Quiz FM111
Turbomachines
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PROBLEMS

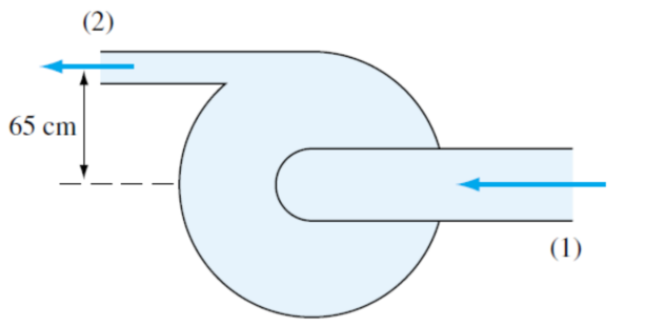
Problem 1 (White, 2003)

A pump delivers 1500 L/min of water at 20°C against a pressure rise of 270 kPa. Kinetic and potential energy losses are negligible. If the driving motor supplies 9 kW, what is the overall efficiency?

A) η = 55%
B) η = 65%
C) η = 75%
D) η = 85%

Problem 2 (White, 2003)

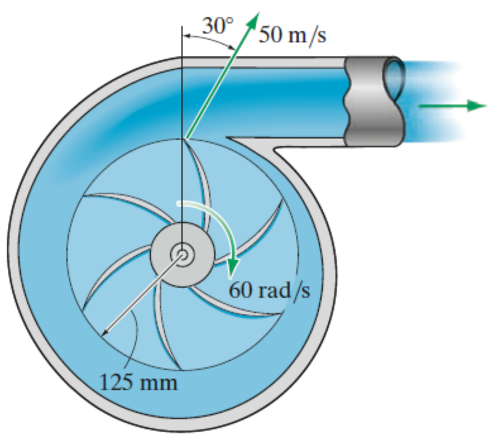
In a test of the pump in the figure, the data are: p_1 = 100 mmHg (vacuum), p_2 = 500 mmHg (gage), D_1 = 12 cm, and D_2 = 5 cm. The flow rate is 180 gal/min of light oil (SG = 0.91). Estimate the input power at 75% efficiency.



A) P = 1120 W
B) P = 1360 W
C) P = 1600 W
D) P = 1850 W

Problem 3 (Hibbeler, 2017, w/ permission)

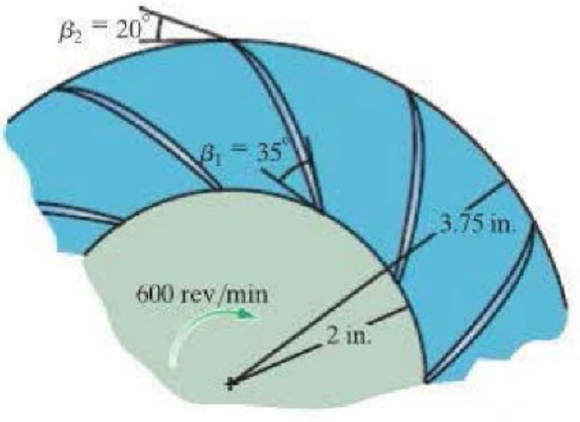
The radial ventilation fan is used to force air into the ducts of a building. If the air is at a temperature of 15°C and the shaft is rotating at 60 rad/s, determine the power output of the motor. Air enters the blades in the radial direction and is discharged with a velocity of 50 m/s at the angle shown.



A) P = 114.5 W
B) P = 154.6 W
C) P = 194.7 W
D) P = 234.6 W

Problem 4 (Hibbeler, 2017, w/ permission)

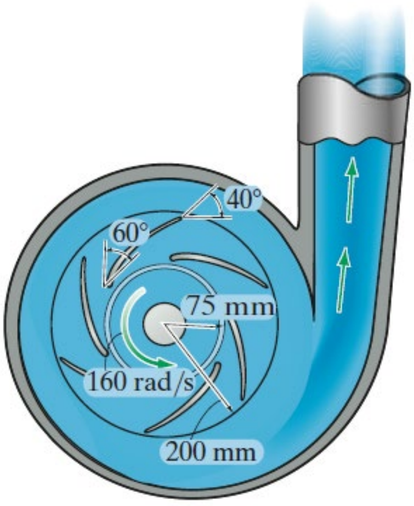
The radial-flow pump impeller rotates at 600 rev/min. If the width of the blades is 2.5 in., and the blade head and tail angles are as shown, determine the ideal power the pump supplies to the water. The water has a temperature of 20°C and is initially guided radially onto the impeller blades.



A) P = 0.615 hp
B) P = 0.985 hp
C) P = 1.44 hp
D) P = 1.82 hp

Problem 5 (Hibbeler, 2017, w/ permission)

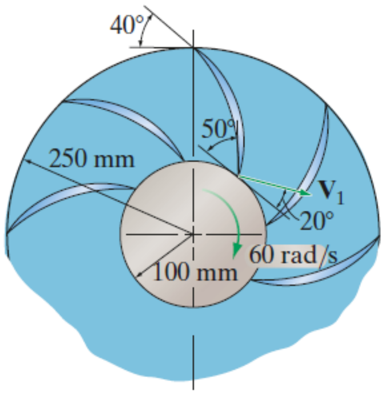
The radial flow pump has a 60-mm wide impeller with the radial dimensions shown. If the blades rotate at 160 rad/s and the discharge is 0.3 m³/s, determine the ideal head developed by the pump.



A) h_{pump} = 60.6 m
B) h_{pump} = 72.7 m
C) h_{pump} = 81.9 m
D) h_{pump} = 90.4 m

Problem 6 (Hibbeler, 2017, w/ permission)

The velocity of water at 15°C flowing onto the 40-mm wide impeller blades of the radial-flow pump is directed at 20° as shown. If the flow leaves the blades at the blade angle of 40°, determine the total head developed by the pump.



A) $h_{pump} = 9.5$ m
B) $h_{pump} = 12.7$ m
C) $h_{pump} = 15.6$ m
D) $h_{pump} = 18.9$ m

Problem 7A (Çengel & Cimbala, 2014, w/ permission)

The performance data for a centrifugal water pump are shown in the table below for water at 20°C. Which of the following values best approximates the head at the best efficiency point (BEP)?

Q (liters/min)	*H* (m)	bhp (W)
0.0	47.5	133
6.0	46.2	142
12.0	42.5	153
18.0	36.2	164
24.0	26.2	172
30.0	15.0	174
36.0	0.0	174

A) $H = 31.1$ m
B) $H = 36.2$ m
C) $H = 40.2$ m
D) $H = 43.2$ m

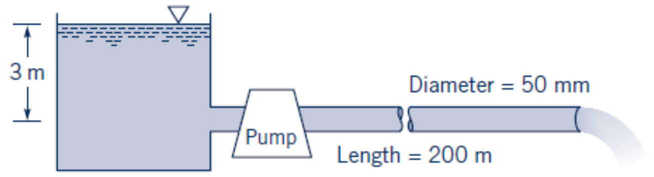
Problem 7B (Çengel & Cimbala, 2014, w/ permission)

Suppose the pump of the previous problem is used in a piping system that has the system requirement $H_{reqd} = (z_2 - z_1) + bQ^2$, where the elevation difference $z_2 - z_1 = 21.7$ m and coefficient $b = 0.0185$ m/(Lpm)^2. Estimate the operating point of the system, namely, Q and H.

A) $Q = 21.7$ L/min and $H = 19.6$ m
B) $Q = 21.7$ L/min and $H = 30.4$ m
C) $Q = 26.9$ L/min and $H = 19.6$ m
D) $Q = 26.9$ L/min and $H = 30.4$ m

Problem 8A (Munson et al., 2009, w/ permission)

Water at 40°C is pumped from an open tank through 200 m of 50-mm diameter smooth horizontal pipe as shown and discharges into the atmosphere with a velocity of 3 m/s. Minor losses are negligible. If the efficiency of the pump is 70%, how much power is being applied to the pump? Neglect losses in the short section of pipe connecting the pump to the tank.



A) P_{shaft} = 0.54 kW

B) P_{shaft} = 1.08 kW

C) P_{shaft} = 1.51 kW

D) P_{shaft} = 2.09 kW

Problem 8B

Considering the system in the previous problem, what is the available NPSH at the pump inlet? Assume standard atmospheric pressure and take the vapor pressure of water to be p_v = 7376 Pa.

A) $NPSH_A$ = 6.72 m

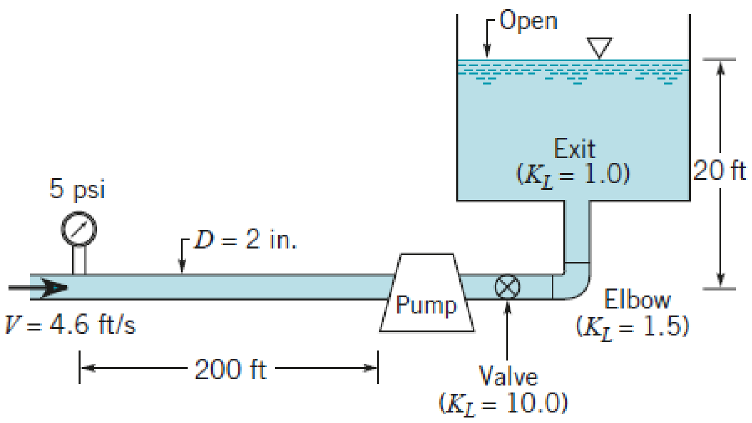
B) $NPSH_A$ = 9.31 m

C) $NPSH_A$ = 12.5 m

D) $NPSH_A$ = 16.8 m

Problem 9 (Munson et al., 2009, w/ permission)

Fuel oil (sp. wt. = 48.0 lb/ft^3, viscosity = 2.0×10^{-5} lb-s/ft^2) is pumped through the piping system of the figure below with a velocity of 4.6 ft/s. The pressure 200 ft upstream of the pump is 5 psi. Pipe losses downstream of the pump are negligible, but minor losses are not (minor loss coefficients are given in the figure). For a pump operating speed of 1750 rpm, what type of pump would you recommend for this application? Use Figure 1 as an aid. The pipe diameter is 2 in. and the relative roughness is ε/D = 0.001.



A) Use a radial flow pump.

B) Use a mixed flow pump.

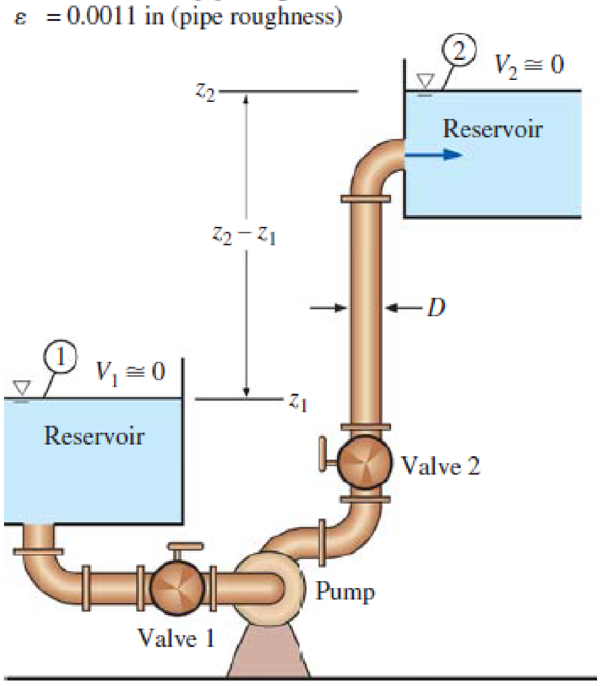
C) Use an axial flow pump.

D) The information is not sufficient to determine the best type of pump.

Problem 10 (Çengel & Cimbala, 2014, w/ permission)

A water pump is used to pump water from one large reservoir to another large reservoir that is at a higher elevation. The free surfaces of both reservoirs are exposed to atmospheric pressure, as sketched below. The dimensions and minor loss coefficients are provided in the figure. The pump's performance is approximated by the expression $H_{\text{available}} = H_o - aQ^2$, where the shutoff head H_o = 125 ft of water column, and capacity Q is in units of gallons per minute (gpm). Estimate the capacity delivered by the pump. Use 6.57×10^{-4} lbm/ft·s as the viscosity of water.

$z_2 - z_1$ = 22.0 ft (elevation difference)
D = 1.20 in (pipe diameter)
$K_{L,\text{ entrance}}$ = 0.50 (pipe entrance)
$K_{L,\text{ valve 1}}$ = 2.0 (valve 1)
$K_{L,\text{ valve 2}}$ = 6.8 (valve 2)
$K_{L,\text{ elbow}}$ = 0.34 (each elbow—there are 3)
$K_{L,\text{ exit}}$ = 1.05 (pipe exit)
L = 124 ft (total pipe length)
ε = 0.0011 in (pipe roughness)



A) Q = 6.34 gpm

B) Q = 8.45 gpm

C) Q = 10.51 gpm

D) Q = 12.60 gpm

Problem 11 (Hibbeler, 2017, w/ permission)

A 200-mm diameter impeller of a radial-flow water pump rotates at 150 rad/s and produces a change in ideal head of 0.3 m. Determine the change in head for a geometrically similar pump that has an impeller diameter of 100 mm and operates at 80 rad/s.

A) ΔH_2 = 0.021 m

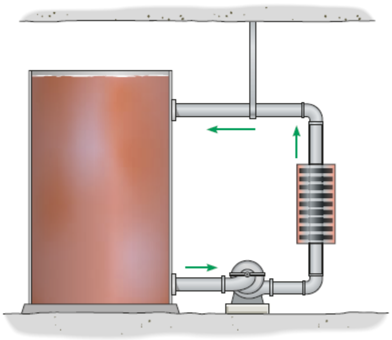
B) ΔH_2 = 0.055 m

C) ΔH_2 = 0.087 m

D) ΔH_2 = 0.103 m

Problem 12 (Hibbeler, 2017, w/ permission)

The temperature of benzene in a processing tank is maintained by recycling this liquid through a heat exchanger, using a pump that has an impeller speed of 1750 rpm and produces a flow of 900 gal/min. If it is found that the heat exchanger can maintain the temperature only when the flow is 650 gal/min, determine the required angular speed of the impeller.



A) ω_2 = 877 rpm

B) ω_2 = 1089 rpm

C) ω_2 = 1264 rpm

D) ω_2 = 1428 rpm

Problem 13 (Hibbeler, 2017, w/ permission)

The model of a water pump has an impeller with a diameter of 4 in. that discharges 80 gal/min. If the power required is 1.5 hp, determine the power required for the prototype having an impeller diameter of 12 in. that will discharge 600 gal/min.

A) P_2 = 5.5 hp

B) P_2 = 7.8 hp

C) P_2 = 10.0 hp

D) P_2 = 12.6 hp

Problem 14 (Hibbeler, 2017, w/ permission)

The model of a water pump has an impeller with a diameter of 4 in. that discharges 80 gal/min with a pressure head of 4 ft. Determine the diameter of the impeller of the prototype that will discharge 600 gal/min with a pressure head of 24 ft.

A) D_2 = 5 in.

B) D_2 = 7 in.

C) D_2 = 9 in.

D) D_2 = 11 in.

ADDITIONAL INFORMATION

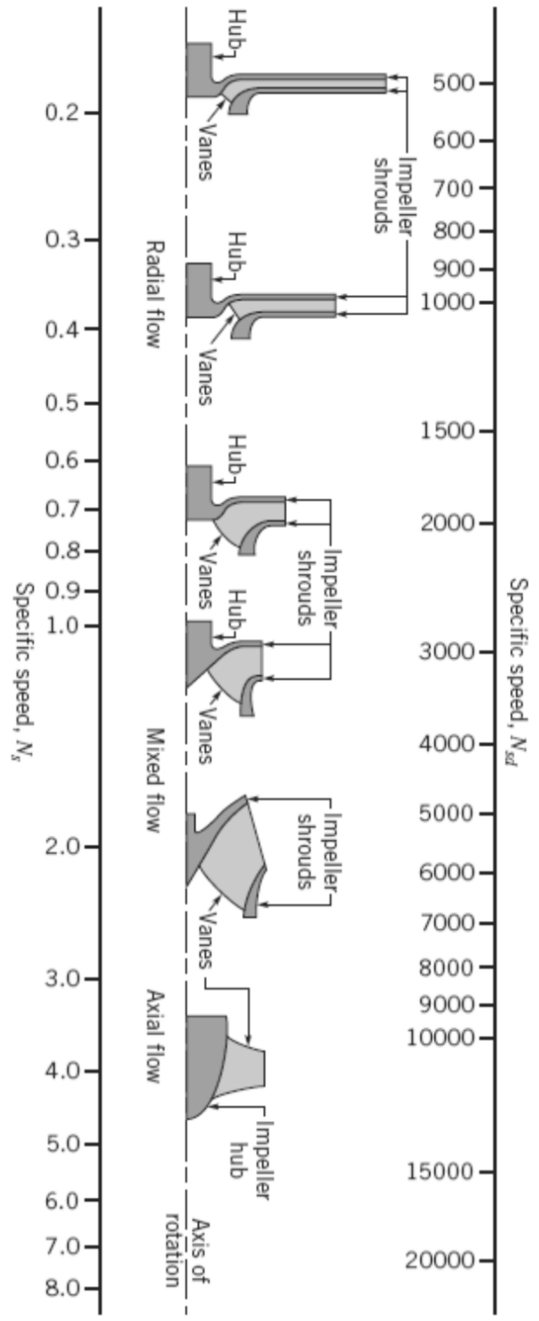
Table 1 Density of water as a function of temperature at p = 1 atm

Temperature (°C)	Density (kg/m³)	Density (sl/ft³)
5	999.90	1.940
10	999.97	1.940
15	999.10	1.940
20	998.21	1.940
25	997.05	1.935
30	995.65	1.932
35	994.03	1.929
40	992.22	1.925

Table 2 Density of air as a function of temperature at p = 1 atm

Temperature (°C)	Density (kg/m³)	Density (sl/ft³)
5	1.268	2.461
10	1.246	2.418
15	1.225	2.376
20	1.204	2.336
25	1.184	2.297
30	1.164	2.259
40	1.127	2.188
50	1.093	2.120

Figure 1 Variation in specific speed at maximum efficiency with type of pump.



(N_s is the dimensionless specific speed $N_s = \omega\sqrt{Q}/\left(gh_p\right)^{3/4}$, while N_{sd} is the dimensional form of specific speed commonly used in the United States,

$$N_{sd} = \omega[\text{rpm}]\sqrt{Q[\text{gpm}]}/\left[h_p[\text{ft}]\right]^{3/4}$$

Both N_s and N_{sd} have the same physical meaning, but their magnitudes will differ by a constant conversion factor ($N_{\text{sd}} = 2733N_s$) when ω is expressed in rad/s.)

SOLUTIONS

P.1 ● Solution

The power can be determined with the simple relation $P = Q\Delta p$; that is,

$$P = \rho g Q H = Q\Delta p = \left(\frac{1.5\ \text{m}^3}{1\,\text{min}} \times \frac{1\ \text{min}}{60\,\text{s}}\right) \times 270 = 6.75\ \text{kW}$$

The efficiency is then

$$\eta = \frac{6.75}{9} = \boxed{75\%}$$

➡ The correct answer is **C**.

P.2 ● Solution

We begin by converting 100 mmHg = 13,332 Pa, 500 mmHg = 66,661 Pa, and 180 gal/min = 0.01136 m³/s. Velocities V_1 and V_2 are such that $V_1 = Q/A_1 = 0.01136/[(\pi/4)(0.12)^2] = 1.00$ m/s and $V_2 = Q/A_2 = 0.01136/[(\pi/4)(0.05)^2] = 5.79$ m/s. The specific weight of the oil is $\gamma = 0.91 \times 9810 = 8927$ N/m². The head H can be obtained from the Bernoulli equation,

$$H = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 - \frac{p_1}{\gamma} - \frac{V_1^2}{2g} - z_1 = \frac{66{,}661}{8927} + \frac{5.79^2}{2 \times 9.81} + 0.65 - \frac{13{,}332}{8927} - \frac{1.00^2}{2 \times 9.81} - 0$$

$$\therefore H = 8.28\,\text{m}$$

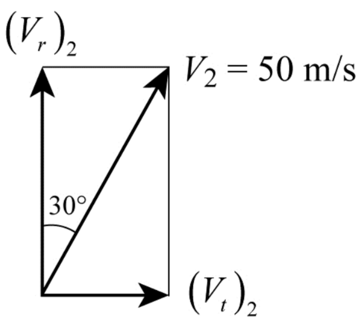
The input power at 75% efficiency is then

$$P = \frac{\gamma Q H}{\eta} = \frac{8927 \times 0.01136 \times 8.28}{0.75} = \boxed{1120\,\text{W}}$$

➡ The correct answer is **A**.

P.3 ● Solution

The air is considered to be incompressible and flow is assumed to be steady. Consider the velocity triangle drawn below.



The tangential and radial velocity components can be obtained with simple trigonometry,

$$(V_t)_2 = V_2 \sin\theta = 50 \times \sin 30^\circ = 25\,\text{m/s}$$
$$(V_r)_2 = V_2 \cos\theta = 50 \times \cos 30^\circ = 43.3\,\text{m/s}$$

Given the radius r_2 of the impeller blade and its width h, the surface area of the pump is determined as

$$A_2 = 2\pi r_2 h = 2\pi \times 0.125 \times 0.03 = 0.0236\ \text{m}^2$$

The flow rate is

$$Q = (V_r)_2 A_2 = 43.3 \times 0.0236 = 1.022\ \text{m}^3/\text{s}$$

The torque applied by the motor follows as

$$T = \rho Q\left[r_2 (V_t)_2 - r_1 (V_t)_1\right] = 1.225 \times 1.022 \times \left[0.125 \times 25 - r_1 \times 0\right] = 3.91\ \text{N} \cdot \text{m}$$

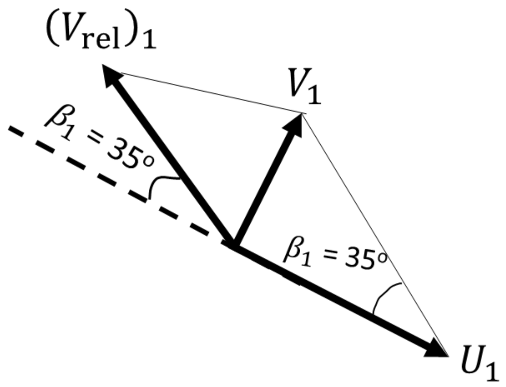
where we have used 1.225 kg/m³ as the density of air, in accordance with Table 2. Note that we have $(V_t)_1 = 0$, because air enters the blade radially. We proceed to compute the power output to the motor, P, which is given by the product of torque and angular velocity,

$$P = T\omega = 3.91 \times 60 = \boxed{234.6\ \text{W}}$$

➡ The correct answer is **D**.

P.4 ● Solution

We draw the velocity diagram of the water on the head of the blade as shown.



The water is considered to be incompressible, and the flow is assumed to be steady. The speed of the point on the blade head, U_1, is determined as

$$U_1 = \omega r_1 = \left(600 \times \frac{1}{60} \times 2\pi\right) \times \left(\frac{2}{12}\right) = 10.47\ \text{ft/s}$$

The speed of the point on the blade tail, in turn, is

$$U_2 = \left(600 \times \frac{1}{60} \times 2\pi\right) \times \left(\frac{3.75}{12}\right) = 19.64\ \text{ft/s}$$

The water is initially guided radially onto the propeller blades, hence $V_1 = (V_r)_1$ and $(V_t)_1 = 0$. The velocity of the blade, V_1, can be obtained with trigonometry,

$$\tan\beta_1 = \frac{V_1}{U_1} \rightarrow V_1 = U_1 \tan\beta_1$$

$$\therefore V_1 = 10.47 \times \tan 35^\circ = 7.33\ \text{ft/s}$$

The surface area of the pump impeller is

$$A_1 = 2\pi r_1 b = 2\pi \times \left(2 \times \frac{1}{12}\right) \times \left(2.5 \times \frac{1}{12}\right) = 0.218\ \text{ft}^2$$

The flow rate is

$$Q = V_1 A_1 = 7.33 \times 0.218 = 1.598\ \text{m}^3/\text{s}$$

We proceed to calculate the ideal pump head,

$$h_{\text{pump}} = \frac{U_2^2}{g} - \frac{U_2 Q \cot\beta_2}{2\pi r_2 b g} = \frac{19.64^2}{32.2} - \frac{19.64 \times 1.598 \times \cot 20^\circ}{2\pi \times \left(3.75 \times \frac{1}{12}\right) \times \left(2.5 \times \frac{1}{12}\right) \times 32.2} = 5.43\ \text{ft}$$

Finally, the ideal power is determined to be

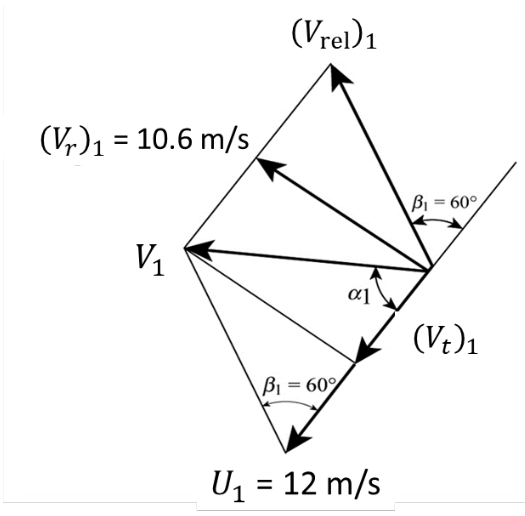
$$P = \rho g Q h_{\text{pump}} = 1.94 \times 32.2 \times 1.598 \times 5.43 = 542\ \text{ft}\cdot\text{lb/s}$$

$$\therefore P = 542 \frac{\text{ft}\cdot\text{lb}}{\text{s}} \times \frac{1\,\text{hp}}{550 \frac{\text{ft}\cdot\text{lb}}{\text{s}}} = \boxed{0.985\,\text{hp}}$$

➡ The correct answer is **B**.

P.5 ● Solution

The velocity diagram for this system is provided below.



Water is assumed to be incompressible and the relative flow is steady. The speed of the point on the blade head is

$$U_1 = \omega r_1 = 0.075 \times 160 = 12 \text{ m/s}$$

The surface area of the pump impeller is

$$A_1 = 2\pi r_1 b = 2\pi \times 0.075 \times 0.06 = 0.0283\,\text{m}^2$$

The radial velocity component of the water at the head is given by

$$Q = (V_r)_1 A_1 \rightarrow 0.3 = (V_r)_1 \times 0.0283$$

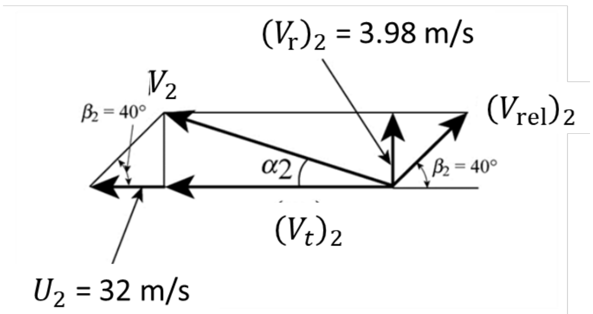
$$\therefore (V_r)_1 = \frac{0.3}{0.0283} = 10.6 \text{ m/s}$$

The tangential component of velocity at the head, in turn, is such that

$$\tan\beta_1 = \frac{(V_r)_1}{U_1 - (V_t)_1} \rightarrow \tan 60^\circ = \frac{10.6}{12 - (V_t)_1}$$

$$\therefore (V_t)_1 = 5.88\,\text{m/s}$$

We then outline the velocity diagram for the tail of the blade.



The speed U_2 of the point on the blade tail is

$$U_2 = \omega r_2 = 0.2 \times 160 = 32 \text{ m/s}$$

The surface area of the pump impeller is

$$A_2 = 2\pi r_2 b = 2\pi \times 0.2 \times 0.06 = 0.0754 \text{ m}^2$$

The radial velocity component of the water at the tail $(V_r)_2$ follows as

$$Q = (V_r)_2 A_2 \rightarrow (V_r)_2 = \frac{Q}{A_2}$$

$$\therefore (V_r)_2 = \frac{0.3}{0.0754} = 3.98\,\text{m/s}$$

Using trigonometry as before, we can determine the tangential velocity of water at the tail,

$$\tan\beta_2 = \frac{(V_r)_2}{U_2 - (V_t)_2} \rightarrow \tan 40^\circ = \frac{3.98}{32 - (V_t)_2}$$

$$\therefore (V_t)_2 = 27.3 \text{ m/s}$$

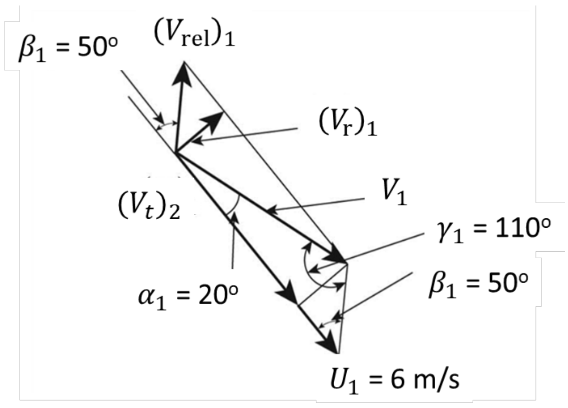
Lastly, the ideal pump head is calculated to be

$$h_{\text{pump}} = \frac{U_2 (V_t)_2 - U_1 (V_t)_1}{g} = \frac{32 \times 27.3 - 12 \times 5.88}{9.81} = \boxed{81.9 \text{ m}}$$

➡ The correct answer is **C**.

P.6 ● Solution

We begin by drawing the velocity diagram of the water on the head of the blade.



The water is assumed to be incompressible, and the flow is assumed to be steady. The speed of the point located on the blade head can be determined as

$$U_1 = \omega r_1 = 60 \times 0.1 = 6 \text{ m/s}$$

Flow velocity V_1 can be obtained with the sine rule,

$$\frac{V_1}{\sin\beta_1} = \frac{U_1}{\sin\gamma_1} \rightarrow V_1 = \frac{\sin\beta_1}{\sin\gamma_1} U_1$$

$$\therefore V_1 = \frac{\sin 50^\circ}{\sin 110^\circ} \times 6 = 4.89 \text{ m/s}$$

Using trigonometry, the radial velocity component of the water at the head, $(V_r)_1$, is computed as

$$\sin\alpha_1 = \frac{(V_r)_1}{V_1} \rightarrow (V_r)_1 = V_1 \sin\alpha_1$$

$$\therefore (V_r)_1 = 4.89 \times \sin 20^\circ = 1.67 \text{ m/s}$$

Similarly, the tangential velocity component of the water at the head, $(V_t)_1$, follows as

$$\cos\alpha_1 = \frac{(V_t)_1}{V_1} \rightarrow (V_t)_1 = V_1 \cos\alpha_1$$

$$\therefore (V_t)_1 = 4.89 \times \cos 20^\circ = 4.60 \text{ m/s}$$

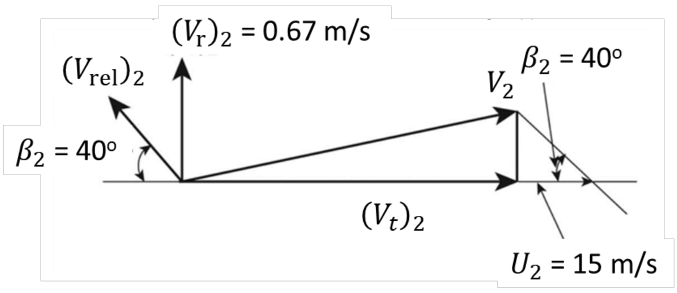
We then proceed to determine the surface area of the pump impeller,

$$A_1 = 2\pi r_1 b = 2\pi \times 0.1 \times 0.04 = 0.0251 \text{ m}^2$$

The flow rate of the pump is

$$Q = (V_r)_1 A_1 = 1.67 \times 0.0251 = 0.0419 \text{ m}^3/\text{s}$$

Next, consider the velocity diagram in the tail of the blade.



The speed U_2 of the point located on the blade tail is

$$U_2 = \omega r_2 = 60 \times 0.25 = 15 \text{ m/s}$$

The surface area of the pump impeller, A_2, is

$$A_2 = 2\pi r_2 b = 2\pi \times 0.25 \times 0.04 = 0.0628 \text{ m}^2$$

where r_2 is the radius of the impeller blade at the tail. The radial component of velocity of the water at the tail follows as

$$Q = (V_r)_2 A_2 \rightarrow (V_r)_2 = \frac{Q}{A_2}$$

$$\therefore (V_r)_2 = \frac{0.0419}{0.0628} = 0.67 \text{ m/s}$$

The tangential velocity component of the water at the tail, $(V_t)_2$, is obtained with trigonometry,

$$\tan\beta_2 = \frac{(V_r)_2}{U_2 - (V_t)_2} \rightarrow \tan 40^\circ = \frac{0.67}{15 - (V_t)_2}$$

$$\therefore (V_t)_2 = 14.2 \text{ m/s}$$

Finally, the ideal pump head is found as

$$h_{\text{pump}} = \frac{U_2 (V_t)_2 - U_1 (V_t)_1}{g} = \frac{15 \times 14.2 - 6 \times 4.60}{9.81} = \boxed{18.9 \text{ m}}$$

➡ The correct answer is **D**.

P.7 ● Solution

Part A: The pump efficiency is determined with the relation

$$\eta = \frac{\rho g Q H}{P}$$

Using Table 1, the density ρ of water at 20°C is taken as ρ = 998.21 kg/m³. The best efficiency point is the one that corresponds to the highest efficiency. Computations can be done with a spreadsheet. As an example, we compute the efficiency for the second row of data, namely,

$$\eta_{\text{pump}} = \frac{\rho g Q H}{P} = \frac{998.21 \times 9.81 \times \left(6 \times 10^{-3} \times \frac{1}{60}\right) \times 46.2}{142} = 31.9\%$$

The calculations are summarized below.

Q (liters/min)	H (m)	bhp (W)	*Efficiency*
0.0	47.5	133	0
6.0	46.2	142	31.9%
12.0	42.5	153	54.4%
18.0	36.2	164	64.8%
24.0	26.2	172	59.7%
30.0	15.0	174	42.2%
36.0	0.0	174	0

The best efficiency point occurs approximately at the fourth row of data, in which η = 64.8%. The corresponding head is H = 36.2 m.

➡ The correct answer is **B**.

Part B: In order to compute the operation point, we require a curve fit for the available head, $H_{available}$, which has the form

$$H_{\text{available}} = H_o - aQ^2$$

In Mathematica, this can be accomplished with the *Fit* function. We first type the data,

$$\text{data} = \{\{0,47.5\},\{6,46.2\},\{12,42.5\},\{18,36.2\},\{24,26.2\},\{30,15.\},\{36.,0\}\}$$

Then, we apply *Fit*,

$$\text{line} = \text{Fit}[\text{data},\{1,Q^2\},Q]$$

This yields an expression with the form

$$H_{\text{available}} = 47.66 - 0.0366Q^2$$

Then, we equate the expression for available head to that for required head, giving

$$H_{\text{available}} = H_{\text{reqd}}$$

$$47.66 - 0.0366Q^2 = 21.7 + 0.0185Q^2$$

$$\therefore Q = \sqrt{\frac{47.66-21.7}{0.0551}} = \boxed{21.7\text{ L/min}}$$

The corresponding head can be obtained by substituting this volume flow rate into the expression for $H_{available}$,

$$H = 47.66 - 0.0366\times 21.7^2 = \boxed{30.4\text{ m}}$$

➡ The correct answer is **B**.

P.8 ● Solution

Part A: We begin by computing the Reynolds number for the pipe flow,

$$\text{Re} = \frac{V_2 D}{\nu} = \frac{3\times 0.05}{6.58\times 10^{-7}} = 2.28\times 10^5$$

To obtain the corresponding friction factor for the pipe, we substitute $\varepsilon/D = 0$ (i.e., the pipe is smooth) and $Re = 2.28 \times 10^5$ in the Colebrook equation and solve it for f, giving

$$\frac{1}{\sqrt{f}} = -2.0\log\left(\frac{\varepsilon/D}{3.7}+\frac{2.51}{\text{Re}\sqrt{f}}\right) \rightarrow \frac{1}{\sqrt{f}} = -2.0\log\left(0+\frac{2.51}{2.28\times 10^5\sqrt{f}}\right)$$

$$\therefore f = 0.0152$$

Next, we compute the head loss due to friction, h_f,

$$h_f = \frac{fLV_2^2}{2gD} = \frac{0.0152\times 200\times 3^2}{2\times 9.81\times 0.05} = 27.89\text{ m}$$

The head gain h_a by the pump is established with the Bernoulli equation,

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1+h_a = \frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+h_f$$

$$\therefore h_a = \frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+h_f-\frac{p_1}{\gamma}-\frac{V_1^2}{2g}-z_1$$

$$\therefore h_a = \frac{0}{9810}+\frac{3^2}{2\times 9.81}+0+27.89-\frac{0}{9810}-\frac{0^2}{2\times 9.81}-3 = 25.35\text{ m}$$

The cross-sectional area of the pipe is

$$A = \frac{\pi}{4}\times D^2 = \frac{\pi}{4}\times 0.05^2 = 0.00196\text{ m}^2$$

and the flow rate is

$$Q = AV_2 = 0.00196 \times 3 = 0.00589\ \text{m}^3/\text{s}$$

We are then able to determine the power gained by the fluid,

$$P_{\text{fluid}} = \gamma Q h_a = 9.81 \times 0.00589 \times 25.35 = 1.46\ \text{kW}$$

Finally, the shaft power supplied to the pump can be obtained with the relation

$$\eta = \frac{P_{\text{fluid}}}{P_{\text{shaft}}} \rightarrow P_{\text{shaft}} = \frac{P_{\text{fluid}}}{\eta}$$

$$\therefore P_{\text{shaft}} = \frac{1.46}{0.7} = \boxed{2.09\ \text{kW}}$$

➡ The correct answer is **D**.

Part B: It remains to compute the available net positive suction head, $NPSH_A$. Since minor losses are negligible, we can write

$$NPSH_A = \frac{p_{\text{atm}}}{\gamma} + z_1 - \frac{p_v}{\gamma}$$

Substituting $p_{atm} = 1.01 \times 10^5$ Pa, $p_v = 7376$ Pa, $\gamma = 9810$ N/m^3, and $z_1 = 3$ m gives

$$NPSH_A = \frac{1.01 \times 10^5}{9810} + 3 - \frac{7376}{9810} = \boxed{12.5\ \text{m}}$$

➡ The correct answer is **C**.

P.9 ● Solution

We begin by applying the Bernoulli equation to the system,

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + \Sigma h_L$$

Using $\gamma = 48.0$ lb/ft^3, $p_1 = 5$ psi, $p_2 = 0$, $V_1 = 4.6$ ft/s, $V_2 = 0$, and $z_2 - z_1 = 20$ ft, we have

$$\frac{(5 \times 144)}{48} + \frac{4.6^2}{2 \times 32.2} + z_1 + h_p = 0 + 0 + z_2 + \Sigma h_L$$

$$\therefore \frac{(5 \times 144)}{48} + \frac{4.6^2}{2 \times 32.2} + h_p = 0 + 0 + z_2 - z_1 + \Sigma h_L$$

$$\therefore \frac{(5 \times 144)}{48} + \frac{4.6^2}{2 \times 32.2} + h_p = 20 + \Sigma h_L$$

The sum of minor head losses is given by

$$\Sigma h_L = \left[(K_L)_{\text{valve}} + (K_L)_{\text{elbow}} + (K_L)_{\text{exit}} + f\frac{L}{D} \right] \frac{V^2}{2g}$$

$$\therefore \Sigma h_L = \left[10 + 1.5 + 1 + f\frac{200}{(2/12)} \right] \times \frac{4.6^2}{2 \times 32.2}$$

To obtain the friction factor f, we require the Reynolds number,

$$\text{Re} = \frac{\rho V D}{\mu} = \frac{\left(\frac{48}{32.2}\right) \times 4.6 \times \left(\frac{2}{12}\right)}{2 \times 10^{-5}} = 5.71 \times 10^4$$

Using this quantity and the relative roughness $\varepsilon/D = 0.001$, the friction factor can be determined with the Colebrook equation, namely,

$$\frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{0.001}{3.7} + \frac{2.51}{5.71 \times 10^4 \sqrt{f}} \right) \rightarrow f = 0.0236$$

The sum of local head losses is then

$$\Sigma h_L = \left[10+1.5+1+0.0236\times\frac{200}{(2/12)}\right]\times\frac{4.6^2}{2\times 32.2} = 13.08\,\text{ft}$$

Returning to the Bernoulli equation with this result and solving for h_p, it follows that

$$\frac{5\times 144}{48}+\frac{4.6^2}{2\times 32.2}+h_p = 20+13.08$$

$$\therefore h_p = 17.75\ \text{m}$$

The flow rate for this system is

$$Q = VA = 4.6\times\left[\frac{\pi}{4}\times\left(\frac{2}{12}\right)^2\right] = 0.1\ \text{ft}^3/\text{s}$$

Converting this to gallons per minute, we have

$$Q = 0.1\times(7.48\times 60) = 44.9\ \text{gpm}$$

We then proceed to compute the specific speed,

$$N_{sd} = \frac{\omega[\text{rpm}]\sqrt{Q[\text{gpm}]}}{(h_p[\text{ft}])^{3/4}} = \frac{1750\times\sqrt{44.9}}{17.75^{3/4}} = 1356$$

In accordance with Figure 1, the pump recommended for this application is the radial-flow pump.

➡ The correct answer is **A**.

P.10 ● Solution

We apply the energy equation in head form between the inlet reservoir's free surface (1) and the outlet reservoir's free surface (2),

$$H_{\text{reqd}} = h_{\text{pump,u}} = \cancel{\frac{p_2-p_1}{\rho g}}+\frac{\cancel{\alpha_2 V_2^2}-\cancel{\alpha_1 V_1^2}}{2g}+(z_2-z_1)+\cancel{h_{\text{turbine}}}+h_{L,\text{total}}$$

Since both free surfaces are at atmospheric pressure, $p_1 = p_2 = p_{atm}$, and the first term on the right-hand side of the foregoing equation vanishes. Furthermore, since there is no flow, $V_1 = V_2 = 0$, and the second term also vanishes. There is no turbine in the control volume, so the second-to-last term is also zero. Finally, the irreversible head losses are composed of both major and minor losses, but the pipe diameter is constant throughout. The energy equation then reduces to

$$H_{\text{reqd}} = (z_2-z_1)+h_{\text{L,total}} = (z_2-z_1)+\left(f\frac{L}{D}+\Sigma K_L\right)\frac{V^2}{2g}$$

The sum of minor loss coefficients is

$$\Sigma K_L = K_{L,\text{entrance}}+K_{L,\text{valve 1}}+K_{L,\text{valve 2}}+3\times K_{L,\text{elbow}}+K_{L,\text{exit}}$$

$$\therefore \Sigma K_L = 0.50+2.0+6.8+3\times 0.34+1.05 = 11.37$$

The pump/piping system operates at conditions where the available pump head equals the required system head. Thus, we equate the relation we were given for $H_{available}$ and the one of required head to obtain

$$H_o - a\left(\frac{\pi D^2}{4}\right)^2 V^2 = (z_2-z_1)+\left(f\frac{L}{D}+\Sigma K_L\right)\frac{V^2}{2g}$$

$$\therefore 125-2.50\times\left[\frac{\pi\times\left(\frac{1.20}{12}\right)^2}{4}\times 7.48\times 60\right]^2 V^2 = 22.0+\left[f\frac{124}{\left(\frac{1.2}{12}\right)}+11.37\right]\frac{V^2}{2\times 32.2}$$

$$\therefore 125-31.06V^2 = 22.0+(1240f+11.37)\frac{V^2}{64.40}$$

$$\therefore 103-31.06V^2 = (19.25f+0.1765)V^2\quad\text{(I)}$$

Now, consider the Colebrook equation,

$$\frac{1}{\sqrt{f}} = -2.0\log\left(\frac{\varepsilon/D}{3.7}+\frac{2.51}{\text{Re}\sqrt{f}}\right)$$

The relative roughness is ε/D = 0.0011/1.20 = 9.16 × 10^{-4}. As for the Reynolds number, substitute the appropriate values of density (= 62.3 lb/ft³), viscosity (= 6.57×10^{-4} lb/ft-s), and diameter (= 1.20/12 = 0.1 ft) to obtain

$$\text{Re} = \frac{62.3\times(1.20/12)\times V}{6.57\times 10^{-4}} = 9482.5V$$

Substituting this result into the Colebrook equation gives

$$\frac{1}{\sqrt{f}} = -2.0\log\left(\frac{9.16\times 10^{-4}}{3.7}+\frac{2.51}{9482.5V\sqrt{f}}\right) \quad \text{(II)}$$

To obtain the velocity, we shall solve equations (I) and (II) simultaneously. In Mathematica, this can be done via the command *NSolve*,

$$\text{NSolve}[\{103 - 31.06V^2 == (19.25f + 0.1765)V^2, \frac{1}{\sqrt{f}} = = -2.* \text{Log10}[\frac{9.16*10^{-4}}{3.7}+\frac{2.51}{9482.5V\sqrt{f}}]\}, \{V, f\}, \text{Reals}]$$

This returns f = 0.0287 and V = 1.80 ft/s. The latter is the result that we seek. The capacity delivered by the pump can now be computed,

$$Q = VA = 1.8\times\left[\frac{\pi\times\left(\frac{1.20}{12}\right)^2}{4}\right] = 0.0141 \text{ ft}^3/\text{s}$$

$$\therefore Q = 0.0141\times 7.48 \text{ gallon}\times 60 \text{ min} = \boxed{6.34 \text{ gpm}}$$

➡ The correct answer is **A**.

P.11 ● Solution

Given the dynamic similitude of the two pumps, we can surmise that both of them have the same head coefficient. Mathematically,

$$\frac{\Delta H_1}{\omega_1 D_1^2} = \frac{\Delta H_2}{\omega_2 D_2^2}$$

Substituting ΔH_1 = 0.3 m, ω_1 = 150 rad/s, D_1 = 200 mm, ω_2 = 80 rad/s, and D_2 = 100 mm yields

$$\frac{0.3}{150^2\times 200^2} = \frac{\Delta H_2}{80^2\times 100^2}$$

$$\therefore \Delta H_2 = \frac{80^2\times 100^2}{150^2\times 200^2}\times 0.3 = \boxed{0.021 \text{ m}}$$

The change in head for the geometrically similar pump is considerably smaller than that of the original pump. This is expected, since the second pump has a smaller diameter and a smaller rotation.

➡ The correct answer is **A**.

P.12 ● Solution

Given the dynamic similitude of both situations, we postulate that the flow coefficient is the same for both configurations, so that

$$\frac{Q_1}{\omega_1 D^3} = \frac{Q_2}{\omega_2 D^3}$$

$$\therefore \frac{900}{1750D^3} = \frac{650}{\omega_2 D^3}$$

$$\therefore \omega_2 = \frac{650}{900} \times 1750 = \boxed{1264 \text{ rpm}}$$

➡ The correct answer is **C**.

P.13 ● Solution

Given the similitude between the pump and the prototype, we postulate that their power coefficients are the same. Thus,

$$\frac{P_1}{\omega_1^3 D_1^5} = \frac{P_2}{\omega_2^3 D_2^5}$$

$$\frac{1.5}{\omega_1^3 \times 4^5} = \frac{P_2}{\omega_2^3 \times 12^5}$$

$$P_2 = \frac{12^5 \times 1.5}{4^5}\left(\frac{\omega_2}{\omega_1}\right)^3 = 364.5\left(\frac{\omega_2}{\omega_1}\right)^3 \quad \text{(I)}$$

We then equate the flow coefficients of pump and prototype,

$$\frac{Q_1}{\omega_1 D_1^3} = \frac{Q_2}{\omega_2 D_2^3}$$

$$\therefore \frac{80}{\omega_1 \times 4^3} = \frac{600}{\omega_2 \times 12^3}$$

$$\therefore \frac{\omega_2}{\omega_1} = \frac{600 \times 4^3}{80 \times 12^3} = 0.278$$

Substituting this result in equation (I), we obtain

$$P_2 = 364.5 \times (0.278)^3 = \boxed{7.8 \text{ hp}}$$

Since the prototype has greater values of discharge and impeller diameter, its power is naturally greater than that of the first pump.

➡ The correct answer is **B**.

P.14 ● Solution

Since the pump and the prototype are dynamically similar, we can write, for the head coefficients,

$$\frac{\Delta H_1}{\omega_1^2 D_1^2} = \frac{\Delta H_2}{\omega_2^2 D_2^2}$$

$$\frac{4}{\omega_1^2 \times 4^2} = \frac{24}{\omega_2^2 \times D_2^2}$$

$$\therefore \left(\frac{\omega_2}{\omega_1}\right)^2 = \frac{24 \times 4^2}{4 D_2^2} = \frac{96}{D_2^2} \quad \text{(I)}$$

Next, consider the equality of flow coefficient for both devices,

$$\frac{Q_1}{\omega_1 D_1^3} = \frac{Q_2}{\omega_2 D_2^3}$$

$$\frac{80}{\omega_1 \times 4^3} = \frac{600}{\omega_2 \times D_2^3}$$

$$\frac{\omega_2}{\omega_1} = \frac{600 \times 4^3}{80 D_2^3} = \frac{480}{D_2^3}$$

Substituting the ratio of angular velocities above into equation (I) and solving for D_2, it follows that

$$\left(\frac{\omega_2}{\omega_1}\right)^2 = \left(\frac{480}{D_2^3}\right)^2 = \frac{96}{D_2^2}$$

$$\therefore \frac{480^2}{D_2^6} = \frac{96}{D_2^2}$$

$$\therefore D_2^4 = \frac{480^2}{96}$$

$$\therefore D_2 = \sqrt[4]{\frac{480^2}{96}} = \boxed{7 \text{ in.}}$$

➡ The correct answer is **B**.

ANSWER SUMMARY

Problem 1		**C**
Problem 2		**A**
Problem 3		**D**
Problem 4		**B**
Problem 5		**C**
Problem 6		**D**
Problem 7	**7A**	**B**
	7B	**B**
Problem 8	**8A**	**D**
	8B	**C**
Problem 9		**A**
Problem 10		**A**
Problem 11		**A**
Problem 12		**C**
Problem 13		**B**
Problem 14		**B**

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