phase quantities
```

The outputs are listed below. `Z012` is the load sequence impedance matrix.

```
Z012 =

  10.0000 +50.0000i  -0.0000 + 0.0000i   0.0000 + 0.0000i

   0.0000 + 0.0000i  10.0000 +35.0000i  -0.0000 + 0.0000i

  -0.0000 + 0.0000i   0.0000 + 0.0000i  10.0000 +35.0000i
```

`V012p` contains the symmetrical components of voltage in polar form.

```
V012p =

   42.2650 -120.0000
  193.1852 -135.0000
   86.9473  -84.8961
```

`I012p` contains the symmetrical components of current in polar form.

```
I012p =

    0.8289  161.3099
    5.3072  150.9454
    2.3886 -158.9507
```

`Iabcp` contains the load phase currents.

```
Iabcp =

    7.9070  165.4600
    5.8190   14.8676
    2.7011  -96.9315
```

`S3ph` contains the load phase currents as calculated via $3(V_a^0 I_a^{0^*} + V_a^1 I_a^{1^*} + V_a^2 I_a^{2^*})$.

```
S3ph =

   1.0368e+03 + 3.6596e+03i
```

`S3ph` is printed a second time, this time as calculated from $S_{3\phi} = V_a I_a^* + V_b I_b^* + V_c I_c^*$.

```
S3ph =

   1.0368e+03 + 3.6596e+03i
```

P.10 ➡ *Solution*

Problems 10.1 to 10.3: The pertaining code is shown below.

```
Vabbcca = [600,36.87; 800,126.87; 1000,-90];
VL012 = abc2sc(Vabbcca); %Sym. comp. of line voltages, rect.
VL012p = rec2pol(VL012) %Converts sym. comp. of line
voltages to polar
Va012 = [0; VL012(2)/(sqrt(3)*(0.866+1i*.5));
VL012(3)/(sqrt(3)*(0.866-1i*.5))]; %Sym. comp. of phase
volt., rect.
```

```
Va012p = rec2pol(Va012) %Converts sym. comp. of phase
voltages to polar
Vabc = sc2abc(Va012); %Phase voltages, rect.
Vabcp = rec2pol(Vabc) %Converts phase voltages to polar
Iabc = Vabc/37; %Line currents, rect.
Iabcp = rec2pol(Iabc) %Converts the line currents to polar
```

The first output is `VL012p`, which contains the symmetrical components of line voltages in polar form.

```
VL012p =
    0.0006 -179.9999
  237.0762  169.9342
  781.3204   24.0621
```

The second output is `Va012p`, which contains the symmetrical components of phase voltages in polar form.

```
Va012p =
         0         0
  136.8790  139.9335
  451.1055   54.0628
```

The third output is `Vabcp`, which contains the unbalanced phase voltages in polar form.

```
Vabcp =
  480.7542   70.5606
  333.3386  163.7411
  569.6111  -73.6857
```

Lastly, we divide the `Vabcp` entries by 37 Ω to obtain the line currents `Iabc` and convert them to polar form with `rec2pol`. The output is

```
Iabcp =
   12.9934   70.5606
    9.0092  163.7411
   15.3949  -73.6857
```

P.11 ➡ *Solution*

Problem 11.1: We begin by calculating the per-unit reactances of the synchronous generators. For G1,

$$X_1 = X_d'' = 0.18 \;\; ; \;\; X_2 = X_d'' = 0.18 \;\; ; \;\; X_0 = 0.07\,\text{pu}$$

For G2,

$$X_1 = X_d'' = 0.20 \;\; ; \;\; X_2 = X_d'' = 0.20 \;\; ; \;\; X_0 = 0.10\,\text{pu}$$

For G3,

$$X_1 = X_d'' = 0.15 \times \left(\frac{13.8}{15}\right)^2 \times \left(\frac{1000}{500}\right) = 0.2539\,\text{pu}$$

$$X_2 = X_d'' = 0.2539\,\text{pu}$$

$$X_0 = 0.05 \times \left(\frac{13.8}{15}\right)^2 \times \left(\frac{1000}{500}\right) = 0.08464\,\text{pu}$$

Also, $3X_n = 3X_0 = 0.2539$.
For G4,

$$X_1 = X_d'' = 0.30 \times \left(\frac{13.8}{15}\right)^2 \times \left(\frac{1000}{750}\right) = 0.3386\,\text{pu}$$

$$X_2 = 0.40 \times \left(\frac{13.8}{15}\right)^2 \times \left(\frac{1000}{750}\right) = 0.4514\,\text{pu}$$

$$X_0 = 0.10 \times \left(\frac{13.8}{15}\right)^2 \times \left(\frac{1000}{750}\right) = 0.1129\,\text{pu}$$

For the transformers, we have X_{T1} = 0.10 pu, X_{T2} = 0.10 pu, X_{T3} = 0.12(1000/500) = 0.24 pu, and X_{T4} = 0.11(1000/750) = 0.1467 pu. Considering the transmission lines with a base impedance Z_{base} = 765^2/1000 = 585.22 Ω, we have, for the positive and negative sequence reactances,

$$X_{12} = \frac{50}{585.22} = 0.08544\,\text{pu}$$

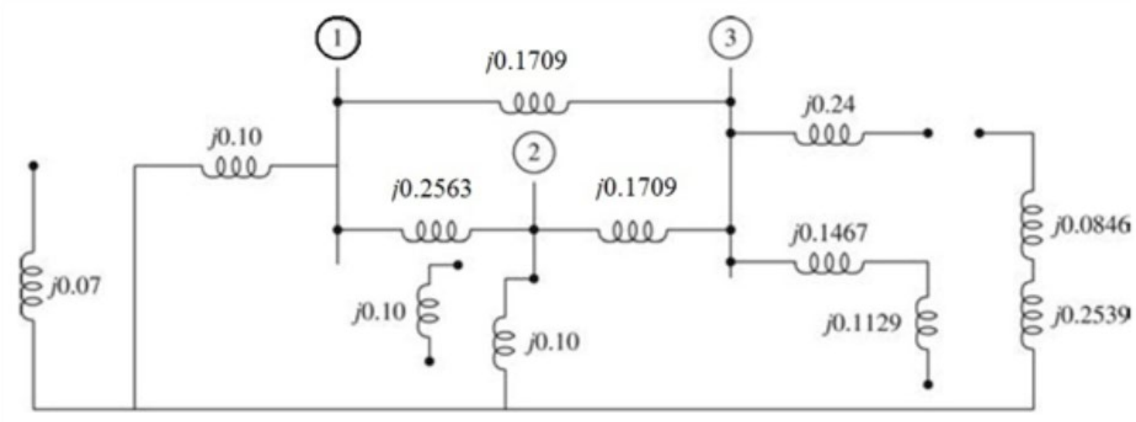
$$X_{13} = X_{23} = \frac{40}{585.22} = 0.06835$$

For the zero sequence reactances,

$$X_{12} = \frac{150}{585.22} = 0.2563\,\text{pu}$$

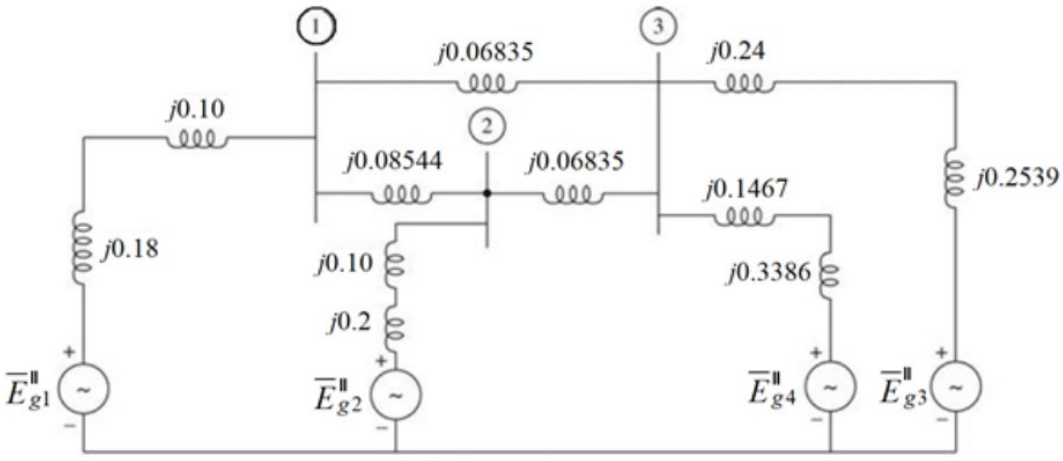
$$X_{13} = X_{23} = \frac{100}{585.22} = 0.1709\,\text{pu}$$

The per-unit zero-sequence network is drawn below.

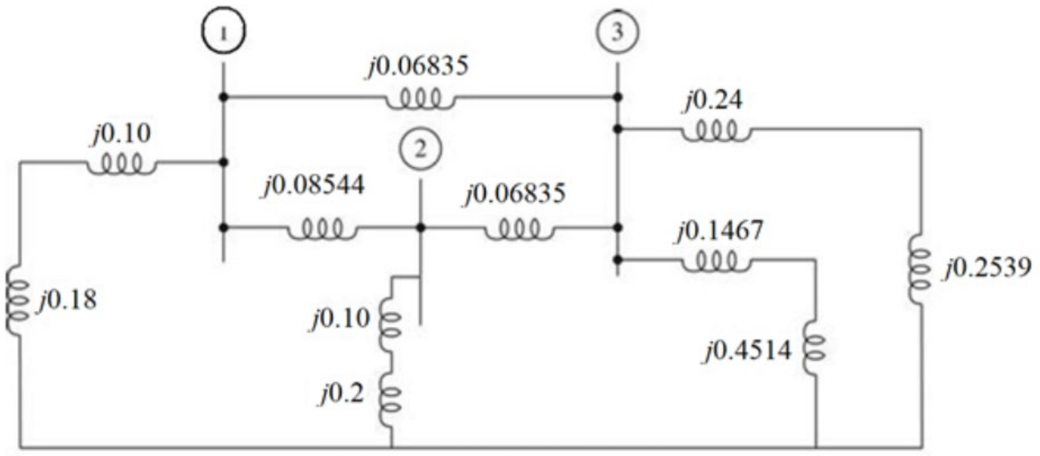


Per unit zero sequence network

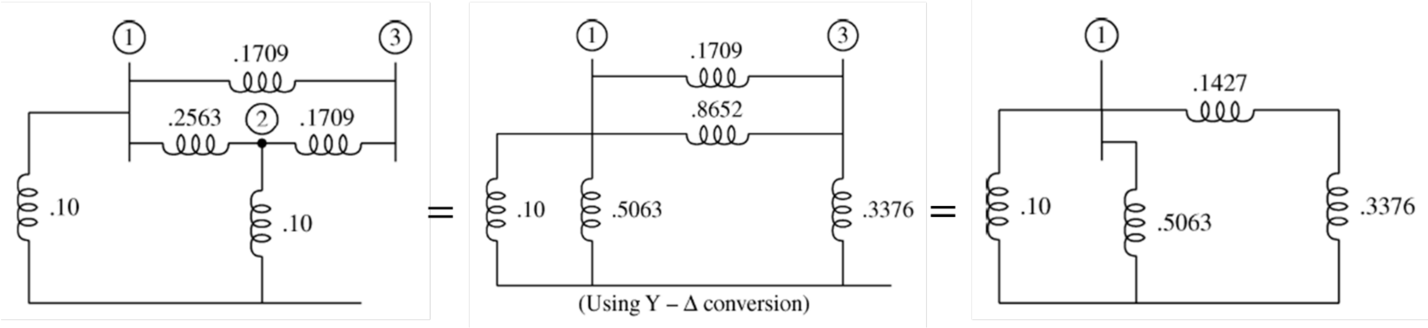
The per-unit positive-sequence network is drawn below.



The per-unit negative-sequence network is drawn below.



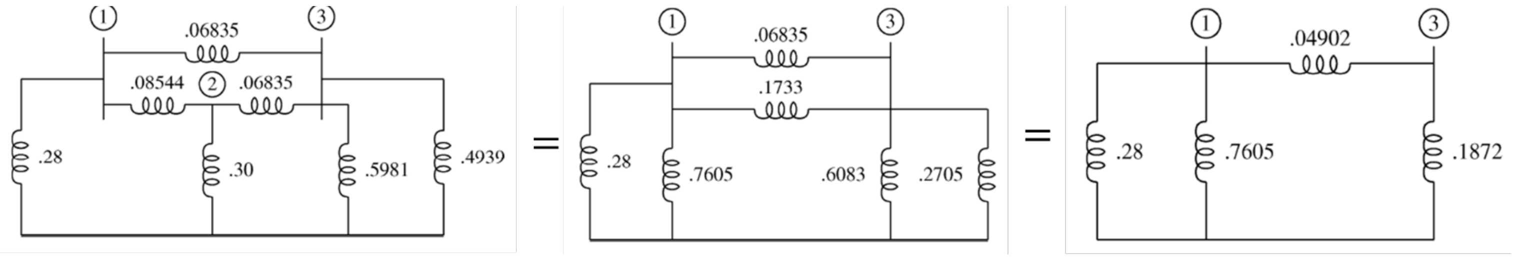
Problem 11.2: The zero-sequence Thévenin equivalent is developed below.



Reducing the latter reactances,

$$X_0 = 0.10 // 0.5063 // \left(0.1427 + 0.3376\right)$$
$$\therefore X_0 = 0.10 // 0.2465 = 0.07114\,\text{pu}$$

The negative-sequence Thévenin equivalent is developed below.



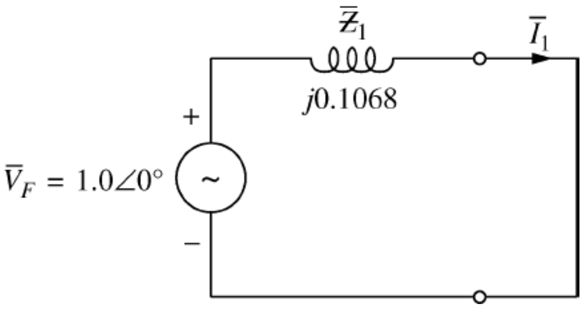
Reducing the remaining reactances,

$$X_2 = 0.28 // 0.7605 // \left(0.04902 + 0.1872\right)$$
$$\therefore X_2 = 0.28 // 0.1802 = 0.1096\,\text{pu}$$

Proceeding similarly with the positive-sequence reactances, you should find

$$X_1 = 0.28 // 0.7605 // \left(0.04902 + 0.1745\right) = 0.1068\,\text{pu}$$

Problem 11.3: Refer to the positive-sequence Thévenin equivalent obtained above.



The base current value is

$$I_{\text{base}} = \frac{S_{\text{base}3\phi}}{\sqrt{3} V_{\text{base}}} = \frac{1000\,\text{MVA}}{\sqrt{3} \times 765\,\text{kV}} = 0.7547\,\text{kA}$$

The current in the positive-sequence circuit is found as

$$I_1 = \frac{V_F}{Z_1} = \frac{1.0\angle 0^\circ}{j0.1068} = -j9.363\,\text{pu}$$

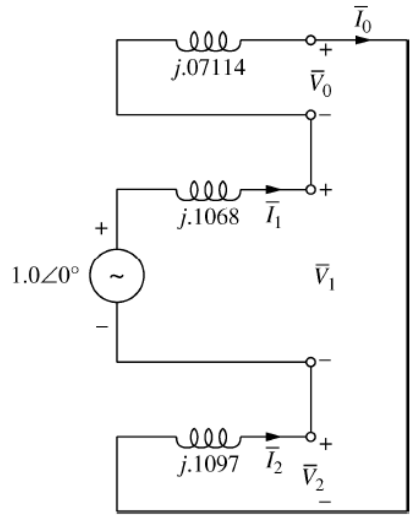
Since the fault is symmetrical, the zero- and negative-sequence currents are zero: $I_0 = I_2 = 0$. We proceed to compute the subtransient fault currents of each phase,

$$\begin{bmatrix} I''_A \\ I''_B \\ I''_C \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ -j9.363 \\ 0 \end{bmatrix} = \begin{bmatrix} -j9.363 \\ -8.1086 + j4.6815 \\ 8.1086 + j4.6815 \end{bmatrix} = \begin{bmatrix} 9.363\angle -90^\circ \\ 9.363\angle 150^\circ \\ 9.363\angle 30^\circ \end{bmatrix} \text{pu}$$

Multiplying by the base current,

$$\begin{bmatrix} I''_A \\ I''_B \\ I''_C \end{bmatrix} = \begin{bmatrix} 9.363\angle -90^{\circ} \\ 9.363\angle 150^{\circ} \\ 9.363\angle 30^{\circ} \end{bmatrix} \times 0.7547 = \begin{bmatrix} 7.067\angle -90^{\circ} \\ 7.067\angle 150^{\circ} \\ 7.067\angle 30^{\circ} \end{bmatrix} \text{kA}$$

Problem 11.4: A bolted single-line-to-ground fault occurs at bus 1.



The pertaining reactances have been determined above. The zero-, positive-, and negative-sequence current components are given by

$$I_0 = I_1 = I_2 = \frac{E_R}{Z_0 + Z_! + Z_2} = \frac{1.0}{j0.07114 + j0.1068 + j0.1096} = -j3.4778\,\text{pu}$$

We proceed to determine the subtransient fault current,

$$\begin{bmatrix} I''_A \\ I''_B \\ I''_C \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} -j3.4778 \\ -j3.4778 \\ -j3.4778 \end{bmatrix} = \begin{bmatrix} -j10.4334 \\ 0 \\ 0 \end{bmatrix} \text{pu}$$

or $-j10.4334 \times 0.7547 = -j7.8741$ kA. To determine the sequence components of voltage at fault current, we write

$$\begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1.0\angle 0^{\circ} \\ 0 \end{bmatrix} - \begin{bmatrix} j0.07114 & 0 & 0 \\ 0 & j0.1068 & 0 \\ 0 & 0 & j0.1096 \end{bmatrix} \begin{bmatrix} -j3.4778 \\ -j3.4778 \\ -j3.4778 \end{bmatrix} = \begin{bmatrix} -0.2474 \\ 0.6286 \\ -0.3812 \end{bmatrix} \text{pu}$$

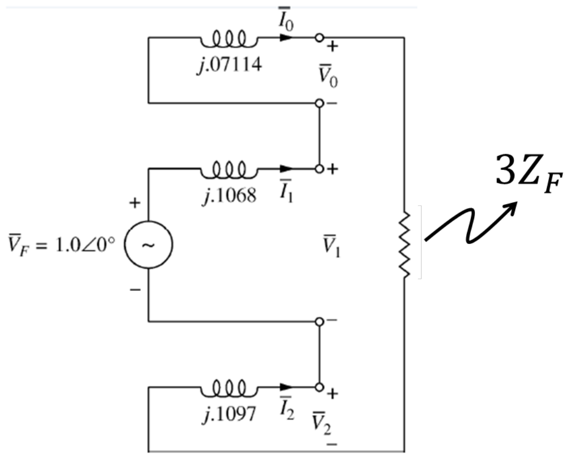
Finally, we convert the sequence components to line-to-phase voltages,

$$\begin{bmatrix} V_{Ag} \\ V_{Bg} \\ V_{Cg} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} -0.2474 \\ 0.6286 \\ -0.3812 \end{bmatrix} = \begin{bmatrix} 0 \\ 0.9500\angle -112.994^{\circ} \\ 0.9500\angle 112.994^{\circ} \end{bmatrix} \text{pu}$$

Problem 11.5: Expressing the added arc impedance in per units, we have

$$Z_F = \frac{15 + j0}{585.22} = 0.02563\,\text{pu}$$

In total, the arc impedance contributes $3Z_F$ = 0.07689 pu to the circuit.



The zero-, positive-, and negative-sequence current components are given by

$$I_0 = I_1 = I_2 = \frac{E_R}{Z_0 + Z_1 + Z_2 + 3Z_F} = \frac{1.0}{j0.07114 + j0.1068 + j0.1096 + 0.07689}$$

$$\therefore I_0 = I_1 = I_2 = 0.8679 - j3.2457 = 3.3597\angle -75.0288^\circ \text{ pu}$$

The phase currents during fault are $I_A'' = 3I_0 = 10.0791\angle -75.0288^\circ$ pu and $I_B'' = I_C'' = 0$ pu. Multiplying by the base current (0.7547 kA) yields $I_A'' = 7.6067\angle -75.0288^\circ$ kA and $I_B'' = I_C'' = 0$. Determining the voltages V_{012} at fault is no different from the previous problem,

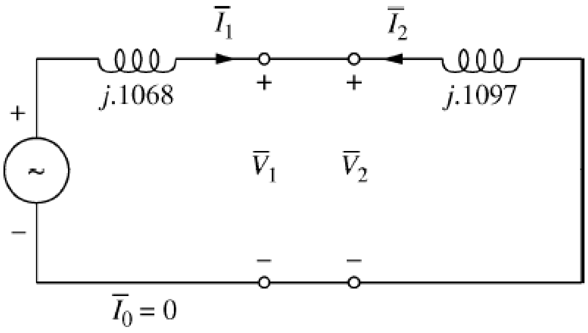
$$\begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1.0\angle 0^\circ \\ 0 \end{bmatrix} - \begin{bmatrix} j0.07114 & 0 & 0 \\ 0 & j0.1068 & 0 \\ 0 & 0 & j0.1096 \end{bmatrix} \begin{bmatrix} 3.3597\angle -75.0288^\circ \\ 3.3597\angle -75.0288^\circ \\ 3.3597\angle -75.0288^\circ \end{bmatrix} = \begin{bmatrix} -0.2309 - j0.0617 \\ 0.6534 - j0.0927 \\ -0.3557 - j0.0951 \end{bmatrix}$$

$$\therefore \begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 0.2390\angle -165.029^\circ \\ 0.6599\angle -8.0748^\circ \\ 0.3682\angle -165.029^\circ \end{bmatrix} \text{pu}$$

Lastly, we convert the sequence components into line-to-phase voltages,

$$\begin{bmatrix} V_{Ag} \\ V_{Bg} \\ V_{Cg} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0.2390\angle -165.029^\circ \\ 0.6599\angle -8.0748^\circ \\ 0.3682\angle -165.029^\circ \end{bmatrix} = \begin{bmatrix} 0.2583\angle -75.0213^\circ \\ 0.9225\angle -114.163^\circ \\ 0.9832\angle 112.851^\circ \end{bmatrix} \text{pu}$$

Problem 11.6: Now, a bolted line-to-line fault occurs at bus 1.



Current components I_1 and I_2 flow in opposite directions and are given by

$$I_1 = -I_2 = \frac{V_F}{Z_1 + Z_2} = \frac{1.0\angle 0^\circ}{j0.1068 + j0.1097} = -j4.6189 \text{ pu}$$

Also, $I_0 = 0$. The fault currents are determined next,

$$\begin{bmatrix} I_A'' \\ I_B'' \\ I_C'' \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ -j4.6189 \\ j4.6189 \end{bmatrix} = \begin{bmatrix} 0 \\ 8.0002\angle 180^\circ \\ 8.0002\angle 0^\circ \end{bmatrix} \text{pu} = \begin{bmatrix} 0 \\ 6.0377\angle 180^\circ \\ 6.0377\angle 0^\circ \end{bmatrix} \text{kA}$$

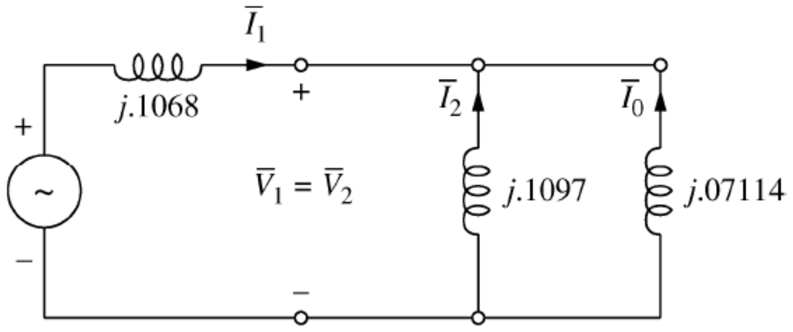
Voltages V_{012} are such that

$$\begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1.0\angle 0^\circ \\ 0 \end{bmatrix} - \begin{bmatrix} j0.07114 & 0 & 0 \\ 0 & j0.1068 & 0 \\ 0 & 0 & j0.1096 \end{bmatrix} \begin{bmatrix} 0 \\ -j4.6189 \\ j4.6189 \end{bmatrix} = \begin{bmatrix} 0 \\ 0.5067 \\ 0.5062 \end{bmatrix} \text{pu}$$

The fact that $|V_1| \neq |V_2|$ is due to round-off error. We finish by determining the phase voltages,

$$\begin{bmatrix} V_{Ag} \\ V_{Bg} \\ V_{Cg} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ 0.5067 \\ 0.5062 \end{bmatrix} = \begin{bmatrix} 1.0219 \\ -0.5065 \\ -0.5065 \end{bmatrix} = \begin{bmatrix} 1.0219\angle 0^\circ \\ 0.5065\angle 180^\circ \\ 0.5065\angle 180^\circ \end{bmatrix} \text{pu}$$

Problem 11.7: Now, a double line-to-ground fault occurs at bus 1.



Referring to the diagram above, current I_1 is determined as

$$I_1 = \frac{V_F}{Z_1 + Z_2 // Z_0} = \frac{1.0\angle 0^{\circ}}{j0.1068 + \dfrac{j0.1097 \times j0.07114}{j0.1097 + j0.07114}} = -j6.6687 = 6.6687\angle -90^{\circ}\,\text{pu}$$

By current division,

$$I_2 = -I_1 \times \left(\frac{Z_0}{Z_0 + Z_2}\right) = -(-j6.6687) \times \frac{0.07114}{0.07114 + 0.1097} = j2.6236\,\text{pu}$$

$$I_0 = -I_1 \times \left(\frac{Z_2}{Z_0 + Z_2}\right) = -(-j6.6687) \times \frac{0.1097}{0.07114 + 0.1097} = j4.0453\,\text{pu}$$

Currents I_{ABC} are such that

$$\begin{bmatrix} I_A'' \\ I_B'' \\ I_C'' \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} j4.0453 \\ -j6.6687 \\ j2.6236 \end{bmatrix} = \begin{bmatrix} 0 \\ 10.0786\angle 142.983^{\circ} \\ 10.0786\angle 37.0169^{\circ} \end{bmatrix} \text{pu} = \begin{bmatrix} 0 \\ 7.6063\angle 142.983^{\circ} \\ 7.6063\angle 37.0169^{\circ} \end{bmatrix} \text{kA}$$

Voltages V_{012} are such that

$$\begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1.0\angle 0^{\circ} \\ 0 \end{bmatrix} - \begin{bmatrix} j0.07114 & 0 & 0 \\ 0 & j0.1068 & 0 \\ 0 & 0 & j0.1096 \end{bmatrix} \begin{bmatrix} j4.0453 \\ -j6.6687 \\ j2.6236 \end{bmatrix} = \begin{bmatrix} 0.2878 \\ 0.2878 \\ 0.2876 \end{bmatrix} \text{pu}$$

The fact that $V_2 \neq V_0, V_1$ is due to round-off error. Finally, phase voltages V_{ABC} are such that

$$\begin{bmatrix} V_{Ag} \\ V_{Bg} \\ V_{Cg} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0.2878 \\ 0.2878 \\ 0.2878 \end{bmatrix} = \begin{bmatrix} 0.8634 \\ 0 \\ 0 \end{bmatrix} \text{pu}$$

P.12 ➡ Solution

Problem 12.1: We begin by calculating the per-unit reactances of the synchronous generators. For G1,

$$X_1 = X_d'' = 0.2 \times \left(\frac{1000}{500}\right) = 0.4 \;\; ; \;\; X_2 = X_d'' = 0.4 \;\; ; \;\; X_0 = 0.10 \times \left(\frac{1000}{500}\right) = 0.2\,\text{pu}$$

For G2,

$$X_1 = X_d'' = 0.18 \times \left(\frac{1000}{750}\right) = 0.24 \;\; ; \;\; X_2 = X_d'' = 0.24 \;\; ; \;\; X_0 = 0.09 \times \left(\frac{1000}{750}\right) = 0.12\,\text{pu}$$

For G3,

$$X_1 = 0.17 \times \left(\frac{1000}{1000}\right) = 0.17\,\text{pu} \;\; ; \;\; X_2 = 0.20 \times \left(\frac{1000}{1000}\right) = 0.20\,\text{pu} \;\; ; \;\; X_0 = 0.09 \times \left(\frac{1000}{1000}\right) = 0.09\,\text{pu}$$

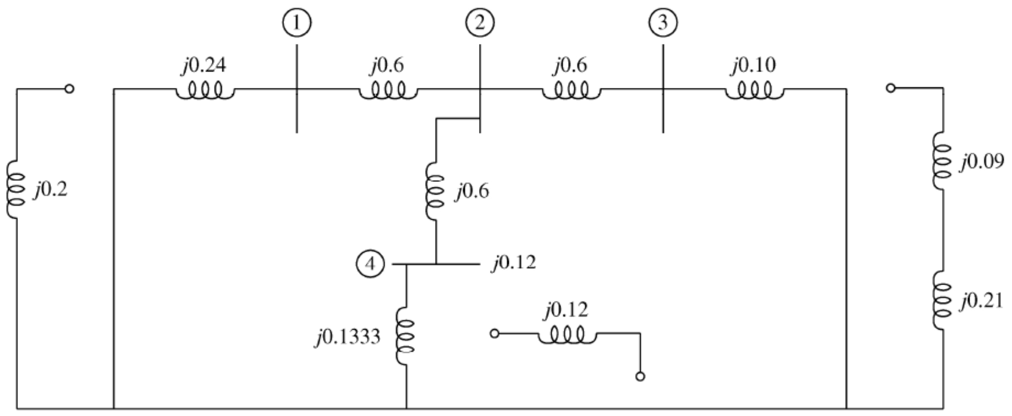
For generator 3, the neutral reactance added to the generator has reactance of 0.028 Ω. Using as a reference $X_{\text{base}3} = 20^2/1000 = 0.4\ \Omega$, we have $3X_n = 3 \times 0.028/0.4 = 0.21$ pu. For the transformers, we have $X_{T1} = 0.12(1000/500) = 0.24$ pu, $X_{T2} = 0.10(1000/750) = 0.1333$ pu, and $X_{T3} = 0.10(1000/1000) = 0.10$ pu. Considering the transmission lines with a base impedance $Z_{\text{base}} = 500^2/1000 = 250\ \Omega$, we have, for the positive and negative sequence reactances,

$$X_1 = X_2 = \frac{50}{250} = 0.20\,\text{pu}$$

For the zero sequence reactance,

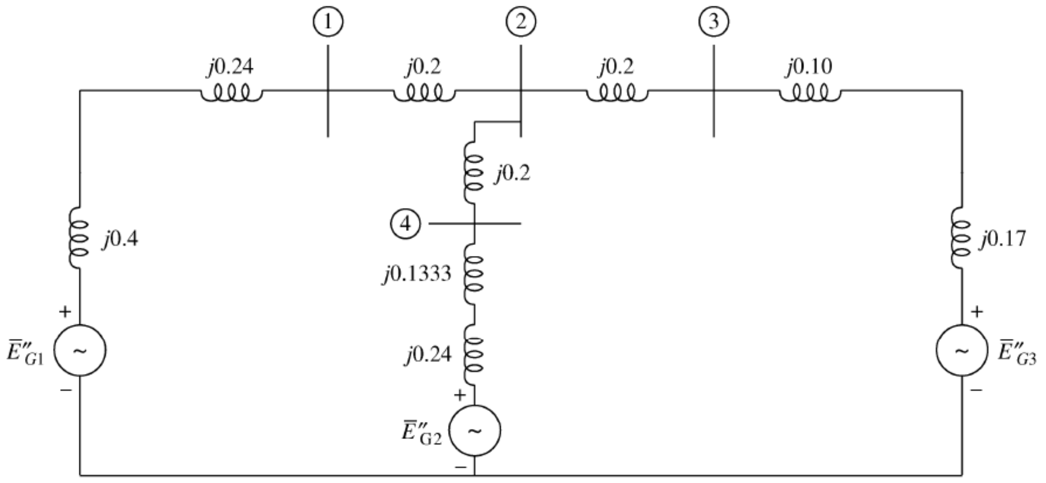
$$X_0 = \frac{150}{250} = 0.60\,\text{pu}$$

The per-unit zero-sequence network is drawn below.



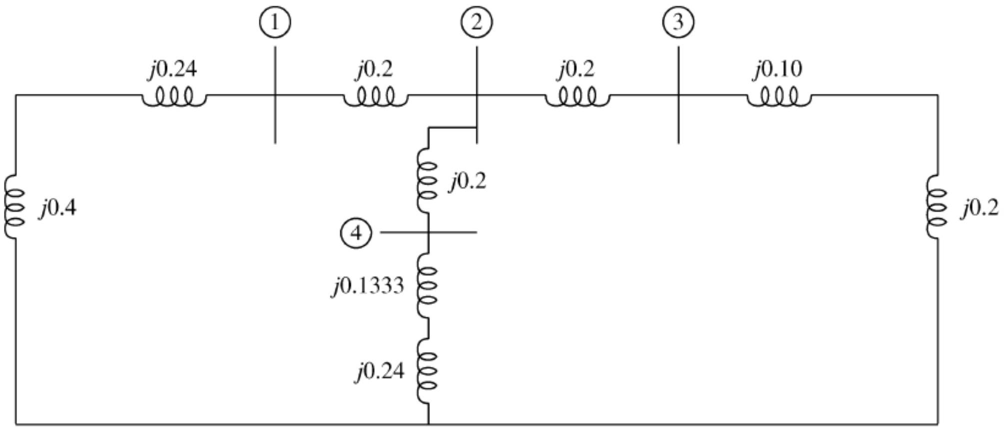
Per unit zero sequence network

The per-unit positive-sequence network is drawn below.



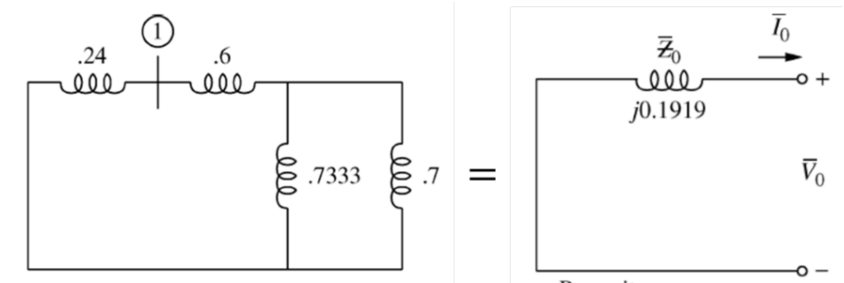
Per unit positive sequence network

The per-unit negative-sequence network is drawn below.



Per unit negative sequence network

Problem 12.2: The zero-sequence Thévenin equivalent is shown below.



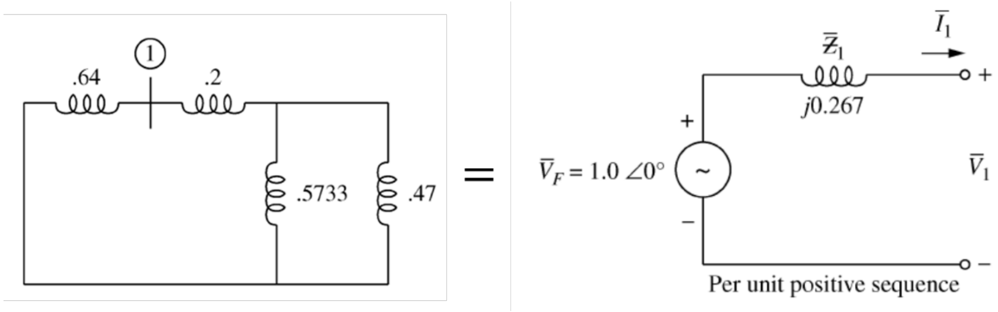
Per unit zero sequence

Reducing the latter reactances,

$$X_0 = 0.24//\left[0.6 + \left(0.7333//0.7\right)\right]$$

$$\therefore X_0 = 0.24//0.9581 = 0.1919\,\text{pu}$$

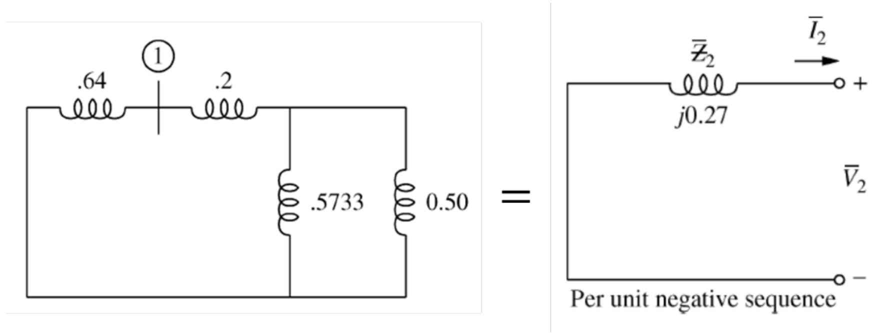
The positive-sequence Thévenin equivalent is shown below.



Reducing the latter impedances,

$$X_1 = 0.64//\left[0.20+\left(0.5733//0.47\right)\right]$$
$$\therefore X_1 = 0.64//0.4583 = 0.2670\,\text{pu}$$

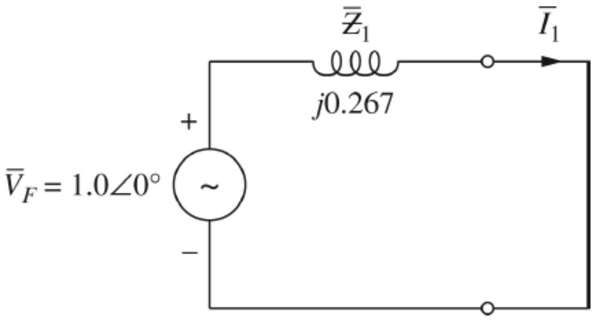
The negative-sequence Thévenin equivalent is shown below.



Reducing the remaining reactances,

$$X_1 = 0.64//\left[0.20+\left(0.5733//0.50\right)\right]$$
$$\therefore X_1 = 0.64//0.4671 = 0.2700\,\text{pu}$$

Problem 12.3: Refer to the positive-sequence Thévenin equivalent obtained above.



The base current value is

$$I_{\text{base}} = \frac{S_{\text{base}3\phi}}{\sqrt{3}V_{\text{base}}} = \frac{1000\,\text{MVA}}{\sqrt{3}\times 500\,\text{kV}} = 1.1547\,\text{kA}$$

The current in the positive-sequence circuit is found as

$$I_1 = \frac{V_F}{Z_1} = \frac{1.0\angle 0^{\circ}}{j0.267} = -j3.7453 = 3.7453\angle -90^{\circ}\,\text{pu}$$

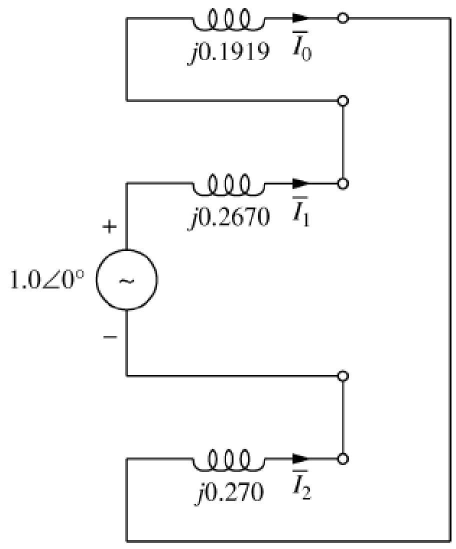
For a symmetrical fault such as the present one, the zero- and negative-sequence currents are zero. The subtransient fault currents of each phase are computed below.

$$\begin{bmatrix} I_A'' \\ I_B'' \\ I_C'' \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ 3.7453\angle -90^{\circ} \\ 0 \end{bmatrix} = \begin{bmatrix} -j3.7453 \\ -3.2435 + j1.8727 \\ 3.2435 + j1.8727 \end{bmatrix} = \begin{bmatrix} 3.7453\angle -90^{\circ} \\ 3.7453\angle 150^{\circ} \\ 3.7453\angle 30^{\circ} \end{bmatrix}\text{pu}$$

Multiplying by the base current,

$$\begin{bmatrix} I_A'' \\ I_B'' \\ I_C'' \end{bmatrix} = \begin{bmatrix} 3.7453\angle -90^{\circ} \\ 3.7453\angle 150^{\circ} \\ 3.7453\angle 30^{\circ} \end{bmatrix} \times 1.1547 = \begin{bmatrix} 4.3247\angle -90^{\circ} \\ 4.3247\angle 150^{\circ} \\ 4.3247\angle 30^{\circ} \end{bmatrix}\text{kA}$$

Problem 12.4: Assume a single line-to-ground fault at bus 1.



The sequence current through each impedance is the same, and can be calculated as

$$I_0 = I_1 = I_2 = \frac{E_g}{Z_0 + Z_1 + Z_2} = \frac{1.0}{j0.1919 + j0.2670 + j0.270} = -j1.3719\,\text{pu}$$

The phase currents are determined next,

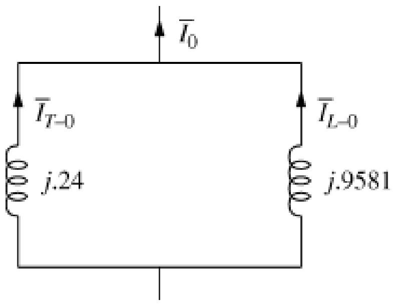
$$\begin{bmatrix} I''_A \\ I''_B \\ I''_C \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} -j1.3719 \\ -j1.3719 \\ -j1.3719 \end{bmatrix} = \begin{bmatrix} -j4.1157 \\ 0 \\ 0 \end{bmatrix} \text{pu}$$

or $-j4.1157 \times 1.1547 = -j4.7524$ kA. For the zero-sequence network, the transformer contribution to current is

$$I_{T-0} = -j1.3719 \times \left(\frac{0.9581}{0.24 + 0.9581} \right) = -j1.0971\,\text{pu}$$

while the line contribution to current is

$$I_{L-0} = -j1.3719 \times \left(\frac{0.24}{0.24 + 0.9581} \right) = -j0.2748\,\text{pu}$$

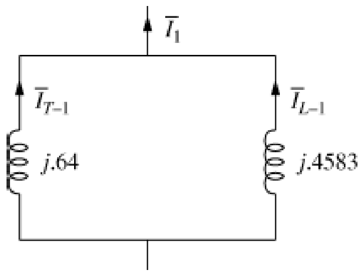


For the positive-sequence network, the transformer contribution to current is

$$I_{T-1} = -j1.3719 \times \left(\frac{0.4583}{0.64 + 0.4583} \right) = -j0.5725\,\text{pu}$$

while the line contribution to current is

$$I_{L-1} = -j1.3719 \times \left(\frac{0.64}{0.64 + 0.4583} \right) = -j0.7994\,\text{pu}$$

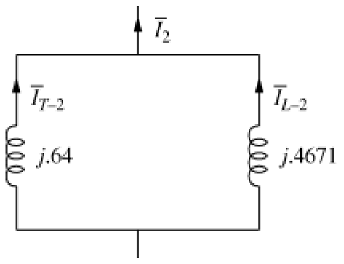


To determine the sequence components of voltage at fault current, we write

$$I_{T-2} = -j1.3719 \times \left(\frac{0.4671}{0.64 + 0.4671} \right) = -j0.5788\,\text{pu}$$

while the line contribution to current is

$$I_{L-2} = -j1.3719 \times \left(\frac{0.64}{0.64 + 0.4671} \right) = -j0.7931\,\text{pu}$$



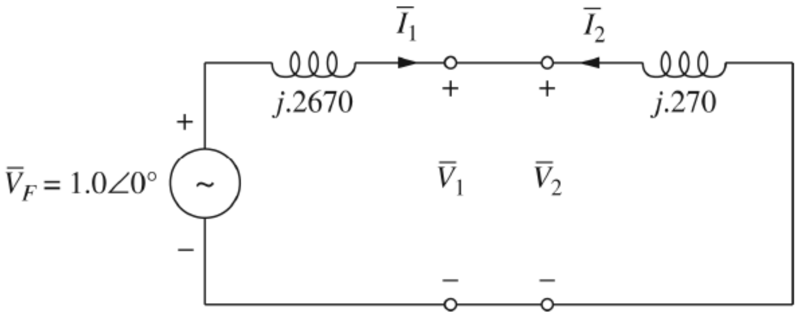
Given the transformer results above, we assemble the matrix equation

$$\begin{bmatrix} I''_{T-A} \\ I''_{T-B} \\ I''_{T-C} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} -j1.0971 \\ -j0.5725 \\ -j0.5788 \end{bmatrix} = \begin{bmatrix} -j2.2484 \\ 0.5215\angle -89.4005^\circ \\ 0.5215\angle -90.5995^\circ \end{bmatrix} \text{pu} = \begin{bmatrix} -j2.5962 \\ 0.6022\angle -89.4005^\circ \\ 0.6022\angle -90.5995^\circ \end{bmatrix} \text{kA}$$

Likewise for the line currents,

$$\begin{bmatrix} I''_{L-A} \\ I''_{L-B} \\ I''_{L-C} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} -j0.2748 \\ -j0.7994 \\ -j0.7931 \end{bmatrix} = \begin{bmatrix} -j1.8673 \\ 0.5215\angle 90.5995^\circ \\ 0.5215\angle 89.4005^\circ \end{bmatrix} \text{pu} = \begin{bmatrix} -j2.1562 \\ 0.6022\angle 90.5995^\circ \\ 0.6022\angle 89.4005^\circ \end{bmatrix} \text{kA}$$

Problem 12.5: A bolted line-to-line fault occurs at bus 1.



Currents I_1 and I_2 flow in opposite directions and are given by

$$I_1 = -I_2 = \frac{V_F}{Z_1 + Z_2} = \frac{1.0\angle 0^\circ}{j0.2670 + j0.270} = -j1.8622\,\text{pu}$$

Also, $I_0 = 0$. The fault currents are determined next,

$$\begin{bmatrix} I''_A \\ I''_B \\ I''_C \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ -j1.8622 \\ j1.8622 \end{bmatrix} = \begin{bmatrix} 0 \\ 3.2254\angle 180^\circ \\ 3.2254\angle 0^\circ \end{bmatrix} \text{pu} = \begin{bmatrix} 0 \\ 3.7244\angle 180^\circ \\ 3.7244\angle 0^\circ \end{bmatrix} \text{kA}$$

In zero sequence, the contributions to current from transformer and line are $I_{T-0} = 0$ and $I_{L-0} = 0$. In positive sequence, the contributions to current are

$$I_{T-1} = -j1.8622 \times \left(\frac{0.4583}{0.64 + 0.4583} \right) = -j0.7771\,\text{pu}$$

and

$$I_{L-1} = -j1.8622 \times \left(\frac{0.64}{0.64 + 0.4583} \right) = -j1.0851\,\text{pu}$$

In negative sequence, the contributions to current are

$$I_{T-2} = j1.8622 \times \left(\frac{0.4671}{0.64 + 0.4671} \right) = j0.7857\,\text{pu}$$

and

$$I_{L-2} = 1.8622 \times \left(\frac{0.64}{0.64 + 0.4671} \right) = j1.0765 \, \text{pu}$$

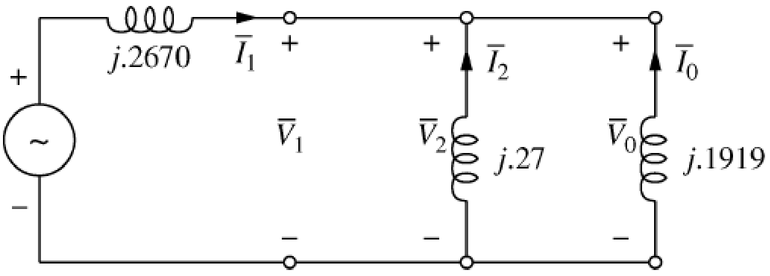
The contributions to fault from transformer are

$$\begin{bmatrix} I''_{T-A} \\ I''_{T-B} \\ I''_{T-C} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ -j0.7771 \\ j0.7857 \end{bmatrix} = \begin{bmatrix} 0.0086\angle 90^\circ \\ 1.3534\angle 179.868^\circ \\ 1.3534\angle 0.132^\circ \end{bmatrix} \text{pu} = \begin{bmatrix} 0.0099\angle 90^\circ \\ 1.5628\angle 179.868^\circ \\ 1.5628\angle 0.132^\circ \end{bmatrix} \text{kA}$$

The contributions to fault from line are

$$\begin{bmatrix} I''_{L-A} \\ I''_{L-B} \\ I''_{L-C} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ -j1.0851 \\ j1.0765 \end{bmatrix} = \begin{bmatrix} 0.0086\angle 90^\circ \\ 1.8720\angle 179.868^\circ \\ 1.8720\angle 0.132^\circ \end{bmatrix} \text{pu} = \begin{bmatrix} 0.0099\angle 90^\circ \\ 2.1616\angle 179.868^\circ \\ 2.1616\angle 0.132^\circ \end{bmatrix} \text{kA}$$

Problem 12.6: Now, a double line-to-ground fault occurs at bus 1.



Referring to the diagram above, current I_1 is determined as

$$I_1 = \frac{V_F}{Z_1 + Z_2 // Z_0} = \frac{1.0\angle 0^\circ}{j0.2670 + \dfrac{j0.27 \times j0.1919}{j0.27 + j0.1919}} = -j2.6373 = 2.6373\angle -90^\circ \, \text{pu}$$

By current division,

$$I_2 = -I_1 \times \left(\frac{Z_0}{Z_0 + Z_2} \right) = -(-j2.6373) \times \frac{0.1919}{0.1919 + 0.27} = j1.0957 \, \text{pu}$$

$$I_0 = -I_1 \times \left(\frac{Z_2}{Z_0 + Z_2} \right) = -(-j2.6373) \times \frac{0.27}{0.1919 + 0.27} = j1.5416 \, \text{pu}$$

Currents I_{ABC} are such that

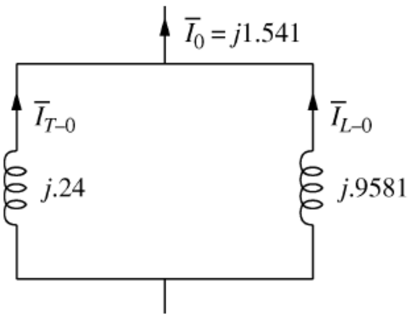
$$\begin{bmatrix} I''_A \\ I''_B \\ I''_C \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} j1.5416 \\ -j2.6373 \\ j1.0957 \end{bmatrix} = \begin{bmatrix} 0 \\ 3.9748\angle 144.425^\circ \\ 3.9748\angle 35.575^\circ \end{bmatrix} \text{pu} = \begin{bmatrix} 0 \\ 4.5897\angle 144.425^\circ \\ 4.5897\angle 35.575^\circ \end{bmatrix} \text{kA}$$

We proceed to compute the contributions to fault current. For zero sequence, noting that $I_0 = j1.5416$ pu, the contributions from transformer and line are

$$I_{T-0} = j1.5416 \times \left(\frac{0.9581}{0.24 + 0.9581} \right) = j1.2328 \, \text{pu}$$

and

$$I_{L-0} = j1.5416 - j1.2328 = j0.3088 \, \text{pu}$$

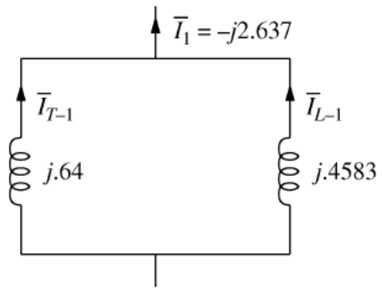


For positive sequence, observing that $I_1 = -j2.6373$ pu, the contributions from transformer and line are

$$I_{T-1} = -j2.6373 \times \left(\frac{0.4583}{0.64 + 0.4583} \right) = -j1.1005 \, \text{pu}$$

and

$$I_{L-1} = -j2.6373 - (-j1.1005) = -j1.5368 \, \text{pu}$$

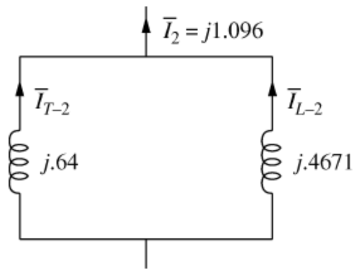


For negative sequence, recalling that $I_2 = j1.0957$ pu, the contributions from transformer and line are

$$I_{T-2} = j1.0957 \times \left(\frac{0.4671}{0.64 + 0.4671} \right) = j0.4623 \, \text{pu}$$

and

$$I_{L-2} = j1.0957 - j0.4623 = j0.6334 \, \text{pu}$$



We proceed to determine the contributions to fault current from transformer,

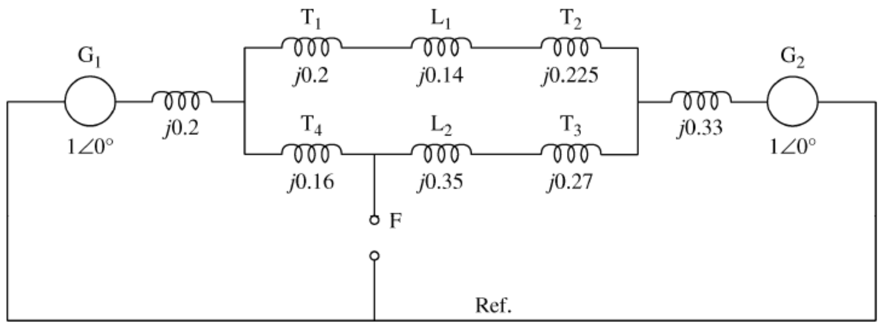
$$\begin{bmatrix} I''_{T-A} \\ I''_{T-B} \\ I''_{T-C} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} j1.2328 \\ -j1.1005 \\ j0.4623 \end{bmatrix} = \begin{bmatrix} 0.5946\angle 90^\circ \\ 2.0592\angle 131.092^\circ \\ 2.0592\angle 48.9081^\circ \end{bmatrix} \text{pu} = \begin{bmatrix} 0.6866\angle 90^\circ \\ 2.3778\angle 131.092^\circ \\ 2.3778\angle 48.901^\circ \end{bmatrix} \text{kA}$$

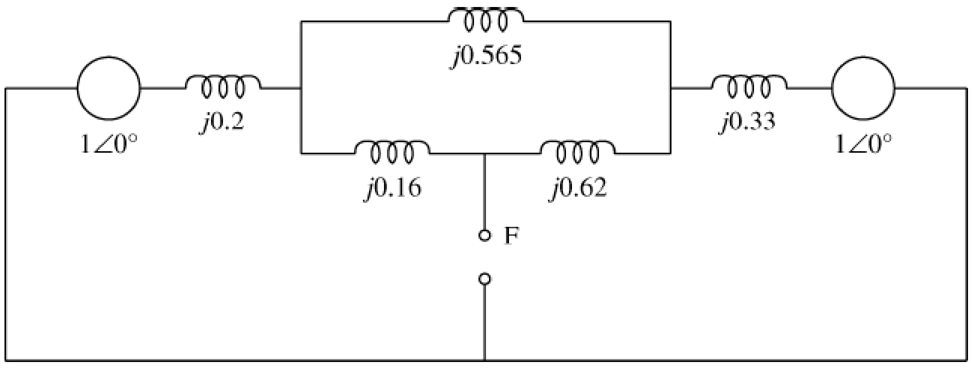
and from line,

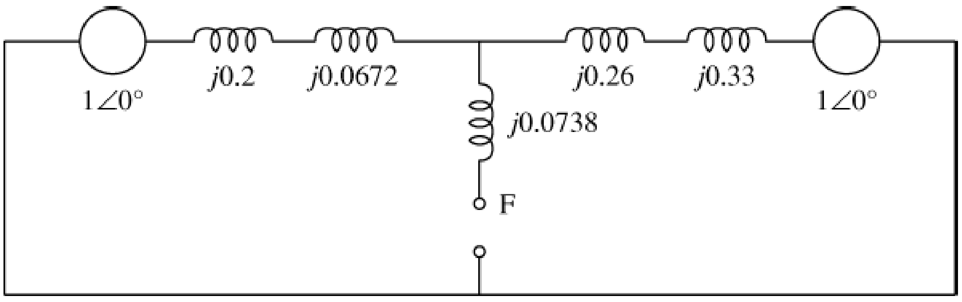
$$\begin{bmatrix} I''_{L-A} \\ I''_{L-B} \\ I''_{L-C} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} j0.3088 \\ -j1.5368 \\ j0.6334 \end{bmatrix} = \begin{bmatrix} 0.5946\angle -90^\circ \\ 2.0275\angle 157.97^\circ \\ 2.0275\angle 22.0302^\circ \end{bmatrix} \text{pu} = \begin{bmatrix} 0.6866\angle -90^\circ \\ 2.3412\angle 157.97^\circ \\ 2.3412\angle 22.0302^\circ \end{bmatrix} \text{kA}$$

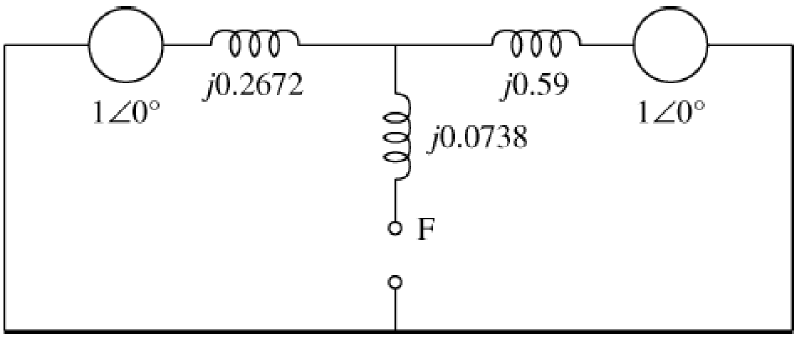
P.13 ➡ *Solution*

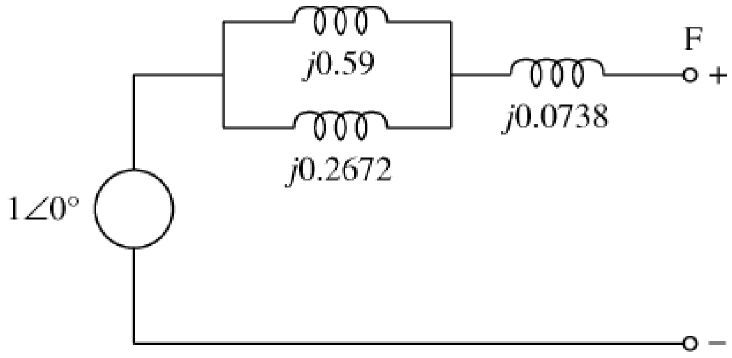
Problem 13.1: The positive sequence network is reduced to its Thévenin equivalent as follows:

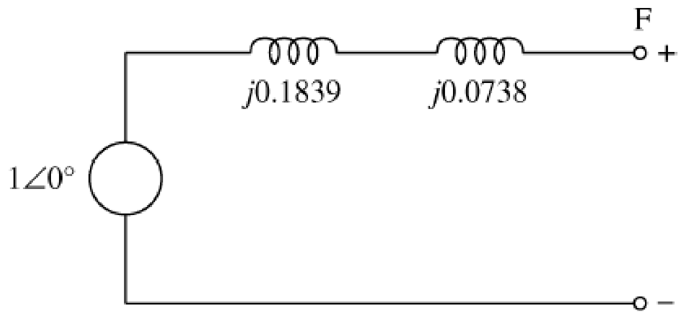


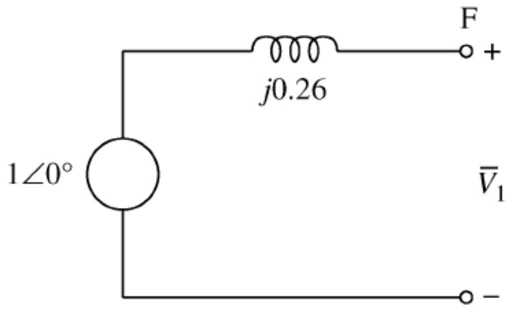




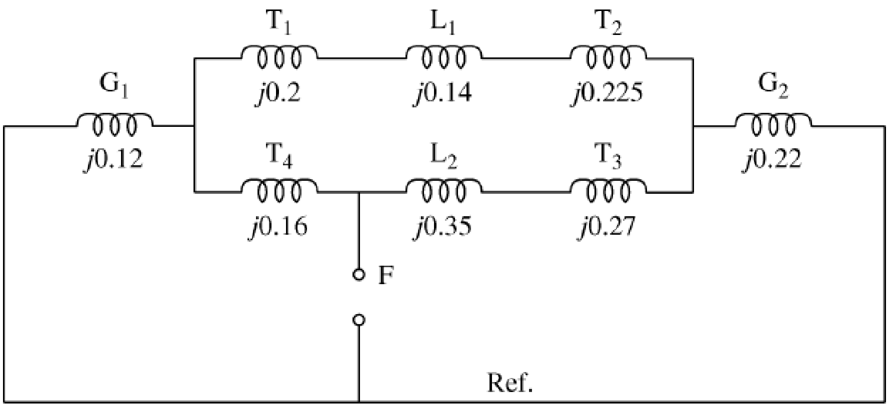


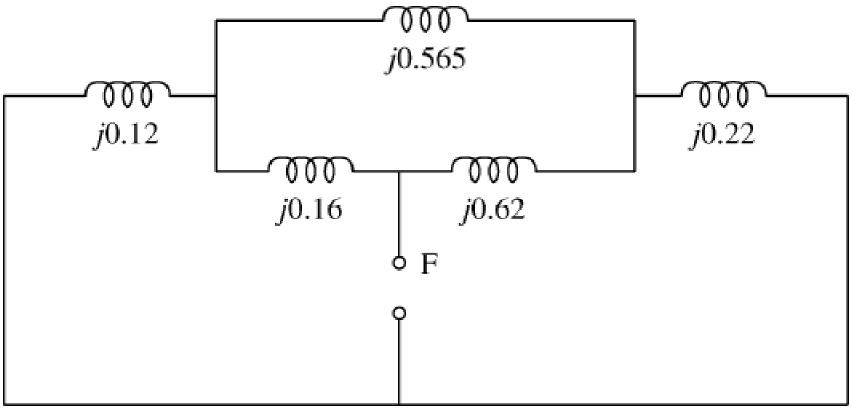


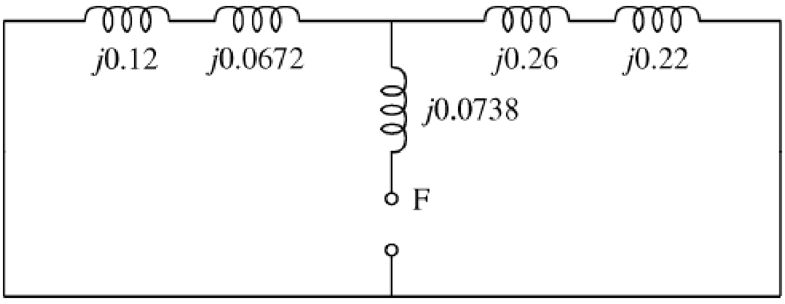


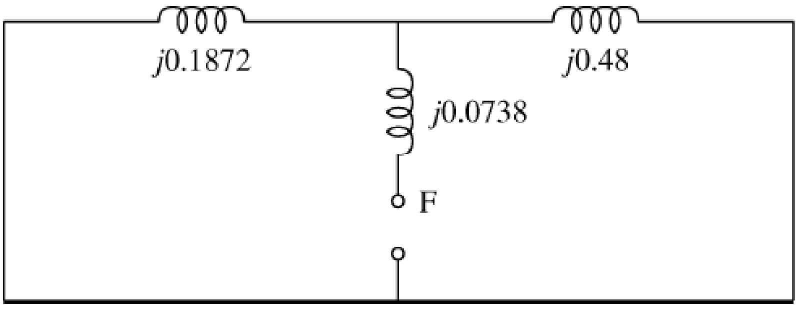


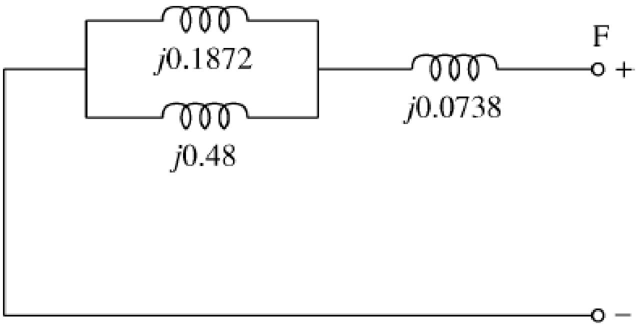
The negative sequence network is reduced to its Thévenin equivalent as follows:

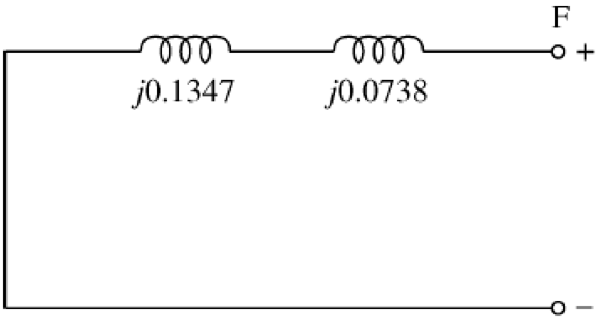


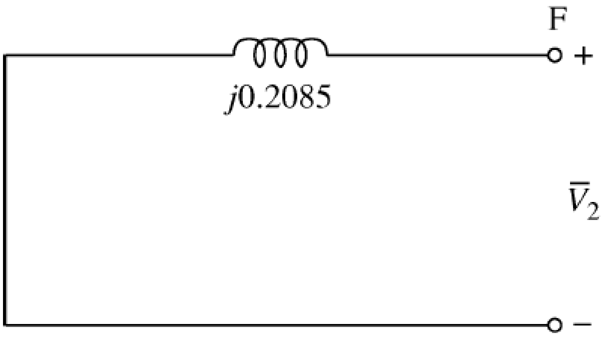




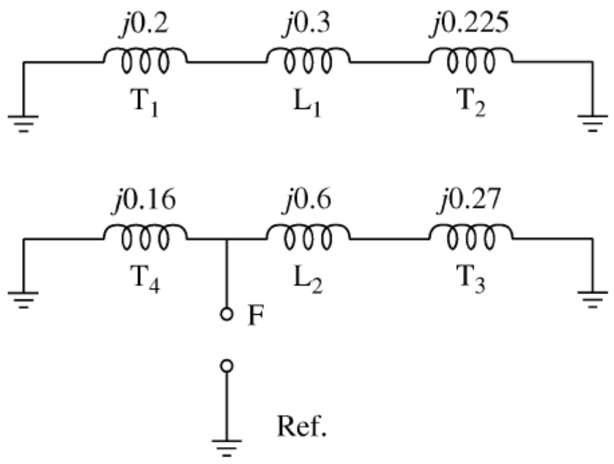


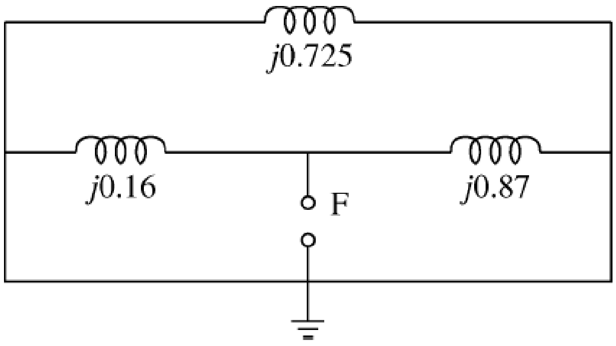


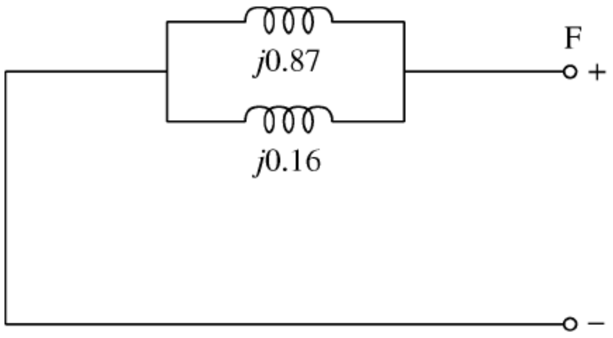


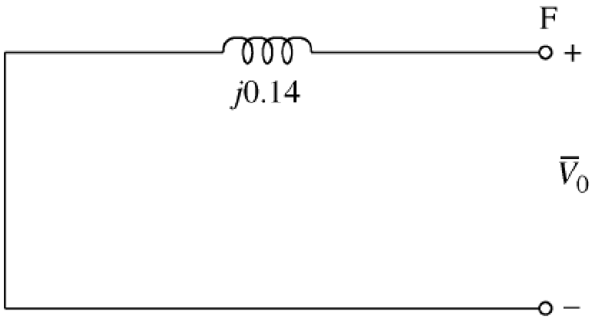


Lastly, the zero sequence network is reduced to its Thévenin equivalent as follows:

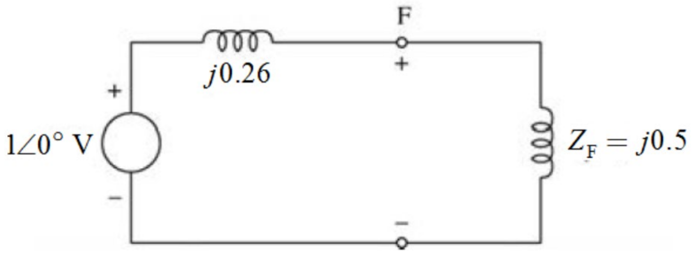








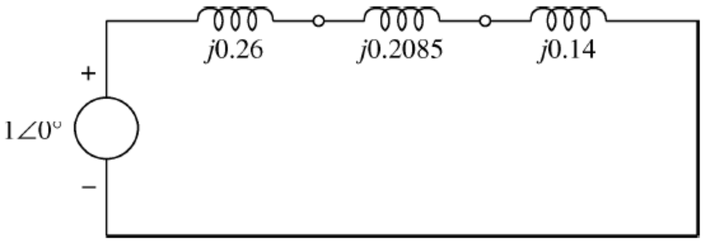
We calculated a final reactance of $j0.14$ pu for the positive sequence network. As stated, a $j0.5$-pu impedance is added to the network.



The fault current for a balanced three-phase fault is calculated to be

$$I_{sc} = \frac{1.0\angle 0^{\circ}}{j(0.26+0.5)} = \boxed{-j1.3158\,\text{pu}}$$

Problem 13.2: For a single line-to-ground fault, the sequence networks from the solution of Problem 13.1 are connected in series, as shown.



The sequence currents are given by

$$I_0 = I_1 = I_2 = \frac{1.0\angle 0^\circ}{j0.26 + j0.2085 + j0.14} = -j1.6434 = 1.6434\angle -90^\circ \text{ pu}$$

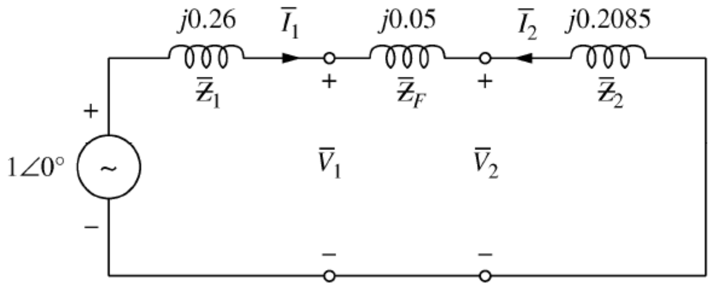
The subtransient fault current can be obtained with the usual matrix equation, which should yield $I''_a = 3I_0 = -j4.9302 = 4.9302\angle -90^\circ$ pu and $I''_b = I''_c = 0$. The sequence voltages are, in turn,

$$V_1 = 1.0\angle 0^\circ - I_1Z_1 = 1.0\angle 0^\circ - (1.6434\angle -90^\circ)\times(0.26\angle 90^\circ) = 0.5727\angle 0^\circ \text{ pu}$$
$$V_2 = -I_2Z_2 = -(1.6434\angle -90^\circ)\times(0.2085\angle 90^\circ) = -0.3426 \text{ pu}$$
$$V_0 = -I_0Z_0 = -(1.6434\angle -90^\circ)\times(0.14\angle 90^\circ) = -0.2301 \text{ pu}$$

It remains to compute the line-to-ground phase voltages at the faulted bus,

$$\begin{bmatrix} V_{ag} \\ V_{bg} \\ V_{cg} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} -0.2301 \\ 0.5727 \\ -0.3426 \end{bmatrix} = \begin{bmatrix} 0 \\ 0.8646\angle -113.53^\circ \\ 0.8646\angle 113.53^\circ \end{bmatrix} \text{pu}$$

For a line-to-line fault through a fault impedance $Z_F = j0.05$, the sequence network connection is as follows.



Current components I_1 and I_2 flow in opposite directions and have intensity given by

$$I_1 = -I_2 = \frac{1\angle 0^\circ}{j0.26 + j0.05 + j0.2085} = -j1.9286 \text{ pu} = 1.9286\angle -90^\circ \text{ pu}$$

Also, $I_0 = 0$. The phase currents are given by

$$I_a = 0$$
$$I_b = -I_c = (a^2 - a)I_1 = \left[(1\angle 120^\circ)^2 - 1\angle 120^\circ\right]\times(1.9286\angle -90^\circ) = 3.3404\angle -180^\circ \text{ pu}$$

The sequence voltages are

$$V_1 = 1.0\angle 0^\circ - I_1Z_1 = 1.0\angle 0^\circ - (1.9286\angle -90^\circ)\times(0.26\angle 90^\circ) = 0.4986 \text{ pu}$$
$$V_2 = -I_2Z_2 = -(1.9286\angle -90^\circ)\times(0.2085\angle 90^\circ) = 0.4021 \text{ pu}$$
$$V_0 = -I_0Z_0 = -(1.9286\angle -90^\circ)\times 0 = 0$$

The phase voltages are then given by

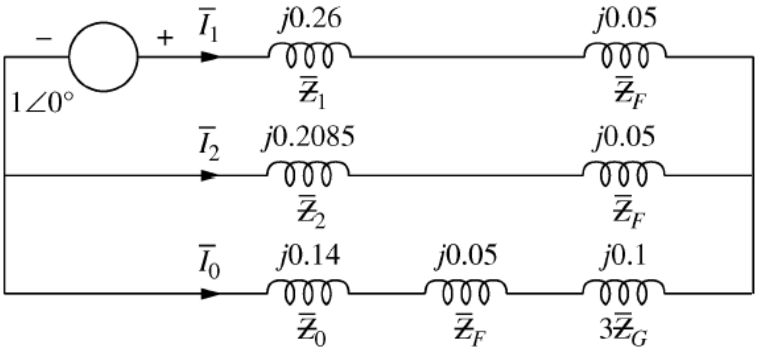
$$V_a = V_1 + V_2 + V_0 = 0.4986 + 0.4021 + 0 = 0.9007\angle 0^\circ \text{ pu}$$
$$V_b = a^2V_1 + aV_2 + V_0 = (1\angle 240^\circ)\times 0.4986 + (1\angle 120^\circ)\times 0.4021 + 0 = 0.4580\angle -169.487^\circ \text{ pu}$$
$$V_c = aV_1 + a^2V_2 + V_0 = (1\angle 120^\circ)\times 0.4986 + (1\angle 240^\circ)\times 0.4021 + 0 = 0.4580\angle 169.487^\circ \text{ pu}$$

As a final check,

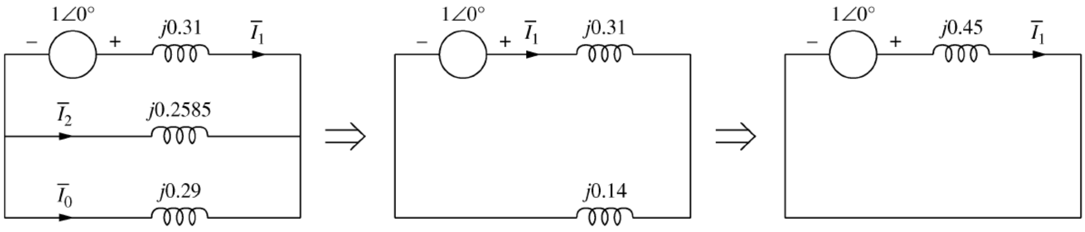
$$V_b - V_c = I_bZ_F \Rightarrow (0.4580\angle -169.487^\circ) - (0.4580\angle 169.487^\circ) = (3.3404\angle -180^\circ)\times j0.05$$
$$\therefore -j0.1671 = -j0.1670$$

The equality checks to three decimal places.

For a double line-to-ground fault with given conditions, the sequence network connection is shown below.



Reducing the system is an elementary task.



The positive-sequence current is

$$I_1 = \frac{1\angle 0^o}{j0.45} = -j2.2222 = 2.2222\angle -90^o$$

Then, I_2 and I_0 are found by current division,

$$I_2 = -I_1 \times \left(\frac{0.29}{0.29+0.2585}\right) = -(-j2.2222)\times\left(\frac{0.29}{0.29+0.2585}\right) = -1.1749\angle -90^o\,\text{pu}$$

$$I_0 = -I_1 \times \left(\frac{0.2585}{0.29+0.2585}\right) = -(-j2.2222)\times\left(\frac{0.2585}{0.29+0.2585}\right) = -1.0472\angle -90^o\,\text{pu}$$

The sequence voltages are given by

$$V_1 = 1\angle 0^o - I_1 Z_1 = 1\angle 0^o - (2.2222\angle -90^o)\times(0.26\angle 90^o) = 0.4223\,\text{pu}$$
$$V_2 = -I_2 Z_2 = 1\angle 0^o - (-1.1749\angle -90^o)\times(0.2085\angle 90^o) = 0.2450\,\text{pu}$$
$$V_0 = -I_0 Z_0 = -(-1.0472\angle -90^o)\times(0.14\angle 90^o) = 0.1466\,\text{pu}$$

The phase currents are given by

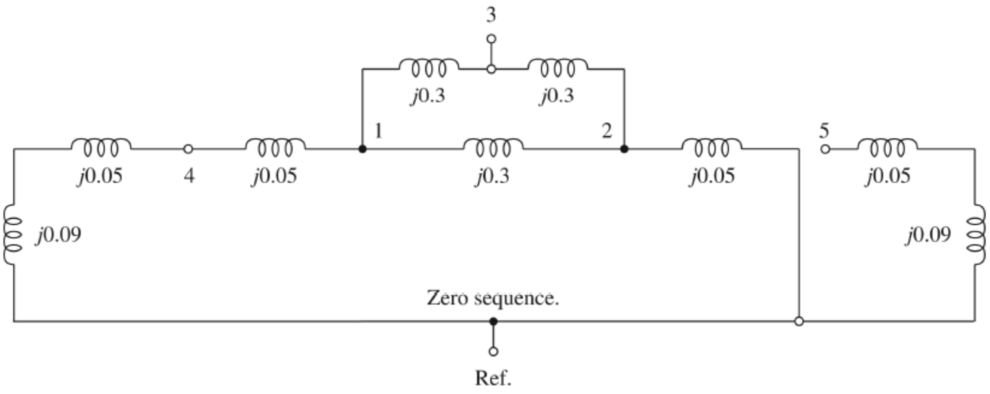
$$I_a = 0$$
$$I_b = a^2 I_1 + aI_2 + I_0 = (1\angle 240^o)\times(2.2222\angle -90^o) + (1\angle 120^o)\times(-1.1749\angle -90^o) - 1.0472\angle -90^o$$
$$\therefore I_b = 3.3351\angle 151.9^o\,\text{pu}$$
$$I_c = aI_1 + a^2 I_2 + I_0 = (1\angle 120^o)\times(2.2222\angle -90^o) + (1\angle 240^o)\times(-1.1749\angle -90^o) - 1.0472\angle -90^o$$
$$\therefore I_c = 3.3351\angle 28.0997^o\,\text{pu}$$

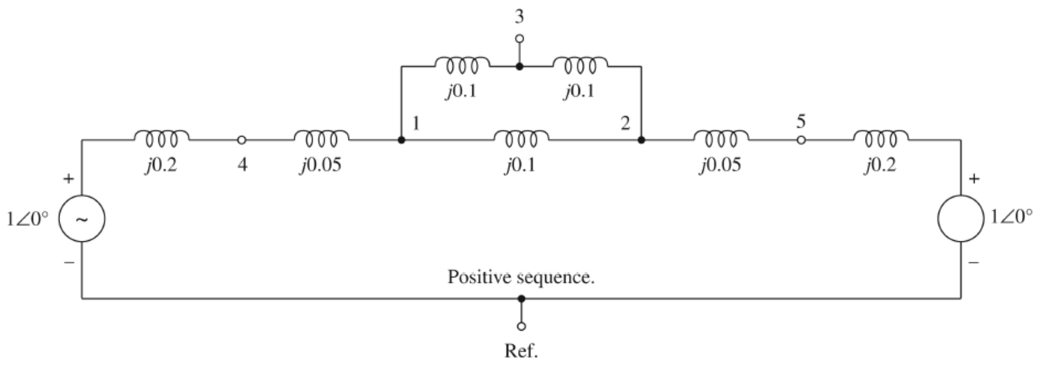
The neutral fault current is $I_b + I_c = 3I_0 = -3.1416\angle -90°$. We conclude by determining the phase voltages,

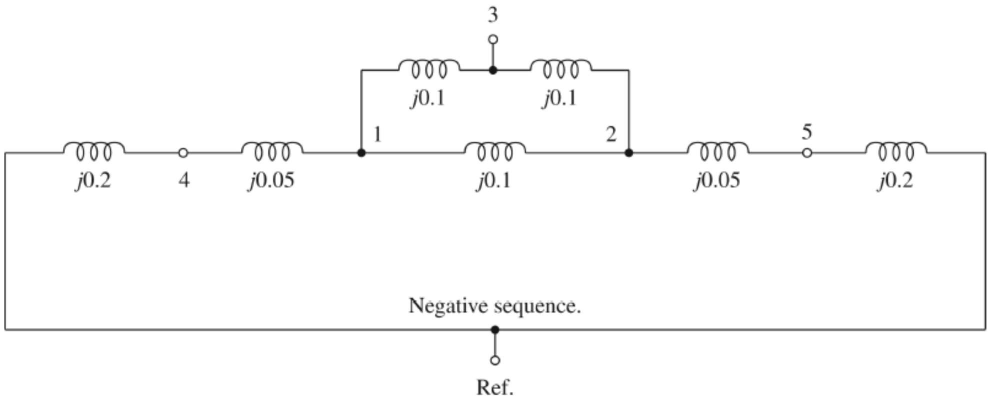
$$V_a = V_1 + V_2 + V_0 = 0.4223 + 0.245 + 0.1466 = 0.8139\angle 0^o\,\text{pu}$$
$$V_b = a^2 V_1 + aV_2 + V_0 = 0.242\angle -140.618^o\,\text{pu}$$
$$V_c = aV_1 + a^2 V_2 + V_0 = 0.242\angle 140.618^o\,\text{pu}$$

P.14 ➡ *Solution*

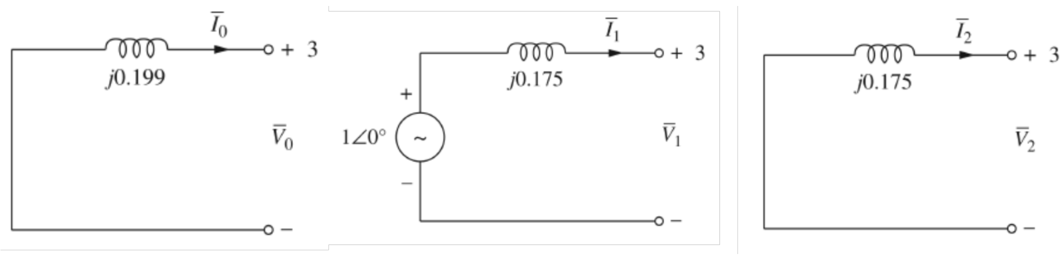
Problem 14.1: The zero, positive, and negative sequence networks are shown below.



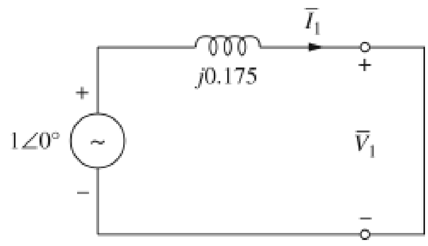




Using delta-wye transformation and series-parallel combinations, Thévenin equivalents looking into bus 3 are shown below.



A bolted three-phase fault occurs at bus 3.



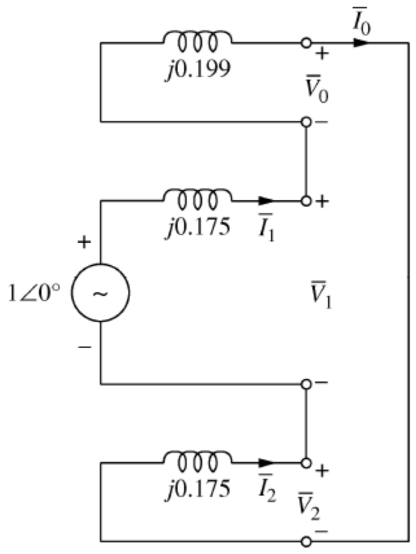
The positive-sequence current is

$$I_1 = \frac{1\angle 0^{\circ}}{j0.175} = -j5.7143 = 5.7143\angle -90^{\circ}\,\mathrm{pu}$$

Also, $V_1 = V_2 = V_0 = 0$. The phase currents are calculated to be

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ -j5.7143 \\ 0 \end{bmatrix} = \begin{bmatrix} 5.7143\angle -90^{\circ} \\ 5.7143\angle 150^{\circ} \\ 5.7143\angle 30^{\circ} \end{bmatrix} \mathrm{pu}$$

Problem 14.2: For a single line-to-ground fault at bus 3, the interconnection of the sequence networks is shown below.



The zero-, positive- and negative-sequence currents are given by

$$I_0 = I_1 = I_2 = \frac{1.0\angle 0^\circ}{j0.199 + j0.175 + j0.175} = -j1.8214\,\text{pu}$$

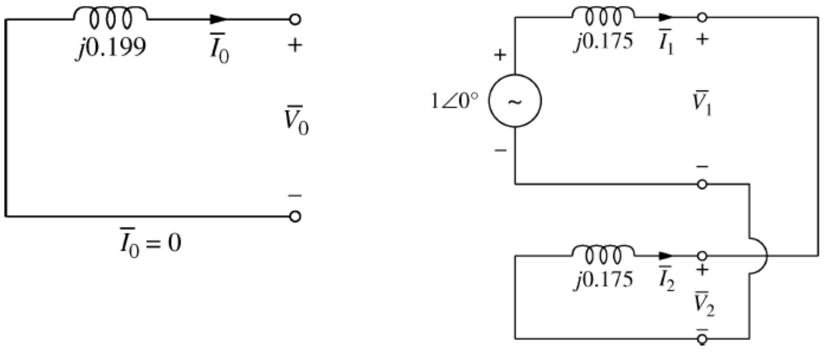
Multiplying by 3 gives the phase current $I_a = -j5.4642$ pu; further, $I_b = I_c = 0$. The sequence voltages are given by

$$V_1 = 1.0\angle 0^\circ - I_1 Z_1 = 1.0\angle 0^\circ - \left(1.8214\angle -90^\circ\right)\times\left(0.175\angle 90^\circ\right) = 0.6813\,\text{pu}$$
$$V_2 = -I_2 Z_2 = -\left(1.8214\angle -90^\circ\right)\times\left(0.175\angle 90^\circ\right) = -0.3187\,\text{pu}$$
$$V_0 = -I_0 Z_0 = -\left(1.8214\angle -90^\circ\right)\times\left(0.199\angle 90^\circ\right) = -0.3625\,\text{pu}$$

The phase voltages are calculated as

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} -0.3625 \\ 0.6813 \\ -0.3187 \end{bmatrix} = \begin{bmatrix} 0 \\ 1.0226\angle -122.126^\circ \\ 1.0226\angle 122.126^\circ \end{bmatrix} \text{pu}$$

For a line-to-line fault at bus 3, the sequence networks are interconnected as shown below.



Current $I_0 = 0$, and

$$I_1 = -I_2 = \frac{1\angle 0^\circ}{j0.175 + j0.175} = -j2.8571 = 2.8571\angle -90^\circ\,\text{pu}$$

The phase currents follow as

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ -j2.8571 \\ j2.8571 \end{bmatrix} = \begin{bmatrix} 0 \\ -4.9486 \\ 4.9486 \end{bmatrix} \text{pu}$$

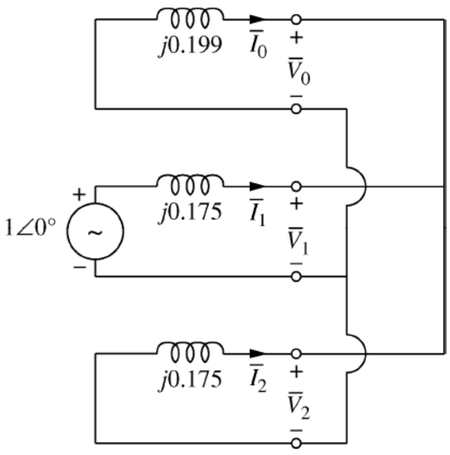
The sequence voltages are

$$V_0 = 0$$
$$V_1 = V_2 = I_1 \times j0.175 = -j2.8571 \times j0.175 = 0.50\,\text{pu}$$

Lastly, the phase voltages are calculated to be

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ 0.5 \\ 0.5 \end{bmatrix} = \begin{bmatrix} 1.0 \\ -0.5 \\ -0.5 \end{bmatrix} \text{pu}$$

For a double line-to-ground fault at bus 3, the sequence network interconnection is shown below.



With reference to the illustration above, the sequence currents are

$$I_1 = \frac{1\angle 0^{\circ}}{j0.175 + j0.175//j0.199} = -j3.7297\,\text{pu}$$

By dint of current division,

$$I_2 = -(-j3.7297)\times\left(\frac{0.199}{0.175+0.199}\right) = j1.9845\,\text{pu}$$

$$I_0 = -(-j3.7297)\times\left(\frac{0.175}{0.175+0.199}\right) = j1.7452\,\text{pu}$$

The phase currents are given by

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}\begin{bmatrix} j1.7452 \\ -j3.7297 \\ j1.9845 \end{bmatrix} = \begin{bmatrix} 0 \\ 5.5984\angle 152.121^{\circ} \\ 5.5984\angle 27.8786^{\circ} \end{bmatrix}\text{pu}$$

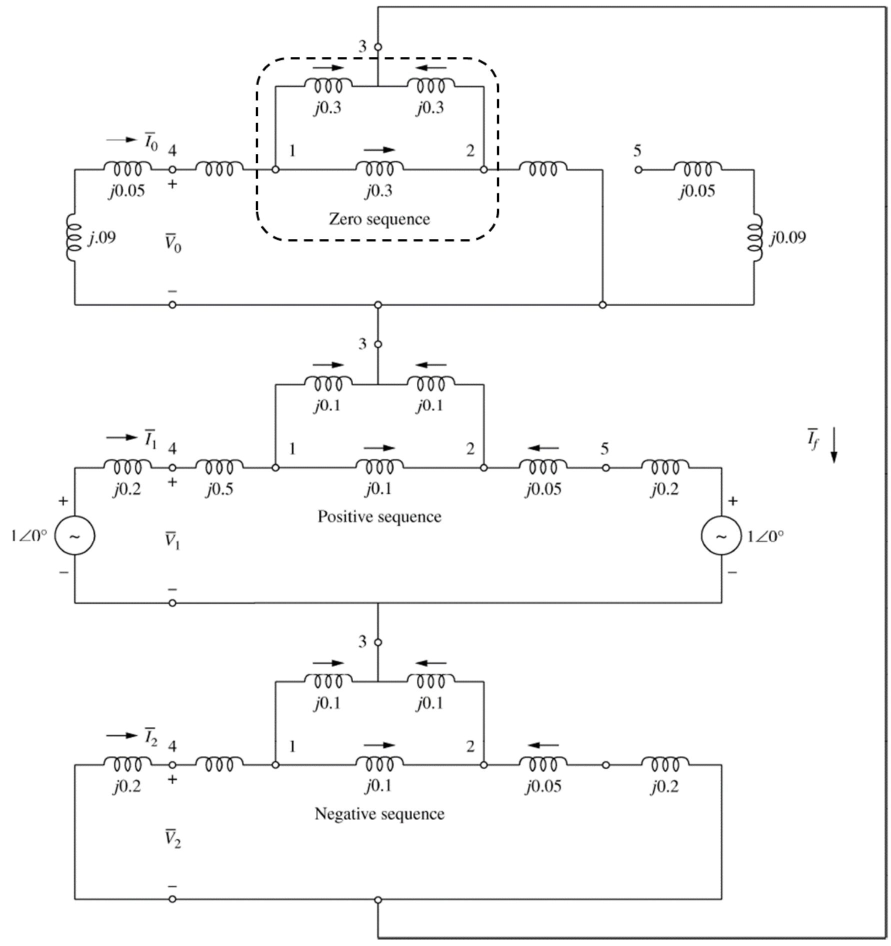
The neutral fault current is $I_b + I_c = 3I_0 = j5.2356$ pu. The sequence voltages are obtained as

$$V_0 = V_1 = V_2 = -j1.7452 \times j0.199 = 0.3473\,\text{pu}$$

Lastly, the phase voltages are

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}\begin{bmatrix} 0.3473 \\ 0.3473 \\ 0.3473 \end{bmatrix} = \begin{bmatrix} 1.0419 \\ 0 \\ 0 \end{bmatrix}\text{pu}$$

In order to compute currents and voltages at the terminals of generators G1 and G2, we need to return to the original sequence circuits in the solution to Problem 14.1. Consider first generator G4 (bus 4). For a single line-to-ground fault, the sequence network interconnection is shown below.



From the solution to Problem 14.1, $I_f = -j1.8214$ pu. From the circuit above, $I_1 = I_2 = I_f/2 = -j0.9107$ pu. Transforming the Δ of $j0.3$ in the zero sequence network (highlighted by dashed lines above) into an equivalent Y of $j0.1$ and using current division,

$$I_0 = \frac{0.15}{0.29+0.15}\times(-j1.8214) = -j0.6209\,\text{pu}$$

The phase currents are then

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} -j0.6209 \\ -j0.9107 \\ -j0.9107 \end{bmatrix} = \begin{bmatrix} 2.4423\angle -90^{\circ} \\ 0.2898\angle 90^{\circ} \\ 0.2898\angle 90^{\circ} \end{bmatrix} \text{pu}$$

The sequence voltages are, in turn,

$$V_0 = -(-j0.6209) \times j0.14 = -0.0869 \,\text{pu}$$
$$V_1 = 1 - (-j0.9107) \times j0.2 = 0.8179 \,\text{pu}$$
$$V_2 = -(-j0.9107) \times j0.2 = -0.1821 \,\text{pu}$$

The phase voltages follow as

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} -0.0869 \\ 0.8179 \\ -0.1821 \end{bmatrix} = \begin{bmatrix} 0.5489\angle 0^{\circ} \\ 0.956\angle -115.052^{\circ} \\ 0.956\angle 115.052^{\circ} \end{bmatrix} \text{pu}$$

Consider now generator G2 (bus 5). From the interconnected sequence networks and the results above,

$$I_f = -j1.8214 \;\; ; \;\; I_1 = I_2 = \frac{1}{2} I_f = -j0.9107 \;\; ; \;\; I_0 = 0$$

Recall that Y-Δ transformer connections produce 30° phase shifts in sequence quantities. The HV quantities are to be shifted 30° ahead of the corresponding LV quantities for positive sequence, and vice versa for negative sequence. One may however neglect phase shifts. Since bus 5 is the LV side, considering phase shifts,

$$I_1 = 0.9107\angle(-90^{\circ} - 30^{\circ}) = 0.9107\angle -120^{\circ} \,\text{pu}$$
$$I_2 = 0.9107\angle(-90^{\circ} + 30^{\circ}) = 0.9107\angle -60^{\circ} \,\text{pu}$$

Then, phase currents are

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ 0.9107\angle -120^{\circ} \\ 0.9107\angle -60^{\circ} \end{bmatrix} = \begin{bmatrix} 1.5774\angle -90^{\circ} \\ 1.5774\angle 90^{\circ} \\ 0 \end{bmatrix} \text{pu}$$

Positive and negative sequence voltages are the same as on the G1 side,

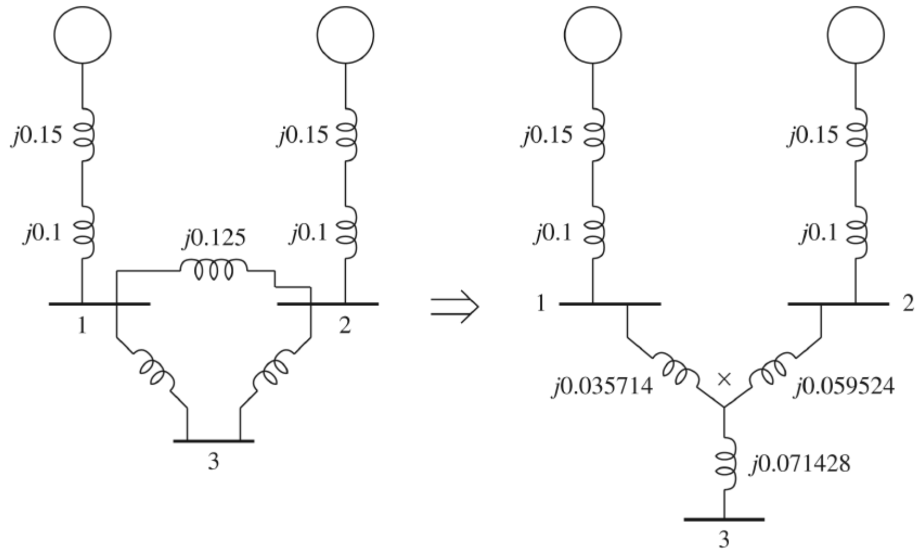
$$V_1 = 0.8179 \,\text{pu} \;\; ; \;\; V_2 = -0.1821 \,\text{pu} \;\; ; \;\; V_0 = 0$$

With phase shift, $V_1 = 0.8179\angle -30^{\circ}$, $V_2 = 0.1821\angle 210^{\circ}$. The phase voltages are calculated as

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ 0.8179\angle -30^{\circ} \\ -0.1821\angle 210^{\circ} \end{bmatrix} = \begin{bmatrix} 0.7438\angle -42.2416^{\circ} \\ 0.7438\angle -137.758^{\circ} \\ 1.0\angle 90^{\circ} \end{bmatrix} \text{pu}$$

P.15 ➡ *Solution*

Problem 15.1: For a balanced three-phase fault at bus 3, we need the positive sequence impedance network reduced to its Thévenin equivalent viewed from bus 3. The development is shown on the next page.



Converting the Δ formed by buses 1, 2 and 3 to an equivalent Y, we have (my calculations use less decimal places than the illustration above)

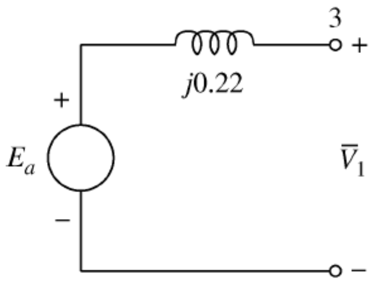
$$Z_{1X} = \frac{j0.125 \times j0.15}{j0.525} = 0.0357\,\text{pu}$$

$$Z_{2X} = \frac{j0.125 \times j0.25}{j0.525} = 0.0595\,\text{pu}$$

$$Z_{3X} = \frac{j0.15 \times j0.25}{j0.525} = 0.0714\,\text{pu}$$

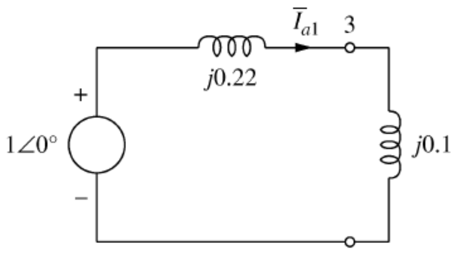
Using series-parallel combinations, the positive-sequence Thévenin impedance is given by, viewed from bus 3,

$$j\left[\frac{(0.15+0.1+0.0357)\times(0.15+0.1+0.0595)}{(0.15+0.1+0.0357)+(0.15+0.1+0.0595)}+0.0714\right] = j0.21996 \approx j0.22$$



Accounting for the fault impedance Z_F = 0.1 pu, the fault current is (with the no-load generated EMF to be 1.0∠0° pu),

$$I_a = I_{a1} = \frac{1.0\angle 0^\circ}{j0.22 + j0.1} = -j3.125\,\text{pu}$$



The base current is

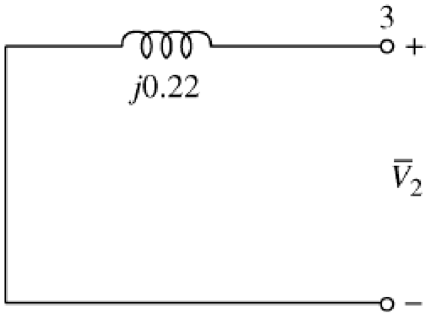
$$I_{\text{base}} = \frac{S_{\text{base}}}{\sqrt{3}V_{\text{base}}} = \frac{100}{\sqrt{3}\times 220} = 0.2625\,\text{kA} = 262.5\ \text{A}$$

so that

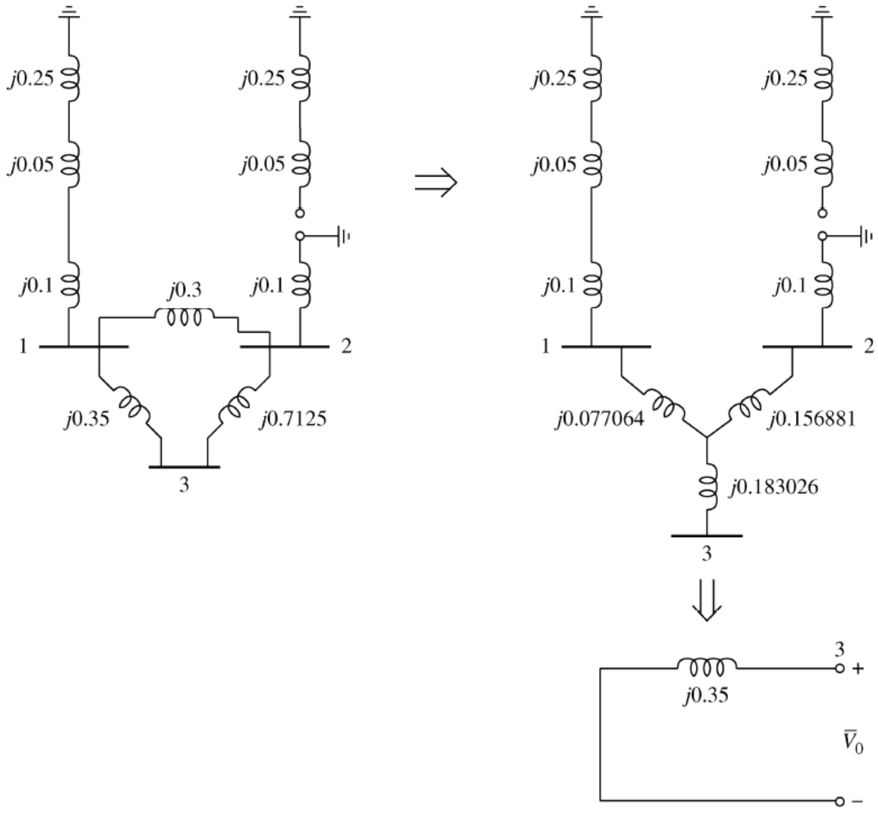
$$I_a = -j3.125 \times 262.5 = \boxed{820.3125\angle -90^\circ\,\text{A}}$$

The fault current is close to 820.3 A.

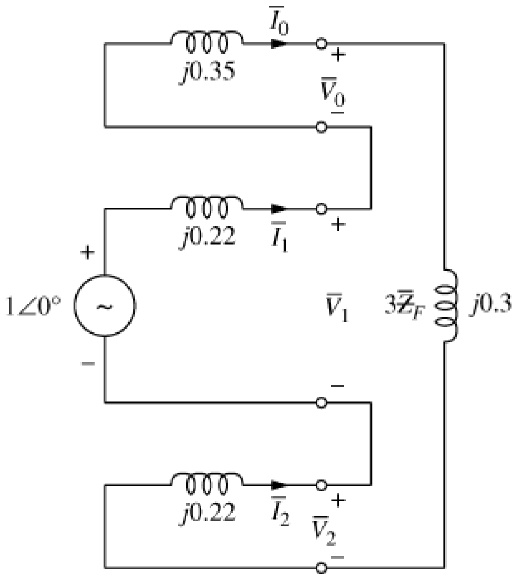
Problem 15.2: The negative sequence network is similar to the positive sequence network, but without the source.



The zero-sequence network is shown below considering the transformer winding connections (as before, I'd suggest converting the Δ to a Y with four decimal places only, instead of the six used in the illustration).



For a single line-to-ground fault through a fault impedance Z_F = 0.1 pu, we draw up the following interconnections.

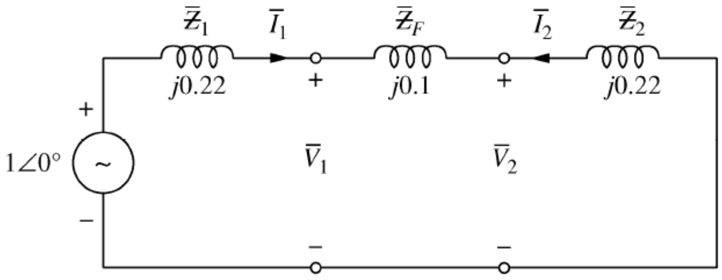


At bus 3 through a fault impedance $Z_F = j0.1$,

$$I_0 = I_1 = I_2 = \frac{1\angle 0°}{j0.35 + j0.22 + j0.22 + j0.3} = -j0.9174\,\text{pu}$$

The fault currents are $I_a = 3I_0 = -j2.7522$ pu and $I_b = I_c = 0$.

For a line-to-line fault at bus 3 through a fault impedance of $j0.1$, the sequence network interconnection is as follows.



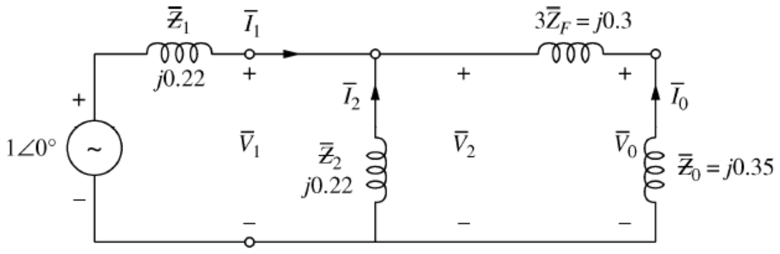
The zero-sequence current I_0 = 0. The other two currents follow from the diagram above,

$$I_1 = -I_2 = \frac{1.0\angle 0^{\circ}}{j0.22 + j0.1 + j0.22} = -j1.8519\,\text{pu}$$

The fault currents follow as

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ -j1.8519 \\ j1.8519 \end{bmatrix} = \begin{bmatrix} 0 \\ -3.2076 \\ 3.2076 \end{bmatrix} \text{pu}$$

For a double line-to-ground fault at bus 3 through a common fault impedance to ground $Z_F = j0.1$, the sequence network interconnection is as follows.



With reference to the diagram above, the positive sequence current I_1 is calculated as

$$I_1 = \frac{1.0\angle 0^{\circ}}{j0.22 + \dfrac{j0.22 \times (j0.35 + j0.3)}{j0.22 + (j0.35 + j0.3)}} = -j2.6017\text{ pu}$$

Applying KVL to the circuit above, we find that

$$I_2 = -\frac{1 - j0.22 \times (-j2.6017)}{j0.22} = j1.9438\,\text{pu}$$

and

$$I_0 = -\frac{1 - j0.22 \times (-j2.6017)}{j0.35 + j0.3} = j0.6579\,\text{pu}$$

The fault phase currents are then

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} j0.6579 \\ -j2.6017 \\ j1.9438 \end{bmatrix} = \begin{bmatrix} 0 \\ 4.0583\angle 165.926^{\circ} \\ 4.0583\angle 14.0732^{\circ} \end{bmatrix} \text{pu}$$

The neutral fault current at bus 3 is $I_b + I_c = 3I_0 = 1.9737\angle 90^{\circ}$.

REFERENCES

- GLOVER, J.D., OVERBYE, T.J. and SARMA, M.S. (2017). *Power System Analysis and Design*. 6th edition. Boston: Cengage Learning.
- SAADAT, H. (1999). *Power System Analysis*. 2nd edition. New York: McGraw-Hill.