Montogue

Quiz EL204

Transistor Amplifiers

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PROBLEM DISTRIBUTION

Problems	Subject
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8 – 11	Common-gate configurations
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PROBLEMS

Problem 1 (Sedra and Smith, 2015, w/ permission)

Calculate the overall voltage gain of a common-source amplifier that is fed a 1.2-MΩ source and connected to a 15-kΩ load. The MOSFET has transconductance g_m = 2 mA/V, and a drain resistance R_D = 10 kΩ is utilized.

Related equation: eq. 1

Problem 2 (Sedra and Smith, 2015, w/ permission)

A MOSFET connected in the common-source configuration has a transconductance g_m = 5 mA/V. When a resistance R_s is connected in the source lead, the effective transconductance is reduced to 2 mA/V. What do you estimate the value of R_s to be?

Related equation: eq. 2

Problem 3 (Sedra and Smith, 2015, w/ permission)

A common-source amplifier utilizes a MOSFET operated at overdrive voltage V_{OV} = 0.25 V. The amplifier feeds a load resistance R_L = 15 kΩ. The designer selects a drain resistance $R_D = 2R_L$. If it is required to realize an overall voltage gain G_v of –10 V/V, what transconductance g_m is needed? Also specify the bias current I_D. If, to increase the output signal swing, R_D is reduced to $R_D = R_L$, what does G_v become?

Related equation: eq. 1

Problem 4 (Sedra and Smith, 2015, w/ permission)

The overall voltage gain of a CS amplifier with a resistance R_s = 0.5 kΩ in the source lead was measured and found to be −10 V/V. When R_s was shorted, but the circuit operation remained linear, the gain doubled. What must the transconductance g_m be? What value of R_s is needed to obtain an overall voltage gain to −5 V/V?

Related equation: eq. 2

Problem 5 (Sedra and Smith, 2015, w/ permission)

A common-emitter amplifier utilizes a BJT with β = 100 biased at collector current I_C = 0.5 mA and has a collector resistance R_C = 12 kΩ and is connected to an emitter lead resistance R_e = 250 Ω. Find the input resistance R_{in}, the open-circuit voltage gain A_{vo}, and the output lead resistance R_o. If the amplifier is fed with a signal source having a resistance of 10 kΩ, and a load resistance R_L = 12 kΩ is connected to the output terminal, find the resulting gain A_v with load resistance and the overall voltage gain G_v. If the peak voltage of the sine wave appearing between base and emitter is to be limited to 5 mV, what signal amplitude $\hat{v}_{\text{sig}}$ is allowed, and what output voltage signal appears across the load?

Related equations: eqs. 3, 4, and 5

Problem 6 (Sedra and Smith, 2015, w/ permission)

Inclusion of an emitter resistance R_e reduces the variability of the gain G_v due to the inevitable wide variance in the value of current gain parameter β. Consider a common-emitter amplifier operating between a signal source with resistance R_{sig} = 10 kΩ and a total collector resistance $R_C||R_L$ of 10 kΩ. The BJT is biased at collector current I_C = 1 mA and its β is specified to be nominally 100 but can lie in the range of 50 to 150. First determine the nominal value and the range of overall voltage gain $|G_v|$ without resistance R_e. Then select a value of R_e that will ensure that $|G_v|$ be within ±20% of its new nominal value. Specify the value of R_e, the new nominal value of $|G_v|$, and the expected range of $|G_v|$.

Related equation: eq. 5

Problem 7 (Sedra and Smith, 2015, w/ permission)

In this problem we investigate the effect of the inevitable variability of β on the realized gain of the common-emitter amplifier. For this purpose, we write the overall voltage gain in a modified form of equation 5,

$$|G_v| = \frac{R'_L}{\frac{R_{\text{sig}}}{\beta} + \frac{1}{g_m}}$$

where $R'_L = R_L||R_C$. Consider the case R'_L = 10 kΩ and R_{sig} = 10 kΩ, and let the BJT be biased at I_C = 1 mA. The BJT has a nominal β of 100. Use 25 mV as the thermal voltage.

Problem 7.1: What is the nominal value of $|G_v|$?

Problem 7.2: If β can be anywhere between 50 and 150, what is the corresponding range of $|G_v|$?

Problem 7.3: If in a particular design, it is required to maintain $|G_v|$ within ±20% of its nominal value, what is the maximum allowable range of β?

Problem 7.4: If it is not possible to restrict β to the range found in Problem 7.3 , and the designer has to contend with β in the range 50 to 150, what value of bias current I_C would result in $|G_v|$ falling in $|G_v|$ falling in a range of ±20% of a new nominal value? What is the nominal value of $|G_v|$ in this case?

Related equation: eq. 5

Problem 8 (Razavi, 2008, w/ permission)

A common-gate amplifier using an NMOS transistor for which g_m = 2 mA/V has a 5-kΩ drain resistance R_D and a 5-kΩ load resistance R_L. The amplifier is driven by a voltage source having a 750-Ω resistance. What is the input resistance of the amplifier? What is the overall voltage gain G_v? By what factor must the bias current I_D of the MOSFET be changed so that input resistance R_{in} matches signal-source resistance R_{sig}?

Related equation: eq. 6

Problem 9 (Sedra and Smith, 2015, w/ permission)

A common-gate amplifier when fed with a signal source having R_{sig} = 100 Ω is found to have an overall voltage gain of 12 V/V. When a 100-Ω resistance was added in series with the signal generator the overall voltage gain decreased to 10 V/V. What must the transconductance g_m of the MOSFET be? If the MOSFET is biased at I_D = 0.25 mA, at what overdrive voltage must it be operating?

Related equation: eq. 6

Problem 10 (Sedra and Smith, 2015, w/ permission)

A common-gate amplifier operating with transconductance g_m = 2 mA/V and transistor output resistance r_o = 20 kΩ is fed with a signal source having resistance R_s = 1 kΩ and is loaded in a resistance R_L = 20 kΩ. Find input resistance R_{in}, output resistance R_{out}, and the voltage gain v_o/v_{sig} (output voltage/signal voltage).

Related equations: eqs. 7 and 8

Problem 11 (Sedra and Smith, 2015, w/ permission)

A common-gate amplifier operating with transconductance g_m = 2 mA/V and transistor output resistance r_o = 20 kΩ is fed with a signal source having a Norton equivalent composed of a current signal i_{sig} and a signal source resistance R_s = 20 kΩ. The amplifier is loaded in a resistance R_L = 20 kΩ. Find the input resistance R_{in} and i_o/i_{sig}, where i_o is the current through the load R_L. If R_L increases by a factor of 10, by what percentage does the current gain change?

Related equation: eq. 7

Problem 12 (Sedra and Smith, 2015, w/ permission)

A common-base amplifier is operating with load resistance R_L = 10 kΩ, collector resistance R_C = 10 kΩ, and signal-source resistance R_{sig} = 50 Ω. At what current I_C should the transistor be biased for the input resistance R_{in} to equal that of the signal source? What is the resulting overall voltage gain? Assume a common-base current gain $\alpha \approx 1$.

Related equation: eq. 8

Problem 13 (Sedra and Smith, 2015, w/ permission)

What value of load resistance R_L causes the input resistance of the common-base amplifier to be approximately double the value of emitter resistance r_e?

Related equation: eq. 9

Problem 14 (Sedra and Smith, 2015, w/ permission)

Show that for a CB amplifier,

$$\frac{R_{\text{out}}}{r_e} \approx 1 + \frac{\beta(R_e/r_e)}{\beta + 1 + (R_e/r_e)}$$

Generate a table for output resistance R_{out} as a multiple of transistor emitter resistance r_e with entries for circuit emitter lead resistance R_e = 0, r_e, $2r_e$, $10r_e$, $(\beta/2)r_e$, βr_e, and $1000r_e$. Let β = 100.

Related equation: eq. 10

Problem 15 (Sedra and Smith, 2015, w/ permission)

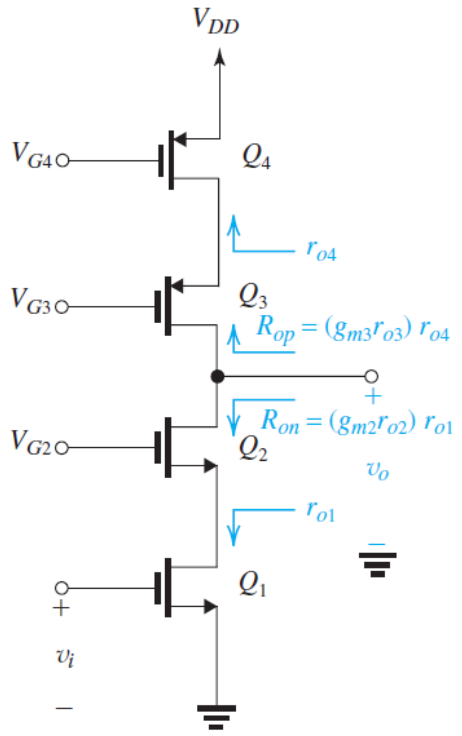
Consider a MOS cascode amplifier for which the CS and CG transistors are identical and are biased to operate at bias current I_D = 0.15 mA with overdrive voltage V_{OV} = 0.2 V. Also let Early voltage V_A = 1.5 V. Find A_{v1}, A_{v2}, and A_v for two cases:

Problem 5.1: R_L = 10 kΩ

Problem 5.2: R_L = 150 kΩ

Problem 16 (Sedra and Smith, 2015, w/ permission)

Consider the cascade amplifier illustrated below with the dc component of the input, V_I = 0.7 V, V_{G2} = 1.0 V, V_{G3} = 0.8 V, V_{G4} = 1.1 V, and V_{DD} = 1.8 V. If all devices are matched (i.e., such that conduction parameters $k_{n1} = k_{n2} = k_{p3} = k_{p4}$) and have ideal threshold voltages $|V_t|$ of 0.5 V, what is the overdrive voltage at which the transistors are operating? What is the allowable voltage range at the output?



Problem 17 (Sedra and Smith, 2015, w/ permission)

Suppose the cascade amplifier illustrated in Problem 16 is operated at a current of 0.2 mA with all devices operating at an overdrive voltage $|V_{OV}|$ = 0.2 V. All devices have Early voltage $|V_A|$ = 2 V. Find g_{m1} (the transconductance of transistor Q_1), the output resistance of the amplifier, R_{on}, and the output resistance of the current source, R_{op}. Also find the overall output resistance and the voltage gain realized.

Problem 18 (Sedra and Smith, 2015, w/ permission)

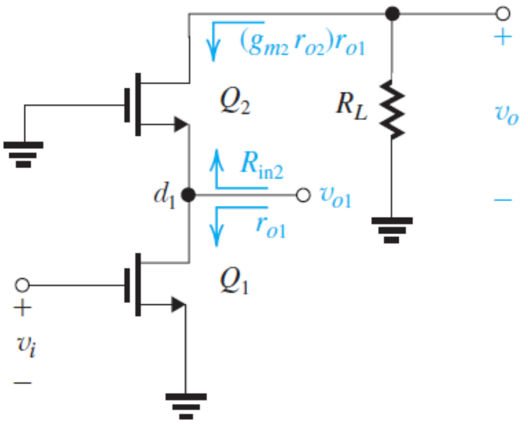
Reconsider the cascode amplifier introduced in Problem 16, taking V_I = 0.6 V as the dc component of the input, V_{G2} = 0.9 V, V_{G3} = 0.4 V, V_{G4} = 0.7 V, and V_{DD} = 1.3 V. If all devices are matched, that is, $k_{n1} = k_{n2} = k_{p3} = k_{p4}$, and have equal ideal threshold voltage $|V_t|$ = 0.4 V, what is the overdrive voltage at which the four transistors are operating? What is the allowable voltage range at the output?

Problem 19 (Sedra and Smith, 2015, w/ permission)

Design the CMOS cascode amplifier illustrated in Problem 16 for the following specifications: transconductance g_{m1} = 1 mA/V and voltage gain A_v = −280 V/V. Assume that for the available fabrication process, Early voltage $|V_A'|$ = 5 V/µm for both NMOS and PMOS devices and process parameter $\mu_n C_{ox} = 4\mu_p C_{ox}$ = 400 µA/V². Use the same channel length L for all devices and operate all four devices at $|V_{OV}|$ = 0.25 V. Determine the required channel length L, the bias current I, and the aspect ratio W/L for each of the four transistors. Assume that suitable bias voltages have been chosen, and neglect the Early effect in determining the W/L ratios.

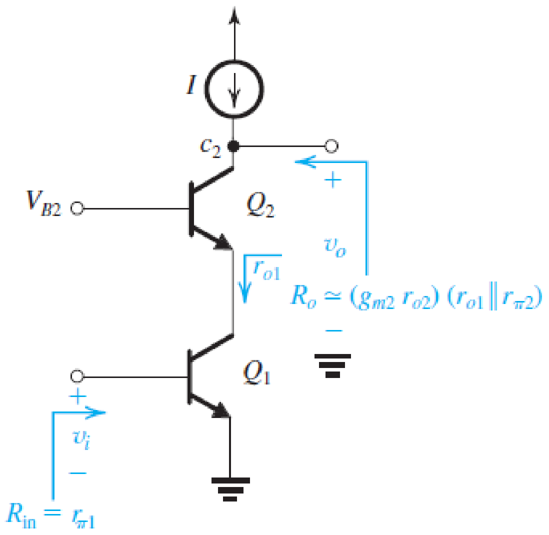
Problem 20 (Sedra and Smith, 2015, w/ permission)

A CMOS cascode amplifier such as the one illustrated below has identical common-source and common-gate transistors that have aspect ratio W/L = 5.4 µm/0.36 µm and are biased at I = 0.2 mA. The fabrication process has process parameter $\mu_n C_{ox}$ = 400 µA/V² and Early voltage V_A' = 5 V/µm. At what value of load resistance R_L does the gain become −100 V/V? What is the voltage gain of the common-source stage?



Problem 21

A cascode current source formed of two *pnp* bipolar transistors for which current gain parameter β = 50 and Early voltage V_A = 5 V supplies a current of 0.2 mA. What is the output resistance?



Problem 22

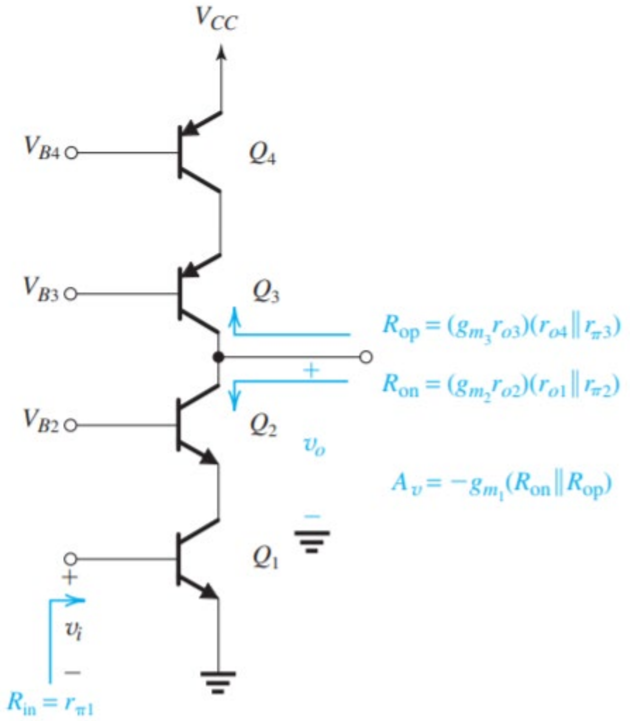
Consider the BJT cascode amplifier illustrated below when biased at a current of 0.2 mA.

Problem 22.1: Assuming that the *npn* transistors have current gain parameter $\beta = 100$ and Early voltage $V_A = 5$ V, and that the *pnp* transistors have $\beta = 50$ and $|V_A| = 4$ V, find the output resistance of the amplifier, R_{on}, the output resistance of the current source, R_{op}. and the voltage gain, A_v.

Problem 22.2: Show that the maximum voltage gain achieved by the BJT cascode illustrated below is given by

$$\left|A_{v,\max}\right| = g_{m1}\left(\beta_2 r_{o2} \parallel \beta_3 r_{o3}\right)$$

Using this relationship, compute $A_{v,max}$ for the amplifier introduced in Problem 22.1.

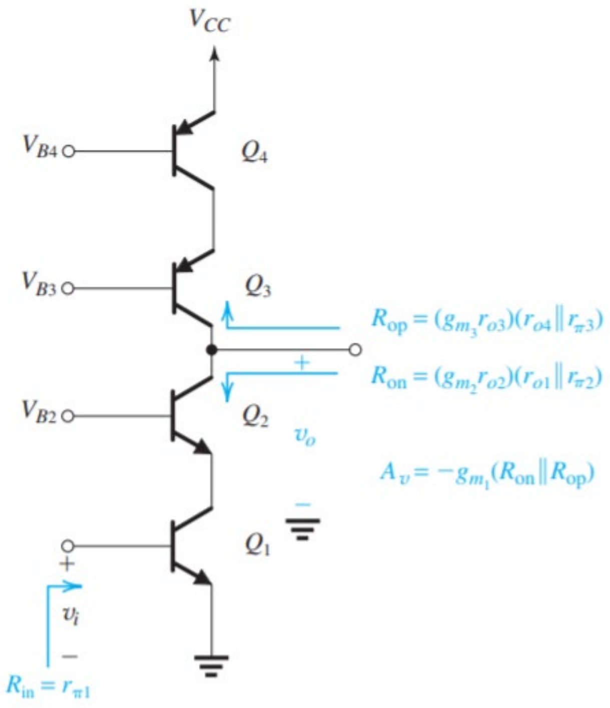


Problem 23

Consider the BJT cascode amplifier illustrated in Problem 22 for the case all transistors have equal current gain parameter β and transistor output resistance r_o. Show that the voltage gain A_v can be expressed in the form

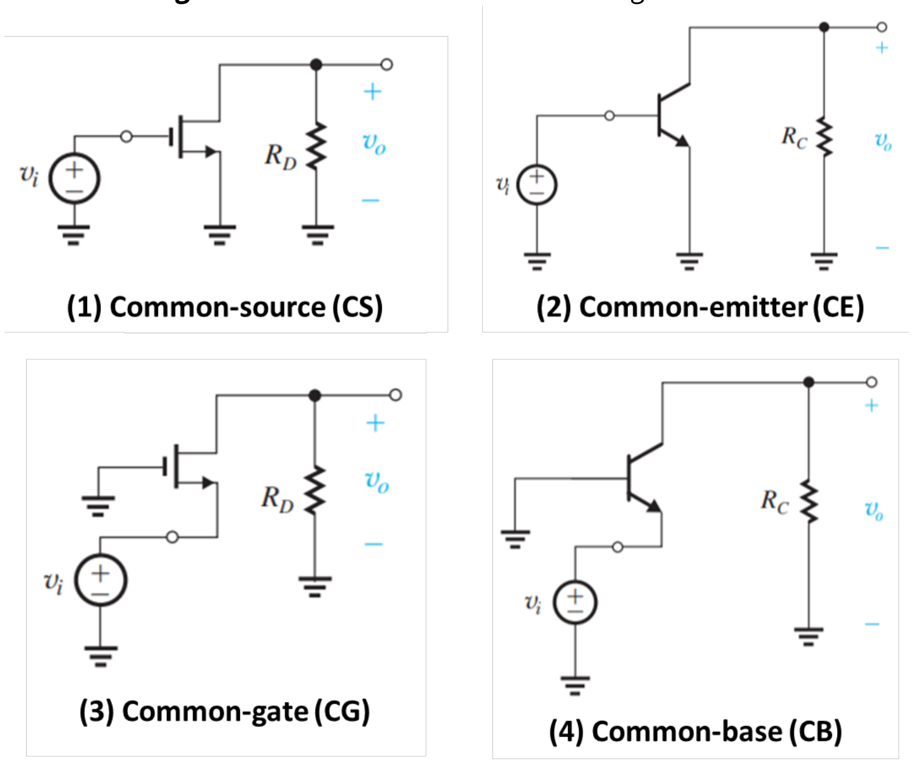
$$A_v = -\frac{1}{2}\frac{|V_A|/V_T}{\left(V_T/|V_A|\right)+1/\beta}$$

Evaluate A_v for the case $|V_A| = 5$ V and $\beta = 50$. Note that except for the fact that β depends on I as a second-order effect, the gain is independent of the bias current I!



ADDITIONAL INFORMATION

Figure 1 Basic transistor circuit configurations.



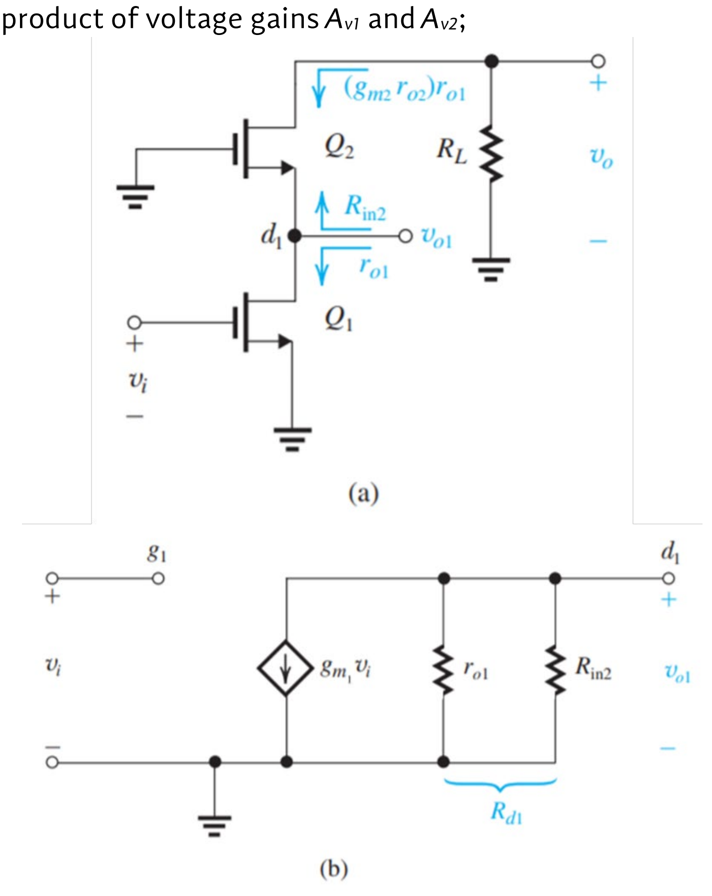
(1) Common-source (CS)   (2) Common-emitter (CE)

(3) Common-gate (CG)   (4) Common-base (CB)

Table 1 Gain distribution in the MOS cascode for various values of load resistance R_L.

Case	R_L	R_{in2}	R_{d1}	A_{v1}	A_{v2}	A_v
1	∞	∞	r_o	$-g_m r_o$	$g_m r_o$	$-(g_m r_o)^2$
2	$(g_m r_o) r_o$	r_o	$r_o/2$	$-\frac{1}{2}(g_m r_o)$	$g_m r_o$	$-\frac{1}{2}(g_m r_o)^2$
3	r_o	$\frac{2}{g_m}$	$\frac{2}{g_m}$	-2	$\frac{1}{2}(g_m r_o)$	$-(g_m r_o)$
4	0	$\frac{1}{g_m}$	$\frac{1}{g_m}$	-1	0	0

Notation:
R_L is the load resistance;
R_{in2} is the input resistance of the CG transistor Q_2;
R_{d1} is the parallel resistance of r_{o1} (the transistor output resistance of Q_1) and R_{in2} (the input resistance of Q_2);
A_{v1} is the voltage gain of Q_1;
A_{v2} is the voltage gain of Q_2;
A_v is the product of voltage gains A_{v1} and A_{v2};



Equations

1 → Overall voltage gain of a common-source amplifier

$$G_v = -g_m\left(R_D \parallel R_L\right)$$

where g_m is device transconductance, R_D is drain resistance, and R_L is load resistance.

2 → Voltage gain in a common-source amplifier with a resistance R_S connected to the source lead

$$G_{v(\text{with added source res.})} = -\frac{g_m\left(R_D \parallel R_L\right)}{1+g_m R_s}$$

where g_m is device transconductance, R_D is drain resistance, R_L is load resistance, and R_s is the added source-lead resistance.

3 → Resistance-reflection rule

$$R_{\text{in}} = \left(1+\beta\right)\left(r_e + R_e\right)$$

where β is the BJT current gain parameter, r_e is the emitter resistance, and R_e is the added emitter resistance.

4 → Open-circuit voltage gain in a common-emitter amplifier

$$A_{\text{vo}} = -\frac{g_m R_C}{1+g_m R_e}$$

where g_m is transconductance parameter, R_C is collector resistance, and R_e is the added emitter resistance.

5 → Overall voltage gain in a common-emitter amplifier

$$G_{\text{v}} = -\beta\frac{R_C \parallel R_L}{R_{\text{sig}} + \left(1+\beta\right)\left(r_e + R_e\right)}$$

where β is the BJT current gain parameter, R_C is collector resistance, R_L is load resistance, R_{sig} is the signal-source resistance, r_e is emitter resistance, R_e is added emitter resistance.

6 → Overall voltage gain in a common-gate amplifier

$$G_v = \frac{R_D \parallel R_L}{R_{\text{sig}} + \dfrac{1}{g_m}}$$

where R_D is drain resistance, R_L is load resistance, R_{sig} is signal-source resistance, and g_m is transconductance parameter.

7 → Input resistance in a common-gate amplifier

$$R_{\text{in}} = \frac{r_o + R_L}{1+g_m r_o}$$

where r_o is transistor output resistance, R_L is load resistance, and g_m is transconductance parameter.

8 → Overall voltage gain in a common-base amplifier as a function of input resistance

$$G_v = \frac{R_{\text{in}}}{R_{\text{in}} + R_{\text{sig}}} g_m\left(R_C \parallel R_L\right)$$

where R_{in} is input resistance, R_{sig} is signal-source resistance, g_m is transconductance parameter, R_C is collector resistance, and R_L is load resistance.

9 → Input resistance in a common-base amplifier

$$R_{\text{in}} \approx r_e\frac{r_o + R_L}{r_o + \dfrac{R_L}{\beta+1}}$$

where r_o is transistor output resistance, R_L is load resistance, and β is the BJT current gain parameter.

10 → Approximate output resistance in a common-gate amplifier

$$R_{\text{out}} \approx r_o\left[1+g_m\left(R_e \parallel r_\pi\right)\right]$$

where r_o is transistor output resistance, g_m is transconductance parameter, R_e is added emitter resistance, and r_π is the internal base-emitter resistance.

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SOLUTIONS

P.1 Solution

The overall voltage gain G_v of a common-source amplifier is given by equation 1,

$$G_v = -g_m\left(R_D \| R_L\right) = -2.0\times\left(10\|15\right) = -2.0\times\left(\frac{10\times15}{10+15}\right) = \boxed{-12.0\text{ V/V}}$$

P.2 Solution

Adding a resistance R_s to the source lead decreases the effective transconductance, and by extension the voltage gain, by a factor $1+g_mR_s$ (see equation 2). If g_{m1} is decreased to 2 mA/V from an initial g_m of 5 mA/V, the resistance connected to the source lead must be

$$g_{m1} = \frac{g_m}{1+g_mR_s} \rightarrow g_{m1}\left(1+g_mR_s\right) = g_m$$

$$\therefore g_{m1} + g_{m1}g_mR_s = g_m$$

$$\therefore R_s = \frac{g_m - g_{m1}}{g_mg_{m1}} = \frac{5-2}{5\times2} = 0.3\,\text{k}\Omega = \boxed{300\,\Omega}$$

P.3 Solution

Noting that R_L = 15 kΩ and $R_D = 2R_L$ = 30 kΩ, we can establish the required transconductance from the voltage gain G_v = –10,

$$G_v = -g_m\left(R_D\|R_L\right) = -10 \rightarrow g_m = \frac{10}{R_D\|R_L}$$

$$\therefore g_m = \frac{10}{30\|15} = \frac{10}{\frac{30\times15}{30+15}} = \boxed{1.0\text{ mA/V}}$$

Referring to the definition of transconductance for a MOSFET, we write

$$g_m = \frac{2I_D}{V_{OV}} \rightarrow I_D = \frac{g_mV_{OV}}{2}$$

$$\therefore I_D = \frac{g_mV_{OV}}{2} = \frac{1.0\times0.25}{2} = \boxed{0.125\text{ mA}}$$

If drain resistance R_D is halved to 15 kΩ, the overall voltage gain becomes

$$G_v = -g_m\left(R_D\|R_L\right) = -1.0\times\left(15\|15\right) = -1.0\times7.5 = \boxed{-7.5\text{ V/V}}$$

P.4 Solution

The overall voltage gain of a CS amplifier in the presence of a source lead resistance is expressed as (equation 2)

$$G_{v(\text{with added source res.})} = -\frac{g_m\left(R_D\|R_L\right)}{1+g_mR_s} \rightarrow -10 = -\frac{g_m\left(R_D\|R_L\right)}{1+g_m\times0.5} \quad \text{(I)}$$

The overall voltage gain with no added source lead resistance is given by the now obvious relation

$$G_{v(\text{no added source res.})} = -g_m\left(R_D\|R_L\right) = 2G_{v(\text{with added source res.})} = -20\text{ V}$$

Substituting in (I) and solving for transconductance,

$$-10 = \frac{-20}{1+0.5g_m} \rightarrow -10-5g_m = -20$$

$$\therefore g_m = \frac{20-10}{5} = \boxed{2\text{ mA/V}}$$

Equipped with the value of g_m, the source lead resistance R_s needed to produce G_v = –16 V/V easily follows,

$$-16 = -\frac{g_m\left(R_D\|R_L\right)}{1+g_mR_s} \rightarrow -16 = \frac{-20}{1+2R_s}$$

$$\therefore -16-32R_s = -20$$

$$\therefore R_s = \frac{-20+16}{-32} = 0.125\,\text{k}\Omega = \boxed{125\,\Omega}$$

P.5 ➡ *Solution*

The transconductance of the device is

$$g_m = \frac{I_C}{V_T} = \frac{0.5}{25\times10^{-3}} = 20\,\text{mA/V}$$

The emitter resistance is then

$$r_e = \frac{1}{g_m} = \frac{1}{20\times10^{-3}} = 50\,\Omega$$

The input resistance is calculated to be (equation 3)

$$R_{\text{in}} = (1+\beta)(r_e+R_e) = (1+100)\times(0.05+0.25) = \boxed{30.3\,\text{k}\Omega}$$

For this simple CE amplifier, the output resistance coincides with the collector resistance,

$$\boxed{R_C = 12\,\text{k}\Omega}$$

The open-circuit voltage gain is (equation 4)

$$A_{\text{vo}} = -\frac{g_m R_C}{1+g_m R_e} = -\frac{(20\times10^{-3})\times(12\times10^{3})}{1+(20\times10^{-3})\times250} = \boxed{-40\,\text{V/V}}$$

To determine the gain A_v with load resistance, we write

$$A_v = A_{vo}\left(\frac{R_L}{R_L+R_o}\right) = -40\times\left(\frac{12}{12+12}\right) = \boxed{-20\,\text{V/V}}$$

As for the overall voltage gain G_v (equation 5),

$$G_v = -\beta\frac{R_C \| R_L}{R_{\text{sig}}+(1+\beta)(r_e+R_e)} = -100\times\frac{12\|12}{10+(1+100)\times(0.05+0.25)} = \boxed{-14.9\,\text{V/V}}$$

If the peak voltage of the sine wave is to be no greater than 5 mV, the corresponding input signal voltage is, at most,

$$\frac{v_\pi}{v_i} = \frac{r_e}{r_e+R_e} \rightarrow v_i = \left(\frac{r_e+R_e}{r_e}\right)v_\pi$$

$$\therefore v_i = \left(\frac{0.05+0.25}{0.05}\right)\times5 = 30\,\text{mV}$$

so that, for the allowable signal voltage amplitude $\hat{v}_{\text{sig}}$,

$$\hat{v}_{\text{sig}} = \left(\frac{R_{\text{in}}+R_{\text{sig}}}{R_{\text{in}}}\right)v_i = \left(\frac{30.3+10}{30.3}\right)\times30 = \boxed{39.9\,\text{mV}}$$

Lastly, the output voltage signal that appears across the load is

$$\hat{v}_{\text{o}} = \hat{v}_{\text{sig}}|G_v| = 39.9\times14.9 = 595\,\text{mV} = \boxed{0.595\,\text{V}}$$

P.6 ➡ *Solution*

Let us first state the usual relationship for overall current gain in a CE configuration (equation 5),

$$G_v = -\beta\frac{R_C \| R_L}{R_{\text{sig}}+(1+\beta)(r_e+R_e)} = -\beta\frac{R_C \| R_L}{R_{\text{sig}}+(1+\beta)\left(\frac{V_T}{I_E}+R_e\right)} \quad \text{(I)}$$

Emitter current I_E can be determined from the nominal common-base current gain α, namely

$$\alpha = \frac{\beta}{\beta+1} = \frac{100}{100+1} = 0.990$$

so that

$$I_E = \frac{I_C}{\alpha} = \frac{1.0}{0.990} = 1.01\,\text{mA}$$

The overall voltage gain with no added emitter resistance R_e is, substituting in (I),

$$G_v = -100\times\frac{10\times10^3}{\left(10\times10^3\right)+\left(1+100\right)\times\left(\frac{25\times10^{-3}}{1.01\times10^{-3}}+0\right)} = -80\text{ V/V}$$

Now, if β is set to vary between 50 and 150, the corresponding α will vary from 0.98 to 0.99. Current I_E will attain a minimum value of

$$I_{E,\min} = \frac{I_C}{\alpha} = \frac{1.0}{0.990} = 1.01\,\text{mA}$$

and a maximum value of

$$I_{E,\max} = \frac{I_C}{\alpha} = \frac{1.0}{0.980} = 1.02\,\text{mA}$$

Taking β = 50, the nominal value of $|G_v|$ without resistance R_e is

$$G_v = -50\times\frac{10\times10^3}{\left(10\times10^3\right)+\left(1+50\right)\times\left(\frac{25\times10^{-3}}{1.02\times10^{-3}}+0\right)} = -44.4\text{ V/V}$$

while for β = 150,

$$G_v = -150\times\frac{10\times10^3}{\left(10\times10^3\right)+\left(1+150\right)\times\left(\frac{25\times10^{-3}}{1.01\times10^{-3}}+0\right)} = -109\text{ V/V}$$

Accordingly, with no added emitter resistance the absolute value of the overall current gain will lie in the interval [44.4, 109] V/V. Now, we aim to find an added emitter resistance R_e that will ensure that the $|G_v|$ be within 20% of its new nominal value $G_{v,nom}$. At the lower limit, we set $|G_v| = 0.8G_{v,nom}$ and write

$$\left|G_v\right| = 50\times\frac{10\times10^3}{\left(10\times10^3\right)+\left(1+50\right)\times\left(\frac{25\times10^{-3}}{1.02\times10^{-3}}+R_e\right)} = 0.8G_{v,\text{nom}}$$

At the upper limit, with $|G_v| = 1.2G_{v,nom}$,

$$\left|G_v\right| = 150\times\frac{10\times10^3}{\left(10\times10^3\right)+\left(1+150\right)\times\left(\frac{25\times10^{-3}}{1.01\times10^{-3}}+R_e\right)} = 1.2G_{v,\text{nom}}$$

Dividing one equation by the other and solving for R_e,

$$\frac{150\times\dfrac{10\times10^3}{\left(10\times10^3\right)+151\times\left(24.8+R_e\right)}}{50\times\dfrac{10\times10^3}{\left(10\times10^3\right)+51\times\left(24.5+R_e\right)}} = \frac{1.2\cancel{G_{v,\text{nom}}}}{0.8\cancel{G_{v,\text{nom}}}}$$

$$\therefore 3\times\frac{\left(10\times10^3\right)+51\times\left(24.5+R_e\right)}{\left(10\times10^3\right)+151\times\left(24.8+R_e\right)} = 1.5$$

$$\therefore \frac{11,250+51R_e}{13,740+151R_e} = 0.5$$

$$\therefore 11,250+51R_e = 6870+75.5R_e$$

$$\therefore R_e = \frac{11,250-6870}{75.5-51} = \boxed{179\,\Omega}$$

Substituting this R_e into (I) yields the nominal voltage gain

$$G_{v,\text{nom}} = -\beta\frac{R_C \parallel R_L}{R_{\text{sig}}+\left(1+\beta\right)\left(r_e+R_e\right)} = -100\times\frac{10\times10^3}{\left(10\times10^3\right)+\left(1+100\right)\times\left(\frac{25\times10^{-3}}{1.01\times10^{-3}}+179\right)} = \boxed{-32.7\text{ V/V}}$$

Using the R_e obtained above, we can establish the expected range of G_v,

$$G_v = -50 \times \frac{10\times10^3}{\left(10\times10^3\right)+\left(1+50\right)\times\left(\dfrac{25\times10^{-3}}{1.02\times10^{-3}}+179\right)} = -24.5\ \text{V/V}$$

$$G_v = -150 \times \frac{10\times10^3}{\left(10\times10^3\right)+\left(1+150\right)\times\left(\dfrac{25\times10^{-3}}{1.02\times10^{-3}}+179\right)} = -36.8\ \text{V/V}$$

The overall voltage gain is expected to vary between −36.8 V/V and −24.5 V/V.

P.7 ➡ *Solution*

Problem 7.1: The nominal value of G_v is that which corresponds to the device's nominal β, which is 100. Noting that $g_m = I_C/V_T$ and substituting the pertaining variables into G_v, we obtain

$$\left|G_v\right| = \frac{10\times10^3}{\dfrac{10\times10^3}{100}+\dfrac{1}{\left(\dfrac{1.0\times10^{-3}}{25\times10^{-3}}\right)}} = \boxed{80\ \text{V/V}}$$

Problem 7.2: Assuming $|G_v|$ is monotonically increasing with $\beta \in [50, 150]$, we have, at one end,

$$\left|G_v\right| = \frac{10\times10^3}{\dfrac{10\times10^3}{50}+\dfrac{1}{\left(\dfrac{1.0\times10^{-3}}{25\times10^{-3}}\right)}} = \boxed{44.44\ \text{V/V}}$$

while at the other,

$$\left|G_v\right| = \frac{10\times10^3}{\dfrac{10\times10^3}{150}+\dfrac{1}{\left(\dfrac{1.0\times10^{-3}}{25\times10^{-3}}\right)}} = \boxed{109.10\ \text{V/V}}$$

Thus, the overall voltage gain varies from about 44 V/V to about 109 V/V in the range of β-values considered.

Problem 7.3: The nominal value of G_v was calculated to be 80 V/V; if the device is allowed to vary within ±20% of this specification, we have $G_{v,min}$ = 64 V/V and $G_{v,max}$ = 96 V/V. In one extreme, the corresponding β is

$$\left|G_{v,\min}\right| = 64 = \frac{10\times10^3}{\dfrac{10\times10^3}{\beta}+\dfrac{1}{\left(\dfrac{1.0\times10^{-3}}{25\times10^{-3}}\right)}} \rightarrow 64 = \frac{10{,}000}{\dfrac{10{,}000}{\beta}+25}$$

$$\therefore 64\left(\frac{10{,}000}{\beta}+\frac{25\beta}{\beta}\right) = 10{,}000$$

$$\therefore 640{,}000 + 1600\beta = 10{,}000\beta$$

$$\therefore 640{,}000 = 8400\beta$$

$$\therefore \beta_{\min} = \frac{640{,}000}{8400} = 76.19$$

At the other extreme, using Mathematica to speed things up,

```
In[48]:= Solve[96 == 10000./(10000/β + 1/(1/25)), β]

Out[48]= {{β → 126.316}}
```

That is, $\beta_{\max}$ = 126.32. The allowable range of β is $76.19 \le \beta \le 126.32$.

Problem 7.4: Let the new nominal G_v be $|G_v|_{nom}$. With β = 50 and $|G_v| = 0.8|G_v|_{nom}$, we write

$$\frac{10\times10^3}{\frac{10\times10^3}{50}+\frac{1}{\left(\frac{I_C}{25\times10^{-3}}\right)}}=0.8|G_v|_{\text{nom}}\quad\text{(I)}$$

With β = 150 and $|G_v| = 1.2|G_v|_{\text{nom}}$, we have

$$\frac{10\times10^3}{\frac{10\times10^3}{150}+\frac{1}{\left(\frac{I_C}{25\times10^{-3}}\right)}}=1.2|G_v|_{\text{nom}}\quad\text{(II)}$$

Dividing (II) by (I) and solving for bias current, we get

$$\frac{\frac{10\times10^3}{\frac{10\times10^3}{150}+\frac{1}{\left(\frac{I_C}{25\times10^{-3}}\right)}}}{\frac{10\times10^3}{\frac{10\times10^3}{50}+\frac{1}{\left(\frac{I_C}{25\times10^{-3}}\right)}}}=\frac{1.2\cancel{|G_v|_{\text{nom}}}}{0.8\cancel{|G_v|_{\text{nom}}}}$$

```
In[52]:= Solve[1.2/0.8 == (10000./(10000/150 + 25*10^-3/iC))/(10000./(10000/50 + 25*10^-3/iC)), iC]

Out[52]= {{iC → 0.000125}}
```

That is, the bias current that would have $|G_v|$ fall in a range of ±20% of the new nominal value is I_C = 0.125 mA. This new nominal voltage gain is

$$|G_v|=\frac{10\times10^3}{\frac{10\times10^3}{100}+\frac{25}{0.125}}=\boxed{33.33\text{ V/V}}$$

P.8 ➡ *Solution*

The input resistance of a typical common-gate amplifier equals the reciprocal of the FET's transconductance:

$$R_{\text{in}}=\frac{1}{g_m}=\frac{1}{2.0\times10^{-3}}=\boxed{500\,\Omega}$$

To determine the overall voltage gain, we apply equation 6,

$$G_v=\frac{R_D\,||\,R_L}{R_{\text{sig}}+\frac{1}{g_m}}=\frac{5.0\,||\,5.0}{0.75+0.5}=\frac{\frac{5.0\times5.0}{5.0+5.0}}{1.25}=\frac{2.5}{1.25}=\boxed{2\text{ V/V}}$$

Now, using the definition of transconductance, we may write

$$g_m=\sqrt{2k_n'I_{D,1}}=2.0\times10^{-3}\quad\text{(I)}$$

For the signal-source resistance R_{sig} to match the input resistance R_{in}, we must have

$$R_{\text{sig}}=R_{\text{in}}=\frac{1}{g_m}\rightarrow g_m=\frac{1}{R_{\text{in}}}=\frac{1}{750}$$

$$\therefore\sqrt{2k_n'I_{D,2}}=\frac{1}{750}\quad\text{(II)}$$

Dividing (II) by (I), we obtain the ratio

$$\sqrt{\frac{\cancel{2k_n'}I_{D,2}}{\cancel{2k_n'}I_{D,1}}}=\frac{\frac{1}{750}}{2.0\times10^{-3}}=\frac{\frac{1}{750}}{\frac{1}{500}}=\frac{2}{3}$$

$$\therefore \frac{I_{D,2}}{I_{D,1}} = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$$

$$\therefore \boxed{I_{D,2} = \frac{4}{9} I_{D,1}}$$

That is, the bias current must be multiplied by a factor of four-ninths in order to have the input resistance R_{in} match the signal-source resistance R_{sig}.

P.9 ➡ Solution

Recall that the overall gain of a common-gate amplifier is expressed as (equation 6)

$$G_v = \frac{R_D \parallel R_C}{R_{\text{sig}} + \dfrac{1}{g_m}}$$

At first, R_{sig} = 100 Ω and G_v = 12 V/V, that is,

$$12 = \frac{R_D \parallel R_C}{100 + \dfrac{1}{g_m}} \quad \text{(I)}$$

After 100 Ω of resistance is added in series to the signal generator, R'_{sig} = 200 Ω and G'_v = 10 V/V, so that

$$10 = \frac{R_D \parallel R_C}{200 + \dfrac{1}{g_m}} \quad \text{(II)}$$

Dividing (I) by (II) and solving for transconductance,

$$\frac{12}{10} = \frac{\dfrac{\cancel{R_D \parallel R_C}}{100 + \dfrac{1}{g_m}}}{\dfrac{\cancel{R_D \parallel R_C}}{200 + \dfrac{1}{g_m}}} = \frac{200 + \dfrac{1}{g_m}}{100 + \dfrac{1}{g_m}}$$

$$\therefore 12 \times \left(100 + \frac{1}{g_m}\right) = 10 \times \left(200 + \frac{1}{g_m}\right)$$

$$\therefore 1200 + \frac{12}{g_m} = 2000 + \frac{10}{g_m}$$

$$\therefore \frac{2}{g_m} = 800$$

$$\therefore g_m = \frac{2}{800} = \frac{1}{400}\,\text{A/V} = \frac{1000}{400}\,\text{mA/V}$$

$$\therefore \boxed{g_m = 2.5\,\text{mA/V}}$$

If the FET is biased at I_D = 0.25 mA, the overdrive voltage V_{OV} must be

$$g_m = \frac{2I_D}{V_{OV}} \rightarrow V_{OV} = \frac{2I_D}{g_m}$$

$$\therefore V_{OV} = \frac{2 \times 0.25}{2.5} = \boxed{0.2\,\text{V}}$$

P.10 ➡ Solution

The input resistance is given by equation 7,

$$R_{\text{in}} = \frac{r_o + R_L}{1 + g_m r_o} = \frac{(20+20) \times 10^3}{1 + (2.0 \times 10^{-3}) \times (20 \times 10^3)} = \boxed{976\,\Omega}$$

The output resistance is, in turn (equation 8),

$$R_{out} = r_o + (1 + g_m r_o) R_s = (20\times10^3) + \left[1 + (2.0\times10^{-3})\times(20\times10^3)\right]\times(1.0\times10^3) = 61{,}000\,\Omega$$

$$\therefore \boxed{R_{out} = 61.0\,\text{k}\Omega}$$

Lastly, the voltage gain is

$$\frac{v_o}{v_{sig}} = \frac{R_L}{R_s + R_{in}} = \frac{20}{1.0 + 0.976} = \boxed{10.1\,\text{V/V}}$$

P.11 ➡ Solution

We have all the data needed to compute input resistance R_{in} (equation 7),

$$R_{in} = \frac{r_o + R_L}{1 + g_m r_o} = \frac{(20+20)\times10^3}{1 + (2.0\times10^{-3})\times(20\times10^3)} = \boxed{976\,\Omega}$$

Now, current gain i_o/i_{sig} can be expressed as

$$\left(\frac{i_o}{i_{sig}}\right)_0 = \frac{R_S}{R_{in} + R_S} = \frac{20}{0.976 + 20} = 0.953\,\text{A/A}$$

Once the load resistance is increased to $R_L' = 10R_L$, the current gain becomes

$$\left(\frac{i_o}{i_{sig}}\right)_1 = \frac{R_S}{R_{in}' + R_S} = \frac{20\times10^3}{\dfrac{(20+200)\times10^3}{1 + (2.0\times10^{-3})\times(20\times10^3)} + 20\times10^3} = 0.788\,\text{A/A}$$

This amounts to a percentage change in current gain given by

$$\Delta = \frac{(i_o/i_{sig})_1 - (i_o/i_{sig})_0}{(i_o/i_{sig})_0}\times100\% = \frac{0.788 - 0.953}{0.953}\times100\% = -17.3\%$$

P.12 ➡ Solution

First, note that the input resistance R_{in} of a common-base amp can be estimated as

$$R_{in} \approx \frac{1}{g_m}$$

For the input resistance R_{in} to equal the signal-source resistance R_{sig} = 50 Ω, the transconductance must be

$$\frac{1}{g_m} = R_{in} = R_{sig} = 50 \rightarrow g_m = \frac{1}{50} = 20\,\text{mA/V}$$

Using the definition of g_m for a BJT, we establish the collector current

$$g_m = \frac{I_C}{V_T} \rightarrow I_C = g_m V_T$$

$$\therefore I_C = (20\times10^{-3})\times(25\times10^{-3}) = 5.0\times10^{-4}\,\text{A} = \boxed{0.5\,\text{mA}}$$

The overall voltage gain G_v is, in turn (equation 8),

$$G_v = \frac{R_{in}}{R_{in} + R_{sig}} g_m (R_C \parallel R_L) = \frac{50}{50+50}\times(20\times10^{-3})\times\left[(10\times10^3)\parallel(10\times10^3)\right]$$

$$\therefore \boxed{G_v = 50\,\text{V/V}}$$

P.13 ➡ Solution

First, note that the input resistance of a CB configuration is related to load resistance R_L and other resistance components by the expression (equation 9)

$$R_{in} \approx r_e \frac{r_o + R_L}{r_o + \dfrac{R_L}{\beta + 1}}$$

Setting $R_{in} = 2r_e$ and solving for R_L, we obtain

$$\not{r_e}\frac{r_o+R_L}{r_o+\dfrac{R_L}{\beta+1}}=2\not{r_e}\rightarrow r_o+R_L=2\left(r_o+\frac{R_L}{\beta+1}\right)$$

$$\therefore r_o+R_L=2r_o+\frac{2R_L}{\beta+1}$$

$$\therefore R_L-\frac{2R_L}{\beta+1}=r_o$$

$$\therefore\frac{R_L(\beta+1)-2R_L}{\beta+1}=r_o$$

$$\therefore R_L(\beta+1)-2R_L=(\beta+1)r_o$$

$$\therefore R_L(\beta-1)=(\beta+1)r_o$$

$$\therefore \boxed{R_L=\left(\frac{\beta+1}{\beta-1}\right)r_o}$$

Thus, if the load resistance were set to $(\beta + 1)/(\beta - 1)$ times the transistor output resistance r_o, the input resistance R_{in} would become twice the emitter resistance r_e. With $\beta = 50$, for example, the load resistance would have to be 51/49 ≈ 1.04 times the value of r_e.

P.14 ➡ *Solution*

Starting with the equation for output resistance R_{out} (equation 10), we write

$$R_{\text{out}}\approx r_o+g_m r_o(R_e\parallel r_\pi)\rightarrow R_{\text{out}}\approx r_o\left[1+g_m(R_e\parallel r_\pi)\right]$$

$$\therefore R_{\text{out}}=r_o\left(1+\frac{\beta}{r_\pi}\frac{R_e r_\pi}{R_e+r_\pi}\right)$$

$$\therefore R_{\text{out}}=r_o\left(1+\frac{\beta R_e}{R_e+r_\pi}\right)$$

$$\therefore R_{\text{out}}=r_o\left[1+\frac{\beta R_e}{R_e+(\beta+1)r_e}\right]$$

$$\therefore\frac{R_{\text{out}}}{r_o}=\left(1+\frac{\beta R_e}{\beta r_e+r_e+R_e}\right)$$

$$\therefore\frac{R_{\text{out}}}{r_o}=\left[1+\frac{\beta(R_e/r_e)}{\beta+1+(R_e/r_e)}\right]$$

The desired relationship has been demonstrated. We proceed to tabulate values of R_{out}/r_o as a function of different emitter lead resistances R_e. One way to go is to apply Mathematica's *Table* function,

```
In[524]:= SetPrecision[Table[1 + (100. * r)/(101 + r), {r, {0, 1, 2, 10, 50, 100, 1000.}}], 3]

Out[524]= {1.00, 1.98, 2.94, 10.0, 34.1, 50.8, 91.8}
```

The results are tabulated below.

R_e	R_{out}/r_o
0	1.0
r_e	1.98
$2r_e$	2.94
$10r_e$	10.0
$50r_e$	34.1
$100r_e$	50.8
$1000r_e$	91.8

P.15 ➡ *Solution*

Problem 5.1: We first determine the transconductance g_m,

$$g_m=\frac{2I_D}{V_{OV}}=\frac{2\times0.15}{0.2}=1.5\text{ mA/V}$$

The transistor output resistance r_o is given by

$$r_o = \frac{|V_A|}{I_D} = \frac{1.5}{0.15\times10^{-3}} = 10\,\text{k}\Omega$$

Note that $r_o = R_L = 10$ kΩ; referring to Table 1, the present system fits into MOS cascode amplifier case 3. The voltage gain of transistor Q_1 is fixed as

$$\boxed{A_{v1} = -2.0\,\text{V/V}}$$

The voltage gain of Q_2 is, in turn,

$$A_{v2} = \frac{1}{2}g_m r_o = -\frac{1}{2}\times\left(1.5\times10^{-3}\right)\times\left(10\times10^{3}\right) = \boxed{7.5\,\text{V/V}}$$

The overall gain can be expressed as the product of the voltage gains of Q_1 and Q_2,

$$A_v = A_{v1}A_{v2} = -2.0\times7.5 = \boxed{-15\,\text{V/V}}$$

Problem 5.2: Note that

$$\left(g_m r_o\right)r_o = \left(1.5\times10^{-3}\right)\times\left(10\times10^{3}\right)\times\left(10\times10^{3}\right) = 150\text{ k}\Omega$$

which happens to be the value of R_L; accordingly, we are now in gain distribution case 2. The voltage gain of Q_1 then becomes

$$A_{v1} = -\frac{1}{2}\left(g_m r_o\right) = -\frac{1}{2}\times\left[\left(1.5\times10^{-3}\right)\times\left(10\times10^{3}\right)\right] = \boxed{-7.5\,\text{V/V}}$$

while the gain of Q_2 is found as

$$A_{v2} = g_m r_o = \left(1.5\times10^{-3}\right)\times\left(10\times10^{3}\right) = \boxed{15\,\text{V/V}}$$

Lastly, the overall gain is

$$A_v = A_{v1}A_{v2} = -7.5\times15 = \boxed{-113\,\text{V/V}}$$

P.16 ➡ *Solution*

The overdrive voltage V_{OV} for a PMOS transistor is of course

$$V_{OV} = V_{SG} - V_t = V_S - V_G - V_t$$

With reference to transistor Q_4, we may write

$$V_{OV} = V_{DD} - V_{G,4} - V_t = 1.8 - 1.1 - 0.5 = \boxed{0.2\,\text{V}}$$

Now, the minimum output voltage is given by

$$V_{o,\min} = V_{D,1} + V_{OV} = \left(V_{G,2} - V_{GS,2}\right) + V_{OV} \quad \text{(I)}$$

Noting that

$$V_{GS,1} = V_{GS,2} = V_{SG,3} = V_{SG,4} = V_{OV} + |V_t| = 0.2 + 0.5 = 0.7\text{ V}$$

we can substitute in (I) to obtain

$$V_{o,\min} = V_{G,2} - V_{GS,2} + V_{OV} = 1.0 - 0.7 + 0.2 = \underline{0.5\,\text{V}}$$

In turn, the maximum output voltage is

$$V_{o,\max} = V_{DD} - V_t = 1.8 - 0.5 = \underline{1.3\,\text{V}}$$

The allowable voltage range at the output is $0.5 \le V_o \le 1.3$ V.

P.17 ➡ *Solution*

To find the transconductance of the transistors, simply substitute the operating conditions $I_D = 0.2$ mA and $|V_{OV}| = 0.2$ V into the usual definition,

$$g_{m1} = \frac{2I_D}{V_{OV}} = \frac{2\times0.2}{0.2} = \boxed{2.0\,\text{mA/V}}$$

To establish the output resistance R_{on} of the amplifier, we first compute the transistor output resistance r_o, which is assumed to be the same for all four FETs,

$$r_o = \frac{|V_A|}{I_D} = \frac{2.0}{0.2\times10^{-3}} = 10\,\text{k}\Omega$$

Accordingly,

$$R_{on} = \left(g_{m1}r_{o1}\right)r_{o2} = \left(2.0\times10^{-3}\right)\times\left(10\times10^{3}\right)\times\left(10\times10^{3}\right) = \boxed{200\,\text{k}\Omega}$$

Likewise, the output resistance R_{op} of the current source is

$$R_{op} = (g_{m1} r_{o3}) r_{o4} = (2.0\times10^{-3})\times(10\times10^{3})\times(10\times10^{3}) = \boxed{200\,\text{k}\Omega}$$

The overall output resistance is then

$$R_o = R_{on} \,||\, R_{op} = \frac{200\times200}{200+200} = \boxed{100\,\text{k}\Omega}$$

Lastly, we compute the voltage gain realized by the cascode amp,

$$A_v = -g_{m1}\left(R_{\text{on}} \,||\, R_{\text{op}}\right) = -(2.0\times10^{-3})\times(100\times10^{3}) = \boxed{-200\text{ V/V}}$$

P.18 ➡ *Solution*

To establish the overdrive voltage of transistor operation, simply subtract the threshold voltage, $|V_t|$ = 0.4 V, from the dc component of input voltage, V_I = 0.6 V,

$$V_{OV} = V_I - V_t = 0.6 - 0.4 = \boxed{0.2\text{ V}}$$

Next, we determine the minimum output voltage on the basis of transistor Q_2; that is,

$$V_{o,\min} = V_{S,2} + V_{OV,2} \text{ (I)}$$

Here, $V_{S,2}$ is given by

$$V_{S,2} = V_{G,2} - V_{GS,2} = V_{G,2} - (V_{OV} + V_t) = V_{G,2} - V_{OV} - V_t$$

$$\therefore V_{S,2} = 0.9 - 0.2 - 0.4 = 0.3\text{ V}$$

Substituting in (I),

$$V_{o,\min} = 0.3 + 0.2 = \underline{0.5\text{ V}}$$

The *maximum* output voltage, in turn, is calculated on the basis of transistor Q_3,

$$V_{o,\max} = V_{S,3} - V_{OV,3} \text{ (II)}$$

To determine source voltage $V_{S,3}$, we write

$$V_{S,3} = V_{G,3} + V_{GS,3} = V_{G,3} + (V_{OV} + V_t)$$

$$\therefore V_{S,2} = 0.4 + 0.2 + 0.4 = 1.0\text{ V}$$

Substituting in (II),

$$V_{o,\max} = 1.0 - 0.2 = \underline{0.8\text{ V}}$$

Thus, the output voltage range is $V_o \in [0.5, 0.8]$ V.

P.19 ➡ *Solution*

Using the specified gain A_v = –280 V/V and transconductance g_{m1} = 1 mA/V, we can estimate the circuit output resistance R_o of the cascode network,

$$A_v = -g_m R_o \rightarrow R_o = -\frac{A_v}{g_{m1}}$$

$$\therefore R_o = -\frac{(-280)}{1.0\times10^{-3}} = 280\,\text{k}\Omega$$

Using R_o, we can determine the transistor output resistances r_o, which are assumed equal for the four transistors,

$$R_o = \left[(g_{o2} r_{o2}) r_{o1} \,||\, (g_{o3} r_{o3}) r_{o4}\right] = \left[(1.0\times10^{-3})\times r_o^2\right] || \left[(1.0\times10^{-3})\times r_o^2\right] = 280\times10^{3}$$

$$\therefore \frac{\left[(1.0\times10^{-3})\times r_o^2\right]\times\left[(1.0\times10^{-3})\times r_o^2\right]}{(1.0\times10^{-3})\times r_o^2 + (1.0\times10^{-3})\times r_o^2} = 280\times10^{3}$$

$$\therefore r_o = \sqrt{\frac{280\times10^{3}}{5.0\times10^{-4}}} = 23.7\,\text{k}\Omega$$

Now, recall that r_o for a FET can be expressed as

$$r_o = \frac{V_A}{I} = \frac{V_A' L}{I} \rightarrow L = \frac{r_o I}{V_A'}$$

In order to determine the channel length L, we require the bias current I,

$$g_m = \frac{2I}{V_{OV}} \rightarrow I = \frac{g_m |V_{OV}|}{2}$$

$$\therefore I = \frac{(1.0\times10^{-3})\times0.25}{2} = \boxed{0.125\,\text{mA}}$$

Thus,

$$L = \frac{r_o I}{V'_A} = \frac{(23.7\times10^3)\times(0.125\times10^{-3})}{5.0\times10^6} = \boxed{0.593\,\mu\text{m}}$$

Next, we write the usual relationship for bias current in a FET and solve for width-to-length ratio,

$$I = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 V_{OV}^2 \rightarrow \left(\frac{W}{L}\right)_1 = \frac{2I}{\mu_n C_{ox} V_{OV}^2}$$

$$\therefore \left(\frac{W}{L}\right)_1 = \frac{2\times(0.125\times10^{-3})}{(400\times10^{-6})\times0.25^2} = \boxed{10}$$

The width-to-length ratio of the NMOS labeled as 2 is the same as that of Q_1,

$$\left(\frac{W}{L}\right)_2 = \left(\frac{W}{L}\right)_1 = 10$$

The width-to-length ratio of the PMOS transistors is, noting that $\mu_p C_{ox} = 100\ \mu\text{A/V}^2$,

$$I = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_3 V_{OV}^2 \rightarrow \left(\frac{W}{L}\right)_3 = \frac{2I}{\mu_p C_{ox} V_{OV}^2}$$

$$\therefore \left(\frac{W}{L}\right)_3 = \frac{2\times(0.125\times10^{-3})}{(100\times10^{-6})\times0.25^2} = \boxed{40}$$

Finally,

$$\left(\frac{W}{L}\right)_4 = \left(\frac{W}{L}\right)_3 = 40$$

P.20 ➡ *Solution*

The information given suffices for us to compute the transconductance g_{m2},

$$g_{m2} = \sqrt{2k_p\left(\frac{W}{L}\right)I_D} = \sqrt{2\times(400\times10^{-6})\times\frac{5.4}{0.36}\times(0.2\times10^{-3})} = 1.55\,\text{mA/V}$$

Also, the device has 0.36-μm effective length, therefore $V_A = 5 \times 0.36 = 1.8$ V. We proceed to determine the transistor output resistance r_o,

$$r_o = \frac{V_A}{I_D} = \frac{1.8}{0.2\times10^{-3}} = 9\,\text{k}\Omega$$

and from there the circuit output resistance R_o,

$$R_o = (g_{m2}r_{o2})r_{o1} = (1.55\times10^{-3})\times(9.0\times10^3)\times(9.0\times10^3) = 126\,\text{k}\Omega$$

Now, setting the voltage gain to −100 V/V,

$$A_v = -g_{m1}(R_o \parallel R_L) = -100 \rightarrow R_o \parallel R_L = \frac{100}{1.55\times10^{-3}}$$

$$\therefore \frac{(126\times10^3)R_L}{(126\times10^3)+R_L} = \underbrace{\frac{100}{1.55\times10^{-3}}}_{=64.5\times10^3}$$

$$\therefore (126\times10^3)R_L = (64.5\times10^3)\times\left[(126\times10^3)+R_L\right]$$

$$\therefore (126\times10^3)R_L = 8.13\times10^9 + (64.5\times10^3)R_L$$

$$\therefore R_L = \frac{8.13\times10^9}{126\times10^3 - 64.5\times10^3} = \boxed{132\,\text{k}\Omega}$$

The gain attained will equal −100 V/V if the load resistance utilized is close to 130 kΩ. We finish by determining the voltage gain of the CS amplifier,

$$A_v = g_m r_o = \left(1.55\times10^{-3}\right)\times\left(9.0\times10^{3}\right) = \boxed{14.0\ \text{V/V}}$$

P.21 ➡ Solution

The output resistance of a BJT cascode is given by

$$R_o \approx \left(g_{m2}r_{o2}\right)\left(r_{o1} \,\|\, r_{\pi2}\right)$$

Before proceeding, we need the transconductance g_m, the transistor output resistance r_o, and the input resistance r_π. The value of g_m is

$$g_m = \frac{I}{V_T} = \frac{0.2\times10^{-3}}{0.025} = 8\ \text{mA/V}$$

The value of r_o is

$$r_o = \frac{V_A}{I} = \frac{5.0}{0.2\times10^{-3}} = 25\ \text{k}\Omega$$

The value of r_π is

$$r_\pi = \frac{\beta}{g_m} = \frac{50}{8.0\times10^{-3}} = 6.25\ \text{k}\Omega$$

Gleaning our results, the value of R_o is calculated to be

$$R_o = \left[\left(8.0\times10^{-3}\right)\times\left(25\times10^{3}\right)\right]\times\left(25\times10^{3} \,\|\, 6.25\times10^{3}\right)$$

$$\therefore R_o = 200\times\frac{\left(25\times10^{3}\right)\times\left(6.25\times10^{3}\right)}{\left(25\times10^{3}\right)+\left(6.25\times10^{3}\right)} = 1.0\times10^{6}\ \Omega = \boxed{1.0\ \text{M}\Omega}$$

P.22 ➡ Solution

Problem 22.1: The output resistance R_{on} of the amplifier is given by

$$R_{\text{on}} = \left(g_{m2}r_{o2}\right)\left(r_{o1} \,\|\, r_{\pi2}\right) \quad \text{(I)}$$

where subscripts 1 and 2 refer to the *npn* transistors in the cascode. Transconductance g_{m2} is

$$g_{m2} = \frac{I_C}{V_T} = \frac{0.2\times10^{-3}}{25\times10^{-3}} = 8.0\ \text{mA/V}$$

The transistor output resistance $r_{o1} = r_{o2}$ is

$$r_{o1} = r_{o2} = \frac{V_A}{I_C} = \frac{5.0}{0.2\times10^{-3}} = 25\ \text{k}\Omega$$

Input resistance $r_{\pi2}$ is

$$r_{\pi2} = \frac{\beta_2}{g_{m2}} = \frac{100}{8.0\times10^{-3}} = 12.5\ \text{k}\Omega$$

Substituting in (I), we get

$$R_{\text{on}} = \left[\left(8.0\times10^{-3}\right)\times\left(25\times10^{3}\right)\right]\times\left[\left(25\times10^{3}\right) \,\|\, \left(12.5\times10^{3}\right)\right]$$

$$\therefore R_{\text{on}} = 200\times\frac{\left(25\times10^{3}\right)\times\left(12.5\times10^{3}\right)}{\left(25\times10^{3}\right)+\left(12.5\times10^{3}\right)} = \boxed{1.67\ \text{M}\Omega}$$

The output resistance R_{op} of the current source is stated by the similar formula

$$R_{\text{op}} = \left(g_{m3}r_{o3}\right)\left(r_{o4} \,\|\, r_{\pi3}\right) \quad \text{(II)}$$

Here, transconductance g_{m3} is

$$g_{m3} = \frac{I_C}{V_T} = \frac{0.2\times10^{-3}}{25\times10^{-3}} = 8.0\ \text{mA/V}$$

The transistor output resistance $r_{o3} = r_{o4}$ is

$$r_{o3} = r_{o4} = \frac{V_A}{I_C} = \frac{4.0}{0.2\times10^{-3}} = 20\ \text{k}\Omega$$

Input resistance $r_{\pi3}$ is

$$r_{\pi 3}=\frac{\beta_3}{g_{m3}}=\frac{50}{8.0\times 10^{-3}}=6.25\,\text{k}\Omega$$

Substituting in (II), we get

$$R_{\text{op}}=\left[\left(8.0\times 10^{-3}\right)\times\left(20\times 10^3\right)\right]\times\left[\left(20\times 10^3\right)\|\left(6.25\times 10^3\right)\right]$$

$$\therefore R_{\text{op}}=200\times\frac{\left(20\times 10^3\right)\times\left(6.25\times 10^3\right)}{\left(20\times 10^3\right)+\left(6.25\times 10^3\right)}=\boxed{762\,\text{k}\Omega}$$

We proceed to compute voltage gain A_v,

$$A_v=-g_{m1}\left(R_{\text{on}}\|R_{\text{op}}\right)=-8.0\times\left(1670\|762\right)$$

$$\therefore A_v=-8.0\times\frac{1670\times 762}{1670+762}=\boxed{-4190\text{ V/V}}$$

Problem 22.2: To show the relationship posited in the problem statement, we write, using the definitions of R_{on} and R_{op} for a BJT cascode amp,

$$A_v=-g_{m1}\left(R_{\text{on}}\|R_{\text{op}}\right)=-g_{m1}\left\{\left[\left(g_{m2}r_{o2}\right)\left(r_{o1}\|r_{\pi 2}\right)\right]\|\left[\left(g_{m3}r_{o3}\right)\left(r_{o4}\|r_{\pi 3}\right)\right]\right\}$$

Here, $r_{o4}||r_{\pi 3}\to r_{\pi 3}$ because $r_{o4}\gg r_{\pi 3}$ and $r_{o1}||r_{\pi 2}\to r_{\pi 2}$ because $r_{o1}\gg r_{\pi 2}$, giving

$$A_v=-g_{m1}\left\{\left[\left(g_{m2}r_{o2}\right)r_{\pi 2}\right]\|\left[\left(g_{m3}r_{o3}\right)r_{\pi 3}\right]\right\}$$

$$\therefore A_v=-g_{m1}\left\{\left[\left(g_{m2}r_{\pi 2}\right)r_{o2}\right]\|\left[\left(g_{m3}r_{\pi 3}\right)r_{o3}\right]\right\}$$

Note that the products in parentheses $g_{m2}r_{\pi 2}=\beta_2$ and $g_{m3}r_{\pi 3}=\beta_3$, hence

$$\left|A_{v,\max}\right|=-g_{m1}\left(\beta_2 r_{o2}\|\beta_3 r_{o3}\right)$$

as we intended to show.

In the case at hand, $g_{m1}=g_{m2}=8.0$ mA/V, $\beta_2=100$, $r_{o2}=25$ kΩ, $\beta_3=50$, and $r_{o3}=20$ kΩ, so that

$$\left|A_{v,\max}\right|=-\left(8.0\times 10^{-3}\right)\times\left\{\left[100\times\left(25\times 10^3\right)\right]\|\left[50\times\left(20\times 10^3\right)\right]\right\}$$

$$\therefore\left|A_{v,\max}\right|=-\left(8.0\times 10^{-3}\right)\times\left[\left(2.5\times 10^6\right)\|\left(1.0\times 10^6\right)\right]$$

$$\therefore\left|A_{v,\max}\right|=-\left(8.0\times 10^{-3}\right)\times\frac{\left(2.5\times 10^6\right)\times\left(1.0\times 10^6\right)}{\left(2.5\times 10^6\right)+\left(1.0\times 10^6\right)}=\boxed{-5710\text{ V/V}}$$

P.23 ➡ *Solution*

The voltage gain of a BJT cascode is given by

$$A_v=-g_m\left(R_{\text{on}}\|R_{\text{op}}\right)\quad\text{(I)}$$

Here, the output resistance R_{on} of the amplifier and the output resistance R_{op} of the current source are given by

$$R_{\text{on}}=\left(g_{m2}r_{o2}\right)\left(r_{o1}\|r_{\pi 2}\right)=\left(g_m r_o\right)\left(r_o\|r_\pi\right)$$

$$R_{\text{op}}=\left(g_{m3}r_{o3}\right)\left(r_{o4}\|r_{\pi 3}\right)=\left(g_m r_o\right)\left(r_o\|r_\pi\right)$$

$$\therefore R_{\text{on}}=R_{\text{op}}=\left(g_m r_o\right)\times\frac{r_o\times r_\pi}{r_o+r_\pi}=\frac{I_C}{V_T}\times\frac{|V_A|}{I_C}\times\frac{\frac{|V_A|}{I_C}\times\frac{\beta V_T}{I_C}}{\frac{|V_A|}{I_C}+\frac{\beta V_T}{I_C}}$$

$$\therefore R_{\text{on}}=R_{\text{op}}=\frac{|V_A|}{\cancel{V_T}}\times\frac{|V_A|\beta\cancel{V_T}}{I_C\left(|V_A|+\beta V_T\right)}=\frac{\beta|V_A|^2}{I_C\left(|V_A|+\beta V_T\right)}$$

$$\therefore R_{\text{on}}=R_{\text{op}}=\frac{\beta|V_A|^2}{I_C\beta|V_A|\left(\frac{1}{\beta}+\frac{V_T}{|V_A|}\right)}=\frac{|V_A|}{I_C\left(\frac{V_T}{|V_A|}+\frac{1}{\beta}\right)}$$

Substituting in (I), we get

$$A_v = -g_m \left(R_{\text{on}} \,||\, R_{\text{op}}\right) = -\frac{I_C}{V_T} \times \left\{ \left[\frac{|V_A|}{I_C\left(\frac{V_T}{|V_A|} + \frac{1}{\beta}\right)} \right] \,||\, \left[\frac{|V_A|}{I_C\left(\frac{V_T}{|V_A|} + \frac{1}{\beta}\right)} \right] \right\}$$

$$\therefore A_v = -\frac{\cancel{I_C}}{V_T} \times \frac{|V_A|}{2\cancel{I_C}\left(\frac{V_T}{|V_A|} + \frac{1}{\beta}\right)} = \boxed{-\frac{1}{2}\frac{|V_A|/V_T}{\left(V_T/|V_A|\right) + 1/\beta}}$$

Substituting $|V_A|$ = 5 V and β = 50 brings to

$$\therefore A_v = -\frac{1}{2}\frac{5.0/0.025}{\left(0.025/5.0\right) + 1/50} = \boxed{-4000 \text{ V/V}}$$

REFERENCE

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