Montogue

Quiz EL102
Transmission Lines: Steady-State Operation

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PROBLEMS

Problem 1

Problem 1.1: A three-phase voltage of 11 kV is applied to a line of resistance R = 10 Ω and reactance X = 12 Ω per conductor. At the end of the line is a balanced load of P kW at a leading power factor. At what value of P is the voltage regulation zero when the power factor of the load is 0.707?

A) P = 994 kW
B) P = 1100 kW
C) P = 1240 kW
D) No such value exists.

Problem 1.2: At what value of P would the voltage regulation be zero if the power factor of the load were 0.85?

A) P = 650 kW
B) P = 842 kW
C) P = 970 kW
D) No such value exists.

Problem 2

Problem 2.1: A 600-km, 500-kV, 60-Hz uncompensated three-phase line has a positive-sequence series impedance z = 0.04 + j0.34 Ω/km and a positive-sequence shunt admittance $y = j4.5\times10^{-6}$ S/km. For this line, true or false?

1.(   ) The characteristic impedance Z_c has phasor amplitude greater than 270 Ω.
2.(   ) The product of propagation constant and line length is greater (real and imaginary parts) than 0.025 + j0.9 pu.
3.(   ) Parameter A has phasor angle greater than 2.5°.
4.(   ) Parameter B has phasor amplitude greater than 175 Ω.
5.(   ) Parameter C has phasor angle greater than 82°.
6.(   ) Parameter D has phasor amplitude greater than 0.9.

Problem 2.2: At full load, the line introduced in Problem 2.1 delivers 850 MW at unity power factor and at 475 kV. True or false?

7.(   ) The sending end voltage is greater than 525 kV.
8.(   ) The sending end current is greater than 0.92 kA.
9.(   ) The sending end power factor is leading and has an absolute value lower than 0.98.
10.(   ) The full-load line losses are greater than 50 MW.
11.(   ) The percentage voltage regulation has absolute value greater than 35%.

Problem 2 was fairly straightforward because the series impedance and shunt admittance were given beforehand. Problem 3 makes things a bit more complicated because these parameters must be calculated by the student.

Problem 3

Problem 3.1: A bundled 500-kV, 60-Hz, three-phase line has a 250 km length. The line consists of three ACSR 25-mm OD, 0.06 Ω/km conductors per bundle, with 0.45 m between conductors in the bundle. The horizontal phase spacings between bundle centers are 12, 12, and 20 m.

1.(   ) The characteristic impedance Z_c has phasor amplitude greater than 300 Ω.
2.(   ) The product of propagation constant and line length is greater (real and imaginary parts) than 0.007 + j0.2 pu.
3.(   ) Parameter A has a phasor angle greater than 1°.
4.(   ) Parameter B has phasor amplitude greater than 80 Ω.
5.(   ) Parameter C has phasor angle greater than 85°.
6.(   ) Parameter D has phasor amplitude greater than 0.92.

Problem 3.2: At full load, the line in the previous part delivers 1500 MVA at 480 kV to the receiving end load. Calculate the sending end voltage when the receiving end power factor is 0.9 lagging. Then, compute the percent voltage regulation.
Problem 3.3: Repeat the previous calculation if the power factor is 0.9 leading.

Problem 4

Problem 4.1: A 275 kV transmission line has line constants $A = 0.84\angle 5^\circ$ and $B = 210\angle 74^\circ$. Determine the power at unity power factor that can be received if the voltage profile at each end is to be maintained at 275 kV.
A) $P_R = 108$ MW
B) $P_R = 120$ MW
C) $P_R = 138$ MW
D) $P_R = 153$ MW
Problem 4.2: If the load is 158 MW at unity power factor, with the same voltage profile as in Problem 4.1, a compensation equipment would have to feed:
A) +20 MVARs into the line.
B) +30 MVARs into the line.
C) +40 MVARs into the line.
D) +50 MVARs into the line.
Problem 4.3: With the load specified in Problem 4.2 but no compensating equipment, what would be the receiving end voltage?
A) $V_R = 109$ kV
B) $V_R = 156$ kV
C) $V_R = 202$ kV
D) $V_R = 251$ kV

Problem 5

The generalized circuit constants of a transmission line are $A = 0.94 + j0.017$ and $B = 25 + j140$. The load at the receiving end is 60 MVA, 50 Hz, 0.8 power factor lagging. The voltage at the supply end is 220 kV. Calculate the load voltage.
A) $V_R = 158$ kV
B) $V_R = 168$ kV
C) $V_R = 178$ kV
D) $V_R = 198$ kV

Problem 6

Problem 6.1: A three-phase, 60-Hz, 525-kV transmission line is 330 km long. The line inductance is 0.90 mH/km per phase and its capacitance is 0.0114 μF/km per phase. Assume a lossless line. Determine the line phase constant β, the surge impedance Z_C, the velocity of propagation v, and the line wavelength λ. True or false?
1.( ) The line phase constant is greater than 0.00128 rad/km.
2.( ) The surge impedance is greater than 286 Ω.
3.( ) The velocity of propagation is greater than 300,000 km/s.
4.( ) The line wavelength is greater than 5000 km.
Problem 6.2: Now, determine the voltage, current, real and reactive power at the sending end. Also compute the percent voltage regulation of the line if the receiving end load is 805 MW at 0.8 power factor lagging and at 520 kV. True or false?
5.( ) The sending end voltage has phasor amplitude greater than 358 kV.
6.( ) The sending end current has phasor amplitude greater than 860 A.
7.( ) The sending end reactive power is greater than 465 MVAR.
8.( ) The percent voltage regulation is greater (absolute value) than 28%.

Problem 7

Problem 7.1: A 500-kV, 320-km, 60-Hz, three-phase overhead transmission line, assumed to be lossless, has a series inductance of 0.96 mH/km per phase and a shunt capacitance of 0.0118 μF/km per phase. Determine the phase constant β, the surge impedance Z_c, the velocity of propagation v, and the wavelength λ of the line. True or false?
1.( ) The phase constant is greater than 0.00122 rad/km.
2.( ) The surge impedance is greater than 290 Ω.
3.( ) The velocity of propagation is greater than 298,000 km/s.
4.( ) The wavelength of the line is greater than 4850 km.

Problem 7.2: Now, determine the voltage, current, real and reactive power at the sending end, and the percent voltage regulation of the line if the receiving end load is 770 MW at 0.8 power factor lagging and at 500 kV. True or false?

5.( ) The sending end voltage has phasor amplitude greater than 365 kV.
6.( ) The sending end current has phasor amplitude greater than 825 A.
7.( ) The sending end reactive power is greater than 450 MVAR.
8.( ) The percent voltage regulation is greater (absolute value) than 38%.

Problem 8

The following parameters are based on a preliminary line design.

Sending end voltage	V_S = 1.0 pu
Receiving end voltage	V_R = 0.9 pu
Line wavelength	λ = 4800 km
Characteristic impedance	Z_C = 345 Ω
Phase angle	δ = 36.5°

A three-phase power of 720 MW is to be transmitted to a substation located 300 km from the source of power. Determine a nominal voltage level for the three-phase transmission line, based on the practical line-loadability equation. For the voltage level determined, what is the theoretical maximum power that can be transferred by the line?

A) V_{rated} = 325 kV and P_{max} = 990 MW
B) V_{rated} = 325 kV and P_{max} = 1090 MW
C) V_{rated} = 355 kV and P_{max} = 990 MW
D) V_{rated} = 355 kV and P_{max} = 1090 MW

Problem 9

Noting that the steady-state stability limit for power in a lossless line is given by

$$P_{R,\max} = \frac{V_R V_S}{Z'} - \frac{A V_R^2}{Z'} \cos(\theta_Z - \theta_A)$$

where V_R is receiving end voltage, V_S is sending end voltage, Z' = B, and θ_Z and θ_A refer to Z' and parameter A, respectively, revisit Problem 2 and determine $P_{R,\max}$. Assume V_S = V_R = 1.0 per unit and unit power factor at the receiving end. The line consists of three ACSR 954 kcmil conductors per phase, which have a current carrying capacity of 1010 A per conductor at the temperature of interest. Does the line in question violate the thermal limit?

A) $P_{R,\max}$ = 1230 MW and the line does not violate the thermal limit.
B) $P_{R,\max}$ = 1230 MW and the line violates the thermal limit.
C) $P_{R,\max}$ = 1380 MW and the line does not violate the thermal limit.
D) $P_{R,\max}$ = 1380 MW and the line violates the thermal limit.

Problem 10

Revisit the transmission line of Problem 6. Calculate the receiving end voltage when the line is terminated in an open circuit and is energized with 520 kV at the sending end. Determine the reactance and the MVAR of a three-phase shunt reactor to be installed at the receiving end to keep the no-load receiving end voltage at the rated value.

A) $X_{L,\text{sh}}$ = 1090 Ω and $Q_{3\phi}$ = 145 MVAR
B) $X_{L,\text{sh}}$ = 1090 Ω and $Q_{3\phi}$ = 195 MVAR
C) $X_{L,\text{sh}}$ = 1390 Ω and $Q_{3\phi}$ = 145 MVAR
D) $X_{L,\text{sh}}$ = 1390 Ω and $Q_{3\phi}$ = 195 MVAR

Problem 11

The transmission line in Problem 6 supplies a load of 1000 MVA, 0.8 power factor lagging at 520 kV. Determine the MVAR and the capacitance of the shunt capacitors to be installed at the receiving end to keep the receiving end voltage at 520 kV when the line is energized with 520 kV at the sending end.

A) Required capacitor MVAR = 488; Capacitance = 2.63 μF
B) Required capacitor MVAR = 538; Capacitance = 5.27 μF
C) Required capacitor MVAR = 488; Capacitance = 2.63 μF
D) Required capacitor MVAR = 538; Capacitance = 5.27 μF

SOLUTIONS

P.1 Solution

Problem 1.1: The receiving end voltage is 11/√3 = 6.35 kV. Theta is the angle of the transmission line impedance, namely

$$\theta = \tan^{-1}\frac{10}{12} = 50.2^{\circ}$$

With |Z| = $\sqrt{10^2 + 12^2}$ = 15.6. Next, $\cos\phi_R$ is the power factor itself, and ϕ_R is an angle given by

$$\phi_R = \cos^{-1} 0.707 = 45^{\circ}$$

The current is determined as

$$|I| = \frac{2|V_R|}{|Z|}\sin\left(\phi_R + \theta - 90^{\circ}\right) = \frac{2\times 6350}{15.6}\sin\left(45^{\circ} + 50.2^{\circ} - 90^{\circ}\right) = 73.8\,\text{A} \quad \text{(I)}$$

The power required to attain zero voltage regulation is found as

$$P = \sqrt{3}V_R I\cos\phi_R = \sqrt{3}\times 11\times 73.8\times 0.707 = \boxed{994\text{ kW}}$$

If the line had a real power of 994 kW, voltage regulation would be zero.

▶ The correct answer is **A**.

Problem 1.2: We first determine ϕ_R,

$$\phi_R = \cos^{-1} 0.85 = 31.8^{\circ}$$

Note, however, that

$$\phi_R + \theta - 90^{\circ} = 31.8^{\circ} + 50.2^{\circ} - 90^{\circ} = -8^{\circ}$$

With $\phi_R + \theta - 90^{\circ} < 0$ a negative, meaningless current would be obtained from (I), and no corresponding power value exists. We surmise that, at a power factor of 0.85, the line cannot achieve zero voltage regulation as it is.

▶ The correct answer is **D**.

P.2 Solution

Problem 2.1: The characteristic impedance is given by

$$Z_c = \sqrt{\frac{z}{y}} = \sqrt{\frac{0.04 + j0.34}{j4.5\times 10^{-6}}}$$

$$\therefore Z_c = \sqrt{\frac{0.342\angle 85.1^{\circ}}{4.5\times 10^{-6}\angle 90^{\circ}}} = \sqrt{76{,}000\angle -4.9^{\circ}} = \boxed{276\angle -2.45^{\circ}\,\Omega}$$

The propagation constant is given by

$$\gamma = \sqrt{zy} = \sqrt{\left(0.342\angle 85.1^{\circ}\right)\times\left(4.5\times 10^{-6}\angle 90^{\circ}\right)}$$

$$\therefore \gamma = \sqrt{1.54\times 10^{-6}\angle 175.1^{\circ}} = 1.24\times 10^{-3}\angle 87.6^{\circ}$$

Multiplying by line length,

$$\gamma\ell = \left(1.24\times 10^{-3}\angle 87.6^{\circ}\right)\times 600 = 0.744\angle 87.6^{\circ}$$

$$\therefore \boxed{\gamma\ell = 0.0312 + j0.743\,\text{pu}}$$

Parameter *A* is determined next,

$$A = \cosh\left(\gamma\ell\right) = 0.737 + j0.0211\,\text{pu}$$

$$\therefore \boxed{A = 0.737\angle 1.64^{\circ}\,\text{pu}}$$

Parameter *B* is found as

$$B = Z_c\sinh\left(\gamma\ell\right) = \left(276\angle -2.45^{\circ}\right)\times\sinh\left(0.0312 + j0.743\right)$$

$$\therefore B = \left(276\angle -2.45^{\circ}\right)\times\left(0.0230 + j0.679\right)$$

$$\therefore B = \left(276\angle -2.45^{\circ}\right)\times\left(0.679\angle 88.1^{\circ}\right)$$

$$\therefore \boxed{B = 187\angle 85.7^{\circ}\,\Omega}$$

Parameter *C* is computed as

$$C = \frac{1}{Z_c}\sinh(\gamma\ell) = \frac{1}{276\angle -2.45^\circ}\sinh(0.0312 + j0.743)$$

$$\therefore C = \frac{1}{276\angle -2.45^\circ}\times(0.679\angle 88.1^\circ)$$

$$\therefore \boxed{C = 2.46\times 10^{-3}\angle 90.6^\circ\ \text{S}}$$

Parameter *D* equals *A*,

$$D = A = \boxed{0.737\angle 1.64^\circ\,\text{pu}}$$

▶ Statements **1**, **4**, and **5** are true, whereas statements **2**, **3**, and **6** are false.

Problem 2.2: We first determine the per-phase receiving end voltage,

$$V_{R,L-N} = \frac{475\times 10^3}{\sqrt{3}}\angle 0^\circ = 274\angle 0^\circ\,\text{kV}$$

and the line-to-line receiving end current,

$$I_{R,L-L} = \frac{P_R\angle\cos^{-1}(pf)}{\sqrt{3}V_{R,L-L}(pf)} = \frac{850\angle\cos^{-1}1}{\sqrt{3}\times 475\times 1.0} = 1.03\angle 0^\circ\,\text{kA}$$

Parameters *A* = 0.737∠1.64° and *B* = 187∠85.7° have already been determined. The sending end voltage follows as

$$\overline{V}_s = \overline{A}\overline{V}_R + \overline{B}\overline{I}_R = (0.737\angle 1.64^\circ)(274\angle 0^\circ) + (187\angle 85.7^\circ)(1.03\angle 0^\circ)$$

$$\therefore \overline{V}_s = (202\angle 1.64^\circ) + (193\angle 85.7^\circ)$$

$$\therefore \overline{V}_s = 293\angle 42.5^\circ\,\text{kV}_{LN}$$

$$\therefore \overline{V}_{s,L-L} = 293\sqrt{3} = \boxed{507\,\text{kV}_{LL}}$$

Parameters $C = 2.46\times 10^{-3}\angle 90.6^\circ$ and *D* = 0.737∠1.64° have been calculated above. The sending end current is then

$$\overline{I}_S = \overline{C}\overline{V}_R + \overline{D}\overline{I}_R = (2.46\times 10^{-3}\angle 90.6^\circ)(274\angle 0^\circ) + (0.737\angle 1.64^\circ)(1.03\angle 0^\circ)$$

$$\therefore \overline{I}_S = (0.752\angle 90.6^\circ) + (0.696\angle 1.64^\circ)$$

$$\therefore \overline{I}_S = 1.02\angle 42.8^\circ\ \text{kA}$$

$$\therefore \boxed{\overline{I}_S = 1.02\,\text{kA}}$$

Given θ_{V_S} = 42.5° and θ_{I_S} = 42.8°, the sending end power factor is calculated to be

$$pf_s = \cos\left(\theta_{V_s} - \theta_{I_s}\right) = \cos(42.5^\circ - 42.8^\circ) \approx 1.0$$

In the three-digit precision used in our calculations, the power factor equals unity. Increasing the precision in previous calculations to six digits would yield a *pf* of 0.999986, indicating that the *pf* at the sending end may be lagging.

The sending end power is

$$P_S = \sqrt{3}V_S I_S(pf) = \sqrt{3}\times 507\times 1.02\times 1.0 = 896\ \text{MW}$$

The full-load line losses are then

$$\text{Full-load line losses} = P_S - P_R = 896 - 850 = \boxed{46\,\text{MW}}$$

The no-load receiving end voltage is

$$V_{R,NL} = \frac{V_S}{A} = \frac{507}{0.737} = 688\,\text{kV}$$

and the percentage voltage regulation is calculated to be

$$|\%\text{VR}| = \left|\frac{V_{R,NL} - V_{R,FL}}{V_{R,FL}}\right|\times 100\% = \left|\frac{688 - 475}{475}\right|\times 100\% = \boxed{44.8\%}$$

▶ Statements **8** and **11** are true, whereas statements **7**, **9**, and **10** are false.

P.3 ➡ Solution

Problem 3.1: The GMR of the subconductors is D_S = 25 mm/2 = 0.0125 m and the spacing between conductors in the bundle is 0.6 m, so that

$$D_{SL} = \sqrt[3]{D_S d^2} = \sqrt[3]{0.0125 \times 0.45^2} = 0.136\,\text{m}$$

The geometric mean distance separating the conductor bundles is

$$D_{\text{eq}} = GMD = \sqrt[3]{12 \times 12 \times 20} = 14.2\,\text{m}$$

The inductance L is then

$$L = 2\times10^{-7}\ln\frac{D_{\text{eq}}}{D_{\text{SL}}} = 2\times10^{-7}\ln\left(\frac{14.2}{0.136}\right) = 9.30\times10^{-7}\text{ H/m}$$

Equipped with L, we can establish the inductive reactance X_L,

$$X_L = j\omega L = j\times(2\pi\times60)\times\left(9.30\times10^{-7}\right) = j3.51\times10^{-4}\ \Omega/\text{m}$$

$$\therefore X_L = j0.351\,\Omega/\text{km}$$

Assuming $D_{SC} = D_{SL}$ = 0.136 m, the capacitance C of the system is determined next,

$$C = \frac{2\pi\varepsilon_0}{\ln\left(D_{\text{eq}}/D_{SC}\right)} = \frac{2\pi\times\left(8.85\times10^{-12}\right)}{\ln(14.2/0.136)} = 1.20\times10^{-11}\text{F/m}$$

The admittance per line per km is then

$$y = j\omega C = j\times(2\pi\times60)\times\left(1.20\times10^{-11}\right) = 4.52\times10^{-9}\text{ S/m}$$

$$\therefore y = 4.52\times10^{-6} = 4.52\times10^{-6}\angle 90^\circ\text{ S/km}$$

Since one kilometer of line has 0.06-Ω resistance and there are three conductors per phase, we may write

$$R = \frac{0.06\,\Omega/\text{km}}{3} = 0.02\,\Omega/\text{km}$$

The impedance per km is then

$$z = R + jX_L = 0.02 + j0.351 = 0.352\angle 86.7^\circ\ \Omega/\text{km}$$

The characteristic impedance is determined next,

$$Z_c = \sqrt{\frac{z}{y}} = \sqrt{\frac{0.352\angle 86.7^\circ}{4.52\times10^{-6}\angle 90^\circ}} = \boxed{279\angle -1.65^\circ\,\Omega}$$

The product of propagation constant and line length is

$$\gamma\ell = \left(\sqrt{zy}\right)\ell = \sqrt{(0.352\angle 86.7^\circ)\left(4.52\times10^{-6}\angle 90^\circ\right)}\times250$$

$$\therefore \gamma\ell = \sqrt{\left(1.59\times10^{-6}\angle 177^\circ\right)}\times250$$

$$\therefore \gamma\ell = \left(1.26\times10^{-3}\angle 88.5^\circ\right)\times250$$

$$\therefore \gamma\ell = 0.315\angle 88.5^\circ$$

$$\therefore \boxed{\gamma\ell = 0.00825 + j0.315\,\text{pu}}$$

Equipped with product $\gamma\ell$, determining the A parameter is straightforward,

$$A = \cosh(\gamma\ell) = \cosh(0.00825 + j0.315) = 0.951 + j0.00256$$

$$\therefore \boxed{A = 0.951\angle 0.154^\circ\,\text{pu}}$$

Parameter B is

$$B = Z_c\sinh(\gamma\ell) = (279\angle -1.65^\circ)\sinh(0.00825 + j0.315)$$

$$\therefore B = (279\angle -1.65^\circ)(0.00784 + j0.310)$$

$$\therefore B = (279\angle -1.65^\circ)(0.310\angle 88.6^\circ)$$

$$\therefore \boxed{B = 86.5\angle 87^\circ}$$

Parameter C is computed as

$$C = \frac{1}{Z_c}\sinh(\gamma\ell) = \frac{1}{279\angle -1.65^\circ}\sinh(0.00825 + j0.315)$$

$$\therefore C = \frac{1}{279\angle -1.65^\circ} \times (0.310\angle 88.6^\circ)$$

$$\therefore \boxed{C = 1.11\times10^{-3}\angle 90.3^\circ \text{ S}}$$

Finally, parameter D equals A,

$$D = A = \boxed{0.951\angle 0.154^\circ \text{pu}}$$

▶ Statements **2**, **4**, **5** and **6** are true, whereas statements **1** and **3** are false.

Problem 3.2: The line-to-neutral receiving end voltage is

$$V_{R,L-N} = \frac{480\times10^3}{\sqrt{3}}\angle 0^\circ = 277\angle 0^\circ \text{kV}$$

The receiving end current, in turn, is

$$\bar{I}_R = \frac{1500}{480\sqrt{3}}\angle -\cos^{-1}0.9 = 1.80\angle -25.8^\circ \text{kA}$$

The sending end voltage is then

$$\bar{V}_S = \bar{A}\bar{V}_R + \bar{B}\bar{I}_R = (0.951\angle 0.154^\circ)\times 277 + (86.5\angle 87^\circ)(1.80\angle -25.8^\circ)$$

$$\therefore \bar{V}_S = 263\angle 0.154^\circ + 156\angle 61.2^\circ$$

$$\therefore \bar{V}_S = 365\angle 22.1^\circ \text{ kV}_{LN}$$

Multiplying by $\sqrt{3}$ gives

$$\bar{V}_{S,L-L} = 365\times\sqrt{3} = \boxed{632 \text{ kV}_{LL}}$$

To determine the percent voltage regulation, we first compute the no-load receiving end voltage,

$$V_{R,NL} = \frac{V_S}{A} = \frac{632}{0.951} = 665 \text{kV}$$

Accordingly, the percent voltage regulation follows as

$$\%VR = \left(\frac{V_{R,NL} - V_{R,FL}}{V_{R,FL}}\right)\times 100\% = \left(\frac{665-480}{480}\right)\times 100\% = \boxed{38.5\%}$$

Problem 3.3: The line-to-neutral receiving end voltage continues to be 277∠0° kV. The receiving end current is now

$$\bar{I}_R = \frac{1500}{480\sqrt{3}}\angle \cos^{-1}0.9 = 1.80\angle 25.8^\circ$$

The sending end voltage is then

$$\bar{V}_S = \bar{A}\bar{V}_R + \bar{B}\bar{I}_R = (0.951\angle 0.154^\circ)\times 277 + (86.5\angle 87^\circ)(1.80\angle 25.8^\circ)$$

$$\therefore \bar{V}_S = 263\angle 0.154^\circ + 156\angle 113^\circ$$

$$\therefore \bar{V}_S = 248\angle 35.5^\circ \text{ kV}_{LN}$$

so that

$$V_S = 248\times\sqrt{3} = \boxed{430 \text{ kV}_{LL}}$$

and

$$V_{R,NL} = \frac{V_S}{A} = \frac{430}{0.951} = 452 \text{kV}$$

Finally,

$$\%VR = \left(\frac{V_{R,NL} - V_{R,FL}}{V_{R,FL}}\right)\times 100\% = \left(\frac{452-480}{480}\right)\times 100\% = \boxed{-5.83\%}$$

P.4 ➡ *Solution*

Problem 4.1: Given $|V_S| = |V_R| = 275$ kV, $\theta_A = 5^\circ$, $\theta_B = 74^\circ$, the power factor is unity and hence the receiving end reactive power is zero. It follows that

$$Q_R = \frac{V_S V_R}{B}\sin(\theta_B - \delta) - \frac{AV_R^2}{B}\sin(\theta_B - \theta_A) = 0$$

$$\therefore \frac{275\times275}{210}\sin\left(74^{\circ}-\delta\right)-\frac{0.84\times275^{2}}{210}\sin\left(74^{\circ}-5^{\circ}\right)=0$$
$$\therefore 360\sin\left(74^{\circ}-\delta\right)-282=0$$
$$\therefore \sin\left(74^{\circ}-\delta\right)=0.783$$
$$\therefore 74^{\circ}-\delta=51.5^{\circ}$$
$$\therefore \delta=22.5^{\circ}$$

The real power is then

$$P_R=\frac{275\times275}{210}\cos\left(74^{\circ}-22.5^{\circ}\right)-\frac{0.84\times275^{2}}{210}\cos 74^{\circ}=\boxed{138\text{ MW}}$$

▶ The correct answer is **C**.

Problem 4.2: The power demanded by the load is P_D = 158 MW. Setting P_R = P_D = 175 MW and solving for δ, we find that

$$158=\frac{275\times275}{210}\cos\left(74^{\circ}-\delta\right)-\frac{0.84\times275^{2}}{210}\cos\left(69^{\circ}\right)$$
$$\therefore \delta=31.7^{\circ}$$

The reactive power Q_R is then

$$Q_R=\frac{275\times275}{210}\sin\left(74^{\circ}-33.4^{\circ}\right)-\frac{0.84\times275^{2}}{210}\sin\left(69^{\circ}\right)=-40.0\text{ MVAR}$$

Thus, in order to maintain a steady 275 kV at receiving end, Q_R = −40.0 MVAR must be drawn along with a real power of 165 MW. The load being 165 MW at unity power factor, i.e. Q_D = 0, compensation equipment must be installed at the receiving end. Accordingly,

$$-40.0+Q_C=0\rightarrow\boxed{Q_C=+40.0\text{ MVAR}}$$

That is, the compensation equipment must feed about 40 positive MVARs into the line.

▶ The correct answer is **C**.

Problem 4.3: With P_R = 165 MW, we may write

$$158=\frac{275\times V_R}{210}\cos\left(74^{\circ}-\delta\right)-\frac{0.84V_R^{2}}{210}\cos 69^{\circ}$$
$$\therefore 158=1.31V_R\cos\left(74^{\circ}-\delta\right)-0.00143V_R^{2}\quad\text{(I)}$$

Likewise, with Q_R = 0,

$$0=\frac{275V_R}{210}\sin\left(74^{\circ}-\delta\right)-\frac{0.84V_R^{2}}{210}\sin 69^{\circ}$$
$$0=1.31V_R\sin\left(74^{\circ}-\delta\right)-0.00373V_R^{2}$$
$$\therefore 0=V_R\left[1.31\sin\left(74^{\circ}-\delta\right)-0.00373V_R\right]$$
$$\therefore 1.31\sin\left(74^{\circ}-\delta\right)-0.00373V_R=0$$
$$\therefore \sin\left(74^{\circ}-\delta\right)=0.00285V_R$$
$$\therefore \cos\left(74^{\circ}-\delta\right)=\sqrt{1-\left(0.00285V_R\right)^{2}}$$

Substituting in (I), we find that

$$\therefore 158=1.31V_R\sqrt{1-\left(0.00285V_R\right)^{2}}-0.00143V_R^{2}$$

This equation can be easily solved with Mathematica's *Solve* command,

```
In[1139]:= Solve[158 == 1.31 V Sqrt[1 - (0.00285 V)^2] - 0.00143 V^2, V]
Out[1139]= {{V → 195.172}, {V → 202.487}}
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The first, lower solution, though feasible, is rejected because it leads to abnormal operation. Thus, we take V_R = 202 kV.

▶ The correct answer is **C**.

P.5 ➡ *Solution*

Writing the *A* and *B* phasors in polar form yields *A* = 0.940∠1.04° and *B* = 142∠79.9°. The real power is P_R = 60 × 0.8 = 48 MW and the reactive power is Q_R = 60 × 0.6 = 36 MVAR. Accordingly, we write, for real power,

$$48 = \frac{220 \times V_R}{142}\cos\left(79.9^\circ - \delta\right) - \frac{0.94 \times V_R^2}{142}\cos\left(79.9^\circ - 1.04^\circ\right)$$

$$\therefore 48 = 1.55 V_R \cos\left(79.9^\circ - \delta\right) - 0.00128 V_R^2 \quad \text{(I)}$$

and for reactive power,

$$36 = \frac{220 \times V_R}{142}\sin\left(79.9^\circ - \delta\right) - \frac{0.94 \times V_R^2}{142}\sin\left(79.9^\circ - 1.04^\circ\right)$$

$$\therefore 36 = 1.55 V_R \sin\left(79.9^\circ - \delta\right) - 0.00650 V_R^2 \quad \text{(II)}$$

Relations (I) and (II) constitute a system of transcendental equations with unknowns V_R and δ. One way to go is to apply Mathematica's *Solve* command,

```
In[1150]:= Solve[{48 == 1.55 * VR * Cos[79.9 ° - δ] - 0.00128 VR^2, 36 == 1.55 * VR * Sin[79.9 ° - δ] - 0.00650 VR^2}, {VR, δ}]

··· Solve: Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information.

Out[1150]= {{VR → -197.951, δ → 3.29095}, {VR → -45.7529, δ → 3.76141}, {VR → 45.7529, δ → 0.619817}, {VR → 197.951, δ → 0.149357}}
```

Rejecting the solutions with negative V_R and the one with a low positive V_R, we conclude that $V_R \approx$ 198 kV.

▶ The correct answer is **D**.

P.6 ➡ *Solution*

Problem 6.1: The line phase constant for a lossless line is

$$\beta = \omega\sqrt{LC} = 2\pi \times 60 \times \sqrt{\left(0.90 \times 10^{-3}\right) \times \left(0.0114 \times 10^{-6}\right)} = \boxed{0.00121\,\text{rad/km}}$$

The surge impedance is determined as

$$Z_C = \sqrt{\frac{L}{C}} = \sqrt{\frac{0.90 \times 10^{-3}}{0.0114 \times 10^{-6}}} = \boxed{281\Omega}$$

The velocity of propagation is computed as

$$v = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\left(0.9 \times 10^{-3}\right) \times \left(0.0114 \times 10^{-6}\right)}} = \boxed{312{,}000\text{ km/s}}$$

The line wavelength is

$$\lambda = \frac{v}{f} = \frac{312{,}000}{60} = \boxed{5200\,\text{km}}$$

▶ Statements **3** and **4** are true, whereas statements **1** and **2** are false.

Problem 6.2: The sending end voltage is $\bar{V}_R = 520/\sqrt{3} = 300\angle 0^\circ$ kV. Given the power = 805 MW and the power factor = 0.8, the three-phase complex power is given by

$$\bar{S}_{R,3\phi} = \frac{805}{0.8}\angle\cos^{-1}0.8 = 1010\angle 36.9^\circ\,\text{MVA}$$

The receiving end current is

$$\bar{I}_R = \frac{\bar{S}^*_{R,3\phi}}{3\bar{V}^*_R} = \frac{\left(1010 \times 10^6\angle -36.9^\circ\right)}{3 \times \left(300 \times 10^3\angle 0^\circ\right)} = 1120\angle -36.9^\circ\,\text{A}$$

Next, noting that $\beta\ell$ = 0.00121 × 330 = 0.399 rad, the sending end voltage is calculated to be

$$\bar{V}_S = \cos\left(\beta\ell\right)\bar{V}_R + j\bar{Z}_C\sin\left(\beta\ell\right)\bar{I}_R$$

$$\therefore \bar{V}_S = \cos\left(0.399\right) \times \left(300\angle 0^\circ\right) + j \times 281 \times \sin\left(0.399\right) \times \left(1120\angle -36.9^\circ\right) \times 10^{-3}$$

$$\therefore \bar{V}_S = 276\angle 0^\circ + j122\angle -36.9^\circ$$

$$\therefore \boxed{\bar{V}_S = 362\angle 15.7^\circ\,\text{kV}}$$

The sending end current is calculated as

$$\bar{I}_S = j\frac{1}{\bar{Z}_C}\sin(\beta\ell)\bar{V}_R + \cos(\beta\ell)\bar{I}_R$$

$$\therefore \bar{I}_S = j\frac{1}{281}\times\sin(0.399)\times(300\times10^3) + \cos(0.399)\times(1120\angle-36.9^\circ)$$

$$\therefore \bar{I}_S = j415 + 1030\angle-36.9^\circ$$

$$\therefore \boxed{\bar{I}_S = 849\angle-13.8^\circ\,\text{A}}$$

The line current is 849 A. The sending end complex power is

$$S_{S,3\phi} = 3\bar{V}_S\bar{I}_S^* = 3\times(362\angle15.7^\circ)\times(849\angle13.8^\circ)\times10^{-3}$$

$$\therefore S_{S,3\phi} = 922\angle29.5^\circ = \boxed{803\,\text{MW} + j454\,\text{MVAR}}$$

The fact that the real component of this power result is not equal to 805 MW can be attributed to round-off error. Lastly, the percentage voltage regulation of the line is

$$\%VR = \left(\frac{V_{R,NL}-V_{R,FL}}{V_{R,FL}}\right)\times100\% = \left(\frac{V_S/A - V_{R,FL}}{V_{R,FL}}\right)\times100\% = \left(\frac{V_S/\cos(\beta\ell) - V_{R,FL}}{V_{R,FL}}\right)\times100\%$$

$$\therefore \%VR = \left(\frac{362/\cos(0.399)-300}{300}\right)\times100\% = \boxed{31.0\%}$$

▶ Statements **5** and **8** are true, whereas statements **6** and **7** are false.

P.7 ➡ *Solution*

Problem 7.1: For a lossless line, the phase constant is given by

$$\beta = 2\pi f\sqrt{LC} = 2\pi\times60\times\sqrt{(0.96\times10^{-3})\times(0.0118\times10^{-6})} = \boxed{0.00127\,\text{rad/km}}$$

The surge impedance is determined as

$$Z_C = \sqrt{\frac{L}{C}} = \sqrt{\frac{0.96\times10^{-3}}{0.0118\times10^{-6}}} = \boxed{285\,\Omega}$$

The velocity of propagation is computed as

$$v = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(0.96\times10^{-3})\times(0.0118\times10^{-6})}} = \boxed{297{,}000\text{ km/s}}$$

Dividing v by f = 60 Hz gives the line wavelength,

$$\lambda = \frac{v}{f} = \frac{297{,}000}{60} = \boxed{4950\,\text{km}}$$

▶ Statements **1** and **4** are true, whereas statements **2** and **3** are false.

Problem 7.2: The receiving end voltage is $\bar{V}_R$ = 500/$\sqrt{3}$ = 289∠0° kV. Given the power = 770 MW and the power factor = 0.8, the three-phase complex power is given by

$$\bar{S}_{R,3\phi} = \frac{770}{0.8}\angle\cos^{-1}0.8 = 963\angle36.9^\circ\,\text{MVA}$$

The receiving end current is

$$\bar{I}_R = \frac{\bar{S}_{R,3\phi}^*}{3\bar{V}_R^*} = \frac{(963\times10^6\angle-36.9^\circ)}{3\times(289\times10^3\angle0^\circ)} = 1110\angle-36.9^\circ\,\text{A}$$

Next, noting that $\beta\ell$ = 0.00127 × 320 = 0.406 rad, the sending end voltage is calculated to be

$$\bar{V}_S = \cos(\beta\ell)\bar{V}_R + j\bar{Z}_C\sin(\beta\ell)\bar{I}_R$$

$$\therefore \bar{V}_S = \cos(0.406)\times(289\angle0^\circ) + j\times285\times\sin(0.406)\times(1110\angle-36.9^\circ)\times10^{-3}$$

$$\therefore \bar{V}_S = 266\angle0^\circ + j125\angle-36.9^\circ$$

$$\therefore \boxed{\bar{V}_S = 355\angle16.3^\circ\,\text{kV}}$$

The sending end current is calculated as

$$\overline{I}_S = j\frac{1}{\overline{Z}_C}\sin(\beta\ell)\overline{V}_R + \cos(\beta\ell)\overline{I}_R$$

$$\therefore \overline{I}_S = j\frac{1}{285}\times\sin(0.406)\times(289\times10^3) + \cos(0.406)\times(1110\angle -36.9^\circ)$$

$$\therefore \overline{I}_S = j400 + 1020\angle -36.9^\circ$$

$$\therefore \boxed{\overline{I}_S = 843\angle -14.6^\circ\ \text{A}}$$

The line current is 843 A. The sending end complex power is

$$S_{S,3\phi} = 3\overline{V}_S\overline{I}_S^* = 3\times(355\angle 16.3^\circ)\times(843\angle 14.6^\circ)\times10^{-3}$$

$$\therefore S_{S,3\phi} = 898\angle 30.9^\circ = \boxed{771\,\text{MW} + j461\,\text{MVAR}}$$

The sending end real power is 771 MW (should've been 770 MW were it not for the round-off error) and the reactive power is 461 MVAR. It remains to determine the percent voltage regulation of the line,

$$\%VR = \left(\frac{V_{R,NL} - V_{R,FL}}{V_{R,FL}}\right)\times100\% \left(\frac{V_S/A - V_{R,FL}}{V_{R,FL}}\right)\times100\% = \left(\frac{V_S/\cos(\beta\ell) - V_{R,FL}}{V_{R,FL}}\right)\times100\%$$

$$\therefore \%VR = \left(\frac{355/\cos(0.406) - 289}{289}\right)\times100\% = \boxed{33.7\%}$$

▶ Statements **6** and **7** are true, whereas statements **5** and **8** are false.

P.8 ➡ *Solution*

The rated power can be established from the surge impedance loading (*SIL*), namely

$$SIL = \frac{V_{\text{rated}}^2}{Z_C}$$

The *SIL* can be determined from the power relationship

$$P_{\max} = \frac{V_{S,\text{pu}}V_{R,\text{pu}}(SIL)}{\sin(\beta\ell)}$$

Here, P_{max} = 720 MW, $V_{S,\text{pu}}$ = 1.0 pu, $V_{R,\text{pu}}$ = 0.9 pu, $\beta = 2\pi/\lambda = 2\pi/4800$ = 1.31×10^{-3} km, and ℓ = 300 km, giving

$$P_{\max} = \frac{V_{S,\text{pu}}V_{R,\text{pu}}(SIL)}{\sin(\beta\ell)} \rightarrow 720\times10^6 = \frac{1.0\times0.9\times(SIL)}{\sin\left[(1.31\times10^{-3})\times300\right]}$$

$$\therefore SIL = 306\,\text{MW}$$

so that

$$SIL = \frac{V_{\text{rated}}^2}{Z_C} \rightarrow 306\times10^6 = \frac{V_{\text{rated}}^2}{345}$$

$$\therefore V_{\text{rated}} = \sqrt{345\times(306\times10^6)} = \boxed{325\ \text{kV}}$$

The equivalent line reactance for a lossless line is

$$X' = \overline{Z}_C\sin(\beta\ell) = 345\sin\left[(1.31\times10^{-3})\times300\right] = 132\ \Omega$$

The power transmitted by the line is given by

$$P_R = \frac{V_SV_R}{X'}\sin\delta$$

The maximum power that can be transmitted under steady-state condition occurs for a load angle of 90°. Thus, substituting and recalling that in the present case 1 pu = 400 $kV_{L\text{-}L}$, the theoretical maximum power is calculated to be

$$P_{R,\max} = \frac{V_SV_R}{X'}\underbrace{\sin 90^\circ}_{=1}$$

$$\therefore P_{R,\max} = \frac{(1.0\times400)\times(0.9\times400)}{132}\times1 = \boxed{1090\,\text{MW}}$$

▶ The correct answer is **B**.

P.9 ➡ Solution

Simply substitute $V_R = V_S = 500$ kV, $Z' = 187\ \Omega$, $A = 0.737$ pu, $\theta_Z = 85.7°$, and $\theta_A = 1.64°$, giving

$$P_{R,\max} = \frac{500\times 500}{187} - \frac{0.737\times 500^2}{187}\cos(85.7°-1.64°) = \boxed{1230\,\text{MW}}$$

For this loading at unity power factor,

$$I_R = \frac{P_{R,\max}}{\sqrt{3}V_{R,LL}(pf)} = \frac{1230}{\sqrt{3}\times 500\times 1.0} = 1.42\ \text{kA/phase}$$

The thermal limit of three ACSR 954 kcmil conductors is 3 × 1.01 = 3.03 kA/phase. The current 1.42 kA corresponding to the theoretical steady-state stability limit is well below this thermal limit.

▶ The correct answer is **A**.

P.10 ➡ Solution

The line is energized with 520 kV at the sending end. The sending end voltage per phase is

$$V_S = \frac{520\angle 0°}{\sqrt{3}} = 300\ \text{kV}$$

We need two quantities from the calculations of Problem 6, namely $\beta\ell = 0.00121 \times 330 = 0.399$ rad and $Z_C = 281\ \Omega$. When the line is open, $I_R = 0$ and the no-load receiving end voltage is given by

$$V_{R,nl} = \frac{V_S}{\cos\beta\ell} = \frac{300}{\cos(0.399)} = 326\,\text{kV}$$

The no-load receiving end line-to-line voltage is

$$V_{R,L-L(nl)} = \sqrt{3}V_{R,nl} = \sqrt{3}\times 326 = 565\,\text{kV}$$

For $V_S = V_R$, the required inductor reactance is given by

$$X_{L,\text{sh}} = \frac{\sin\beta\ell}{1-\cos\beta\ell}Z_C = \frac{\sin(0.399)}{1-\cos(0.399)}\times 281 = \boxed{1390\ \Omega}$$

The three-phase shunt reactor rating is

$$Q_{3\phi} = \frac{(kV_{L,\text{rated}})^2}{X_{L,\text{sh}}} = \frac{520^2}{1390} = \boxed{195\ \text{MVAR}}$$

▶ The correct answer is **D**.

P.11 ➡ Solution

From Problem 6, $Z_C = 281\ \Omega$ and $\beta\ell = 0.399$ rad. Thus, the equivalent line reactance for a lossless line is determined as

$$X' = Z_C\sin\beta\ell = 281\sin 0.399 = 109\,\Omega$$

The receiving end power is

$$S_{R(3\phi)} = 1000\angle\cos^{-1}0.8 = 800 + j600\ \text{MVA}$$

We have all quantities needed to determine the power angle δ,

$$P_{3\phi} = \frac{|V_{S(L-L)}||V_{R(L-L)}|}{X'}\sin\delta \to 800 = \frac{520\times 520}{109}\sin\delta$$

$$\therefore \sin\delta = 0.322$$

$$\therefore \delta = 18.8°$$

It follows that the reactive power at the receiving end is

$$Q_{R,3\phi} = \frac{|V_{S(L=L)}||V_{R(L-L)}|}{X'}\cos\delta - \frac{|V_{R(L-L)}|^2}{X'}\cos\beta\ell$$

$$\therefore Q_{R,3\phi} = \frac{520\times 520}{109}\cos 18.8° - \frac{520^2}{109}\cos 0.399 = 62.5\,\text{MVAR}$$

Thus, the required capacitor MVAR is $S_C = j62.5 - j600 = -j538$. The capacitive reactance is given by

$$X_C = \frac{|V_L|^2}{S_C^*} = \frac{520^2}{j538} = -j503\ \Omega$$

so that

$$C = \frac{10^6}{(2\pi \times 60) \times 503} = \boxed{5.27\ \mu\text{F}}$$

▶ The correct answer is **D**.

ANSWER SUMMARY

Problem 1	**1.1**	A
	1.2	D
Problem 2	**2.1**	T/F
	2.2	T/F
Problem 3	**3.1**	T/F
	3.2	Open-ended
	3.3	Open-ended
Problem 4	**4.1**	C
	4.2	C
	4.3	C
Problem 5		D
Problem 6	**6.1**	T/F
	6.2	T/F
Problem 7	**7.1**	T/F
	7.2	T/F
Problem 8		B
Problem 9		A
Problem 10		D
Problem 11		D

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