Quiz FM108
Viscous Flow in Pipes
Lucas Montogue

PROBLEMS

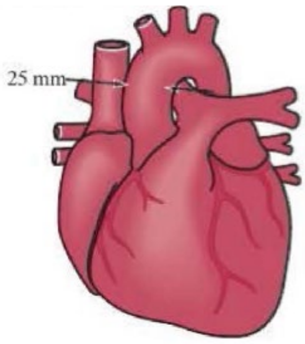
● Problem 1A (Hibbeler, 2015, w/ permission)

The retinal arterioles supply the retina of the eye with blood flow. The inner diameter of an arteriole is 0.08 mm, and the mean velocity of flow is 28 mm/s. Is this flow laminar, transitional, or turbulent? Assume that blood has a density of 1060 kg/m³ and an apparent viscosity of 0.0036 N·s/m².

A) The flow is laminar.
B) The flow is transitional.
C) The flow is turbulent.
D) There is not enough information to establish the type of flow.

● Problem 1B

Most blood flow in humans is laminar, and apart from pathological conditions, turbulence can occur in the descending portion of the aorta at high flow rates as when exercising. Assuming that the diameter of the aorta is 25 mm, and that blood has the same density and viscosity as in the previous part, the largest velocity blood flow can have before entering the transitional region, which begins at $Re \approx 2100$, is



A) V_{max} = 0.104 m/s
B) V_{max} = 0.285 m/s
C) V_{max} = 0.466 m/s
D) V_{max} = 0.647 m/s

Problem 2

For laminar flow in a round pipe of diameter D, at what distance from the centerline is the velocity equal to the average velocity?

A) $r^* = 0.121D$
B) $r^* = 0.258D$
C) $r^* = 0.354D$
D) $r^* = 0.500D$

Problem 3 (Munson et al., 2009, w/ permission)

A person with no experience in fluid mechanics wants to estimate the friction factor for a 1-in.-diameter galvanized iron pipe at a Reynolds number of 8000. They stumble across the simple equation $f = 64/Re$ and use it to calculate the friction factor. Estimate the error in this estimate. The roughness height for galvanized iron pipe is $\varepsilon = 5 \times 10^{-4}$ m.

A) The actual friction factor is 10 times lower than the estimate obtained by the person.
B) The actual friction factor is 5 times lower than the estimate obtained by the person.
C) The actual friction factor is 5 times greater than the estimate obtained by the person.
D) The actual friction factor is 10 times greater than the estimate obtained by the person.

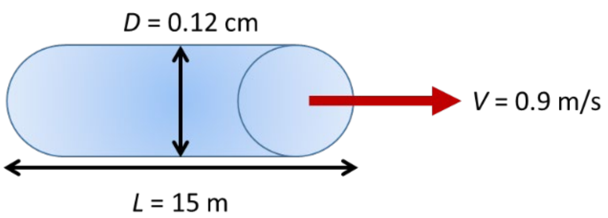
Problem 4

Oil with a kinematic viscosity of 0.007 ft²/s flows in a 3-in.-diameter pipe at 0.01 ft³/s. Determine the head loss per unit length of this flow.

A) h_L = 0.0227 ft/ft of pipe
B) h_L = 0.0454 ft/ft of pipe
C) h_L = 0.0675 ft/ft of pipe
D) h_L = 0.0832 ft/ft of pipe

Problem 5

Water at 10°C (ρ = 999.7 kg/m³, $\mu = 1.307 \times 10^{-3}$ kg/m·s) is flowing steadily in a 0.12-cm-diameter, 15-m-long pipe at an average velocity of 0.9 m/s. Determine the head loss for this flow.



A) h_L = 10.2 m
B) h_L = 23.6 m
C) h_L = 30.6 m
D) h_L = 40.0 m

Problem 6 (Munson et al., 2009, w/ permission)

Air at standard conditions flows through an 8-in. diameter, 14.6-ft long, straight duct with the velocity versus pressure drop data indicated in the following table. Determine the average friction factor over this range of data.

V (ft/min)	Δp (in. water)
3950	0.35
3730	0.32
3610	0.30
3430	0.27
3280	0.24
3000	0.20
2700	0.16

A) f_{avg} = 0.0125
B) f_{avg} = 0.0162
C) f_{avg} = 0.0201
D) f_{avg} = 0.0243

Problem 7 (Munson et al., 2009, w/ permission)

Water flows downward through a vertical 10-mm-diameter galvanized iron pipe with an average velocity of 5.0 m/s and exits as a free jet. There is a small hole in the pipe 4 m above the outlet. Will water leak out of the pipe through this hole, or will air enter into the pipe through the hole? What would happen if the flow velocity were 0.5 m/s? The roughness height of galvanized iron is 0.15 mm. Use $\nu = 1.12 \times 10^{-6}$ m²/s.

A) Water will leak out of the pipe in both cases.

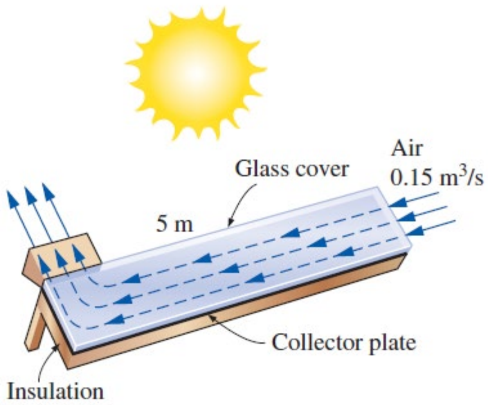
B) Water will leak out of the pipe if V = 5.0 m/s and air will enter into the pipe if V = 0.5 m/s.

C) Air will enter the pipe if V = 5.0 m/s and water will enter into the pipe if V = 0.5 m/s.

D) Air will enter the pipe in both cases.

Problem 8 (Çengel & Cimbala, 2014, w/ permission)

Consider an air solar collector that is 1 m wide and 5 m long and has a constant spacing of 3 cm between the glass cover and the collector plate. Air flows at an average temperature of 45ºC (ρ = 1.11 kg/m³, $\nu = 1.75 \times 10^{-5}$ m²/s) at a rate of 0.15 m³/s through the 1-m-wide edge of the collector along the 5-m-long passageway. Disregarding the entrance and roughness effects and the 90° bend, determine the pressure drop in the collector.



A) Δp = 32.3 kPa

B) Δp = 47.1 kPa

C) Δp = 60.2 kPa

D) Δp = 70.9 kPa

Problem 9A (Çengel & Cimbala, 2014, w/ permission)

Heated air at 1 atm and 35ºC ($\nu = 1.655 \times 10^{-5}$ m²/s) is to be transported in a 150-m-long circular plastic duct at a rate of 0.45 m³/s. If the head loss in the pipe is not to exceed 25 m, determine the minimum diameter of the duct. Neglect surface roughness.

A) D = 0.081 m

B) D = 0.15 m

C) D = 0.28 m

D) D = 0.45 m

Problem 9B

Reconsider the previous example. Now the duct length is doubled while its diameter is maintained constant. If the total head loss is to be the same, determine the drop in the flow rate through the duct relatively to the previous situation.

A) ΔQ = −0.145 m³/s

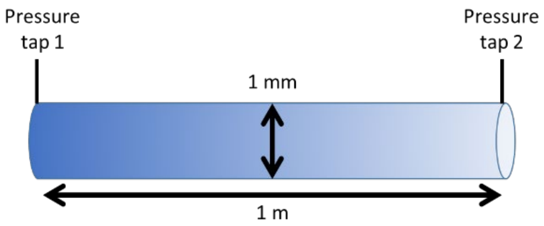
B) ΔQ = −0.195 m³/s

C) ΔQ = −0.235 m³/s

D) ΔQ = −0.265 m³/s

• Problem 10A

Water flows through a horizontal 1-mm-diameter tube to which are attached two pressure taps a distance 1 m apart. What is the maximum pressure drop allowed if the flow is to be laminar? Use $\mu = 10^{-3}$ N·s/m² and $\nu = 10^{-6}$ m²/s.



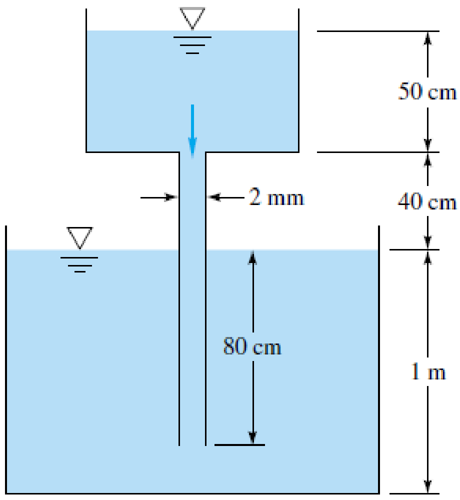
A) $(\Delta p)_{max}$ = 32.1 kPa
B) $(\Delta p)_{max}$ = 37.3 kPa
C) $(\Delta p)_{max}$ = 52.1 kPa
D) $(\Delta p)_{max}$ = 67.2 kPa

• Problem 10B

Assume the manufacturing tolerance on the tube diameter is $D = 1.0 \pm 0.1$ mm. Given this uncertainty in the tube diameter, what is the maximum pressure drop allowed if it must be assured that the flow is laminar?
A) $(\Delta p)_{max}$ = 20.8 kPa
B) $(\Delta p)_{max}$ = 30.5 kPa
C) $(\Delta p)_{max}$ = 40.7 kPa
D) $(\Delta p)_{max}$ = 50.5 kPa

• Problem 11A (White, 2003, w/ permission)

For the configuration shown in the figure below, the fluid is ethyl alcohol at 20ºC (μ = 0.0012 kg/m·s, ρ = 789 kg/m³) and the tanks are very wide. Find the flow rate which occurs in m³/hour and establish whether the flow is laminar or not.



A) Q = 0.00395 m³/h and the flow is laminar.
B) Q = 0.00395 m³/h and the flow is turbulent.
C) Q = 0.00684 m³/h and the flow is laminar.
D) Q = 0.00684 m³/h and the flow is turbulent.

• Problem 11B

For the system in the previous problem, if the fluid has density of 920 kg/m³ and the flow rate is unknown, for what value of viscosity will the capillary Reynolds number exactly equal the critical value of 2100?
A) $\mu = 2.23 \times 10^{-4}$ kg/m·s.
B) $\mu = 4.41 \times 10^{-4}$ kg/m·s
C) $\mu = 6.02 \times 10^{-4}$ kg/m·s
D) $\mu = 8.61 \times 10^{-4}$ kg/m·s

Problem 12 (Çengel & Cimbala, 2014, w/ permission)

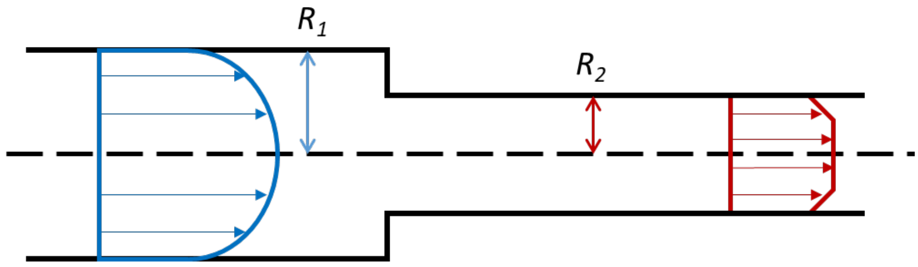
Water flows steadily through a reducing pipe section. The flow upstream with a radius of R_1 is laminar with a velocity profile given by

$$u_1(r) = (u_0)_1\left(1 - \frac{r^2}{R_1^2}\right)$$

The flow downstream is turbulent with a velocity profile given by

$$u_2(r) = (u_0)_2\left(1 - \frac{r}{R_2}\right)^{\frac{1}{7}}$$

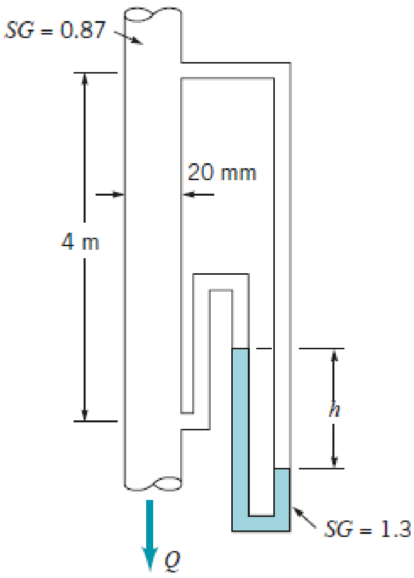
For incompressible flow with $R_2/R_1 = 4/7$, determine the ratio of centerline velocities $(u_0)_1/(u_0)_2$.



A) ${}^{(u_0)_1}\!/_{(u_0)_2} = {}^{2}\!/_{15}$
B) ${}^{(u_0)_1}\!/_{(u_0)_2} = {}^{4}\!/_{15}$
C) ${}^{(u_0)_1}\!/_{(u_0)_2} = {}^{8}\!/_{15}$
D) ${}^{(u_0)_1}\!/_{(u_0)_2} = {}^{4}\!/_{5}$

Problem 13 (Munson et al., 2009, w/ permission)

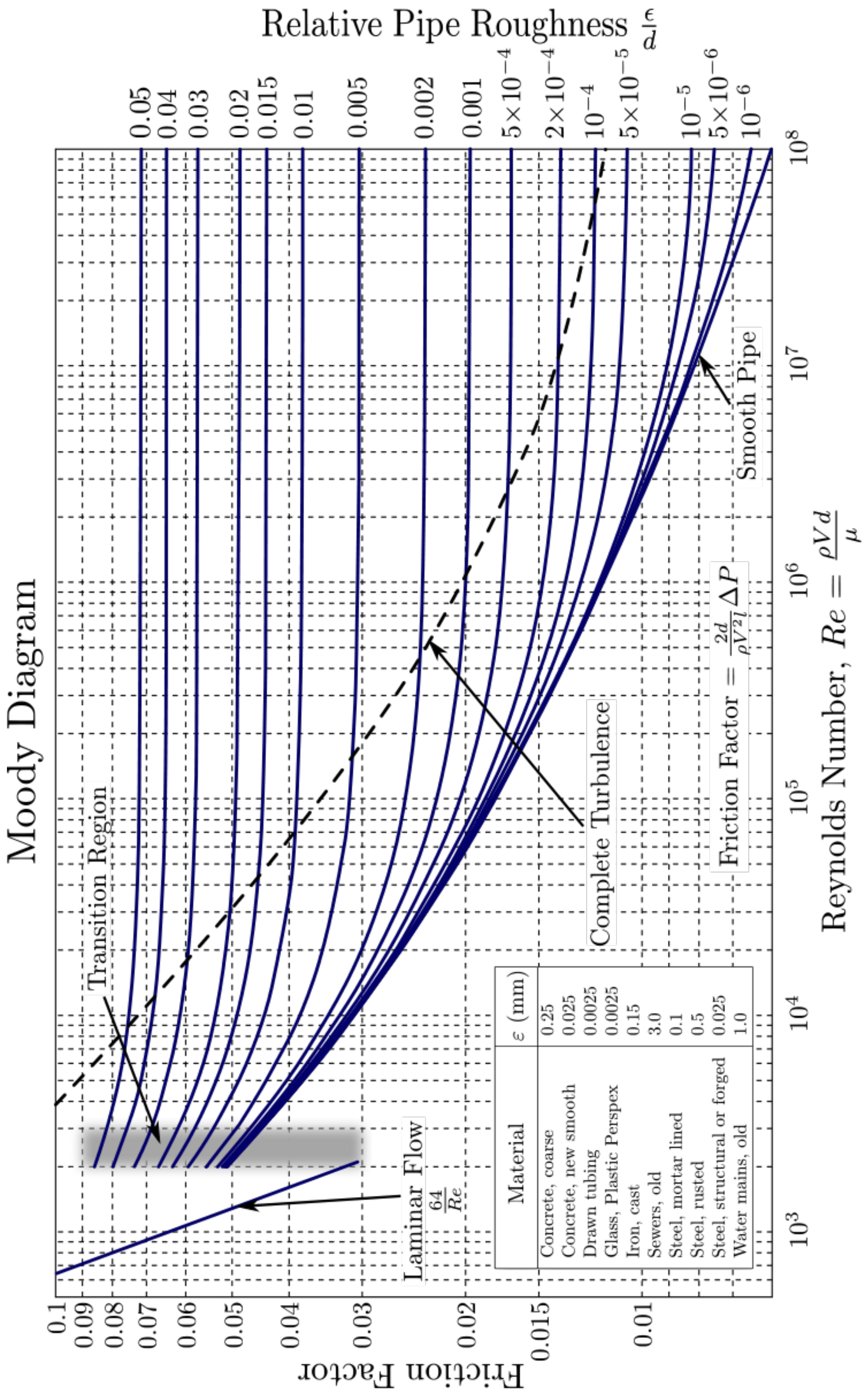
Oil of specific gravity equal to 0.87 and a kinematic viscosity $\nu = 2.2 \times 10^{-4} \text{m}^2/\text{s}$ flows through the vertical pipe shown below at a rate of $4 \times 10^{-4}\ \text{m}^3/\text{s}$. Determine the manometer reading.



A) h = 9.6 m
B) h = 12.4 m
C) h = 15.2 m
D) h = 18.5 m

ADDITIONAL INFORMATION

Figure 1 Moody diagram.



SOLUTIONS

P.1 ● Solution

Part A: All we have to do is determine the Reynolds number,

$$\mathrm{Re} = \frac{\rho V D}{\mu}$$

where ρ = 1060 kg/m^3, V = 28 × 10^{-3} m/s, D = 0.08 mm = 8 × 10^{-5} m, and μ = 3.6 × 10^{-3} N·s/m^2, so that

$$\mathrm{Re} = \frac{1060 \times (28 \times 10^{-3}) \times (8 \times 10^{-5})}{3.6 \times 10^{-3}} = 0.66$$

Flow in a cylindrical conduit such as an arteriole is laminar if $\mathrm{Re} < 2100$. Accordingly, we conclude that flow is laminar in this situation.

➡ The correct answer is **A**.

Part B: We appeal to the definition of *Re* a second time,

$$\text{Re} = \frac{\rho V_{\max} D}{\mu}$$

Solving for $V_{\max}$ gives

$$V_{\max} = \frac{\text{Re} \times \mu}{\rho D}$$

Flow in a circular duct such as the aorta enters the transitional region when Re ≈ 2100. Thus,

$$V_{\max} = \frac{2100 \times 0.0036}{1060 \times 25 \times 10^{-3}} = \boxed{0.285\ \text{m/s}}$$

That is to say, flow in the aorta will be laminar if the flow velocity is lower than 0.285 m/s.

➡ The correct answer is **B**.

P.2 ● Solution

For laminar flow in a circular tube, a generic velocity profile is

$$u(r) = V_c\left[1 - \left(\frac{2r}{D}\right)^2\right]$$

We know that the average velocity for flow in a closed pipe is one-half the velocity in the centerline (i.e., the maximum velocity), $V_c/2$. To locate the point at which this velocity occurs, we substitute $u(r^*) = V_c/2$ in the equation above and manipulate, giving

$$u(r^*) = \frac{\cancel{V_c}}{2} = \cancel{V_c}\left[1 - \left(\frac{2r^*}{D}\right)^2\right]$$

$$\therefore \frac{1}{2} = 1 - \left(\frac{2r^*}{D}\right)^2$$

$$\therefore \sqrt{\left(\frac{2r^*}{D}\right)^2} = \sqrt{\frac{1}{2}}$$

$$\therefore \left(\frac{2r^*}{D}\right) = \frac{1}{\sqrt{2}}$$

$$\therefore r^* = \frac{D}{2\sqrt{2}} \approx \boxed{0.354D}$$

Hence, the average velocity occurs at a distance slightly greater than one-third of the duct diameter.

➡ The correct answer is **C**.

P.3 ● Solution

The estimate is wrong because flow is turbulent at *Re* = 8000 and the result $f = 64/Re$ only applies to laminar flow. The friction factor obtained by the person is

$$f_1 = \frac{64}{\text{Re}} = \frac{64}{8000} = 0.008$$

The actual friction factor can be obtained with the Colebrook equation or the Moody chart. The relative roughness is $\varepsilon/D = (5\times10^{-4})/(1/12) = 0.006$. Referring to the Moody chart, we read $f_2 \approx 0.04$. The ratio of f_2 to f_1 is

$$\frac{f_2}{f_1} = \frac{0.04}{0.008} = 5$$

That is, the actual friction factor is 5 times greater than the estimate obtained by the person.

➡ The correct answer is **C**.

P.4 ● Solution

The solution is started by calculating the flow velocity,

$$V = \frac{Q}{A} = \frac{0.01}{\dfrac{\pi\times(3/12)^2}{4}} = 0.204\ \text{ft/s}$$

The Reynolds number is determined next,

$$\text{Re} = \frac{VD}{\nu} = \frac{0.204\times(3/12)}{0.007} = 7.3$$

Since *Re* < 2100, flow is laminar and the friction factor can be computed with the relation

$$f = \frac{64}{\text{Re}} = \frac{64}{7.3} = 8.77$$

The head loss per unit length of pipe follows as

$$h_L = f\frac{L}{D}\frac{V^2}{2g} = 8.77\times\frac{1}{(3/12)}\times\frac{0.204^2}{2\times 32.2} = \boxed{0.0227\ \text{ft/ft of pipe}}$$

➡ The correct answer is **A**.

P.5 ● Solution

The Reynolds number of the flow is

$$\text{Re} = \frac{\rho VD}{\mu} = \frac{999.7\times 0.9\times(0.12\times 10^{-2})}{1.307\times 10^{-3}} = 826$$

which is below the 2100 threshold and therefore implies that the flow is laminar. The friction factor can be obtained with the simple relationship

$$f = \frac{64}{\text{Re}} = \frac{64}{826} = 0.0775$$

To compute the headloss, we substitute the pertaining variables in the Darcy-Weisbach equation, giving

$$h_L = \frac{\Delta p}{\rho g} = f\frac{L}{D}\times\frac{V^2}{2g} = 0.0775\times\frac{15}{0.12\times 10^{-2}}\times\frac{0.9^2}{2\times 9.81} = \boxed{40.0\ \text{m}}$$

➡ The correct answer is **D**.

P.6 ● Solution

Applying the Bernoulli equation, we write

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + f\frac{L}{D}\frac{V^2}{2g}$$

However, $V_1 = V_2$ and $z_1 = z_2$. Solving for the friction factor yields

$$\Delta p = f\frac{L}{D}\frac{\rho V^2}{2} \rightarrow f = \frac{2\Delta p D}{\rho L V^2}$$

where $\Delta p = \gamma_{H_2O}h$. Substituting the pertaining variables, we obtain

$$f = \frac{2\times\left(\dfrac{h}{12}\ \text{ft}\right)\times\left(62.4\ \dfrac{\text{lb}}{\text{ft}^3}\right)\times\left(\dfrac{8}{12}\ \text{ft}\right)}{\left(2.38\times 10^{-3}\ \dfrac{\text{slugs}}{\text{ft}^3}\right)\times(14.6\ \text{ft})\times\left(\dfrac{V}{60}\ \dfrac{\text{ft}}{\text{s}}\right)^2} = \frac{7.18\times 10^5 h}{V}$$

where h is given in inches of water and V in ft/min. The values of f for the data we received are tabulated below.

V (ft/min)	h (in. water)	f
3950	0.35	0.01611
3730	0.32	0.01651
3610	0.3	0.01653
3430	0.27	0.01648
3280	0.24	0.01602
3000	0.2	0.01596
2700	0.16	0.01576
	Average f	0.0162

The average friction factor is 0.0162.

➡ The correct answer is **B**.

P.7 ● Solution

We begin by writing the Bernoulli equation,

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+f\frac{L}{D}\frac{V^2}{2g}$$

Here, we have $p_2 = 0$, $z_2 = 0$, $z_1 = 4$ m, and $V_1 = V_2 = V$. Accordingly,

$$\frac{p_1}{\gamma}=f\frac{L}{D}\frac{V^2}{2g}-z_1 \rightarrow p_1=f\frac{L}{D}\frac{\rho V^2}{2}-\gamma L \text{ (I)}$$

The relative roughness is $\varepsilon/D = 0.15/10 = 0.015$ and the Reynolds number is

$$\text{Re}=\frac{VD}{\nu}=\frac{5.0\times 0.01}{1.12\times 10^{-6}}=4.46\times 10^4$$

Referring to the Moody diagram or appealing to the Colebrook equation, we find $f = 0.045$. Substituting in equation (I) brings to

$$p_1=0.045\times\frac{4}{0.01}\times\frac{1}{2}\times 1000\times 5^2-9800\times 4=1.86\times 10^5 \text{ N/m}^2$$

Since $p_1 > 0$, water will leak out of the pipe. This conclusion applies regardless of the length of the conduit. With $V = 0.5$ m/s, the Reynolds number becomes

$$\text{Re}=\frac{0.5\times 0.01}{1.12\times 10^{-6}}=4.46\times 10^3$$

and the corresponding f is found to be 0.052. Substituting in equation (I) gives

$$p_1=0.052\times\frac{4}{0.01}\times\frac{1}{2}\times 1000\times 0.5^2-9800\times 4=-3.66\times 10^4 \text{ N/m}^2$$

Since $p_1 < 0$, air will enter the pipe when $V = 0.5$ m/s.

➡ The correct answer is **B**.

P.8 ● Solution

The mass flow rate is given by

$$\dot{m}=\rho Q=1.11\times 0.15=0.167 \text{ kg/s}$$

The cross-sectional area is

$$A_c=1\times 0.03=0.03 \text{ m}^2$$

The hydraulic diameter is

$$D_h=\frac{4A_c}{P}=\frac{4\times 0.03}{2(1+0.03)}=0.0583 \text{ m}$$

The flow velocity is found as

$$V=\frac{Q}{A_c}=\frac{0.15}{0.03}=5 \text{ m/s}$$

To determine the type of flow, we compute the Reynolds number,

$$\mathrm{Re} = \frac{VD_h}{\nu} = \frac{5 \times 0.0583}{1.75 \times 10^{-5}} = 16{,}660$$

The flow is well above the threshold for turbulent conditions. The friction factor corresponding to this Reynolds number for a smooth flow section (ε/D = 0) can be obtained from the Moody chart or the Colebrook equation, giving f = 0.0271. Accordingly, the pressure drop follows as

$$\Delta p = f\frac{L}{D_h}\frac{\rho V^2}{2} = 0.0271 \times \frac{5}{0.0583} \times \frac{1.11 \times 5}{2} = \boxed{32.3\ \mathrm{kPa}}$$

➡ The correct answer is **A**.

P.9 ● Solution

Part A: The average velocity, the Reynolds number, the friction factor, and the diameter are the unknowns in the present problem. in order to obtain these quantities, we require four equations. The equation for velocity is

$$V = \frac{Q}{A} = \frac{0.45}{\pi D^2/4} = \frac{1.8}{\pi D^2}$$

The equation for the Reynolds number, in turn, is

$$\mathrm{Re} = \frac{VD}{\nu} = \frac{VD}{1.655 \times 10^{-5}}$$

Next, we prepare the Colebrook equation,

$$\frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right)$$

Neglecting roughness, ε/D = 0 and the equation simplifies to

$$\frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{2.51}{\mathrm{Re}\sqrt{f}}\right)$$

Finally, we have the Darcy-Weisbach equation,

$$h_L = f\frac{L}{D}\frac{V^2}{2g} \rightarrow 20 = f\frac{150}{D}\frac{V^2}{2 \times 9.81} = 7.645\frac{fV^2}{D}$$

This set of equations could be solved by a CAS such as Mathematica. For this program in particular, the command *NSolve* can be used,

$$\mathrm{NSolve}[V == \frac{1.8}{\pi * \mathrm{D}^2}\&\&\mathrm{RE} == \frac{V * \mathrm{D}}{1.655 * 10^{-5}}\&\&\frac{1}{\sqrt{f}} == -2.* \mathrm{Log10}[\frac{2.51}{\mathrm{RE} * \sqrt{f}}]\&\&20 =$$
$$= 7.645\frac{f * V^2}{\mathrm{D}}, \{\mathrm{D}, \mathrm{V}, \mathrm{RE}, f\}, \mathrm{Reals}]$$

This yields D = 0.280 m, V = 7.30 m/s, Re = 125,531, and f = 0.0172. The required duct diameter is 0.28 m. Note that the Reynolds number is well above the turbulent threshold, as expected.

➡ The correct answer is **C**.

Part B: This problem is analogous to the previous one, the difference being that one of the four unknowns – namely, the diameter D – is replaced with the flow rate, Q. The length of the pipe is now equal to 300 m. The average velocity is

$$V = \frac{Q}{A} \rightarrow V = \frac{Q}{\pi D^2/4} = \frac{Q}{\pi(0.28)^2/4} = 16.24Q$$

The Reynolds number is

$$\mathrm{Re} = \frac{VD}{\nu} \rightarrow \mathrm{Re} = \frac{V \times 0.28}{1.655 \times 10^{-5}}$$

The Colebrook equation is prepared as well,

$$\frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right) \rightarrow \frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{2.51}{\mathrm{Re}\sqrt{f}}\right)$$

Lastly, we have the Darcy-Weisbach equation,

$$h_L = f\frac{L}{D}\frac{V^2}{2g} \rightarrow 25 = f\frac{300}{0.28}\frac{V^2}{2(9.81)} = 54.6\frac{fV^2}{D}$$

As before, these equations can be solved with a trial-and-error procedure. Using the *NSolve* command in Mathematica, we enter the code

$$\text{NSolve}[V == \frac{Q}{(\pi * 0.28^2)/4} \&\&\text{RE} == \frac{V * 0.28}{1.655 * 10^{-5}} \&\& \frac{1}{\sqrt{f}} =$$
$$= -2.* \text{Log10}[\frac{2.51}{\text{RE} * \sqrt{f}}]\&\&25 == 54.6f * V^2, \{Q, V, \text{RE}, f\}, \text{Reals}]$$

This returns Q = 0.305 m³/s, V = 4.95 m/s, Re = 83,779, and f = 0.0187. That is, the flow rate is now equal to 0.305 m³/s, which corresponds to a decrease of ΔQ = 0.305 – 0.45 = -0.145 m³/s relatively to the previous configuration.

➡ The correct answer is **A**.

P.10 ● Solution

Part A: To obtain the Δp that applies the maximum laminar velocity, V_{max}, we set the Reynolds number to 2100 and solve for velocity; that is,

$$\text{Re} = \frac{VD}{\nu} = 2100 \rightarrow V = \frac{2100\nu}{D}$$

$$\therefore V = \frac{2100(10^{-6})}{10^{-3}} = 2.1 \text{ m/s}$$

The corresponding pressure drop is

$$V = \frac{\Delta p D^2}{32\mu L} \rightarrow (\Delta p)_{\max} = \frac{32\mu V L}{D^2}$$

$$\therefore (\Delta p)_{\max} = \frac{32 \times 10^{-3} \times 2.1 \times 1}{(10^{-3})^2} = 67{,}200 = \boxed{67.2 \text{ kPa}}$$

➡ The correct answer is **D**.

Part B: Since $V = (2100\nu)/D$ and $\Delta p = 32\mu LV/D^2$, it follows that

$$\Delta p = \frac{32\mu L(2100\nu)}{D^3}$$

From the equality above, note that the larger the diameter, the smaller the Δp allowed to maintain laminar flow. Thus, supposing that $D = 1.1$ mm, the maximum pressure drop is calculated to be

$$(\Delta p)_{\max} = \frac{32 \times 10^{-3} \times 1 \times 2100 \times 10^{-6}}{(1.1 \times 10^{-3})^3} = \boxed{50.5 \text{ kPa}}$$

➡ The correct answer is **D**.

P.11 ● Solution

Part A: We begin by writing the Bernoulli equation from the upper free surface to the lower free surface,

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2}{2g} + z_2 + h_L$$

With $p_1 = p_2$ and $V_1 \approx V_2$, the equation simplifies to

$$h_L = z_1 - z_2 = 0.9 \text{ m}$$

Appealing to the Poiseuille equation and solving for the flow rate Q, we obtain

$$h_L = 0.9 = \frac{128\mu LQ}{\pi\rho g D^4} = \frac{128 \times 0.0012 \times 1.2Q}{\pi \times 789 \times 9.81 \times 0.002^4} = 473{,}758Q$$

$$\therefore Q = \frac{0.9}{473{,}758} = 1.9 \times 10^{-6}\frac{\text{m}^3}{\text{s}} \times 3600\frac{\text{s}}{\text{hour}} = \boxed{0.00684 \text{ m}^3/\text{h}}$$

Let us verify the nature of the flow by computing the Reynolds number,

$$\text{Re} = \frac{\rho VD}{\mu} = \frac{\rho}{\mu} \times \frac{Q}{A} \times D = \frac{\rho}{\mu} \times \frac{4Q}{\pi D^2} \times D = \frac{4\rho Q}{\pi\mu D}$$

Substituting the pertaining variables, we get

$$\mathrm{Re} = \frac{4\rho Q}{\pi\mu D} = \frac{4 \times 789 \times 1.9 \times 10^{-6}}{\pi \times 0.0012 \times 0.002} = 795$$

Hence, the flow is laminar. It is important to note that if the flow were found to be turbulent, the variant of the Poiseuille equation that we have used may not have been valid.

➡ The correct answer is **C**.

Part B: We can solve the equation obtained in the previous part for the flow rate Q,

$$\mathrm{Re} = \frac{4\rho Q}{\pi\mu D} \rightarrow Q = \frac{\pi\mu D\mathrm{Re}}{4\rho}$$

Then, we can insert this result in the Poiseuille equation,

$$h_L = \frac{128\mu L}{\pi\rho g D^4} Q = \frac{128\mu L}{\pi\rho g D^4}\left[\frac{\pi\mu D\mathrm{Re}}{4\rho}\right]$$

Substituting the pertaining variables and solving for μ, it follows that

$$\therefore h_L = 0.9 = \frac{128 \times \mu \times 1.2}{\pi \times 920 \times 9.81 \times 0.002^4}\left[\frac{\pi \times \mu \times 0.002 \times 2100}{4 \times 920}\right]$$

$$\therefore \boxed{\mu = 8.61 \times 10^{-4}\ \mathrm{kg/m\cdot s}}$$

➡ The correct answer is **D**.

P.12 ● Solution

The volume flow rate for both pipe sections must be the same. Hence, we can write

$$Q_1 = Q_2 \rightarrow \int_{A_1} u_1 dA_1 = \int_{A_2} u_2 dA_1$$

Preparing the integrals on each side, we have

$$\int_0^{R_1} (u_0)_1 \left[1 - \frac{r_1^2}{R_2^2}\right] \times 2\pi r_1 dr_1 = \int_0^{R_2} (u_0)_2 \left[1 - \frac{r_2}{R_2}\right]^{\frac{1}{7}} \times 2\pi r_2 dr_2$$

These integrals could be evaluated by means of a CAS such as Mathematica, in which case we apply the command *Integrate*; for the integral on the left-hand side, we type

$$\mathrm{Q}_1 = \mathrm{Integrate}[(u_0)_1\left(1 - \frac{r^2}{R_1^2}\right)2\pi * r, \{r, 0, R_1\}]$$

This produces the output $\pi R_1^2 (u_0)_1/2$. In a similar manner, we have, for the right-hand side,

$$\mathrm{Integrate}[(u_0)_2(1 - \frac{r}{R_2})^{1/7}2\pi * r, \{r, 0, R_2\}]$$

which yields $49\pi R_2^2 (u_0)_2/60$. Using the equality $Q_1 = Q_2$, it follows that

$$Q_1 = Q_2 \rightarrow \frac{\pi R_1^2 (u_0)_1}{2} = \frac{49\pi R_2^2 (u_0)_2}{60}$$

$$\therefore \frac{(u_0)_1}{(u_0)_2} = \frac{49}{60} \times \underbrace{\frac{R_2^2}{R_1^2}}_{=(4/7)^2} \times \frac{2}{1}$$

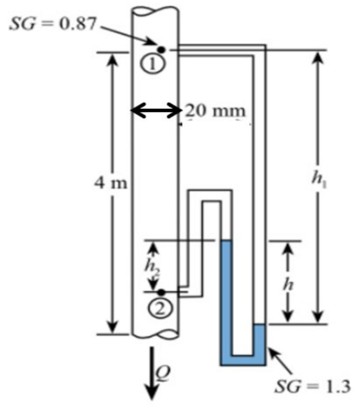
$$\therefore \frac{(u_0)_1}{(u_0)_2} = \frac{49}{60} \times \left(\frac{4}{7}\right)^2 \times 2$$

$$\therefore \boxed{\frac{(u_0)_1}{(u_0)_2} = \frac{8}{15}}$$

➡ The correct answer is **C**.

P.13 ● Solution

Consider sections 1 and 2 as shown below.



The cross-sectional area of the pipe is A = $\pi D^2/4 = \pi(20\times10^{-3})^2/4 = 3.14\times10^{-4}\ \text{m}^2$ and the flow velocity is

$$V = \frac{Q}{A} = \frac{4\times10^{-4}}{3.14\times10^{-4}} = 1.27\ \text{m/s}$$

The Reynolds number for flow in the pipe is

$$\text{Re} = \frac{VD}{\nu} = \frac{1.27\times20\times10^{-3}}{2.2\times10^{-4}} = 115$$

The flow is within the range for laminar conditions. Now, the density ρ of the oil is $\rho = 0.87\times1000 = 870\ \text{kg/m}^3$, and the specific weight is $\gamma = SG\times\gamma_{\text{water}} = 0.87\times9.81 = 8.53\ \text{kN/m}^3$. The dynamic viscosity, in turn, can be obtained from the definition of kinematic viscosity, giving

$$\nu = \frac{\mu}{\rho} \rightarrow \mu = \rho\nu = 870\times2.2\times10^{-4} = 0.1914\ \text{N}\cdot\text{s/m}$$

The pressure difference can be obtained with the Poiseuille equation,

$$Q = \frac{\pi(\Delta p + \gamma L)D^4}{128\mu L}$$

$$\therefore \Delta p + \gamma L = \frac{128Q\mu L}{\pi D^4}$$

$$\therefore \Delta p = \frac{128Q\mu L}{\pi D^4} - \gamma L = \frac{128\times4\times10^{-4}\times0.1914\times4}{\pi\times(20\times10^{-3})^4} - 8.53\times10^3\times4 = 43{,}859\ \text{Pa}$$

The specific weight of the heavier fluid is $\gamma_m = SG_m\gamma_w = 1.3\times9.81 = 12.75\ \text{kN/m}^3$. Applying an equilibrium of pressures in the device, we write

$$p_1 - \gamma h_2 + \gamma_m h - \gamma h_1 = p_2$$

$$\therefore p_1 - p_2 - \gamma h_2 + \gamma_m h - \gamma h_1 = 0$$

$$\therefore\ -(p_2 - p_1) - \gamma h_2 + \gamma_m h - \gamma h_1 = 0$$

$$\therefore\ -\Delta p - \gamma h_2 + \gamma_m h - \gamma h_1 = 0$$

$$\therefore\ -\Delta p - \gamma h_2 + \gamma_m h - \gamma[\ell + (h - h_2)] = 0$$

$$\therefore\ -\Delta p - \gamma h_2 + \gamma_m h - \gamma\ell - \gamma h + \gamma h_2 = 0$$

$$\therefore -\Delta p - \gamma\ell + (\gamma_m - \gamma)h = 0$$

Solving for h, we obtain

$$\therefore h = \frac{\Delta p + \gamma\ell}{(\gamma_m - \gamma)}$$

Substituting Δp = 43,859 Pa, γ = 8.53 kN/m^3, γ_m = 12.75 kN/m^3, and ℓ = 4 m, the manometer reading is calculated to be

$$h = \frac{43.86 + 8.53\times4}{12.75 - 8.53} = \boxed{18.5\ \text{m}}$$

➡ The correct answer is **D**.

ANSWER SUMMARY

Problem 1	1A	A
	1B	B
Problem 2		C
Problem 3		C
Problem 4		A
Problem 5		D
Problem 6		B
Problem 7		B
Problem 8		A
Problem 9	9A	C
	9B	A
Problem 10	10A	D
	10B	D
Problem 11	11A	C
	11B	D
Problem 12		C
Problem 13		D

REFERENCES

- ÇENGEL, Y. and CIMBALA, J. (2014). *Fluid Mechanics: Fundamentals and Applications*. 3rd edition. New York: McGraw-Hill.
- HIBBELER, R. (2015). *Fluid Mechanics*. Upper Saddle River: Pearson.
- MUNSON, B., YOUNG, D., OKIISHI, T., and HUEBSCH, W. (2009). *Fundamentals of Fluid Mechanics*. 6th edition. Hoboken: John Wiley and Sons.

Got any questions related to this quiz? We can help!
Send a message to contact@montogue.com and we'll
answer your question as soon as possible.