Quiz EL101

Transmission Line Parameters

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PROBLEMS

Problem 1

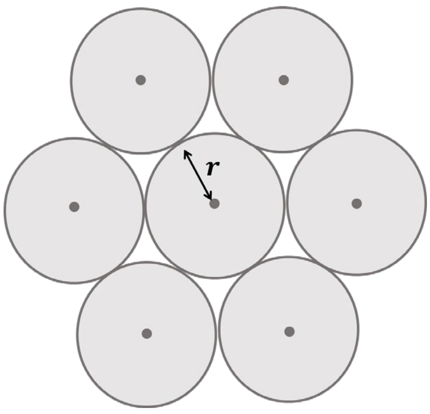
The temperature dependence of resistance can be quantified by the relation

$$R_2 = R_1\left[1+\alpha\left(T_2 - T_1\right)\right]$$

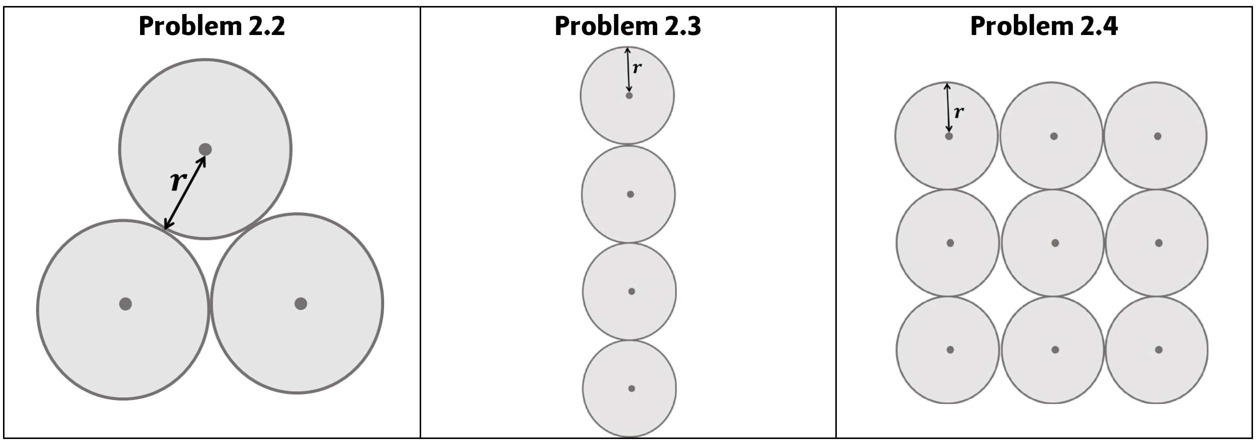
where R_1 and R_2 are the resistances at temperatures at temperatures T_1 and T_2, respectively, and α is known as the temperature coefficient of resistance. If a copper wire has a resistance of 60 Ω at 20°C, find the maximum permissible operating temperature of the wire if its resistance is to increase by at most 18%. Take the temperature coefficient at 20°C to be α = 0.00382 °C/°C.

Problem 2 (Glover *et al.*, 2017, w/ permission)

Problem 2.1: Find the geometric mean radius (GMR) of a stranded conductor consisting of six outer strands surrounding and touching one central strand, all strands having the same radius *r*.

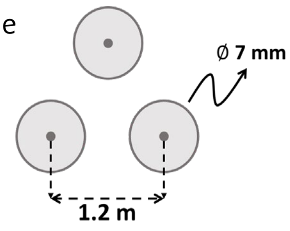


Problems 2.2, 2.3, and 2.4: Find the geometric mean radii (GMR) of the following unconventional stranded conductors. All strands have the same radius *r*.



Problem 3

Problem 3.1: A 60-Hz, three-phase three-wire overhead line has solid cylindrical conductors arranged in the form of an equilateral triangle with 1.2 m spacing. The conductor diameter is 7 mm. Calculate the positive-sequence inductance in H/m and the positive-sequence reactance in Ω/km.



Problem 3.2: Rework the previous problem if the phase spacing is (*a*) increased by 25% to 1.5 m; or (*b*) decreased by 25% to 0.9 m.

Problem 4

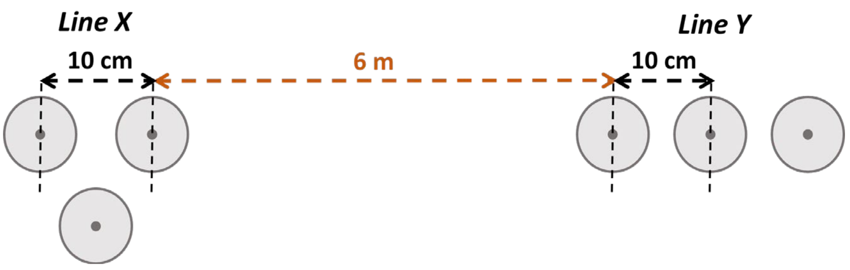
A 230-kV, 60-Hz, three-phase completely transposed overhead line has one 954 kcmil per phase and flat horizontal phase spacing, with 6 m between adjacent conductors. Determine the inductance in H/m and the inductive reactance in Ω/km.

Problem 5

Calculate the inductive reactance in Ω/km of a bundled 500-kV, 60-Hz, three-phase completely transposed overhead line having three 1113 kcmil conductors per bundle, with 0.6 m between conductors in the bundle. The horizontal phase spacings between bundle centers are 8, 8, and 16 m.

Problem 6 (Glover *et al.*, 2017, w/ permission)

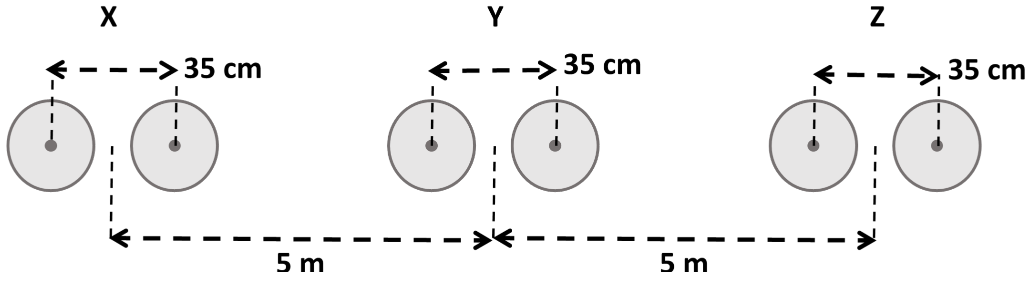
The conductor configuration of a bundled single-phase overhead transmission line is shown in the figure below. Line X has its three conductors situated at the corners of an equilateral triangle with 10 cm spacing. Line Y has its three conductors arranged in a horizontal configuration with 10 cm spacing. All conductors are identical, solid-cylindrical conductors each with a radius of 2 cm. Find the equivalent representation in terms of the geometric mean radius of each bundle and a separation that is the geometric mean distance. True or false?



1.( ) The GMR of line X is greater than 5 cm.
2.( ) The GMR of line Y is greater than 6.5 cm.
3.( ) The GMD between lines X and Y is greater than 6.2 m.

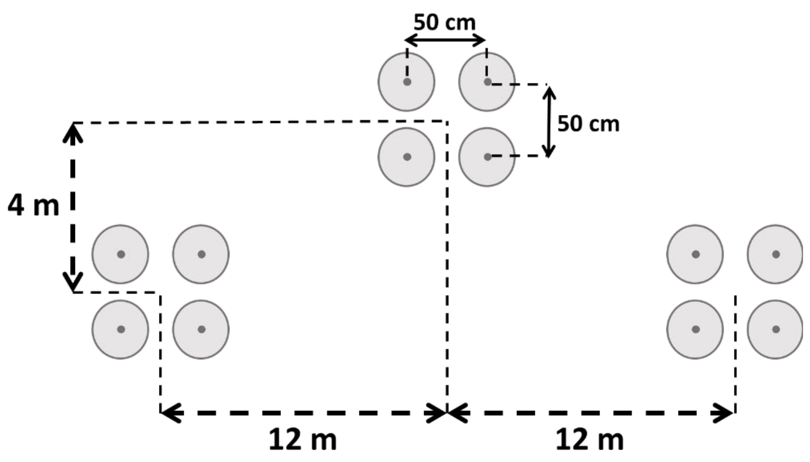
Problem 7 (Glover *et al.*, 2017, w/ permission)

The following figure shows the conductor configuration of a completely transposed three-phase overhead transmission line with bundled phase conductors. All conductors have a radius of 0.75 cm with a 35-cm bundle spacing. Determine the inductance per phase in mH/km and the inductive line reactance per phase in Ω/km at 60 Hz.



Problem 8 (Glover *et al.*, 2017, w/ permission)

For the overhead line of configuration shown below operating at 60 Hz, determine the inductive reactance in ohms/mile/phase. All subconductors have 1.3-cm OD. Note that, for a 4-subconductor square-shaped bundle, $GMR = 1.091\sqrt[4]{D_s d^3}$, where D_s is the radius of one subconductor and d is the distance between subconductors.



Problem 9 (Glover *et al.*, 2017, w/ permission)

Problem 9.1: The following figure shows double-circuit conductors' relative positions in segment 1 of transposition of transposition of a completely transposed three-phase overhead transmission line. The inductance is given by

$$L = 2\times10^{-7}\ln\frac{GMD}{GMR}\ \text{H/m/phase}$$

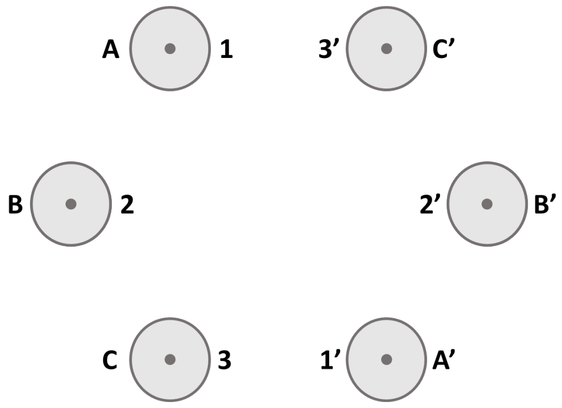
where $GMD = (D_{AB_{eq}}D_{BC_{eq}}D_{AC_{eq}})^{1/3}$, with mean distances defined by equivalent spacings

$$D_{AB_{eq}} = \left(D_{12}D_{1'2'}D_{12'}D_{1'2}\right)^{1/4}$$
$$D_{BC_{eq}} = \left(D_{23}D_{2'3'}D_{2'3}D_{23'}\right)^{1/4}$$
$$D_{AC_{eq}} = \left(D_{13}D_{1'3'}D_{13'}D_{1'3}\right)^{1/4}$$

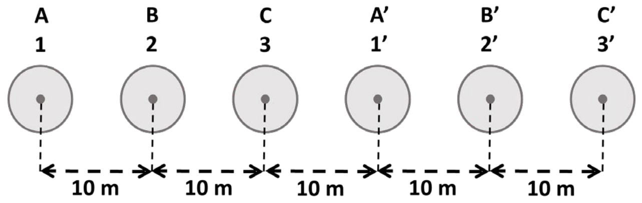
and $GMR = [(GMR)_A(GMR)_B(GMR)_C]^{1/3}$, with phase GMRs defined by

$$(GMR)_A = \left(r'D_{11'}\right)^{1/2}\ ;\ (GMR)_B = \left(r'D_{22'}\right)^{1/2}\ ;\ (GMR)_C = \left(r'D_{33'}\right)^{1/2}$$

and r' is the GMR of phase conductors.



Now consider a 350-kV, three-phase, double-circuit line with phase conductor's GMR of 1.8 cm and the horizontal conductor configuration shown below. Determine the inductance per meter per phase in henries (H).

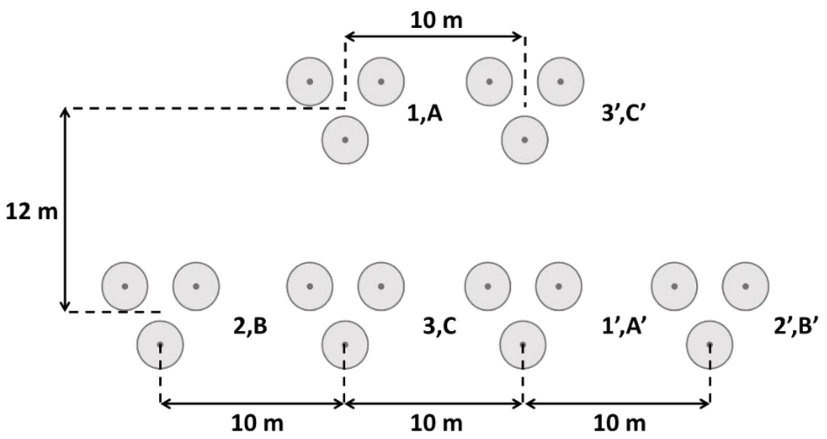


Problem 9.2: Calculate the inductance of just one circuit and then divide by 2 to obtain the inductance of the double circuit. Compare this result with the one in the previous part.

Problem 10 (Glover *et al.*, 2017, w/ permission)

Now consider a double-circuit configuration shown in the following figure. The configuration belongs to a 500-kV, three-phase line with bundle conductors of three subconductors at 50 cm spacing. The GMR of each subconductor is given to be 1.5 cm. Determine the inductive reactance of the line in ohms per kilometer per phase. You may use

$$X_L = 0.1736\log_{10}\frac{GMD}{GMR}\ \Omega/\text{km/phase}$$



Problem 11

Calculate the capacitance-to-neutral in F/m and the admittance-to-neutral in S/km for the three-phase line in Problem 4. Also calculate the line-charging current in kA/phase if the line is 110 km in length and is operated at 230 kV. Neglect the effect of the earth plane. True or false?

1.( ) The capacitance-to-neutral is greater than 9.2×10^{-12} F/m.
2.( ) The admittance-to-neutral is greater than $j3.6\times10^{-6}$ S/km.
3.( ) The line-charging current is greater than 0.04 kA/phase.

Problem 12

Calculate the capacitance-to-neutral in F/m and the admittance-to-neutral in S/km of the line introduced in Problem 5. Also determine the total reactive power in Mvar/km supplied by the line capacitance when it is operated at 500 kV. True or false?

1.( ) The capacitance-to-neutral is greater than 7.8×10^{-12} F/m.
2.( ) The admittance-to-neutral is greater than $j3\times10^{-6}$ S/km.
3.( ) The total reactive power is greater than 0.83 Mvar/km.

Problem 13 (Stevenson Jr. and Grainger, 1994)

A three-phase 60-Hz transmission line has its conductors arranged in a triangular formation so that two of the distances between conductors are 7.5 m and the third is 13 m. The conductors are ACSR with 2.25 cm OD. Determine the capacitance to neutral (μF/km) and the capacitive reactance (Ω · km) of the line.

Problem 14 (Stevenson Jr. and Grainger, 1994)

The 60-Hz capacitive reactance to neutral of a solid conductor, which is one conductor of a single-phase line with 1.5 m spacing, is 280 kΩ-km. What value of reactance, in kΩ · mi, would be specified in a table listing the capacitive reactance in ohm-miles to neutral of the conductor at 2 ft spacing for 25 Hz? What is the cross-sectional area of the conductor in circular mils?

Problem 15 (Stevenson Jr. and Grainger, 1994)

Three ACSR conductors of 28-mm OD are used for a three-phase overhead transmission line operating at 60 Hz. The conductor configuration is in the form of an isosceles triangle with sides of 6 m, 6 m, and 11 m. Calculate the capacitance-to-neutral for each 1-kilometer length of line. Next, determine the capacitive reactance-to-neutral for a line length of 75 mi. Also determine the charging current per mile for a normal operating voltage of 220 kV. Lastly, determine the three-phase reactive power supplied by the line capacitance. True or false?

1.( ) The capacitance-to-neutral of the line is greater than 8.5 nF/km.
2.( ) The capacitive reactance-to-neutral of the line for a 75-mi line is greater than 2200 Ω.
3.( ) The charging current is greater than 0.72 A/mi.
4.( ) The three-phase reactive power for a 75-mi line is greater than 21 Mvar.

SOLUTIONS

P.1 ➡ *Solution*

Since the resistance of the wire is to increase 18% at most, $R_2 = 1.18R_1$. Further, $\alpha = 0.00382$ and $T_1 = 20$°C. Accordingly,

$$R_2 = R_1\left[1+\alpha\left(T_2-T_1\right)\right] \rightarrow 1.18R_1 = R_1\left[1+0.00382\times\left(T_{\max}-20\right)\right]$$

$$\therefore 1.18 = 1+0.00382\times\left(T_{\max}-20\right)$$

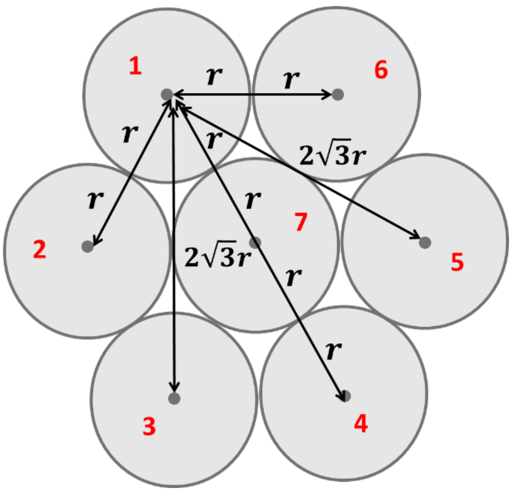
$$\therefore 0.18 = 0.0382T_{\max} - 0.0764$$

$$\therefore T_{\max} = \frac{0.18+0.0764}{0.0382} = \boxed{67.1^{\circ}\text{C}}$$

The wire is to operate at a temperature no greater than 67.1°C.

P.2 ➡ *Solution*

Problem 2.1: Refer to the illustration on the next page.



From the figure, it is easy to see that, for conductors 1 – 6,

$$D_{11} = r' = e^{-\frac{1}{4}} r = 0.779r$$
$$D_{12} = D_{16} = D_{17} = 2r$$
$$D_{13} = D_{15} = 2\sqrt{3}r$$
$$D_{14} = 4r$$

For conductor 7,

$$D_{77} = 0.779r$$
$$D_{71} = D_{72} = D_{73} = D_{74} = D_{75} = D_{76} = 2r$$

The GMR is then

$$GMR = \sqrt[49]{\left(D_{11}D_{12}D_{13}D_{14}D_{15}D_{16}D_{17}\right)^6 \left(D_{71}D_{72}D_{73}D_{74}D_{75}D_{76}D_{77}\right)}$$

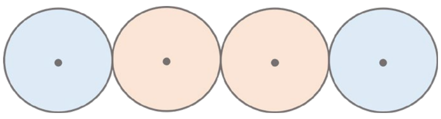
$$\therefore GMR = \sqrt[49]{\left(0.779r \times 2r \times 2\sqrt{3}r \times 4r \times 2\sqrt{3}r \times 2r \times 2r\right)^6 \times \left(2r \times 2r \times 2r \times 2r \times 2r \times 2r \times 0.779r\right)}$$

$$\therefore \boxed{GMR = 2.18r}$$

Problem 2.2: This is a simplified version of the stranded conductor investigated in the previous problem. The GMR is

$$GMR = \sqrt[9]{\left[\left(e^{-1/4}r\right)(2r)(2r)\right] \times \left[\left(e^{-1/4}r\right)(2r)(2r)\right] \times \left[\left(e^{-1/4}r\right)(2r)(2r)\right]} = \boxed{1.46r}$$

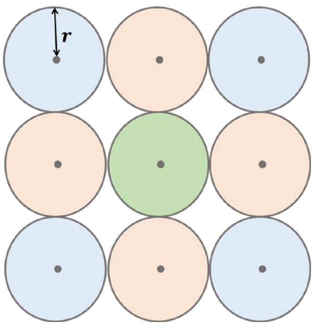
Problem 2.3: In this configuration, there are two "types" of strands, which we highlight below in blue and red.



The GMR of the conductor is given by

$$GMR = \sqrt[16]{\left[\left(e^{-1/4}r\right)(2r)(4r)(6r)\right]^2 \times \left[\left(e^{-1/4}r\right)(2r)(2r)(4r)\right]^2} = \boxed{2.16r}$$

Problem 2.4: In this particular configuration, there are three "types" of strands, which we highlight below in blue, red and green.



The GMR is given by

$$GMR = \sqrt{\begin{array}{c}\left[(0.779r)(2r)^2(4r)^2\left(\sqrt{20}r\right)^2\left(\sqrt{8}r\right)\left(\sqrt{32}r\right)\right]^4 \\ \times\left[(0.779r)(2r)^3\left(\sqrt{8}r\right)^2\left(\sqrt{20}r\right)^2(4r)\right]^4 \times\left[(0.779r)(2r)^4\left(\sqrt{8}r\right)^4\right]\end{array}}$$

$$\therefore \boxed{GMR = 2.64r}$$

P.3 ➡ Solution

Problem 3.1: The positive-sequence inductance is given by

$$L_1 = 2\times10^{-7}\ln\left(\frac{D_1}{r'}\right)\quad \text{H/m}$$

where D_1 = 1.2 m is the distance that separates the conductor centroids and r' is a parameter that depends on conductor diameter according to

$$r' = \frac{e^{-\frac{1}{4}}}{2}d = \frac{e^{-\frac{1}{4}}}{2}\times 0.007 = 0.00273\,\text{m}$$

so that

$$L_1 = 2\times10^{-7}\ln\left(\frac{1.2}{0.00273}\right) = \boxed{1.22\times10^{-6}\ \text{H/m}}$$

Next, for a 60-Hz line, the positive-sequence reactance is given by

$$X_1 = \omega L_1 = (2\pi\times60)\times\left(1.22\times10^{-6}\right) = 4.60\times10^{-4}\ \Omega/\text{m}$$

$$\therefore \boxed{X_1 = 0.460\ \Omega/\text{km}}$$

Problem 3.2: With a spacing D_2 = 1.5 m, the positive-sequence inductance now becomes

$$L_2 = 2\times10^{-7}\ln\left(\frac{D_2}{r'}\right) = 2\times10^{-7}\ln\left(\frac{1.5}{0.00273}\right) = \boxed{1.26\times10^{-6}\ \text{H/m}}$$

which represents an increase of 3.3% relatively to L_1. The updated positive-sequence reactance is

$$X_2 = \omega L_2 = (2\pi\times60)\times\left(1.26\times10^{-7}\right) = 4.75\times10^{-4}\ \Omega/\text{m}$$

$$\therefore \boxed{X_2 = 0.475\ \Omega/\text{km}}$$

Now, decreasing phase spacing to 0.9 m will lower the positive-sequence inductance to L_3 = 1.16×10^{-6} H/m (a 4.9% decrease relatively to L_1) and decrease the positive-sequence reactance to X_3 = 0.437 Ω/km.

P.4 ➡ Solution

A 954-kcmil conductor has GMR equal to 24.8 mm/2 = 12.4 mm (Use Convert-me's converter at https://www.convert-me.com/en/convert/wire_gauge/wireareakcmil.html?u=wireareakcmil&v=954). The geometric mean distance between phases is

$$D_{eq} = \sqrt[3]{D_{12}D_{23}D_{13}} = \sqrt[3]{6\times6\times12} = 7.56\,\text{m}$$

We proceed to determine the inductance L,

$$L = 2\times10^{-7}\ln\left(\frac{D_{eq}}{D_s}\right) = 2\times10^{-7}\ln\left(\frac{7.56}{0.0124}\right) = \boxed{1.28\times10^{-6}\ \text{H/m}}$$

and the reactance X_L,

$$X_L = 2\pi fL = 2\pi\times60\times\left(1.28\times10^{-6}\right) = 4.83\times10^{-4}\ \Omega/\text{m}$$

$$\therefore \boxed{X_L = 0.483\,\Omega/\text{km}}$$

P.5 ➡ Solution

A 1113-kcmil conductor has GMR equal to 26.8 mm/2 = 13.4 mm. The geometric mean distance between phases is

$$D_{eq} = \sqrt[3]{D_{12}D_{23}D_{13}} = \sqrt[3]{8\times8\times16} = 10.1\,\text{m}$$

The inductive reactance is calculated to be

$$X_L = \omega L_1 = (2\pi\times60)\times\left(2\times10^{-7}\ln\frac{10.1}{0.0134}\right) = 5.0\times10^{-4}\,\Omega/\text{m}$$

$$\therefore \boxed{X_L = 0.500\ \Omega/\text{km}}$$

P.6 ➡ Solution

Conductor X has N = 3 identical subconductors, each with radius r_X = 2 cm. Likewise, conductor Y has M = 3 identical subconductors, each with radius r_Y = 2 cm. Let D_{XX} denote the geometric mean radius of conductor X, D_{YY} denote the geometric mean radius of conductor Y, and D_{XY} denote the geometric mean distance between conductors X and Y. Firstly, the value of D_{XX} is

$$D_{XX} = \sqrt[N^2]{\prod_{k=1}^{N}\prod_{m=1}^{N} D_{km}} = \sqrt[9]{\prod_{k=1}^{3}\prod_{m=1}^{3} D_{km}} = \sqrt[9]{\prod_{k=1}^{3} D_{k1}D_{k2}D_{k3}}$$

$$\therefore D_{XX} = \sqrt[9]{(D_{11}D_{12}D_{13})(D_{21}D_{22}D_{23})(D_{31}D_{32}D_{33})}$$

$$\therefore D_{XX} = \sqrt[9]{(0.778r_X \times 0.1 \times 0.1)\times(0.1\times 0.778 r_X \times 0.1)\times(0.1\times 0.1\times 0.778 r_X)}$$

$$\therefore D_{XX} = \sqrt[9]{\left[(0.778\times 0.02)\times 0.1\times 0.1\right]\times\left[0.1\times(0.778\times 0.02)\times 0.1\right]\times\left[0.1\times 0.1\times(0.778\times 0.02)\right]}$$

$$\therefore D_{XX} = 0.0538\ \text{m} = \boxed{5.38\ \text{cm}}$$

The value of D_{YY} is determined next,

$$D_{YY} = \sqrt[M^2]{\prod_{k=1'}^{M}\prod_{m=1'}^{M} D_{km}} = \sqrt[9]{\prod_{k=1'}^{3'}\prod_{m=1'}^{3'} D_{km}} = \sqrt[9]{\prod_{k=1'}^{3'} D_{k1'}D_{k2'}D_{k3'}}$$

$$\therefore D_{YY} = \sqrt[9]{(D_{1'1'}D_{1'2'}D_{1'3'})(D_{2'1'}D_{2'2'}D_{2'3'})(D_{3'1'}D_{3'2'}D_{3'3'})}$$

$$\therefore D_{YY} = \sqrt[9]{\left[(0.778\times 0.02)\times 0.1\times 0.2\right]\times\left[0.1\times(0.778\times 0.02)\times 0.1\right]\times\left[0.2\times 0.1\times(0.778\times 0.02)\right]}$$

$$\therefore D_{YY} = 0.0627\ \text{m} = \boxed{6.27\ \text{cm}}$$

We proceed to determine the GMD between conductors X and Y,

$$D_{XY} = \sqrt[NM]{\prod_{k=1}^{N}\prod_{m=1'}^{M} D_{km}} = \sqrt[3\times 3]{\prod_{k=1}^{N} D_{k1'}D_{k2'}D_{k3'}}$$

$$\therefore D_{XY} = \sqrt[9]{(D_{11'}D_{12'}D_{13'})(D_{21'}D_{22'}D_{23'})(D_{31'}D_{32'}D_{33'})}$$

Distances $D_{31'}$,$D_{32'}$ and $D_{33'}$ are somewhat trickier to visualize but, due to the tight spacing between subconductors, turn out to yield elementary results:

$$D_{31'} = \sqrt{(0.1^2 - 0.05^2)^2 + 6.05^2} \approx 6.05\ \text{m}$$

$$D_{32'} = \sqrt{(0.1^2 - 0.05^2)^2 + 6.15^2} \approx 6.15\ \text{m}$$

$$D_{33'} = \sqrt{(0.1^2 - 0.05^2)^2 + 6.25^2} \approx 6.25\ \text{m}$$

Finally,

$$D_{XY} = \sqrt[9]{(6.1\times 6.2\times 6.3)\times(6\times 6.1\times 6.2)\times(6.05\times 6.15\times 6.25)} = \boxed{6.15\ \text{m}}$$

▶ Statement **1** is **true**, while statements **2** and **3** are **false**.

P.7 ➡ Solution

The geometric mean radius of each phase is calculated as

$$R = \sqrt[4]{(r')^2 (0.35)^2} = \sqrt[4]{(0.778\times 0.0075)^2 \times 0.35^2} = 0.0452\ \text{m}$$

The geometric mean distance between the conductors of phases X and Y is given by

$$D_{XY} = \sqrt[4]{5^2 \times 4.65 \times 5.35} = 4.99\ \text{m}$$

Similarly,

$$D_{YZ} = \sqrt[4]{5^2 \times 4.65 \times 5.35} = 4.99\,\text{m}$$

and

$$D_{XZ} = \sqrt[4]{10^2 \times 10.35 \times 9.65} \approx 10\,\text{m}$$

The GMD between phases is given by the cube root of the product of the three-phase spacings,

$$D_{eq} = \sqrt[3]{D_{XY}D_{YZ}D_{XZ}} = \sqrt[3]{4.99 \times 4.99 \times 10} = 6.29\,\text{m}$$

The inductance per phase is found as

$$L = 2\times10^{-7}\ln\left(\frac{6.29}{0.0452}\right)\times1000\times1000 = \boxed{0.987\,\text{mH/km}}$$

The line reactance for each phase is, in turn,

$$X = 2\pi fL = 2\pi\times60\times\left(0.987\times10^{-3}\right) = \boxed{0.372\,\Omega/\text{km}}$$

P.8 ➡ Solution

Using the formula provided, the GMR of one bundle is given by

$$GMR = 1.091\sqrt[4]{D_s d^3} = 1.091\sqrt[4]{0.013\times0.5^3} = 0.219\,\text{m}$$

Next, the GMD of the system is found as

$$GMD = \sqrt[3]{\sqrt{12^2+4^2}\times24\times\sqrt{12^2+4^2}} = 15.7\,\text{m}$$

We proceed to determine the per-phase inductance of the overhead transmission line,

$$L = 2\times10^{-7}\ln\frac{GMD}{GMR} = 2\times10^{-7}\ln\frac{15.7}{0.219} = 8.54\times10^{-7}\ \text{H/m}$$

Lastly, the inductive reactance is calculated to be

$$X_L = 2\pi f\times L$$

$$\therefore X_L = 2\pi\times60\times\left(8.54\times10^{-7}\right) = 3.22\times10^{-4}\ \Omega/\text{m/phase}$$

$$\therefore \boxed{X_L = 0.518\ \Omega/\text{mi/phase}}$$

P.9 ➡ Solution

Problem 9.1: The solution is started by determining $D_{AB_{eq}}$ between conductors A and B.

$$D_{AB_{eq}} = \sqrt[4]{D_{12}D_{1'2'}D_{12'}D_{1'2}} = \sqrt[4]{10\times10\times40\times20} = 16.8\,\text{m}$$

$D_{BC_{eq}}$ is determined next,

$$D_{BC_{eq}} = \sqrt[4]{D_{23}D_{2'3'}D_{2'3}D_{23'}} = \sqrt[4]{10\times10\times20\times40} = 16.8\,\text{m}$$

Then, $D_{AC_{eq}}$,

$$D_{AC_{eq}} = \sqrt[4]{D_{13}D_{1'3'}D_{13'}D_{1'3}} = \sqrt[4]{20\times20\times50\times10} = 21.1\,\text{m}$$

It follows that

$$GMD = \sqrt[3]{D_{AB_{eq}}D_{BC_{eq}}D_{AC_{eq}}} = \sqrt[3]{16.8\times16.8\times21.1} = 18.1\,\text{m}$$

Noting that $r' = 0.778 \times 1.8 = 1.40$ cm = 0.0014 m, we compute

$$(GMR)_A = \sqrt{r'D_{11'}} = \sqrt{0.0014\times30} = 0.205\,\text{m}$$

$$(GMR)_B = \sqrt{r'D_{22'}} = \sqrt{0.0014\times30} = 0.205\,\text{m}$$

$$(GMR)_C = \sqrt{r'D_{33'}} = \sqrt{0.0014\times30} = 0.205\,\text{m}$$

so that

$$GMR = \sqrt[3]{(GMR)_A(GMR)_B(GMR)_C} = \sqrt[3]{0.205\times0.205\times0.205} = 0.205\,\text{m}$$

The total inductance per meter per phase is

$$L = 2\times10^{-7}\ln\frac{GMD}{GMR} = 2\times10^{-7}\ln\left(\frac{18.1}{0.205}\right) = \boxed{8.961\times10^{-7}\text{ H/m}}$$

Problem 9.2: To compute the inductance of just one circuit, we first establish the equivalent GMD D_{eq},

$$D_{eq} = \sqrt[3]{D_{12}D_{23}D_{31}} = \sqrt[3]{20\times20\times40} = 25.2\,\text{m}$$

Further, we have the GMR $D_S = r' = 0.0014$ m, so that

$$L = 2\times10^{-7}\ln\frac{D_{eq}}{D_S} = 2\times10^{-7}\ln\frac{25.2}{0.0014} = 1.960\times10^{-6}\text{ H/m}$$

Dividing this result by two gives the inductance of the double circuit,

$$\text{Inductance of the double circuit} = \frac{1.960\times10^{-6}}{2} = \boxed{9.798\times10^{-7}\text{ H/m}}$$

This approach overestimates the inductance per meter by 9.3% relatively to the previous result.

P.10 ➡ Solution

The GMR of each subconductor is 1.5 cm, and the spacing between subconductors is 50 cm. Accordingly, the bundle GMR is

$$\text{Bundle}\,GMR = \sqrt[3]{0.015\times0.5^2} = 0.155\,\text{m}$$

Now, the overall phase GMR is given by

$$GMR = \sqrt[3]{(GMR)_A(GMR)_B(GMR)_C}$$

Here,

$$(GMR)_A = \sqrt{r'D_{11'}} = \sqrt{0.155\times\sqrt{10^2+12^2}} = 1.56\,\text{m}$$

$$(GMR)_B = \sqrt{r'D_{22'}} = \sqrt{0.155\times30} = 2.16\,\text{m}$$

$$(GMR)_C = \sqrt{r'D_{33'}} = \sqrt{0.155\times\sqrt{10^2+12^2}} = 1.56\,\text{m}$$

so that

$$GMR = \sqrt[3]{1.56\times2.16\times1.56} = 1.74\text{ m}$$

To determine the GMD, we use

$$GMD = \sqrt[3]{D_{AB_{eq}}D_{BC_{eq}}D_{AC_{eq}}}$$

Here,

$$D_{AB_{eq}} = \sqrt[4]{D_{12}D_{1'2'}D_{12'}D_{1'2}}$$

$$\therefore D_{AB_{eq}} = \sqrt[4]{10\times\sqrt{10^2+12^2}\times\sqrt{20^2+12^2}\times20} = 16.4\,\text{m}$$

$$D_{BC_{eq}} = \sqrt[4]{D_{23}D_{2'3'}D_{2'3}D_{23'}}$$

$$\therefore D_{BC_{eq}} = \sqrt[4]{10\times\sqrt{10^2+12^2}\times20\times\sqrt{20^2+12^2}} = 16.4\,\text{m}$$

$$D_{AC_{eq}} = \sqrt[4]{D_{13}D_{1'3'}D_{13'}D_{1'3}}$$

$$\therefore D_{AC_{eq}} = \sqrt[4]{12\times12\times10\times10} = 11.0\,\text{m}$$

giving

$$GMD = \sqrt[3]{D_{AB_{eq}}D_{BC_{eq}}D_{AC_{eq}}} = \sqrt[3]{16.4\times16.4\times11.0} = 14.4\,\text{m}$$

It remains to determine the inductive reactance X_L,

$$X_L = 0.1736\log_{10}\frac{GMD}{GMR} = 0.1736\log_{10}\frac{14.4}{1.74} = \boxed{0.159\ \Omega/\text{km/phase}}$$

P.11 ➡ Solution

In Problem 4, the *GMR* and *GMD* for this system were calculated to be 12.4 mm and 7.56 m, respectively. The capacitance-to-neutral is then

$$C_1 = \frac{2\pi\varepsilon_0}{\ln(D/r)} = \frac{2\pi\times(8.85\times10^{-12})}{\ln(7.56/0.0124)} = \boxed{8.67\times10^{-12}\ \text{F/m}}$$

The admittance-to-neutral, in turn, is

$$\overline{Y}_1 = j\omega C_1 = j(2\pi\times60)\times(8.67\times10^{-12}) = j3.27\times10^{-9}\ \text{S/m}$$

$$\therefore \boxed{\overline{Y}_1 = j3.27\times10^{-6}\ \text{S/km}}$$

For a 125 km line length,

$$I_{\text{chg}} = \overline{Y}_1 V_{LN} = (3.27\times10^{-6}\times125)\times(230/\sqrt{3}) = \boxed{0.0543\ \text{kA/phase}}$$

▶ Statement **3** is true, while statements **1** and **2** are false.

P.12 ➡ Solution

In Problem 5, the *GMR* and *GMD* for this system were calculated to be 13.4 mm and 10.1 m, respectively. The capacitance-to-neutral is then

$$C_1 = \frac{2\pi\varepsilon_0}{\ln(D/r)} = \frac{2\pi\times(8.85\times10^{-12})}{\ln(10.1/0.0134)} = \boxed{8.39\times10^{-12}\ \text{F/m}}$$

The admittance-to-neutral, in turn, is

$$\overline{Y}_1 = j\omega C_1 = j(2\pi\times60)\times(8.39\times10^{-12}) = j3.16\times10^{-9}\ \text{S/m}$$

$$\therefore \boxed{\overline{Y}_1 = j3.16\times10^{-6}\ \text{S/km}}$$

Lastly, the total reactive power is

$$Q_1 = V_{LL}^2 Y_1 = 500^2\times(3.16\times10^{-6}) = \boxed{0.790\ \text{Mvar/km}}$$

▶ Statements **1** and **2** are true, while statement **3** is false.

P.13 ➡ Solution

We first determine D_{eq},

$$D_{\text{eq}} = \sqrt[3]{7.5\times7.5\times13} = 9.01\,\text{m}$$

The capacitance to neutral is calculated as

$$C = \frac{2\pi\times(8.85\times10^{-12})}{\ln\left(\dfrac{9.01}{0.0225/2}\right)} = 1.05\times10^{-11}\ \text{F/m} = \boxed{0.0105\ \mu\text{F/km}}$$

The capacitive reactance is

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi\times60\times(1.05\times10^{-11})} = 2.53\times10^{8}\ \Omega\cdot\text{m} = \boxed{253{,}000\ \Omega\cdot\text{km}}$$

P.14 ➡ Solution

In general, the capacitive reactance is

$$X_C = \frac{1}{2\pi fC} = \frac{2.86\times10^{9}}{f}\ln\frac{D}{r}\quad \Omega\cdot\text{m to neutral}$$

or, equivalently,

$$X_C = \frac{2.86\times10^{3}}{f}\ln\frac{D}{r}\quad \text{k}\Omega\cdot\text{km to neutral}$$

$$\therefore X_C = \frac{2.86\times10^{3}}{60}\ln\frac{1.5}{r} = 280$$

$$\therefore r = 0.00422\ \text{m} = 0.422\,\text{cm}$$

Equivalently, r = 0.0138 ft. Now, for a spacing of 2 ft and an angular frequency of 25 Hz,

$$X_C = \frac{1.779\times 10^6}{f}\ln\frac{D}{r} \quad \Omega\cdot\text{mi}$$

$$\therefore X_C = \frac{1.779\times 10^6}{25}\ln\frac{2}{0.0138} = 354{,}000\ \Omega\cdot\text{mi}$$

$$\therefore X_C = \boxed{354\ \text{k}\Omega\cdot\text{mi}}$$

Noting that $d = 2\times 0.0138$ ft = 0.331 in, and that 1 cmil = 7.854×10^{-7} in^2, the cross-section of the wire is converted as

$$A = \frac{\pi\times 0.331^2}{4}\text{in.}^2\times\frac{1\,\text{cmil}}{7.854\times 10^{-7}\ \text{in.}^2} = \boxed{110{,}000\ \text{cmil}}$$

P.15 ➡ *Solution*

The solution is started by computing the GMR = 0.028/2 = 0.014 m and the GMD

$$D_{\text{eq}} = \sqrt[3]{D_{12}D_{23}D_{13}} = \sqrt[3]{6\times 6\times 11} = 7.34\,\text{m}$$

The capacitance-to-neutral follows as

$$C_n = \frac{2\pi\varepsilon_0}{\ln\left(D_{\text{eq}}/D_s\right)} = \frac{2\pi\times\left(8.85\times 10^{-12}\right)}{\ln\left(7.34/0.014\right)} = 8.88\times 10^{-12}\ \text{F/m}$$

$$\therefore \boxed{C_n = 8.88\ \text{nF/km}}$$

The per-unit-length reactance-to-neutral is

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi\times 60\times\left(8.88\times 10^{-12}\right)} = 2.99\times 10^8\ \Omega\cdot\text{m}$$

$$\therefore X_C = 299{,}000\ \Omega\cdot\text{km}$$

For a line length of 75 mi ≈ 121 km,

$$X_C = \frac{299{,}000\ \Omega\cdot\cancel{\text{km}}}{121\ \cancel{\text{km}}} = \boxed{2470\ \Omega}$$

Next comes the charging current per mile,

$$I_{\text{chg}} = \frac{220{,}000}{\sqrt{3}}\times\frac{1}{X_C} = \frac{220{,}000}{\sqrt{3}}\times\frac{1}{2.99\times 10^8} = 4.25\times 10^{-4}\ \text{A/m}$$

$$\therefore \boxed{I_{\text{chg}} = 0.684\ \text{A/mi}}$$

For a line length of 75 mi, I_{chg} = 51.3 A. Multiplying this current by the voltage gives the three-phase reactive power,

$$\Pi = UI_{\text{chg}} = \left(\sqrt{3}\times 220{,}000\right)\times 0.684 = \boxed{19.6\ \text{MVar}}$$

▶ Statements **1** and **2** are true, while statements **3** and **4** are false.

REFERENCES

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