Quiz VB106
Vibration Measurement
Lucas Montogue

PROBLEMS

Problem 1A (Rao, 2011, w/ permission)

Determine the maximum percent error of a vibrometer in the frequency-ratio range $4 \leq r \leq 8$ with a damping ratio of $\zeta = 0$.

A) Percent Error = 0.11%
B) Percent Error = 2.8%
C) Percent Error = 6.7%
D) Percent Error = 14.4%

Problem 1B

Repeat the previous problem with a damping ratio of $\zeta = 0.69$.

A) Percent Error = 0.11%
B) Percent Error = 2.8%
C) Percent Error = 6.7%
D) Percent Error = 14.4%

Problem 2 (Rao, 2011, w/ permission)

A vibrometer has an undamped natural frequency of 10 Hz and a damped natural frequency of 8 Hz. Find the lowest frequency in the range to infinity at which the amplitude can be directly read from the vibrometer with less than 2 percent error.

A) ω_0 = 14.5 Hz
B) ω_0 = 24.9 Hz
C) ω_0 = 35.1 Hz
D) ω_0 = 44.8 Hz

Problem 3 (Rao, 2011, w/ permission)

A vibration pickup has been designed for operation above a frequency level of 100 Hz without exceeding an error of 2 percent. When mounted on a structure vibrating at a frequency of 100 Hz, the relative amplitude of the mass is found to be 1 mm. Find the suspended mass of the pickup if the stiffness of the spring is 4000 N/m and damping is negligible.

A) m = 0.265 kg
B) m = 0.517 kg
C) m = 0.749 kg
D) m = 0.985 kg

Problem 4 (Rao, 2011, w/ permission)

A spring-mass system, having a static deflection of 10 mm and negligible damping, is used as a vibrometer. When mounted on a machine operating at 4000 rpm, the relative amplitude is recorded as 1 mm. True or false?

1.( ) The maximum displacement of the machine is greater than 0.99 mm.
2.( ) The maximum velocity of the machine is greater than 425 mm/s.
3.( ) The maximum acceleration of the machine is greater than 175,000 mm/s^2.

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Problem 5 (Rao, 2011, w/ permission)

A vibration pickup has a natural frequency of 5 Hz and a damping ratio of ζ = 0.5. Find the lowest frequency that can be measured with a 1 percent error.

A) ω_0 = 5.44 Hz
B) ω_0 = 20.5 Hz
C) ω_0 = 35.3 Hz
D) ω_0 = 50.1 Hz

Problem 6A (Rao, 2011, w/ permission)

A vibrometer is used to measure the vibration of an engine whose operating-speed range is from 500 to 2000 rpm. The vibration consists of two harmonics. The amplitude distortion must be less than 3 percent. Find the natural frequency of the vibrometer if the damping is negligible.

A) ω_n = 1.42 Hz
B) ω_n = 3.06 Hz
C) ω_n = 4.88 Hz
D) ω_n = 6.71 Hz

Problem 6B

Find the natural frequency of the vibrometer if the damping ratio ζ = 0.6.

A) ω_n = 1.42 Hz
B) ω_n = 3.06 Hz
C) ω_n = 4.88 Hz
D) ω_n = 6.71 Hz

Problem 7 (Rao, 2011, w/ permission)

It is proposed that the vibration of the foundation of an internal combustion engine be measured over the speed range 500 rpm to 1500 rpm using a vibrometer. The vibration is composed of two harmonics, the first one caused by the primary inertia forces and the second one by the secondary inertia forces in the engine. Determine the maximum natural frequency of the vibrometer in order to have an amplitude distortion less than 2 percent.

A) ω_n = 0.884 Hz
B) ω_n = 2.37 Hz
C) ω_n = 4.59 Hz
D) ω_n = 7.14 Hz

Problem 8A (Rao, 2011, w/ permission)

Determine the maximum percent error of an accelerometer in the frequency-ratio range $0 \leq r \leq 0.65$ with a damping ratio of ζ = 0.

A) Percent Error = 0.89%
B) Percent Error = 5.44%
C) Percent Error = 11.8%
D) Percent Error = 73%

Problem 8B

Determine the maximum percent error of an accelerometer in the frequency-ratio range $0 \leq r \leq 0.65$ with a damping ratio of ζ = 0.75.

A) Percent Error = 0.89%
B) Percent Error = 5.44%
C) Percent Error = 11.8%
D) Percent Error = 73%

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Problem 9 (Rao, 2011, w/ permission)

Determine the necessary stiffness and damping constant of an accelerometer if the maximum error is to be limited to 3 percent for measurements in the frequency ratio of 0 to 100 Hz. Assume that the suspended mass is 0.05 kg.

A) $k = 1.15 \times 10^4$ N/m and $c = 25.3$ N·s/m
B) $k = 1.15 \times 10^4$ N/m and $c = 50.6$ N·s/m
C) $k = 3.36 \times 10^4$ N/m and $c = 25.3$ N·s/m
D) $k = 3.36 \times 10^4$ N/m and $c = 50.6$ N·s/m

SOLUTIONS

P.1) Solution

Part A: The error factor for the vibrometer is

$$E = \frac{r^2}{\sqrt{\left(1-r^2\right)^2 + \left(2\zeta r\right)^2}}$$

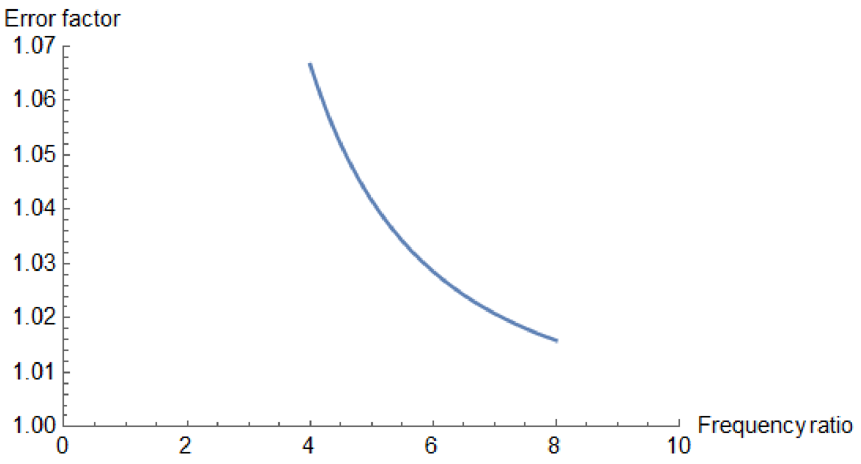
The maximum error occurs at

$$r^* = \frac{1}{\sqrt{1-2\zeta^2}}$$

which, for zero damping ratio, becomes simply $r = 1$. This is outside the range of values proposed in the problem. The error factor with $\zeta = 0$ becomes

$$E = \frac{r^2}{\left|1-r^2\right|}$$

The error factor approaches unity with increasing frequency ratio, as shown.



Accordingly, the maximum error factor occurs for $r = 4$, and is calculated to be

$$E = \frac{4^2}{\left|1-4^2\right|} = 1.067 \rightarrow \boxed{\text{Percent Error} = 6.7\%}$$

▶ The correct answer is **C**.

Part B: The maximum E now occurs at

$$r^* = \frac{1}{\sqrt{1-2\zeta^2}} = \frac{1}{\sqrt{1-2\times 0.69^2}} = 4.57$$

This value is within the operational range of the vibrometer. Inserting it into the equation for E, we obtain

$$E = \frac{4.57^2}{\sqrt{\left(1-4.57^2\right)^2 + \left(2\times 0.69 \times 4.57\right)^2}} = 1.00114 \rightarrow \boxed{\text{Percent Error} = 0.11\%}$$

▶ The correct answer is **A**.

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P.2 › Solution

The damping ratio of the vibrometer is calculated as

$$\omega_d = \omega_n\sqrt{1-\zeta^2} \to \zeta = \sqrt{1-\frac{\omega_d^2}{\omega_n^2}}$$

$$\therefore \zeta = \sqrt{1-\frac{8^2}{10^2}} = 0.6$$

The error factor is equal to 1.02. Substituting and solving for the frequency ratio, we obtain

$$E = \frac{r^2}{\sqrt{\left(1-r^2\right)^2+\left(2\zeta r\right)^2}} \to \frac{r^2}{\sqrt{\left(1-r^2\right)^2+\left(2\times 0.6\times r\right)^2}} = 1.02$$

Solving the equation above, we get r = 1.45 and r = 3.51 as positive results. Denoting by ω_0 the lowest frequency that satisfies the specified conditions, we have

$$\omega_0 = 1.45\times\omega_n = 1.45\times 10 = 14.5\text{ Hz}$$

and

$$\omega_0 = 3.51\times 10 = 35.1\text{ Hz}$$

The first result controls, and the lowest frequency for which amplitude can be read with less than 2 percent error is established as 14.5 Hz.

▸ The correct answer is **A**.

P.3 › Solution

Since damping is negligible, the amplitude ratio is written as

$$E = \frac{X}{Y} = \frac{r^2}{\sqrt{\left(1-r^2\right)^2+\left(2\zeta r\right)^2}} \to E = \frac{r^2}{\left|1-r^2\right|}$$

For an admissible error of 2%, we write

$$\frac{r^2}{\left|1-r^2\right|} = 1.02$$

The positive solutions to the equation above are r = 0.711, which is not feasible, and r = 7.14. The natural frequency follows as

$$r = \frac{\omega}{\omega_n} \to \omega_n = \frac{\omega}{r}$$

$$\therefore \omega_n = \frac{100}{7.14} = 14.0\text{ Hz}$$

or, equivalently, ω_n = 88.0 rad/s. Lastly, the mass of the pickup is calculated as

$$\omega_n = \sqrt{\frac{k}{m}} \to m = \frac{k}{\omega_n^2}$$

$$\therefore m = \frac{4000}{88^2} = \boxed{0.517\text{ kg}}$$

▸ The correct answer is **B**.

P.4 › Solution

1. True. The static deflection is δ_{st} = 10/1000 = 0.01 m, the operating speed of the machine is $\omega = 2\pi \times (4000/60)$ = 418.879 rad/s, and the natural frequency is

$$\omega_n = \sqrt{\frac{g}{\delta_{st}}} = \sqrt{\frac{9.81}{0.01}} = 31.3209\text{ rad/s}$$

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The corresponding frequency ratio is $r = 418.79/31.3209 = 13.3738$. Noting that the system is undamped, the error factor is determined as

$$E = \frac{r^2}{\left|1-r^2\right|} = \frac{13.3738^2}{\left|1-13.3738^2\right|} = 1.00562$$

The maximum displacement follows as

$$Y = \frac{Z}{1.00562} = \frac{1}{1.00562} = 0.99441 \text{ mm}$$

2. False. The maximum velocity is calculated as

$$\dot{x}_{\max} = \omega Y = 418.879 \times 1 = 418.879 \text{ mm/s}$$

3. True. The maximum acceleration is computed as

$$\ddot{x}_{\max} = \omega^2 Y = 418.879^2 \times 1 = 175{,}460 \text{ mm/s}^2$$

P.5 › Solution

The error factor, or amplitude ratio, is given by

$$E = \frac{Z}{Y} = \frac{r^2}{\sqrt{\left(1-r^2\right)^2 + \left(2\zeta r\right)^2}}$$

The maximum error occurs when the frequency ratio attains the value

$$r^* = \frac{1}{\sqrt{1-2\zeta^2}} = \frac{1}{\sqrt{1-2\times 0.5^2}} = 1.41$$

The error in question is

$$E = \frac{Z}{Y} = \frac{1.41^2}{\sqrt{\left(1-1.41^2\right)^2 + \left(2\times 0.5\times 1.41\right)^2}} = 1.15$$

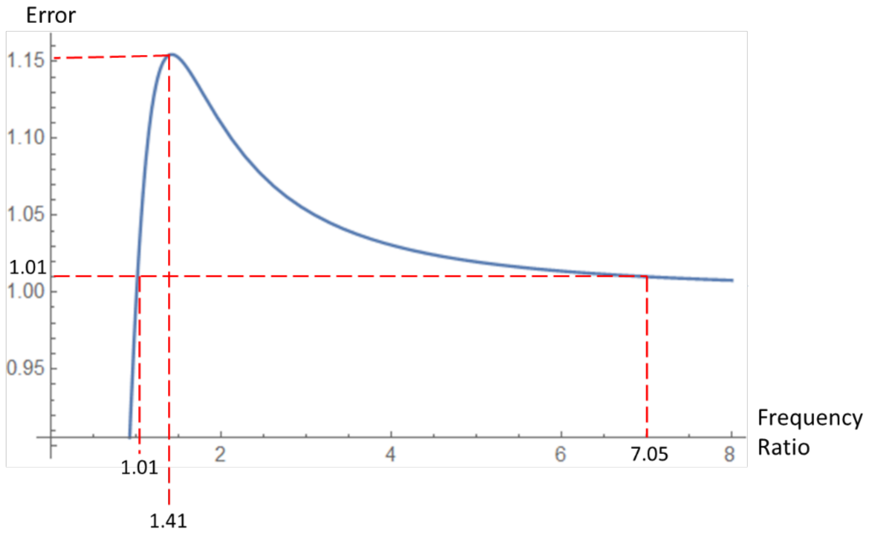
When the error is one percent, we write

$$\frac{r^2}{\sqrt{\left(1-r^2\right)^2 + \left(2\times 0.5\times r\right)^2}} = 1.01$$

$$\therefore \frac{r^2}{\sqrt{\left(1-r^2\right)^2 + r^2}} = 1.01$$

The positive solutions to this equation are $r = 1.01$ and $r = 7.05$. At the outset, we'd take the lower solution, $r = 1.01$, as the ratio that corresponds to the lowest frequency for one percent accuracy. But previous calculations indicate that, for $r > 1.01$, there will be ratios for which the error is larger than 1%. This is illustrated in the following graph. We are left with $r = 7.05$ as our solution, and the lowest frequency is determined to be

$$\omega_0 = 7.05 \times 5 = \boxed{35.3 \text{ Hz}}$$



The correct answer is **C**.

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P.6 › Solution

Part A: The amplitude ratio with zero damping ratio and an error of 3% is given by

$$E = \frac{Z}{Y} \rightarrow 1.03 = \frac{r^2}{\left|1-r^2\right|}$$

of which the positive solutions are r = 0.712, which is unfeasible, and r = 5.86. The natural frequency is chosen on the basis of the lowest frequency being measured. Accordingly, we take ω = 500 rpm and compute ω_n as

$$r = \frac{\omega}{\omega_n} \rightarrow \omega_n = \frac{\omega}{r}$$

$$\therefore \omega_n = \frac{(500/60)}{5.86} = \boxed{1.42\text{ Hz}}$$

▶ The correct answer is **A**.

Part B: As before, the updated value of r is obtained from the equation for amplitude ratio,

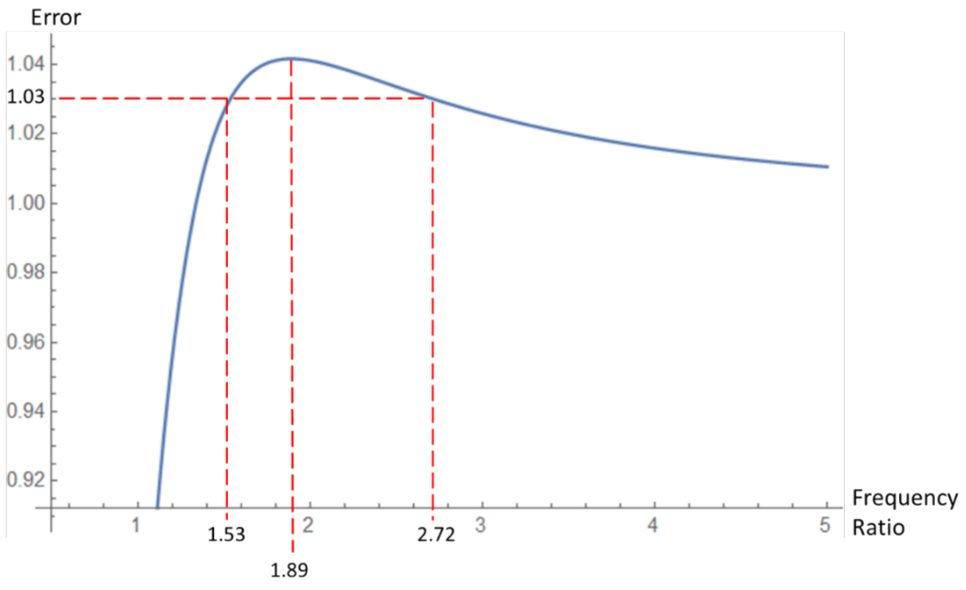
$$\frac{r^2}{\sqrt{\left(1-r^2\right)^2 + \left(2\times 0.6\times r\right)^2}} = 1.03$$

which yields r = 1.53 and r = 2.72 as positive solutions. At first, both results are viable. Note, however, that the error E is maximum for a frequency ratio r such that

$$r* = \frac{1}{\sqrt{1-2\zeta^2}} = \frac{1}{\sqrt{1-2\times 0.6^2}} = 1.89$$

The instrument should be designed for a range of frequency ratios that avoids this error peak, as shown below. Selecting r = 1.53 would include a range of frequency ratios for which the error is greater than 3%. Choosing r = 2.72 would avoid this unfavorable interval. As a result, we take r = 2.72 and determine the natural frequency as

$$\omega_n = \frac{(500/60)}{2.72} = \boxed{3.06\text{ Hz}}$$



▶ The correct answer is **B**.

P.7 › Solution

The engine operating speeds are ω_1 = 500/60 = 8.33 Hz and ω_2 = 1500/60 = 25 Hz. As a first case, let the damping ratio ζ = 0. The amplitude ratio is given by

$$Z = \frac{r^2 Y}{\sqrt{\left(1-r^2\right)^2 + \left(2\zeta r\right)^2}} \rightarrow \frac{Z}{Y} = \frac{r^2}{\left|r^2-1\right|} = 1.02$$

Solving for the frequency ratio, we obtain r = 7.14. Next, as a second case, the damping ratio ζ = 0.6. Substituting in the foregoing equation gives

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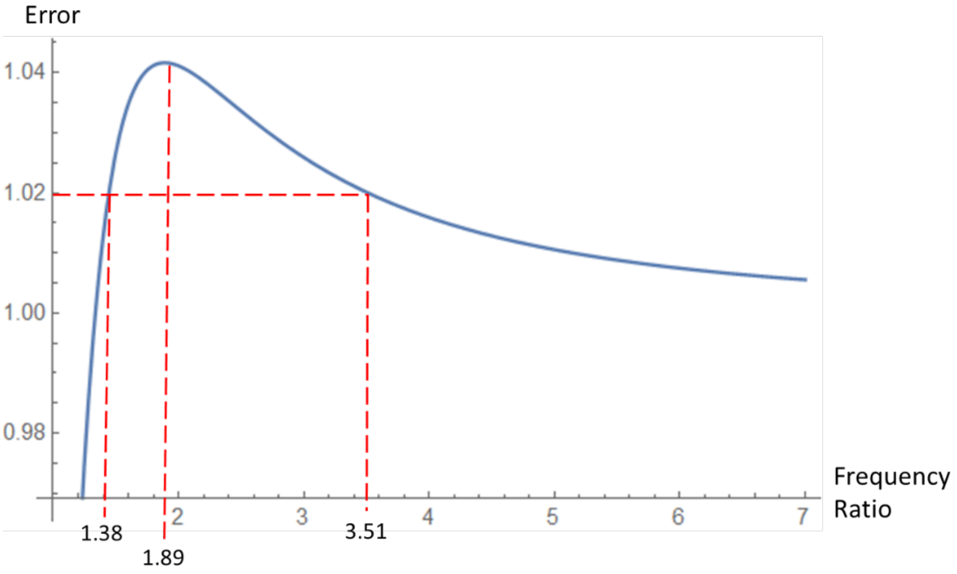
$$Z = \frac{r^2 Y}{\sqrt{\left(1-r^2\right)^2 + \left(2\times 0.6\times r\right)^2}} \rightarrow \frac{Z}{Y} = \frac{r^2}{\sqrt{\left(1-r^2\right)^2 + 1.44r^2}} = 1.02$$

This equation admits two values of r greater than unity, namely r = 1.45 and 3.51. The error is maximum for r such that

$$r* = \frac{1}{\sqrt{1-2\zeta^2}} = \frac{1}{\sqrt{1-2\times 0.6^2}} = 1.89$$

The vibrometer should be designed for a range of frequency ratios that avoids this error peak, as shown below, otherwise the amplitude distortion will be greater than 2% for some frequency ratios. Choosing r = 1.38 would include this unfavorable range, whereas having r = 3.51 or r = 7.14 would not. Hence, we choose r = 3.51 and calculate the maximum natural frequency as

$$\omega_n = \frac{(500/60)}{3.51} = \boxed{2.37\ \text{Hz}}$$



▶ The correct answer is **B**.

P.8 › Solution

Part A: The error factor for an accelerometer is

$$E = \frac{1}{\sqrt{\left(1-r^2\right)^2 + \left(2\zeta r\right)^2}}$$

The maximum error occurs at

$$r* = \sqrt{1-2\zeta^2}$$

which, with zero damping ratio, becomes simply r = 1. This is outside the range of values proposed in the problem. The error factor with ζ = 0 becomes

$$E = \frac{1}{\left|1-r^2\right|}$$

Clearly, the greater the frequency ratio, the greater the error factor will be. Accordingly, we can insert r = 0.65 in the equation above and obtain

$$E = \frac{1}{\left|1-0.65^2\right|} = 1.73 \rightarrow \boxed{\text{Percent Error} = 73\%}$$

▶ The correct answer is **D**.

Part B: With ζ = 0.75, the frequency ratio for maximum error is

$$r* = \sqrt{1-2\zeta^2} = \sqrt{1-2\times 0.75^2} \notin \Re$$

That is, the result above is an imaginary quantity, and therefore is not a suitable candidate for optimal frequency ratio. The error factor is given by

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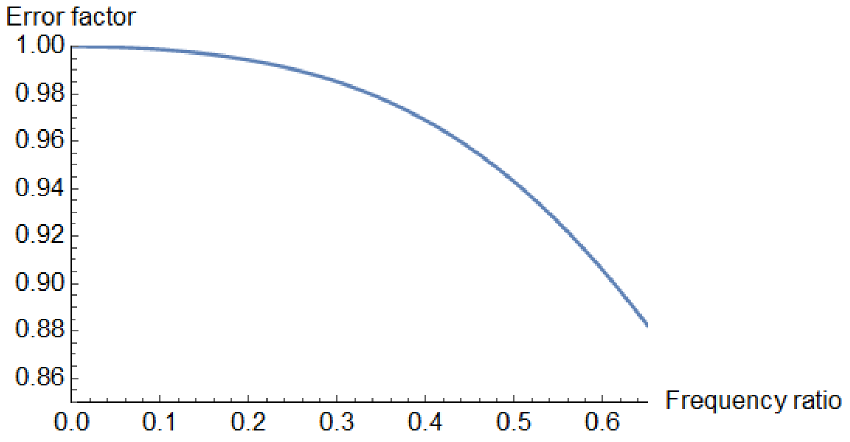
$$E=\frac{1}{\sqrt{\left(1-r^2\right)^2+\left(2\times 0.75\times r\right)^2}}=\frac{1}{\sqrt{\left(1-r^2\right)^2+2.25r^2}}$$

Plotting this relation as a function of frequency ratio reveals that the E deviates further from unity as r increases. Thus, the maximum percent error occurs when r = 0.65, and is determined as

$$E=\frac{1}{\sqrt{\left(1-0.65^2\right)^2+\left(2\times 0.75\times 0.65\right)^2}}=0.882\rightarrow \boxed{\text{Percent Error}=11.8\%}$$



▶ The correct answer is **C**.

P.9 › Solution

The error factor for an accelerometer is calculated with the relation

$$E=\frac{1}{\sqrt{\left(1-r^2\right)^2+\left(2\zeta r\right)^2}}$$

The error is maximum at a frequency ratio of

$$r*=\sqrt{1-2\zeta^2}$$

so that

$$E_{\max}=\frac{1}{2\zeta\sqrt{1-\zeta^2}}$$

To begin, consider a *maximum* error of +0.03. In mathematical terms, deviation e is given by

$$e=E-1=0.03\rightarrow\frac{1}{2\zeta\sqrt{1-\zeta^2}}-1=0.03$$

Solving this equation for ζ, we obtain ζ = 0.617 and ζ = 0.787. The optimum frequency ratio that corresponds to the first solution is

$$r*=\sqrt{1-2\times 0.617^2}=0.488$$

while the optimum r for the second solution is

$$r*=\sqrt{1-2\times 0.787^2}\notin\Re$$

Hence, we take ζ = 0.617. Consider now a *minimum* error of −0.03. Mathematically, deviation e is given by

$$e=E-1=-0.03\rightarrow\frac{1}{\sqrt{\left(1-r^2\right)^2+\left(2\zeta r\right)^2}}-1=-0.03$$

Substituting ζ = 0.617 gives

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$$\frac{1}{\sqrt{\left(1-r^2\right)^2+\left(2\times0.617\times r\right)^2}}-1=-0.03$$

$$\therefore\frac{1}{\sqrt{\left(1-r^2\right)^2+1.52r^2}}-1=-0.03$$

The only positive real solution to this equation is r = 0.766. This frequency ratio is greater than the previous obtained optimum of r = 0.488 and still meets the requirements imposed by the problem, as shown below. Given the maximum $\omega = 2\pi \times 100 = 628$ rad/s, the natural frequency is determined as

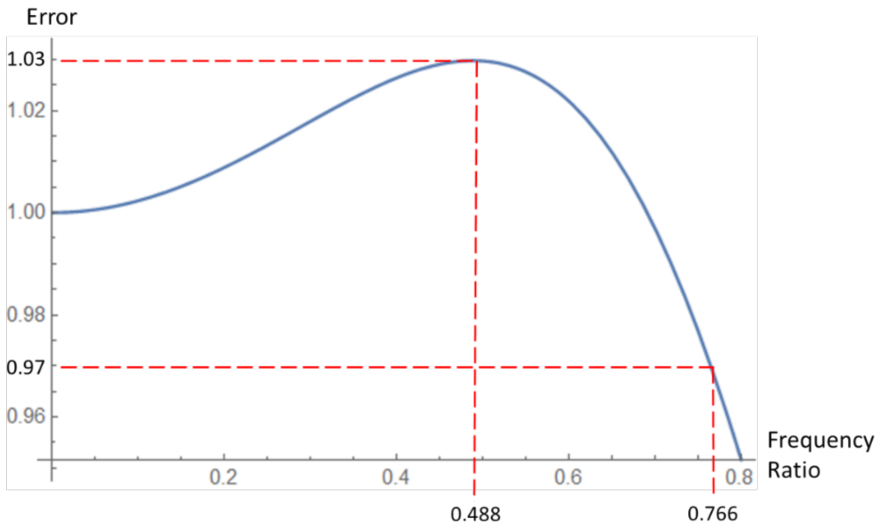
$$r=\frac{\omega}{\omega_n}\rightarrow\omega_n=\frac{\omega}{r}=\frac{628}{0.766}=820\text{ rad/s}$$

The stiffness of the accelerometer is calculated as

$$k=m\omega_n^2=0.05\times820^2=\boxed{3.36\times10^4\text{ N/m}}$$

Finally, the damping constant is computed as

$$c=2m\omega_n\zeta=2\times0.05\times820\times0.617=\boxed{50.6\text{ N}\cdot\text{s/m}}$$



The correct answer is **D**.

ANSWER SUMMARY

Problem 1	**1A**	C
	1B	A
Problem 2		A
Problem 3		B
Problem 4		T/F
Problem 5		C
Problem 6	**6A**	A
	6B	B
Problem 7		B
Problem 8	**8A**	D
	8B	C
Problem 9		D

REFERENCES

- RAO, S. (2011). *Mechanical Vibrations*. 5th edition. Upper Saddle River: Pearson.

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