Quiz VB105
Vibration Isolation and Absorption
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PROBLEMS

Problem 1 (Palm, 2007)

An undamped isolator is designed for a 30-kg mass subjected to a harmonic forcing function whose amplitude and frequency are 1000 N and 20 Hz, respectively. The isolator should transmit to the base no more than 10% of the applied force. What are the stiffness and displacement amplitude of the isolator?

A) $k = 4.31 \times 10^4$ N/m and $X = 2.32 \times 10^{-3}$ m
B) $k = 4.31 \times 10^4$ N/m and $X = 5.64 \times 10^{-3}$ m
C) $k = 8.62 \times 10^4$ N/m and $X = 2.32 \times 10^{-3}$ m
D) $k = 8.62 \times 10^4$ N/m and $X = 5.64 \times 10^{-3}$ m

Problem 2 (Inman, 2014, w/ permission)

A small compressor weighs about 70 lb and runs at 900 rpm. The compressor is mounted on a single support made of metal with negligible damping. Design the stiffness of the support so that only 15% of the harmonic force produced by the compressor is transmitted to the foundation.

A) k = 60.5 lb/in.
B) k = 200 lb/in.
C) k = 350 lb/in.
D) k = 490 lb/in.

Problem 3 (Inman, 2014, w/ permission)

The instrument board of an aircraft is mounted on an isolation pad to protect the panel from vibration of the aircraft frame. The dominant vibration in the aircraft is measured to be at 2000 rpm. Because of size limitation in the aircraft's cabin, the isolators are only allowed to deflect 0.15 in. Find the percent of motion transmitted to the instrument panel if it weighs 50 lb.

A) Percent of motion transmitted = 3%
B) Percent of motion transmitted = 7%
C) Percent of motion transmitted = 15%
D) Percent of motion transmitted = 30%

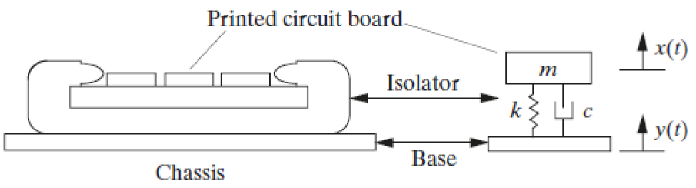
Problem 4 (Inman, 2014, w/ permission)

Design the stiffness for a base isolation system for an electronic module of mass 5 kg so that only 10% of the displacement of the base is transmitted into displacement of the module at 50 Hz. Then, determine the transmissibility if the frequency of the base motion changes to 100 Hz.

A) $k = 1.71 \times 10^4$ N/m and $T_r = 0.0231$
B) $k = 1.71 \times 10^4$ N/m and $T_r = 0.0705$
C) $k = 4.46 \times 10^4$ N/m and $T_r = 0.0231$
D) $k = 4.46 \times 10^4$ N/m and $T_r = 0.0705$

Problem 5 (Inman, 2014, w/ permission)

A 2-kg printed circuit board for a computer is to be isolated from external vibration of frequency 3 rad/s at a maximum amplitude of 1 mm, as illustrated below. Find the stiffness for an undamped isolator such that the transmitted displacement is 10% of the base motion. Also calculate the range of the transmitted force.



A) $k = 2.57 \times 10^3$ N/m and $F_T = 0.0181$ N
B) $k = 2.57 \times 10^3$ N/m and $F_T = 0.0504$ N
C) $k = 6.73 \times 10^3$ N/m and $F_T = 0.0181$ N
D) $k = 6.73 \times 10^3$ N/m and $F_T = 0.0504$ N

Problem 6 (Rao, 2011, w/ permission)

An electronic instrument is to be isolated from a panel that vibrates at frequencies ranging from 25 Hz to 35 Hz. It is estimated that at least 80 percent vibration isolation must be achieved to prevent damage to the instrument. If the instrument weighs 85 N, find the necessary static deflection of the isolator.

A) $\delta_{st} = 8.15 \times 10^{-5}$ m
B) $\delta_{st} = 4.72 \times 10^{-4}$ m
C) $\delta_{st} = 8.93 \times 10^{-4}$ m
D) $\delta_{st} = 2.39 \times 10^{-3}$ m

Problem 7 (Rao, 2011, w/ permission)

An electronic instrument of mass 20 kg is isolated from engine vibrations with frequencies ranging from 1000 rpm to 3000 rpm. Find the stiffness of an undamped isolator to be used to achieve 90% isolation.

A) $k = 2 \times 10^4$ N/m
B) $k = 8.31 \times 10^4$ N/m
C) $k = 1.8 \times 10^5$ N/m
D) $k = 8.62 \times 10^5$ N/m

Problem 8 (Rao, 2011, w/ permission)

A machine weighing 2600 lb is mounted on springs. A piston of weight w = 60 lb moves up and down in the machine at a speed of 600 rpm with a stroke of 15 in. Considering the motion to be harmonic, determine the maximum force transmitted to the foundation if k_1 = 10,000 lb/in. and if k_2 = 30,000 lb/in.

A) $F_{T,1}$ = 652 lb and $F_{T,2}$ = 18,320 lb
B) $F_{T,1}$ = 652 lb and $F_{T,2}$ = 39,460 lb
C) $F_{T,1}$ = 2772 lb and $F_{T,2}$ = 18,320 lb
D) $F_{T,1}$ = 2772 lb and $F_{T,2}$ = 39,460 lb

Problem 9A (Rao, 2011, w/ permission)

A viscously damped single-degree-of-freedom system has a body (mass) weighing 60 lb with a spring constant of 400 lb/in. Its base is subjected to harmonic vibration. When the base vibrates with an amplitude of 2.0 in. at resonance, the steady-state amplitude of the body is found to be 5.0 in. Find the damping ratio of the system.

A) c = 0.89 lb-sec/in.
B) c = 3.44 lb-sec/in.
C) c = 6.01 lb-sec/in.
D) c = 8.62 lb-sec/in.

Problem 9B

When the base vibrates at a frequency of 10 Hz, the steady-state amplitude of the body is found to be 1.5 in. Find the magnitude of the force transmitted to the base.

A) F_T = 173 lb
B) F_T = 425 lb
C) F_T = 671 lb
D) F_T = 923 lb

Problem 10 (Rao, 2011, w/ permission)

An exhaust fan, having a small unbalance, weighs 800 N and operates at a speed of 600 rpm. It is desired to limit the response to a transmissibility of 2.5 as the fan passes through resonance during start-up. In addition, an isolation of 90 percent is to be achieved at the operating speed of the fan. Design a suitable isolator for the fan, specifying its stiffness and damping coefficient.

A) $k = 8.11 \times 10^3$ N/m and $c = 190$ N · s/m

B) $k = 8.11 \times 10^3$ N/m and $c = 448$ N · s/m

C) $k = 1.30 \times 10^4$ N/m and $c = 190$ N · s/m

D) $k = 1.30 \times 10^4$ N/m and $c = 448$ N · s/m

Problem 11 (Rao, 2011, w/ permission)

An internal combustion engine has a rotating unbalance of 1.0 kg-m and operates between 800 and 2000 rpm. When attached directly to the floor, it transmitted a force of 7018 N at 800 rpm and 43,865 N at 2000 rpm. Find the stiffness of the isolator that is necessary to reduce the force transmitted to the floor to 6000 N over the operating-speed range of the engine. Assume that the damping ratio of the isolator is 0.08 and the mass of the engine is 200 kg.

A) $k = 1.11 \times 10^5$ N/m

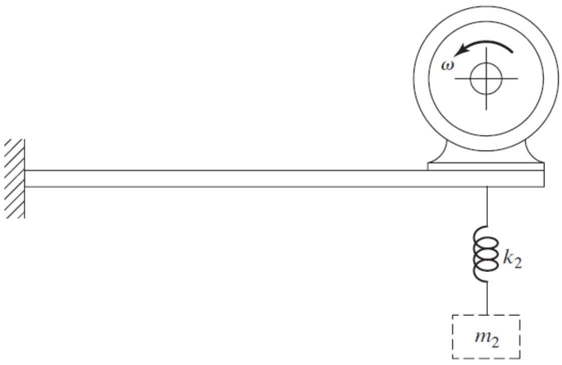
B) $k = 3.76 \times 10^5$ N/m

C) $k = 6.41 \times 10^5$ N/m

D) $k = 9.06 \times 10^5$ N/m

Problem 12 (Rao, 2011, w/ permission)

An electric motor, having an unbalance of 2 kg-cm, is mounted at the end of a steel cantilever beam, as shown in the next figure. The beam is observed to vibrate with large amplitudes at the operating speed of 1500 rpm of the motor. Determine the ratio of the absorber mass to the mass of the motor needed in order to have the lower frequency of the resulting system equal to 75 percent of the operating speed of the motor. If the mass of the motor is 300 kg, determine the stiffness and mass of the absorber. Also find the amplitude of vibration of the absorber mass. True or false?



1.( ) The required mass of the absorber is greater than 80 kg.

2.( ) The required stiffness of the absorber is greater than 3×10^6 N/m.

3.( ) The amplitude of vibration of the absorber mass is greater than 1×10^{-4} m.

Problem 13 (Rao, 2011, w/ permission)

An air compressor of mass 200 kg, with an unbalance of 0.01 kg·m, is found to have a large amplitude of vibration while running at 1200 rpm. Determine the mass and spring constant of the absorber to be added if the natural frequencies of the system are to be at least 20 percent from the impressed frequency.

A) $m_2 = 40.6$ kg and $k_2 = 6.46 \times 10^5$ N/m

B) $m_2 = 40.6$ kg and $k_2 = 2.12 \times 10^6$ N/m

C) $m_2 = 80.5$ kg and $k_2 = 6.46 \times 10^5$ N/m

D) $m_2 = 80.5$ kg and $k_2 = 2.12 \times 10^6$ N/m

Problem 14A (Rao, 2011, w/ permission)

A reciprocating engine is installed on the first floor of a building, which can be modeled as a rigid rectangular plate resting on four elastic columns. The equivalent weight of the engine and the floor is 2000 lb. At the rated speed of the engine, which is 600 rpm, the operators experience large vibration of the floor. It has been decided to reduce these vibrations by suspending a spring-mass system from the bottom surface of the floor. Assume that the spring stiffness is k_2 = 5000 lb/in. Find the weight of the mass to be attached to absorb the vibrations.

A) m_2 = 251 lb
B) m_2 = 491 lb
C) m_2 = 665 lb
D) m_2 = 882 lb

Problem 14B

Determine the resonant frequencies of the system after the absorber is added. Which of the following is one such frequency?

A) Ω = 312 rpm
B) Ω = 768 rpm
C) Ω = 1224 rpm
D) Ω = 1512 rpm

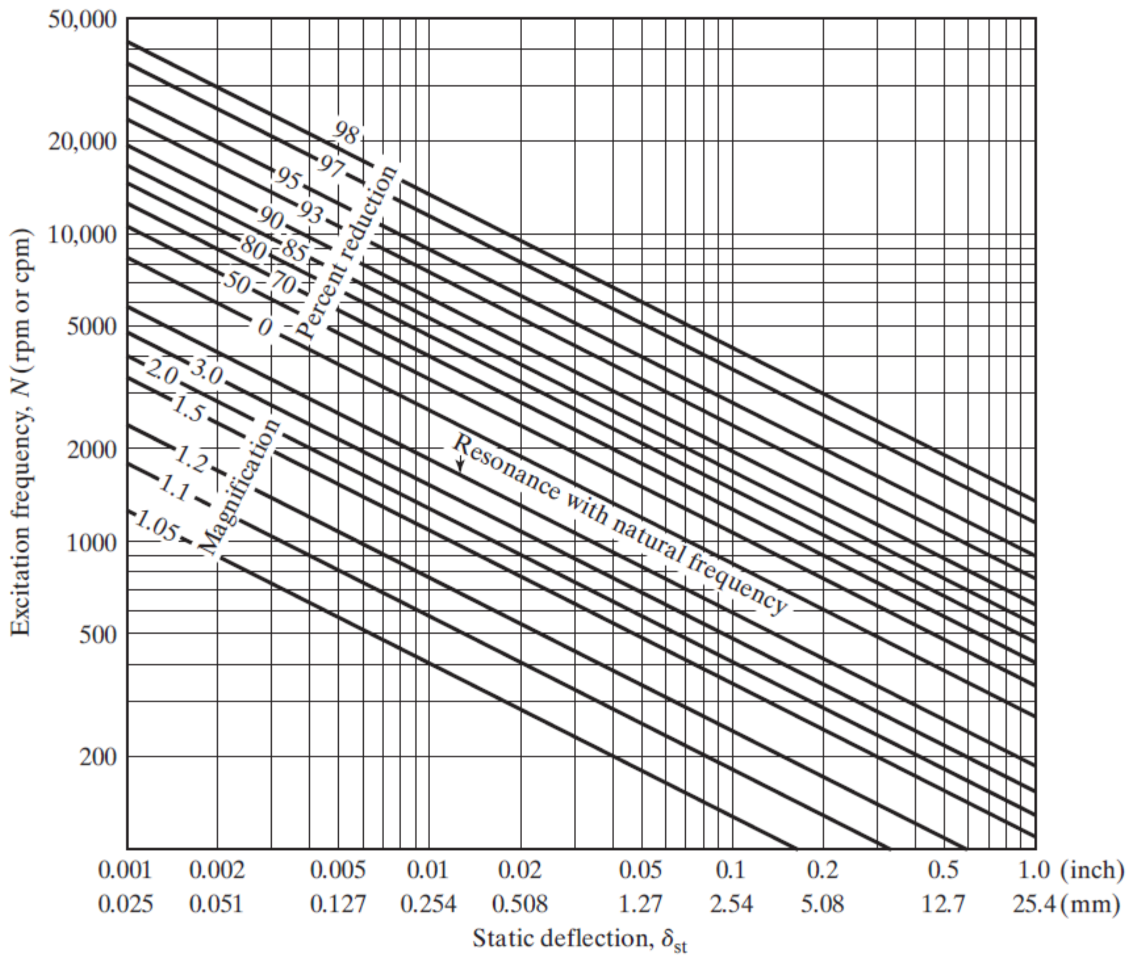
Problem 14C

Find the mass and stiffness of the absorber so that the natural frequencies of the system be at least 30 percent away from the forcing frequency.

A) $m_2 = 531$ lb and $k_2 = 1.08 \times 10^4$ lb/in.
B) $m_2 = 531$ lb and $k_2 = 7.44 \times 10^4$ lb/in.
C) $m_2 = 1062$ lb and $k_2 = 1.08 \times 10^4$ lb/in.
D) $m_2 = 1062$ lb and $k_2 = 7.44 \times 10^4$ lb/in.

ADDITIONAL INFORMATION

Figure 1 Isolation efficiency.



SOLUTIONS

P.1 › Solution

The frequency ratio can be determined from the force transmissibility as

$$T_r = \frac{1}{r^2 - 1} \rightarrow r^2 = \frac{1+T_r}{T_r}$$

$$\therefore r^2 = \frac{1+0.1}{0.1} = 11$$

The isolator stiffness can be calculated with the relation

$$r^2 = \frac{\omega^2}{\omega_n^2} = \frac{\omega^2}{\left(\sqrt{k/m}\right)^2} = \frac{m\omega^2}{k} \rightarrow k = \frac{m\omega^2}{r^2}$$

$$\therefore k = \frac{30\times(2\pi\times 20)^2}{11} = \boxed{4.31\times 10^4\ \text{N/m}}$$

We also require the displacement amplitude, which is given by

$$X = \frac{F}{k}\times\frac{1}{|1-r^2|} = \frac{1000}{(4.31\times 10^4)}\times\frac{1}{|1-11|} = \boxed{2.32\times 10^{-3}\ \text{m}}$$

▶ The correct answer is **A**.

P.2 › Solution

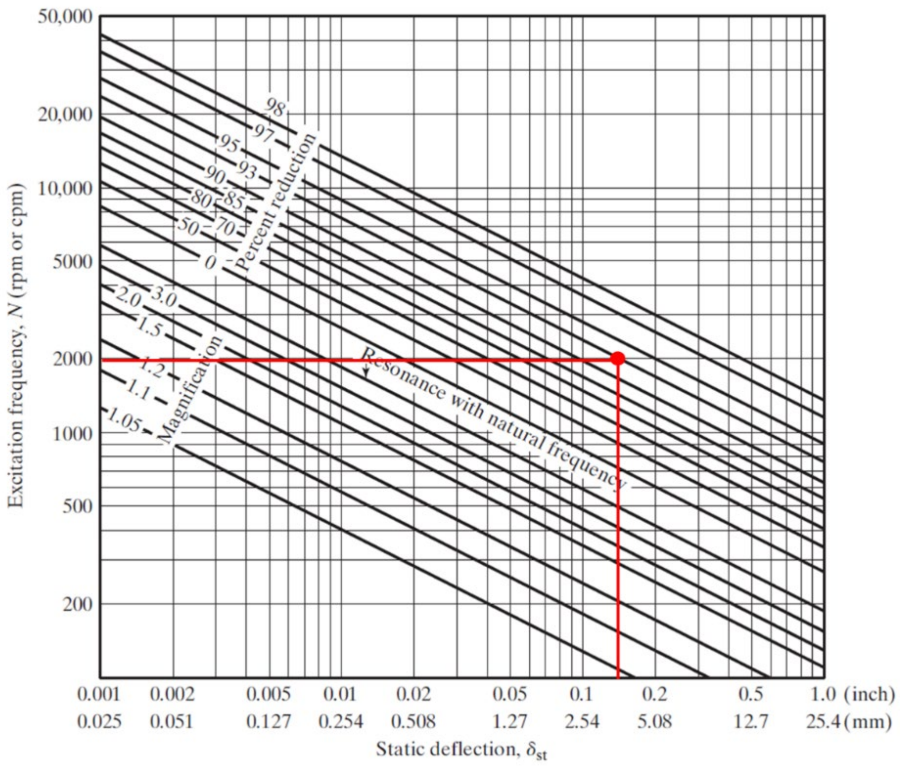
Referring to Figure 1, we verify that, for a 85% reduction in transmitted force and a vibration frequency of 900 rpm, the corresponding static deflection is δ_{st} = 0.35 in. The stiffness of the support is then

$$k = \frac{W}{\delta_{st}} = \frac{70}{0.35} = \boxed{200\ \text{lb/in.}}$$

▶ The correct answer is **B**.

P.3 › Solution

Refer to Figure 1. Entering an excitation frequency of n = 2000 rpm and a static deflection of δ_{st} = 0.15 in. into this chart, we read a corresponding percent reduction in motion of about 93%. Consequently, the percent of motion transmitted to the instrument panel is 7%.



▶ The correct answer is **B**.

P.4 › Solution

The frequency of vibration of the system in rpm is 50×60 = 3000 rpm. Referring to Figure 1, we see that, for such an operating frequency and a 90% reduction in transmitted force, the corresponding static deflection is about 1.1 mm. Thus, equating the elastic force on the module and its weight, we have

$$k\delta_{\mathrm{st}} = mg \rightarrow k = \frac{mg}{\delta_{\mathrm{st}}}$$

$$\therefore k = \frac{5\times 9.81}{0.0011} = \boxed{4.46\times 10^4\ \mathrm{N/m}}$$

The corresponding natural frequency is calculated as

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{4.46\times 10^4}{5}} = 94.4\ \mathrm{rad/s}$$

In circular values, a frequency of 100 Hz is equivalent to 628 rad/s. The frequency ratio then follows as

$$r = \frac{628}{94.4} = 6.65$$

Lastly, we compute the transmissibility ratio

$$T_r = \frac{1}{r^2 - 1} = \frac{1}{6.65^2 - 1} = \boxed{0.0231}$$

▶ The correct answer is **C**.

P.5 › Solution

The frequency of external vibration in rpm is 3×2π ×60 = 1130 rpm. Referring to Figure 1, we see that the static deflection that corresponds this frequency of vibration and a 90% reduction in transmitted force is about 0.3 in. or 0.00762 m. Equating the elastic force imparted on the circuit board to its weight, stiffness k is computed as

$$k\delta_{\mathrm{st}} = mg \rightarrow k = \frac{mg}{\delta_{\mathrm{st}}}$$

$$\therefore k = \frac{2\times 9.81}{0.00762} = \boxed{2.57\times 10^3\ \mathrm{N/m}}$$

We were also asked to determine the range of the transmitted force. For a system with base excitation such as the present one, this is given by

$$F_T = kY\left(\frac{r^2}{1-r^2}\right)$$

The natural frequency of the system is ω_n = $(2575/2)^{1/2}$ = 35.9 rad/s and the frequency ratio is r = 3/35.9 = 0.0836. Substituting in the relation above, we ultimately obtain

$$F_T = \left(2.57\times 10^3\right)\times 10^{-3} \times \frac{0.0836^2}{1-0.0836^2} = \boxed{0.0181\ \mathrm{N}}$$

▶ The correct answer is **A**.

P.6 › Solution

The transmissibility of an undamped isolator is given by

$$T_r = \frac{1}{\left|1-\left(\frac{\omega}{\omega_n}\right)^2\right|}$$

which, with T_r = 1 – 0.8 = 0.2, becomes

$$0.2=\frac{1}{\left|1-\left(\frac{\omega}{\omega_n}\right)^2\right|}\rightarrow\frac{\omega}{\omega_n}=2.45$$

The circular frequencies that correspond to the values we received are ω_1 = 25 × 2π = 157 rad/s and ω_2 = 35 × 2π = 220 rad/s. The ensuing natural frequencies, in turn, are $\omega_{n,1}$ = 157/2.45 = 64.1 rad/s and $\omega_{n,2}$ = 220/2.45 = 89.8 rad/s. The frequency ratio is given by

$$r=\sqrt{\frac{1+T_r}{T_r}}=\sqrt{\frac{1+0.2}{0.2}}=2.45$$

Doing some algebra, we can obtain an expression for the static deflection,

$$\omega_n=\sqrt{\frac{k}{m}}=\sqrt{\frac{kg}{W}}=\sqrt{\frac{g}{\delta_{\text{st}}}}\rightarrow\delta_{\text{st}}=\frac{g}{\omega_n^2}$$

At f_1 = 25 Hz, we have

$$\delta_{\text{st},1}=\frac{9.81}{64.1^2}=2.39\times10^{-3}\text{ m}$$

while at f_2 = 35 Hz,

$$\delta_{\text{st},2}=\frac{9.81}{89.8^2}=1.22\times10^{-3}\text{ m}$$

The greater value dominates, and the static deflection of the isolator is taken as δ_{st} = 2.39 × 10^{-3} m = 2.39 mm. Substituting the pertaining frequencies into the relation for transmissibility, the student can verify that the transmissibility that follows from this displacement is about 0.085, which implies an isolation of 91.5% – a satisfactory amount indeed.

▸ The correct answer is **D**.

P.7 › Solution

Let the stiffness of the isolator be k. The natural frequency is, of course,

$$\omega_n=\sqrt{\frac{k}{m}}=\sqrt{\frac{k}{20}}=0.224\sqrt{k}$$

The engine vibrations range from ω_1 = 2π × 1000/60 = 105 rad/s to ω_2 = 2π × 3000/60 = 314 rad/s. Accordingly, the frequency ratio ranges from r_1 = ω_1/ω_n = 105/(0.224$\sqrt{k}$) = 469/$\sqrt{k}$ to r_2 = ω_2/ω_n = 314/(0.224$\sqrt{k}$) = 1402/$\sqrt{k}$. In order for 90% isolation to be attained, the displacement transmissibility T_d must be 0.1. In mathematical terms,

$$T_d=0.1=\left|\frac{1}{1-r^2}\right|\rightarrow\left|1-r^2\right|=10$$

Substituting the first value we have for frequency ratio and solving for k, it follows that

$$\left|1-\left(\frac{469}{\sqrt{k}}\right)^2\right|=10\rightarrow k=2\times10^4\text{ N/m}$$

Likewise, with the other value of r obtained above,

$$\left|1-\left(\frac{1402}{\sqrt{k}}\right)^2\right|=10\rightarrow k=1.8\times10^5\text{ N/m}$$

The larger value controls, and we take k = 1.8 × 10^5 N/m as the stiffness of the isolator.

▸ The correct answer is **C**.

P.8 › Solution

The frequency of the machine is $\omega = 2\pi \times 600/60 = 62.8$ rad/s. The force due to the motion of the piston is

$$F = m\omega^2 r = \left(\frac{60}{386.4}\right)\times 62.8^2 \times \left(\frac{15}{2}\right) = 4593 \text{ lb}$$

With k = 10,000 lb/in., the natural frequency of the machine is

$$\omega_n = \sqrt{\frac{k_1}{m_{\text{machine}}}} = \sqrt{\frac{10{,}000}{(2600/386.4)}} = 38.6 \text{ rad/s}$$

The frequency ratio follows as

$$r = \frac{\omega}{\omega_n} = \frac{62.8}{38.6} = 1.63$$

Since $\omega > \omega_n$, the force transmitted to the foundation is given by

$$F_{T,1} = \frac{F}{r^2 - 1} = \frac{4593}{1.63^2 - 1} = \boxed{2772 \text{ lb}}$$

Now, with k = 30,000 lb/in., the natural frequency becomes

$$\omega_n = \sqrt{\frac{k_2}{m_{\text{machine}}}} = \sqrt{\frac{30{,}000}{(2600/386.4)}} = 66.8 \text{ rad/s}$$

The frequency ratio follows as

$$r = \frac{\omega}{\omega_n} = \frac{62.8}{66.8} = 0.940$$

Since $\omega < \omega_n$, the force transmitted to the foundation is, in this case,

$$F_{T,2} = \frac{F}{1 - r^2} = \frac{4593}{1 - 0.94^2} = \boxed{39{,}460 \text{ lb}}$$

That is, tripling the stiffness of the springs on which the machine is mounted will cause the transmitted force to increase over eight-fold.

▶ The correct answer is **D**.

P.9 › Solution

Part A: To begin, we compute the natural frequency of the system,

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{kg}{mg}} = \sqrt{\frac{400 \times 386.4}{60}} = 50.8 \text{ rad/s}$$

The amplitude of base vibration is Y = 2.0 in. The steady-state amplitude of the body at resonance is X = 1.0 in. At resonance, the frequency ratio r = 1. The damping ratio ζ can be determined from the relation

$$T_d = \frac{X}{Y} = \left[\frac{1 + (2\zeta r)^2}{(1 - r^2)^2 + (2\zeta r)^2}\right]^{1/2} \rightarrow \frac{5}{2} = \left[\frac{1 + (2\zeta)^2}{(2\zeta)^2}\right]$$

$$\therefore \zeta = 0.218$$

The damping coefficient follows as

$$\zeta = 0.218 = \frac{c}{2m\omega_n} \rightarrow c = 0.218 \times 2 \times (60/386.4) \times 50.8 = \boxed{3.44 \text{ lb-sec/in.}}$$

▶ The correct answer is **B**.

Part B: With a base frequency of ω = 10 Hz = 62.8 rad/s, the frequency ratio becomes $r = \omega/\omega_n$ = 62.8/50.8 = 1.24. The force transmitted to the mass is determined according to

$$\frac{F_T}{kY}=r^2\frac{X}{Y}\rightarrow F_T=kr^2X$$

$$\therefore F_T=400\times1.24^2\times1.5=\boxed{923\text{ lb}}$$

The correct answer is **D**.

P.10 › Solution

The operating frequency of the fan is $\omega = 2\pi \times 600/60 = 62.8$ rad/s. The transmissibility at resonance is given by

$$T_{r,\text{res}}=\left[\frac{1+(2\zeta r)^2}{(1-r^2)^2+(2\zeta r)^2}\right]^{1/2}=2.5$$

Since $r = 1$ at resonance, we can write

$$\left[\frac{1+(2\zeta\times1)^2}{(1-1^2)^2+(2\zeta\times1)^2}\right]^{1/2}=2.5\rightarrow\zeta=0.218$$

The damping ratio of the isolator has been specified. Next, with a required transmissibility at operating speed of $T_r = 1 - 0.9 = 0.1$, we can write

$$T_r=\left[\frac{1+(2\times0.218r)^2}{(1-r^2)^2+(2\times0.218r)^2}\right]^{1/2}=0.1\rightarrow r=4.98$$

This is the frequency ratio of the isolator. We can proceed to determine its stiffness,

$$k=m\omega_n^2=\frac{m\omega^2}{r^2}=\frac{(800/9.81)\times62.8^2}{4.98^2}=\boxed{1.30\times10^4\text{ N/m}}$$

Lastly, with $\omega_n = 62.8/4.98 = 12.6$ rad/s, the damping coefficient of the isolator is

$$c=2m\omega_n\zeta=2\times(800/9.81)\times12.6\times0.218=\boxed{448\text{ N}\cdot\text{s/m}}$$

The correct answer is **D**.

P.11 › Solution

The relation that must be satisfied is

$$\frac{F_T}{me\omega^2}=\left[\frac{1+(2\zeta r)^2}{(1-r^2)^2+(2\zeta r)^2}\right]^{1/2}\le\frac{6000}{me\omega^2}$$

Substituting, we have, when $n = 800$ rpm and $me\omega^2 = 7018$ N,

$$\left[\frac{1+(2\times0.08\times r)^2}{(1-r^2)^2+(2\times0.08\times r)^2}\right]^{1/2}\le\frac{6000}{7018}$$

$$\therefore\left[\frac{1+0.0256r^2}{(1-r^2)^2+0.0256r^2}\right]^{1/2}=0.855$$

$$\therefore\frac{1+0.0256r^2}{(1-r^2)^2+0.0256r^2}=0.731$$

$$\therefore r_1=1.48$$

Likewise, we have, when $n = 2000$ rpm and $me\omega^2 = 43{,}865$ N,

$$\frac{1+0.0256r^2}{(1-r^2)^2+0.0256r^2}=0.137^2=0.0188$$

$$\therefore r_2 = 3.01$$

The operation frequencies are $\omega_1 = 2\pi \times 800/60 = 83.8$ rad/s and $\omega_2 = 2\pi \times 2000/60 = 209$ rad/s. If we select $r_1 = 1.48$, we obtain the natural frequency $\omega_n = 83.8/1.48 = 56.6$ rad/s. If we select $r_2 = 3.01$, we obtain $\omega_n = 209/3.01 = 69.4$ rad/s. Suppose the lower frequency result satisfies the transmitted force requirement at both ends of the operating speed range. This can be easily verified. At the frequency of 2000 rpm, $r = \omega/\omega_n = 209/56.6 = 3.69$, so that

$$\left[\frac{1+0.0256\times 3.69^2}{\left(1-3.69^2\right)^2+0.0256\times 3.69^2}\right]^{1/2} = 0.0919 < 0.137$$

The frequency we chose *does* satisfy both extremes of the engine's operating frequency interval. We can then establish the stiffness of the isolator,

$$k = m\omega_n^2 = 200\times 56.6^2 = \boxed{6.41\times 10^5 \text{ N/m}}$$

▶ The correct answer is **C**.

P.12 › Solution

1. True. The operating frequency of the motor is $\omega_1 = 2\pi \times 1500/60 = 157$ rad/s. At optimal conditions, the frequency of the absorber should be equal to the frequency of the motor, i.e., $\omega_2 = \omega_1$. Further, if the lower frequency Ω_1 of the resulting system is to have 75% of the operating speed of the motor, we can write $r_1 = \Omega_1/\omega_2 = 0.75$. The ratio μ of the mass of the absorber to the mass of the initial system is given by

$$r_1^2 = \left(1+\frac{\mu}{2}\right) - \sqrt{\left(1+\frac{\mu}{2}\right)^2 - 1} = 0.75^2$$

Solving this equation yields $\mu = 0.340$. The absorber mass should be about one-third of the mass of the motor. The mass of the absorber is then

$$m_2 = m_1\mu = 300\times 0.34 = 102 \text{ kg}$$

2. False. The stiffness of the absorber is

$$k_2 = m_2\omega_2^2 = 102\times 157^2 = 2.51\times 10^6 \text{ N/m}$$

3. True. The amplitude of vibration of the absorber mass is

$$|X_2| = \left|-\frac{me\omega^2}{k_2}\right| = \left|-\frac{0.02\times 157^2}{2.51\times 10^6}\right| = 1.96\times 10^{-4} \text{ m}$$

P.13 › Solution

The operating speed of the compressor is $\omega_1 = 2\pi \times 1200/60 = 126$ rad/s, and its stiffness is $k_1 = m_1\omega_1^2 = 200 \times 126^2 = 3.18\times 10^6$ N/m. For optimal conditions, suppose the absorber is tuned so that the frequency ω_1 of the compressor is equal to the frequency ω_2 of the absorber, i.e., $\omega_2/\omega_1 = 1$. The natural frequencies of the combined system are given by the relation

$$\left(\frac{\omega}{\omega_2}\right)^4\left(\frac{\omega_2}{\omega_1}\right)^2 - \left(\frac{\omega}{\omega_2}\right)^2\left[1+\left(1+\frac{m_2}{m_1}\right)\left(\frac{\omega_2}{\omega_1}\right)^2\right] + 1 = 0$$

which, in the present case, simplifies to

$$\left(\frac{\omega}{\omega_2}\right)^4 - \left(2+\frac{m_2}{m_1}\right)\left(\frac{\omega}{\omega_2}\right)^2 + 1 = 0$$

The natural frequency of the system must be at least 20 percent from the impressed frequency. Accordingly, let $r_1 = \Omega_1/\omega_2 = 0.8$. Substituting in the equation above gives

$$0.8^4-\left(2+\frac{m_2}{m_1}\right)\times 0.8^2+1=0 \rightarrow \frac{m_2}{m_1}=0.203$$

$$\therefore m_2=0.203\times 200=\boxed{40.6 \text{ kg}}$$

The corresponding stiffness of the absorber, k_2, follows as

$$\frac{k_2}{k_1}=\frac{m_2}{m_1}\times\left(\frac{\omega_2}{\omega_1}\right)^2=\frac{m_2}{m_1}\rightarrow k_2=0.203\times\left(3.18\times 10^6\right)=\boxed{6.46\times 10^5 \text{ N/m}}$$

Another feasible possibility is to have the natural frequencies of the system be 20% *greater* than the impressed frequency, with the result that r_2 = 1.2. In this case, the ratio of masses becomes

$$1.2^4-\left(2+\frac{m_2}{m_1}\right)\times 1.2^2+1=0 \rightarrow \frac{m_2}{m_1}=0.134$$

Since this mass ratio is smaller, the design parameters that we'd obtain from this value would be inherently less safe than the ones we got with m_2/m_1 = 0.203. The larger mass ratio controls and we need not proceed with this alternative design.

▶ The correct answer is **A**.

P.14 › Solution

Part A: The excitation frequency of the engine is $\omega_1 = 2\pi \times 600/60 = 62.8$ rad/s. Under optimal conditions, the frequency of the absorber is tuned to that of the engine, so that $\omega_2 = \omega_1 = 62.8$ rad/s. Then, the mass of the absorber is calculated as

$$\omega_2=\sqrt{\frac{k_2}{m_2}}\rightarrow m_2=\frac{k_2}{\omega_2^2}=\frac{5000}{62.8^2}=1.27 \text{ lb-sec}^2/\text{in.}$$

$$\therefore \boxed{m_2=491 \text{ lb}}$$

▶ The correct answer is **B**.

Part B: The frequency ratios and the mass ratio $\mu = m_2/m_1$ are associated by the expression

$$r_1^2,r_2^2=\left(1+\frac{\mu}{2}\right)\mp\sqrt{\left(1+\frac{\mu}{2}\right)^2-1}$$

Here, $r_1 = \Omega_1/\omega_2$ and $r_2 = \Omega_2/\omega_2$. The mass ratio μ = 491/2000 = 0.246. Substituting in the relation above, we obtain

$$r_1^2=\left(1+\frac{0.246}{2}\right)-\sqrt{\left(1+\frac{0.246}{2}\right)^2-1}=0.612\rightarrow r_1=0.782$$

and

$$r_2^2=\left(1+\frac{0.246}{2}\right)+\sqrt{\left(1+\frac{0.246}{2}\right)^2-1}=1.63\rightarrow r_2=1.28$$

Accordingly, the first resonant frequency is

$$\Omega_1=r_1\omega_2=0.782\times 62.8=49.1 \text{ rad/s}=469 \text{ rpm}$$

while the second is

$$\Omega_2=r_2\omega_2=1.28\times 62.8=80.4 \text{ rad/s}=\boxed{768 \text{ rpm}}$$

▶ The correct answer is **B**.

Part C: In view of the requirement that the frequencies be 30% away from the forcing frequency, the frequency ratios become r_1 = 0.7 and r_2 = 1.3. Accordingly, the resonant frequencies are now

$$\Omega_1 = 0.7\times\omega_1 = 0.7\times 62.8 = 44 \text{ rad/s}$$

$$\Omega_2 = 1.3\times\omega_2 = 1.3\times 62.8 = 81.6 \text{ rad/s}$$

The stiffness of the absorber is $k_2 = m_2\omega_2^2$. Since $\omega_2 = \omega_1 = 62.8$ rad/s for a tuned absorber, we have $k_2 = m_2\omega_2^2 = 3944m_2$. Now, the frequency ratios and the mass ratio are associated by the relation

$$r_{1,2}^2 = \left(1+\frac{\mu}{2}\right)\mp\sqrt{\left(1+\frac{\mu}{2}\right)^2 - 1}$$

Take $r_1 = 0.7$. Rearranging the relation above and substituting r, it follows that

$$r_1^2 = \left(1+\frac{\mu}{2}\right)-\sqrt{\left(1+\frac{\mu}{2}\right)^2 - 1} \rightarrow \mu = \frac{r_1^4+1}{r_1^2} - 2$$

$$\therefore \mu = \frac{0.7^4+1}{0.7^2} - 2 = 0.531$$

Then, the mass of the absorber is calculated as

$$\mu = \frac{m_2}{m_1} \rightarrow m_2 = \mu\times m_1$$

$$\therefore m_2 = 0.531\times(2000/386.4) = 2.75 \text{ lb-sec}^2\text{/in.} = \boxed{1062 \text{ lb}}$$

Backsubstituting in the equation for k_2, the stiffness of the absorber is determined to be

$$k_2 = 3944\times 2.75 = \boxed{1.08\times 10^4 \text{ lb/in.}}$$

The basic parameters of the absorber have been specified. However, we should check whether the mass ratio we've picked ensures that $r_2 \geq 1.3$ as well. Mathematically,

$$r_2^2 = \left(1+\frac{\mu}{2}\right)+\sqrt{\left(1+\frac{\mu}{2}\right)^2 - 1} = \left(1+\frac{0.531}{2}\right)+\sqrt{\left(1+\frac{0.531}{2}\right)^2 - 1} = 2.04$$

$$\therefore r_2 = 1.43$$

which is greater than 1.30. Our design does guarantee that the natural frequency will be at least 30 percent away from the forcing frequency.

▸ The correct answer is **C**.

ANSWER SUMMARY

Problem 1		A
Problem 2		B
Problem 3		B
Problem 4		C
Problem 5		A
Problem 6		D
Problem 7		C
Problem 8		D
Problem 9	**9A**	B
	9B	D
Problem 10		D
Problem 11		C
Problem 12		T/F
Problem 13		A
Problem 14	**14A**	B
	14B	B
	14C	C

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