Quiz SM107
Virtual Work and Potential Energy

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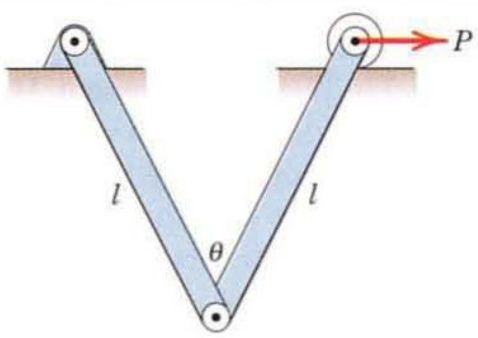
Problems

PROBLEM 1 (Merriam & Kraige, 2002, w/ permission)

Each of the two uniform hinged bars has a mass *m* and a length *l*, and is supported and loaded as shown. For a given force *P*, what is the angle θ for equilibrium?



A) $\theta = \arctan\frac{P}{mg}$

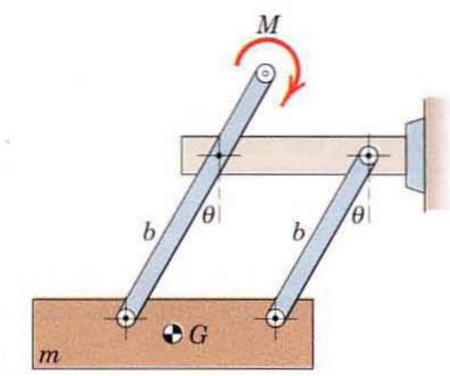
B) $\theta = \arctan\frac{2P}{mg}$

C) $\theta = 2\arctan\frac{P}{mg}$

D) $\theta = 2\arctan\frac{2P}{mg}$

PROBLEM 2 (Merriam & Kraige, 2002, w/ permission)

The mass *m* is brought to an equilibrium position by the application of the couple *M* to the end of one of the two parallel links, which are hinged as shown in the figure below. Determine an expression for the equilibrium angle θ between the links and the vertical for a given value of *M*.



A) $\theta = \arcsin\frac{M}{2mgb}$

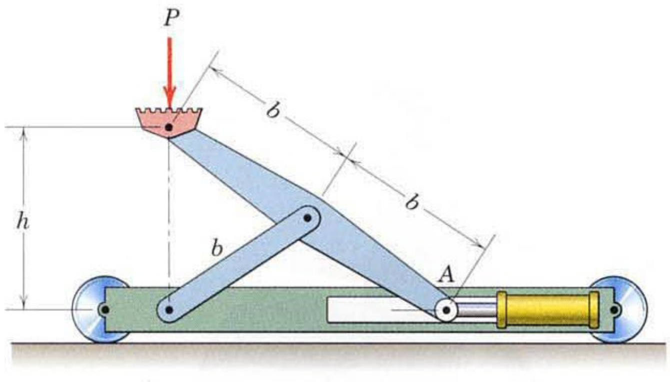
B) $\theta = \arcsin\frac{M}{mgb}$

C) $\theta = 2\arcsin\frac{M}{2mgb}$

D) $\theta = 2\arcsin\frac{M}{mgb}$

PROBLEM 3 (Merriam & Kraige, 2002, w/ permission)

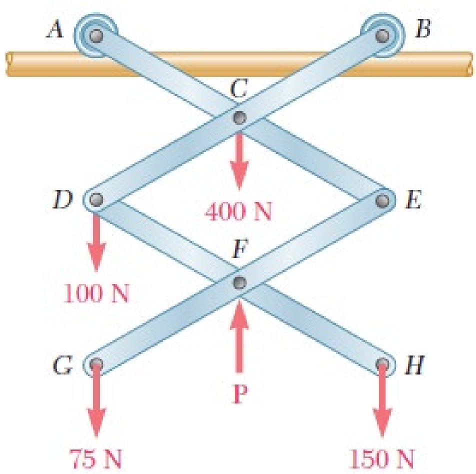
The portable car hoist is operated by the hydraulic cylinder which controls the horizontal movement of end A of the link in the horizontal slot. Determine the compression *C* in the piston rod of the cylinder to support the load *P* at a height *h*.



A) $C = P\sqrt{(2b)^2 - h^2}$
B) $C = P\sqrt{\left(\frac{b}{2h}\right)^2 - 1}$
C) $C = P\sqrt{\left(\frac{b}{h}\right)^2 - 1}$
D) $C = P\sqrt{\left(\frac{2b}{h}\right)^2 - 1}$

PROBLEM 4 (Beer et al., 2013, w/ permission)

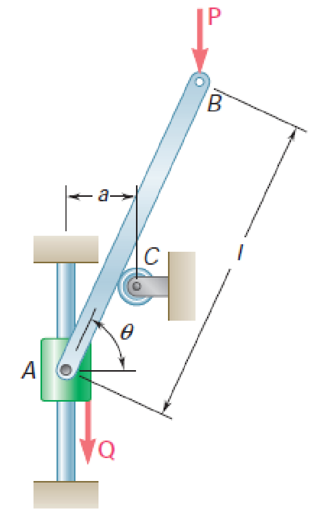
Determine the force *P* required to maintain the equilibrium of the linkage shown. All members are of the same length and the wheels at A and B roll freely on the horizontal rod.



A) P = 375 N
B) P = 500 N
C) P = 625 N
D) P = 750 N

PROBLEM 5 (Beer et al., 2013, w/ permission)

The slender rod AB is attached to a collar A and rests on a small wheel at C. Neglecting the radius of the wheel and the effect of friction, derive an expression for the magnitude of the force *Q* required to maintain the equilibrium of the rod.



A) $Q = P\left(\frac{l}{a}\cos^2\theta - 1\right)$

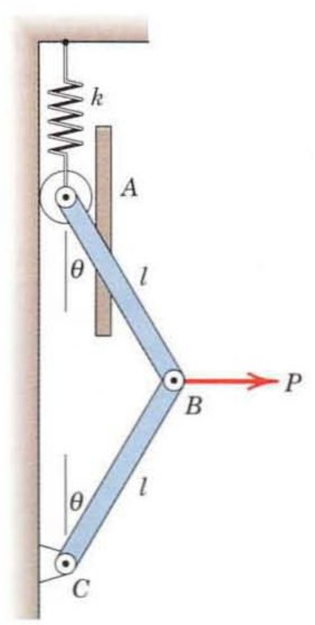
B) $Q = 2P\left(\frac{l}{a}\cos^2\theta - 1\right)$

C) $Q = P\left(\frac{l}{a}\cos^3\theta - 1\right)$

D) $Q = 2P\left(\frac{l}{a}\cos^3\theta - 1\right)$

PROBLEM 6 (Merriam & Kraige, 2002, w/ permission)

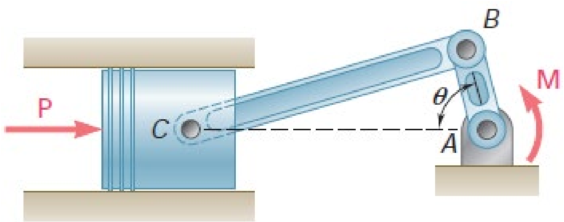
The spring of constant *k* is unstretched when $\theta = 0$. Derive an expression for the force *P* required to deflect the system to an angle θ. The masses of the bars are negligible.



A) $P = 2kl(\tan\theta - \sin\theta)$
B) $P = 4kl(\tan\theta - \sin\theta)$
C) $P = 2kl(\tan\theta - \sin 2\theta)$
D) $P = 4kl(\tan\theta - \sin 2\theta)$

PROBLEM 7 (Beer et al., 2013, w/ permission)

A 4-kN force *P* is applied as shown to the piston of the engine system. Knowing that AB = 50 mm and BC = 200 mm, determine the couple *M* required to maintain the equilibrium of the system when $\theta = 30°$.



A) M = 88.7 N·m

B) M = 100.4 N·m

C) M = 121.8 N·m

D) M = 145.0 N·m

PROBLEM 8 (Merriam & Kraige, 2002, w/ permission)

The potential energy of a mechanical system is given by $V(x) = 6x^4 - 3x^2 + 5$, where *x* is the position coordinate which defined the configuration of the single degree-of-freedom system. Regarding this system, which of the following statements is true?

A) The system is in stable equilibrium when $x = 0$.

B) The system is in unstable equilibrium when $x = 0.5$.

C) The system is in stable equilibrium when $x = -0.5$.

D) The system is in unstable equilibrium when $x = 0.25$.

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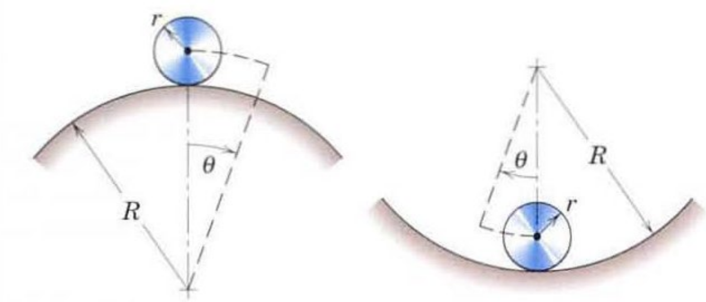
PROBLEM 9 (Merriam & Kraige, 2002, w/ permission)

The small cylinder of mass *m* and radius *r* is confined to roll on the circular surface of radius *R*. Which of the following statements is true?



A) Equilibrium is stable for both systems.

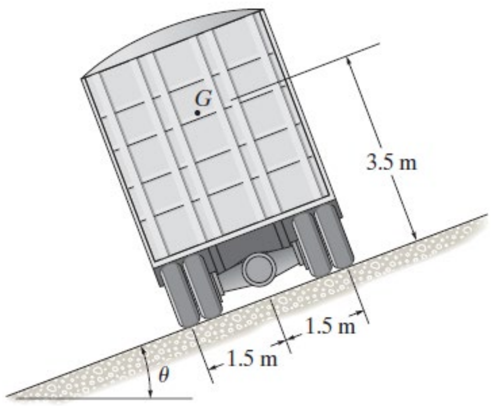
B) Equilibrium is stable for the system to the left and unstable for the system to the right.

C) Equilibrium is stable for the system to the right and unstable for the system to the left.

D) Equilibrium is unstable for both systems.

PROBLEM 10 (Hibbeler, 2010, w/ permission)

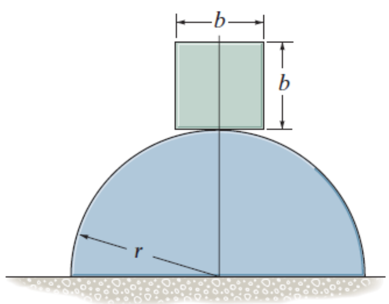
The truck has a mass of 20 Mg and a mass center at G. Determine the steepest grade θ along which it can park without overturning and establish whether the system is in stable or unstable equilibrium.



A) θ = 15.2° and the system is stable.
B) θ = 15.2° and the system is unstable.
C) θ = 23.2° and the system is stable.
D) θ = 23.2° and the system is unstable.

PROBLEM 11 (Hibbeler, 2010, w/ permission)

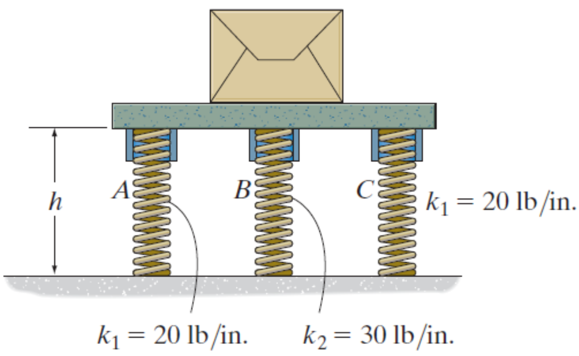
A homogeneous block rests on top of the cylindrical surface. What is the relationship between the radius of the cylinder, *r*, and the dimension of the block, *b*, that ensures stable equilibrium?



A) The system is stable if $b < r$.
B) The system is stable if $b < 2r$.
C) The system is stable if $b < 3r$.
D) The system is stable if $b < 4r$.

PROBLEM 12 (Hibbeler, 2010, w/ permission)

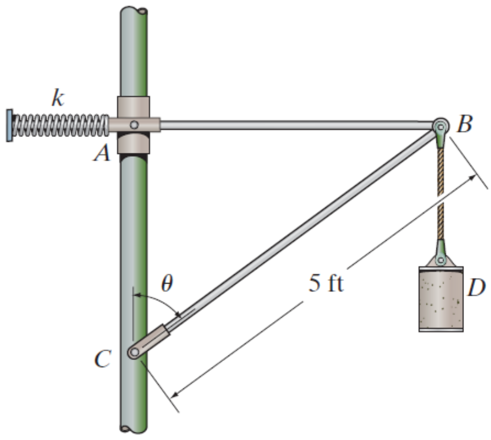
If springs A and C have an unstretched length of 10 in and spring B has an unstretched length of 12 in., determine the height *h* of the platform when the system is in equilibrium. Is the equilibrium stable or unstable?



A) h = 6.5 in. and the system is stable.
B) h = 6.5 in. and the system is unstable.
C) h = 8.7 in. and the system is stable.
D) h = 8.7 in. and the system is unstable.

PROBLEM 13 (Hibbeler, 2010, w/ permission)

The spring has a stiffness k = 600 lb/ft and is unstretched when θ = 45°. If the mechanism is in equilibrium when θ = 60°, determine the weight of cylinder D. Neglect the weight of the members. Rod AB remains horizontal at all times since the collar can slide freely along the vertical guide.

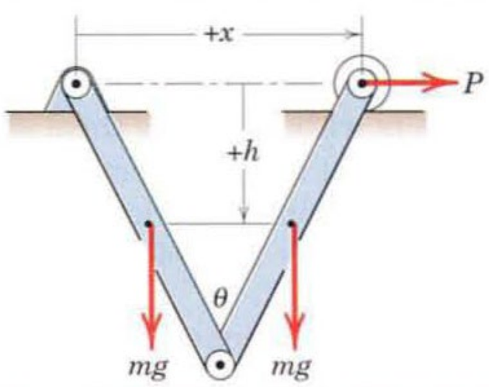


A) W_D = 274 lb
B) W_D = 351 lb
C) W_D = 425 lb
D) W_D = 506 lb

Solutions

P.1 ■ Solution

The active-force diagram for the system composed of the two members is shown separately and includes the weight *mg* of each bar in addition to the force *P*.



The principle of virtual work requires that the total work of all external active forces be zero for any virtual displacement consistent with the constraints. Thus, for a movement δx the virtual work becomes

$$P\delta x + 2mg\delta h = 0$$

We express each of these virtual displacements in terms of the variable θ, which is the required quantity. Mathematically,

$$x = 2l\sin\frac{\theta}{2} \to \delta x = l\cos\frac{\theta}{2}\delta\theta$$

Similarly for h,

$$h = \frac{l}{2}\cos\frac{\theta}{2} \to \delta h = -\frac{l}{4}\sin\frac{\theta}{2}\delta\theta$$

Substitution into the equation of virtual work gives

$$Pl\cos\frac{\theta}{2}\delta\theta - \frac{1}{2}mgl\sin\frac{\theta}{2}\delta\theta = 0$$

Dividing both sides by l gives

$$P\cos\frac{\theta}{2}\delta\theta - \frac{mg}{2}\sin\frac{\theta}{2}\delta\theta = 0$$

$$\therefore \left(P\cos\frac{\theta}{2} - \frac{mg}{2}\sin\frac{\theta}{2}\right)\delta\theta = 0$$

then dividing by $\cos(\theta/2)$ gives

$$P - \frac{mg}{2}\tan\frac{\theta}{2} = 0$$

$$\therefore \tan\frac{\theta}{2} = \frac{2P}{mg}$$

$$\therefore \frac{\theta}{2} = \arctan\frac{2P}{mg}$$

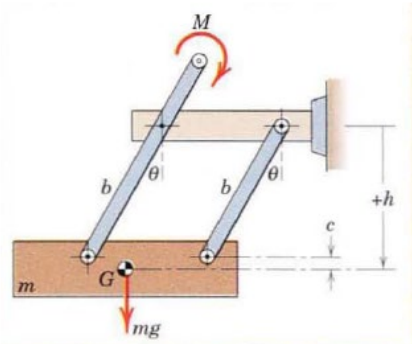
$$\therefore \boxed{\theta = 2\arctan\frac{2P}{mg}}$$

It is noteworthy that, in order to obtain this result by ordinary force and moment equilibrium equations, we would have to dismember the frame and take into account all forces acting on each member. Solution by the method of virtual work involves much simpler calculations.

■ The correct answer is **D**.

P.2 ■ Solution

The active-force diagram below shows the weight *mg* acting through the center of mass G and the couple *M* applied to the end of the link. There are no other external active forces or moments which do work on the system during a change in the angle θ. The vertical position of the center of mass G is designated by the distance *h* below the fixed horizontal reference line and is such that $h = b\cos\theta + c$.



The work done by *mg* during a movement δh in the direction of *mg* is

$$+mg\ \delta h = mg\ \delta(b\cos\theta + c)$$

$$\therefore +mg\ \delta h = mg(-b\sin\theta\ \delta\theta + 0)$$

$$\therefore +mg\ \delta h = -mgb\sin\theta\ \delta\theta$$

The minus sign shows that the work is negative for a positive value of $\delta\theta$. Constant *c* drops out since its derivative is zero. With θ measured positive in the clockwise sense, $\delta\theta$ is also positive clockwise. Thus, the work done by the clockwise couple *M* is $+M\delta\theta$. Substitution into the virtual work equation yields

$$M\delta\theta + mg\ \delta h = 0$$

$$\therefore M\delta\theta = mgb\sin\theta\ \delta\theta$$

$$\therefore \sin\theta = \frac{M}{mgb}$$

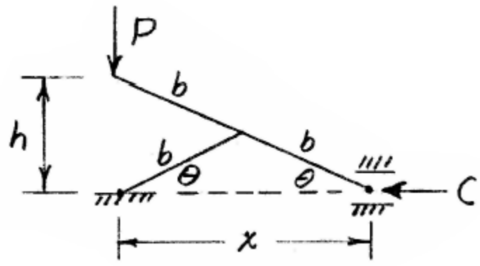
$$\boxed{\therefore \theta = \arcsin\frac{M}{mgb}}$$

Inasmuch as $\sin\theta$ does not exceed unity, we see that, for equilibrium, *M* is limited to values that do not exceed *mgb*.

■ The correct answer is **B**.

P.3 ■ Solution

Consider the following illustration of the hoist system.



Using the principle of virtual work, $\delta U = 0$, we have

$$\delta U = 0 \rightarrow -C\delta x - P\delta h = 0 \quad \text{(I)}$$

where *x* and *h* are such that

$$x = 2b\cos\theta \rightarrow \delta x = -2b\sin\theta\,\delta\theta$$

$$h = 2b\sin\theta \rightarrow \delta h = 2b\cos\theta\,\delta\theta$$

so that equation (I) becomes

$$-C(-2b\sin\theta\,\delta\theta) - P(2b\cos\theta\,\delta\theta) = 0$$

$$\therefore 2bC\sin\theta - 2bP\cos\theta = 0$$

$$\therefore C = P\cot\theta \quad \text{(II)}$$

Applying the Pythagorean theorem with reference to the previous figure, we see that

$$\cot\theta = \frac{x}{h} = \frac{\sqrt{4b^2 - h^2}}{h} = \sqrt{\frac{4b^2 - h^2}{h^2}} = \sqrt{\left(\frac{2b}{h}\right)^2 - 1}$$

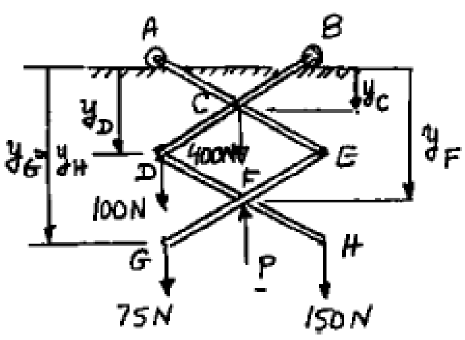
which, upon substitution in the expression for *C*, equation (II), gives

$$\boxed{C = P\sqrt{\left(\frac{2b}{h}\right)^2 - 1}}$$

■ The correct answer is **D**.

P.4 ■ Solution

Consider the following illustration of the present system.



Using y_C as an independent variable, we have

$$y_D = 2y_C \rightarrow \delta y_D = 2\delta y_C$$

$$y_F = 3y_C \rightarrow \delta y_F = 3\delta y_C$$

$$y_G = y_H = 4y_C \rightarrow \delta y_G = \delta y_H = 4\delta y_C$$

Now, applying the principle of virtual work, $\delta U = 0$, it follows that

$$\delta U = 400\delta y_C + 100\delta y_D - P\delta y_F + 75\delta y_G + 150\delta y_H = 0$$

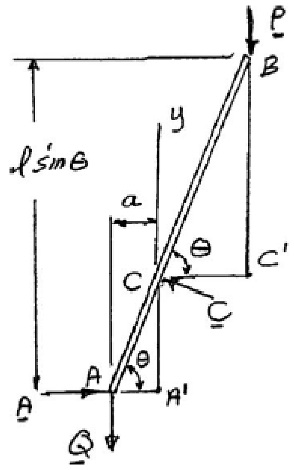
$$\therefore 400\delta y_C + 100(2\delta y_C) - P(3\delta y_C) + (75 + 150)(4\delta y_C) = 0$$

$$\therefore \boxed{P = 500\text{ N}}$$

■ The correct answer is **B**.

P.5 ■ Solution

Consider the following illustration of the rod system.



For triangle AA'C, we can state that

$$A'C = a\tan\theta$$

and

$$y_A = -(A'C) = -a\tan\theta$$

$$\therefore \delta y_A = -\frac{a}{\cos^2\theta}\delta\theta$$

Next, considering triangle CC'B, we can write

$$BC' = l\sin\theta - A'C$$

$$\therefore y_B = BC' = l\sin\theta - a\tan\theta$$

$$\therefore \delta y_B = l\cos\theta\,\delta\theta - \frac{a}{\cos^2\theta}\delta\theta$$

Applying the principle of virtual work, it follows that

$$\delta U = 0 \rightarrow -Q\delta y_A - P\delta y_B = 0$$

$$\therefore -Q\left(-\frac{a}{\cos^2\theta}\cancel{\delta\theta}\right) - P\left(l\cos\theta\,\cancel{\delta\theta} - \frac{a}{\cos^2\theta}\cancel{\delta\theta}\right) = 0$$

$$\therefore \frac{Q}{\cos^2\theta}a - Pl\cos\theta + \frac{Pa}{\cos^2\theta} = 0 \quad (\times\cos^2\theta)$$

$$\therefore Qa - Pl\cos^3\theta + Pa = 0$$

$$\therefore Qa = P(l\cos^3\theta - a)$$

$$\therefore \boxed{Q = P\left(\frac{l}{a}\cos^3\theta - 1\right)}$$

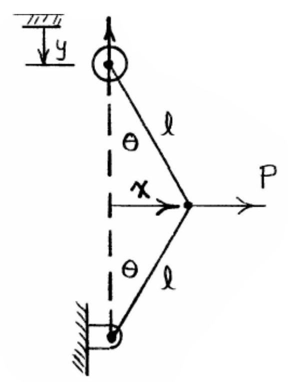
■ The correct answer is **C**.

P.6 ■ Solution

According to the principle of virtual work, we have $\delta U = 0$. Hence,

$$P\delta x - k(2l - 2l\cos\theta)\delta y = 0 \quad \text{(I)}$$

Consider the figure below.



Lengths x and y are such that

$$x = l\sin\theta$$

$$y = 2l - 2l\cos\theta$$

where y is measured from wheel position when the spring is unstretched. Differentiating these with respect to θ, we obtain

$$\delta x = l\cos\theta\,\delta\theta$$

$$\delta y = 2l\sin\theta\,\delta\theta$$

Now, substituting δx and δy in Equation (I) gives

$$P(l\cos\theta\,\delta\theta) - k(2l - 2l\cos\theta)(2l\sin\theta) = 0$$

$$\therefore [Pl\cos\theta - k(4l^2\sin\theta - 4l^2\sin\theta\cos\theta)]\delta\theta = 0$$

$$\therefore [Pl\cos\theta - 4kl^2\sin\theta + 4kl^2\sin\theta\cos\theta]\delta\theta = 0$$

$$\therefore Pl\cos\theta = 4kl^2\sin\theta - 4kl^2\sin\theta\cos\theta = 0$$

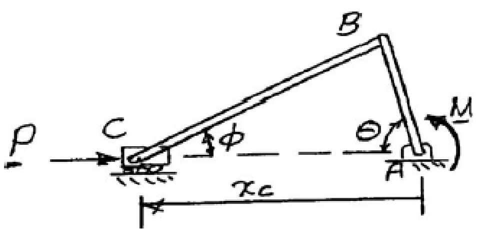
$$\therefore P = \frac{4kl^2\sin\theta - 4kl^2\sin\theta\cos\theta}{l\cos\theta} = 4kl\left(\frac{\sin\theta}{\cos\theta} - \frac{\sin\theta\cos\theta}{\cos\theta}\right)$$

$$\therefore \boxed{P = 4kl(\tan\theta - \sin\theta)}$$

■ The correct answer is **B**.

P.7 ■ Solution

Consider the following illustration of the piston system.



Using the law of sines, we have

$$\frac{\sin\phi}{AB} = \frac{\sin\theta}{BC} \rightarrow \sin\phi = \frac{AB}{BC}\sin\theta \quad \text{(I)}$$

Now, x_C is such that

$$x_C = AB\cos\theta + BC\cos\phi$$

$$\therefore \delta x_C = -AB\sin\theta\,\delta\theta - BC\sin\phi\,\delta\phi \quad \text{(II)}$$

From Equation (I), we have

$$\cos\phi\,\delta\phi = \frac{AB}{BC}\cos\theta\,\delta\theta$$

or

$$\delta\phi = \frac{AB\cos\theta}{BC\cos\phi}\delta\theta$$

$\delta\phi$ can then be replaced in Equation (II), giving

$$\delta x_C = -AB\sin\theta\,\delta\theta - BC\sin\phi\left(\frac{AB\cos\theta}{BC\cos\phi}\delta\theta\right)$$

$$\therefore \delta x_C = -\frac{AB}{\cos\phi}\left(\underbrace{\sin\theta\cos\phi + \sin\phi\cos\theta}_{=\sin(\theta+\phi)}\right)\delta\theta$$

$$\therefore \delta x_C = -\frac{AB\sin(\theta+\phi)}{\cos\phi}\delta\theta$$

Applying the principle of virtual work, $\delta U = 0$, we have

$$\delta U = 0 \rightarrow -P\delta x_C - M\delta\theta = 0$$

$$\therefore -P\left[-\frac{AB\sin(\theta+\phi)}{\cos\phi}\delta\theta\right] - M\delta\theta = 0$$

Therefore,

$$M = \frac{AB\sin(\theta+\phi)}{\cos\phi}P \quad \text{(III)}$$

We can return to equation (I) to determine the value of ϕ,

$$\sin\phi = \frac{AB}{BC}\sin\theta = \frac{50}{200}\sin 30^{\circ} = 0.125 \rightarrow \phi = \arcsin 0.125 = 7.18^{\circ}$$

Finally, substituting AB = 50 mm, P = 4 kN, ϕ = 7.18°, and θ = 30° into Equation (III), we obtain

$$M = \frac{50\sin(30^{\circ}+7.18^{\circ})}{\cos 7.18^{\circ}} \times 4 = \boxed{121.8\ \mathrm{N\cdot m}}$$

■ The correct answer is **C**.

P.8 ■ Solution

For equilibrium to occur, we must have $dV/dx = 24x^3 - 6x = 0$. Hence,

$$\frac{dV}{dx} = 24x^3 - 6x = x(24x^2 - 6) = 0$$

$$\therefore x = 0, \frac{1}{2}, -\frac{1}{2}$$

Differentiating the equation a second time, we obtain $d^2V/dx^2 = 72x^2 - 6$. Substituting the values of *x* just obtained into the second derivative gives the results

$$\left.\frac{d^2V}{dx^2}\right|_{x=0} = 72(0) - 6 = -6 \rightarrow \textit{Unstable equilibrium when } x = 0$$

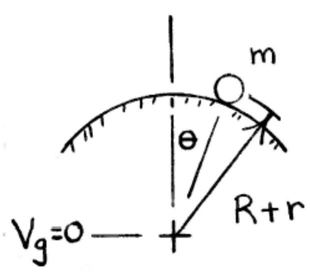
$$\left.\frac{d^2V}{dx^2}\right|_{x=\frac{1}{2}} = 72(0.5)^2 - 6 = 12 \rightarrow \textit{Stable equilibrium when } x = ½$$

$$\left.\frac{d^2V}{dx^2}\right|_{x=-\frac{1}{2}} = 72(-0.5)^2 - 6 = 12 \rightarrow \textit{Stable equilibrium when } x = -½$$

■ The correct answer is **C**.

P.9 ■ Solution

Consider first the configuration to the left.



The potential energy of the system, *V*, which is due to gravity only, is such that

$$V = V_g = (R+r)\cos\theta$$

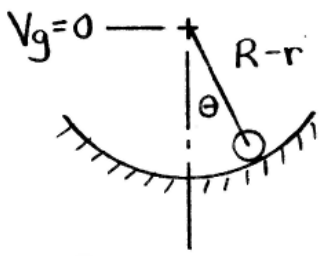
which, upon differentiation with respect to θ, becomes

$$\frac{dV}{d\theta} = -(R+r)\sin\theta = 0$$

For the equality above to hold, we must have θ = 0 or $\theta = \pi$. The second possibility is rejected, and we thus have θ = 0. Differentiating the previous expression a second time gives

$$\frac{d^2V}{d\theta^2} = -(R+r)\cos\theta$$

which is clearly less than zero when $\theta = 0$. Since $d^2V/d\theta^2 < 0$, we conclude that equilibrium is *unstable* for the system to the left. Consider now the hypothetical system to the right.



The potential energy, in this case, is

$$V = V_g = -(R - r)\cos\theta$$

which, upon differentiation, becomes

$$\frac{dV}{d\theta} = (R - r)\sin\theta = 0$$

This expression equals zero if $\theta = 0$ or $\theta = \pi$, the latter case being immediately rejected. Differentiating the equation a second time, we get

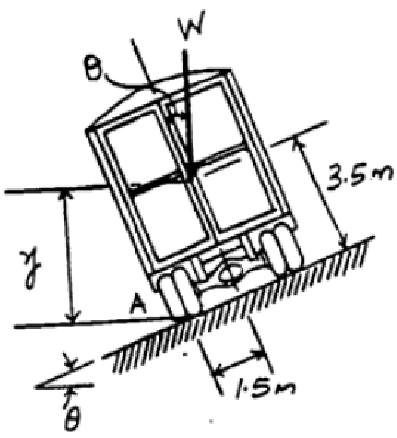
$$\frac{d^2V}{d\theta^2} = (R - r)\cos\theta$$

which, for $\theta = 0$, is clearly greater than zero. Since $d^2V/d\theta^2 > 0$, we conclude that equilibrium is *stable* for the system to the right.

■ The correct answer is **C**.

P.10 ■ Solution

The datum is located at point A, as shown.



Since the center of gravity for the truck is above the datum, its potential energy is positive. Here, $y = (1.5\sin\theta + 3.5\cos\theta)$ m. The potential energy is due to gravity only and equals

$$V = V_g = Wy = W(1.5\sin\theta + 3.5\cos\theta)$$

The system is in equilibrium if $dV/d\theta = 0$. Therefore,

$$\frac{dV}{d\theta} = W(1.5\cos\theta - 3.5\sin\theta) = 0$$

Since $W \neq 0$, the term in parentheses must equal zero; that is,

$$1.5\cos\theta - 3.5\sin\theta = 0$$

$$\therefore 1.5\cos\theta = 3.5\sin\theta$$

$$\therefore \tan\theta = \frac{1.5}{3.5} = 0.429$$

$$\therefore \theta = 23.2^{\circ}$$

To assess the stability of the truck in this position, we differentiate $dV/d\theta$ a second time,

$$\frac{d^2V}{d\theta^2} = -1.5\sin\theta - 3.5\cos\theta$$

which, when θ = 23.2°, yields

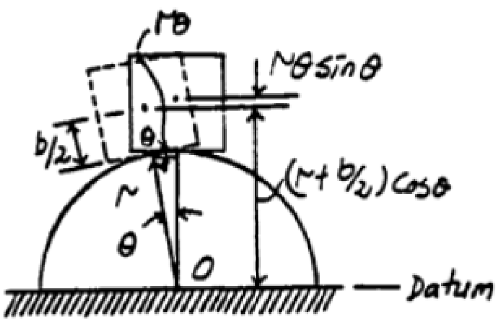
$$\left.\frac{d^2V}{d\theta^2}\right|_{\theta=23.2^{\circ}} = -1.5\sin 23.2^{\circ} - 3.5\cos 23.2^{\circ} = -3.81$$

Since $d^2V/d\theta^2 < 0$, we conclude that the equilibrium is unstable.

■ The correct answer is **D**.

P.11 ■ Solution

Consider the figure below.



The datum is established at point O. Since the center of gravity for the block is above the datum, its potential energy is positive. Here,

$$y = \left(r + \frac{b}{2}\right)\cos\theta + r\theta\sin\theta$$

and the potential energy equals

$$V = W\left[\left(r + \frac{b}{2}\right)\cos\theta + r\theta\sin\theta\right]$$

where W is the weight of the block. For a small angle θ, we may use the approximations $\sin\theta \approx \theta$ and $\cos\theta \approx 1 - \theta^2/2$. Consequently, V becomes

$$V = W\left[\left(r + \frac{b}{2}\right)\left(1 - \frac{\theta^2}{2}\right) + r\theta^2\right]$$

$$\therefore V = W\left[\frac{b}{2} + r + \frac{b\theta^2}{4} + \frac{3r\theta^2}{2}\right]$$

The system is in equilibrium if $dV/d\theta$ = 0. Therefore,

$$\frac{dV}{d\theta} = -\frac{b\theta}{2} + r\theta = 0$$

$$\therefore \frac{dV}{d\theta} = \theta\left(-\frac{b}{2} + r\right) = 0$$

In order for us to have stable equilibrium, the second derivative must be greater than zero; that is,

$$\frac{d^2V}{d\theta^2} = -\frac{b}{2} + r > 0$$

$$\therefore r > \frac{b}{2}$$

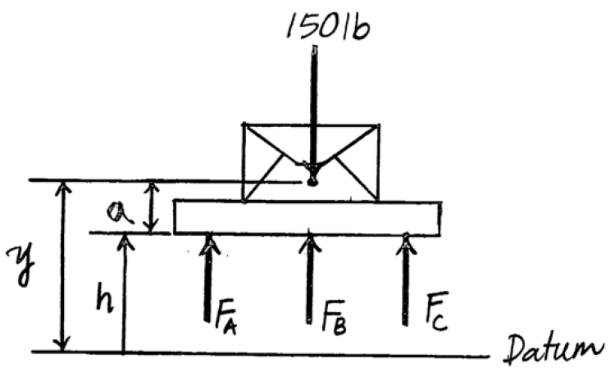
$$\therefore \boxed{b < 2r}$$

Thus, equilibrium will be stable if the dimension of the block is smaller than the diameter of the cylinder.

■ The correct answer is **B**.

P.12 ■ Solution

Consider the following illustration of this system.



With reference to the datum shown above, the gravitational potential energy of the package and the platform is positive. Here, *y* = *h* + *a*, where *a* is a constant. Thus,

$$V_g = Wy = 150(h + a) = 150h + 150a$$

The elastic potential energy for the springs can be computed using the typical relationship $V_e = (1/2)ks^2$. In this equation, the compression of the springs is $s_A = s_C = 10 - h$, and $s_B = 12 - h$. Therefore, the elastic potential energy is, in total,

$$V_e = 2\left[\frac{1}{2}(20)(10 - h)^2\right] + \frac{1}{2}(30)(12 - h)^2$$

$$\therefore V_e = 35h^2 - 760h + 4160$$

The total potential energy of the system also includes the gravitational potential energy, V_g, so that

$$V = V_e + V_g = 35h^2 - 760h + 4160 + 150(h + a)$$

$$\therefore V = 35h^2 - 610h + 4160 + 150a$$

Taking the first derivative of *V* and equating it to zero, we obtain

$$\frac{dV}{dh} = 70h - 610 = 0$$

$$\therefore h = \frac{610}{70} = 8.71 \text{ in.}$$

Thus, the system is in equilibrium when $h = 8.71 \approx 8.7$ in. To verify the stability of equilibrium, we differentiate *dV/dh* a second time, with the result that

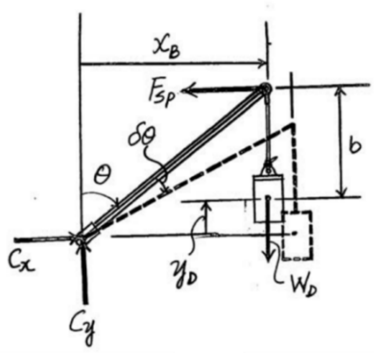
$$\frac{d^2V}{dh^2} = 70$$

Since the result is greater than zero, the system is stable.

■ The correct answer is **C**.

P.13 ■ Solution

Consider the following illustration.



With reference to the datum in the figure shown above, the gravitational potential energy of the cylinder is positive (since its center of gravity is located above the datum). We have $y = (5\cos\theta - b)$ ft. The gravitational potential energy is then

$$V_g = Wy = W_D(5\cos\theta - b) = 5W_D\cos\theta - W_D b$$

The elastic potential energy of the spring can be computed using $V_g = (1/2)ks^2$, where $s = 5\sin\theta - 5\sin 45° = 5\sin\theta - 3.54$ ft. Hence, the elastic potential energy is

$$V_e = \frac{1}{2}(600)(5\sin\theta - 3.54)^2 = 7500\sin^2\theta - 10620\sin\theta + 3759.5$$

The total potential energy of the system is the sum of V_g and V_e, namely,

$$V = V_g + V_e = 5W_D\cos\theta - W_D b + 7500\sin^2\theta - 10620\sin\theta + 3759.5$$

For the equilibrium configuration, we take the first derivative of θ.

$$\frac{dV}{d\theta} = -5W_D\sin\theta + 15000\sin\theta\cos\theta - 10620\cos\theta = 0$$

Equilibrium requires $dV/d\theta = 0$. Isolating W_D in the expression above, it follows that

$$W_D = \frac{15000\sin\theta\cos\theta - 10620\cos\theta}{5\sin\theta}$$

Substituting $\theta = 60°$, the result is $W_D = 274$ lb.

■ The correct answer is **A**.

Answer Summary

Problem 1	D
Problem 2	B
Problem 3	D
Problem 4	B
Problem 5	C
Problem 6	B
Problem 7	C
Problem 8	C
Problem 9	C
Problem 10	D
Problem 11	B
Problem 12	C
Problem 13	A

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