QUIZ GT102
Weight-Volume Relationships

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► PROBLEMS

PROBLEM 1

Which of the following statements is *false*?

A) Porosity is a dimensionless quantity defined as the ratio of the volume of voids to total volume.

B) Void ratio is usually expressed as a decimal number, while porosity is usually defined as a percentage.

C) Unlike the degree of saturation, the water content of a soil can be higher than 100%.

D) The degree of saturation of a soil is given by $S = wG_S/n$, where w is the water content, G_s is the specific gravity, and n is the porosity.

PROBLEM 2 (Das & Sobhan, 2014)

A soil sample has a total mass of 23.3 g, a volume of 12 cm^3, an oven-dry mass of 21.2 g, and a specific gravity of 2.5 for the solids. The void ratio of the specimen is:

A) e = 0.28

B) e = 0.42

C) e = 0.61

D) e = 0.87

PROBLEM 3

A soil sample has a total unit weight of 16.9 kN/m^3 and a void ratio of 0.84. The specific gravity of solids is 2.70. Determine the degree of saturation and the moisture content of the soil.

A) S = 45% and w = 17.4%

B) S = 45% and w = 22.2%

C) S = 56% and w = 17.4%

D) S = 56% and w = 22.2%

PROBLEM 4

The specific gravity of a saturated soil sample is 2.60, with a total unit mass of 2250 kg/m^3. The dry unit mass is:

A) ρ = 1670 kg/m^3

B) ρ = 1860 kg/m^3

C) ρ = 2080 kg/m^3

D) ρ = 2240 kg/m^3

PROBLEM 5 (Prieto-Portar, 2009)

A soil sample has moist unit weight equal to 16.5 kN/m^3, water content of 15%, and specific gravity of 2.70. Find the dry unit weight and the void ratio for this soil.

A) γ_d = 12.1 kN/m^3 and e = 0.62

B) γ_d = 12.1 kN/m^3 and e = 0.85

C) γ_d = 14.3 kN/m^3 and e = 0.62

D) γ_d = 14.3 kN/m^3 and e = 0.85

PROBLEM 6 (Prieto-Portar, 2009)

A soil sample has a unit weight of 105.7 pcf and a degree of saturation of 50%. When the saturation is increased to 75%, its unit weight raises to 112.7 pcf. Determine the void ratio e and the specific gravity G_s for this soil.

A) e = 0.81 and G_s = 2.67

B) e = 0.81 and G_s = 2.85

C) e = 0.95 and G_s = 2.67

D) e = 0.95 and G_s = 2.85

PROBLEM 7 (Prieto-Portar, 2009)

The moist unit weight of a soil is 16.5 kN/m^3. Given that w = 15% and G_s = 2.70, which of the following statements is *false*?

A) The dry unit weight of the soil equals about 12.5 kN/m^3.

B) The porosity of the soil equals about 46%.

C) The degree of saturation of the soil equals about 48%.

D) The mass of water that must be added to achieve full saturation is 2300 kg per cubic meter of soil.

PROBLEM 8 (Craig, 2004)

In its natural condition, a soil sample has a mass of 2290 g and a volume of 1.15×10^{-3} m^3. After being completely dried in an oven, the mass of the sample becomes 2035 g. The value of G_s for the soil is 2.68. Regarding this soil, which of the following statements is *false*?

A) The water content of the soil equals about 12.5%.

B) The porosity of the soil equals about 34%.

C) The degree of saturation of the soil is about 65%.

D) The air content of the soil is about 18%.

PROBLEM 9 (Aysen, 2003)

Dry soil with G_s = 2.71 is mixed with 16% by weight of water and compacted to produce a cylindrical sample with 38 mm diameter and 76 mm height with 6% air content. Calculate the mass of the mixed soil that will be required and the void ratio of the sample.

A) M = 112.8 g and e = 0.53

B) M = 112.8 g and e = 0.81

C) M = 177.7 g and e = 0.53

D) M = 177.7 g and e = 0.81

PROBLEM 10 (Das & Sobhan, 2014)

In a construction project, the field moist unit weight was 115 lb/ft^3 at a moisture content of 8%. If the maximum and minimum dry unit weight determined in the laboratory were 108 lb/ft^3 and 92 lb/ft^3, respectively, what was the field relative density?

A) D_r = 81.3%

B) D_r = 86.8%

C) D_r = 91.8%

D) D_r = 96.5%

PROBLEM 11 (Das & Sobhan, 2014)

A soil sample with a grain specific gravity of 2.67 was filled in a 1000 mL container in the loosest possible state and the dry unit weight of the sample was found to be 14.75 N. It was then filled at the densest obtainable state and the weight was found to be 17.70 N. The void ratio of the soil in the natural state was 0.63. Determine the relative density in the natural state.

A) D_r = 60.0%

B) D_r = 65.0%

C) D_r = 70.0%

D) D_r = 75.0%

PROBLEM 12 (Das & Sobhan, 2014)

For a given sandy soil, the maximum and minimum void ratios are 0.72 and 0.46, respectively. If G_s = 2.71 and w = 11%, what is the moist unit weight of compaction in the field if the relative density equals 82%?

A) γ = 17.5 kN/m^3

B) γ = 19.4 kN/m^3

C) γ = 21.0 kN/m^3

D) γ = 23.1 kN/m^3

▶ SOLUTIONS

P.1 ■ Solution

Statement *C* is false, because in reality the degree of saturation is such that

$$S = \frac{V_w}{V_v}$$

which can be adjusted to give

$$S = \frac{wG_s}{e}$$

where w is the water content, G_s is the specific gravity, and e is the void ratio (not the porosity!)

► The false statement is **D**.

P.2 ■ Solution

The volume of soil solids is given by the following equation,

$$V_s = \frac{m_s}{G_s\rho_w} = \frac{21.2}{2.5 \times 1.0} = 8.48 \text{ cm}^3$$

The volume of the voids is found by subtracting the volume of solids from the total volume,

$$V_v = V_t - V_s = 12 - 8.48 = 3.52 \text{ cm}^3$$

Finally, the void ratio is determined to be

$$e = \frac{V_v}{V_s} = \frac{3.52}{8.48} = 0.415 \approx \boxed{0.42}$$

▶ The correct answer is **B**.

P.3 ■ Solution

The degree of saturation can be obtained with the relation

$$\gamma_t = \frac{\gamma_w(G_s + eS)}{1+e} \rightarrow 16.9 = \frac{9.81(2.70 + 0.84 \times S)}{1 + 0.84}$$

$$\therefore S = 0.559 \approx \boxed{56\%}$$

The moisture content, in turn, can be obtained with the relation

$$Se = wG_s \rightarrow w = \frac{Se}{G_s}$$

$$\therefore w = \frac{0.56 \times 0.84}{2.70} = 0.174 = \boxed{17.4\%}$$

▶ The correct answer is **C**.

P.4 ■ Solution

Assume the total volume to be 1 m³. The total volume, V, is the sum of the volume of water, V_w, and the volume of solids, V_s,

$$V = V_w + V_s$$

In addition, the sum of masses is such that

$$m = m_w + m_s = 2250$$

where $m_s = G_s V_s \rho_w$ and $m_w = V_w \rho_w$. Substituting in the first equation gives

$$1000V_w + 2.5 \times 1000V_s = 2250$$

$$\therefore 1000(1 - V_s) + 2.5 \times 1000V_s = 2250$$

with the result that V_s = 0.833 m³. The volume of water immediately follows as $V_w = 1 - 0.833 = 0.167 \text{ m}^3$. We can then obtain the dry unit mass with the expression

$$m_s = G_s V_s \rho_w = 2.5 \times 0.833 \times 1000 = 2082.5 \text{ kg}$$

That is,

$$\boxed{\rho = 2080 \text{ kg/m}^3}$$

▶ The correct answer is **C**.

P.5 ■ Solution

The dry unit weight is given by the relation

$$\gamma_d = \frac{\gamma}{1+w} = \frac{16.5}{1 + 0.15} = \boxed{14.3 \text{ kN/m}^3}$$

The void ratio, in turn, can be obtained with the expression

$$\gamma_d = \frac{G_s \gamma_w}{1+e}$$

$$\therefore 1 + e = \frac{G_s \gamma_w}{\gamma_d}$$

$$\therefore e = \frac{G_s \gamma_w}{\gamma_d} - 1 = \frac{2.70 \times 9.81}{14.3} - 1 = 0.852 \approx \boxed{0.85}$$

▶ The correct answer is **D**.

P.6 ■ Solution

We resort to the expression

$$\gamma = \frac{\gamma_w (G_s + Se)}{1+e}$$

Substituting the pertaining data when saturation equals 50%, we have

$$105.7 = \frac{62.4(G_s + 0.50e)}{1+e} \quad \text{(I)}$$

whereas, when the saturation equals 75%, the equation becomes

$$112.7 = \frac{62.4(G_s + 0.75e)}{1+e} \quad \text{(II)}$$

Equations (I) and (II) can be solved simultaneously for the specific gravity and void ratio. The results are e = 0.81 and G_s = 2.67.

► The correct answer is **A**.

P.7 ■ Solution

The dry unit weight follows from the relation

$$\gamma_d = \frac{\gamma}{1+w} = \frac{16.5}{1+0.15} = 14.3 \text{ kN/m}^3$$

The porosity is given by

$$n = \frac{e}{1+e}$$

and thus requires the void ratio. This quantity can be extracted from the formula

$$\gamma_d = \frac{G_s \gamma_w}{1+e} \to 14.3 = \frac{2.7 \times 9.81}{1+e}$$

$$\therefore e = 0.852$$

Returning to the expression for n, we have

$$n = \frac{0.852}{1+0.852} = 0.46 = 46\%$$

Next, the degree of saturation is

$$Se = wG_s \to S = \frac{wG_s}{e}$$

$$\therefore S = \frac{0.15 \times 2.7}{0.852} = 0.475 = 47.5\%$$

Thence, the saturated unit weight can be obtained with the expression

$$\gamma_{\text{sat}} = \frac{(G_s + e)\gamma_w}{1+e} = \frac{(2.7+0.852)\times 9.81}{1+0.852} = 18.8 \text{ kN/m}^3$$

The water to be added to attain full saturation can be calculated via the relation $\gamma = \rho g$; that is,

$$\rho = \frac{\gamma}{g} = \frac{(18.8 - 16.5)\text{ kN/m}^3}{9.81 \text{ kgm/s}^2} \times \frac{1000 \text{ N}}{1 \text{ kN}} \times 9.81 \frac{\text{kgm/s}^2}{\text{N}} = 2300 \text{ kgm/m}^3$$

Statement A is false, because $\gamma_d \neq 12.5 \text{ kN/m}^3$.

► The false statement is **A**.

P.8 ■ Solution

To begin, the water content of the soil is given by the ratio M_w/M_s; that is,

$$w = \frac{M_w}{M_s} = \frac{2290 - 2035}{2035} = 0.125 \to w = 12.5\%$$

The bulk density and unit weight can be extracted from the data we were given. Mathematically,

$$\rho = \frac{M}{V} = \frac{2290}{1.15\times10^{-3}} = 1991\ \text{kg/m}^3$$

and

$$\gamma = \frac{Mg}{V} = 1990 \times 9.81 = 19.52\ \text{kN/m}^3$$

Next, the porosity follows from the relationship $n = e/(1+e)$. The void ratio is

$$e = G_s(1+w)\frac{\rho_w}{\rho} - 1 = 2.68 \times (1 + 0.125) \times \frac{1000}{1990} - 1 = 0.515$$

so that

$$n = \frac{e}{1+e} = \frac{0.515}{1+0.515} = 0.340 = 34\%$$

The degree of saturation is given by

$$S = \frac{wG_s}{e} = \frac{0.125 \times 2.68}{0.515} = 0.650 = 65\%$$

Finally, the air content is computed as

$$A = n(1-S) = 0.34 \times (1 - 0.65) = 0.119 = 11.9\%$$

which is evidently different from 18%. Accordingly, statement D is false.

► The false statement is **D**.

P.9 ■ Solution

The overall volume of the specimen is

$$V = 7.6 \times \frac{\pi \times 3.8^2}{4} = 86.19\ \text{cm}^3$$

The density of the soil solids follows as

$$\rho_s = G_s\rho_w = 2.71 \times 1.0 = 2.71\ \text{g/cm}^3$$

The sum of the volume of solids, the volume of water and the volume of air is

$$V_s + V_w + V_a = 86.19$$

Substituting the corresponding expressions for each term, we obtain

$$\frac{M_s}{2.71} + \frac{0.16 \times M_s}{\rho_w} + 0.06 \times 86.19 = 86.19$$

Solving the equation above, one obtains M_s = 153.2 g. The mass of the sample follows as

$$M = 153.2 + 0.16 \times 153.2 = \boxed{177.7\ \text{g}}$$

The volume of solids is, in turn,

$$M_s = 153.2 = \rho_s V_s = G_s \rho_w V_s = 2.71 \times 1.0 \times V_s$$

$$\therefore V_s = 56.49\ \text{cm}^3$$

The volume of voids is

$$V_v = V - V_s = 86.19 - 56.49 = 29.7\ \text{cm}^3$$

Finally, we calculate the void ratio,

$$e = \frac{V_v}{V_s} = \frac{29.7}{56.49} = \boxed{0.53}$$

► The correct answer is **C**.

P.10 ■ Solution

The relative density can be obtained with the expression

$$D_r = \frac{\gamma_d - \gamma_{d,\min}}{\gamma_{d,\max} - \gamma_{d,\min}}\left(\frac{\gamma_{d,\max}}{\gamma_d}\right)$$

With γ = 115 lb/ft³ and w = 8%, the dry unit weight is calculated as

$$\gamma_d = \frac{115}{1+0.08} = 106.48\ \text{lb/ft}^3$$

Then, using $\gamma_{d,\min}$ = 92 lb/ft³, $\gamma_{\max}$ = 108 lb/ft³, and γ_d = 106.48 lb/ft³, the relative density is determined to be

$$D_r = \left(\frac{106.48-92}{108-92}\right)\left(\frac{108}{106.48}\right) = 0.918 = \boxed{91.8\%}$$

► The correct answer is **C**.

P.11 ■ Solution

The volume occupied by the soil is 1000 cm³. In the loosest state, the weight of soil is 14.75 N and the volume of solids is

$$V_s = \frac{14.75}{0.0267} = 552.4\ \text{cm}^3$$

Consequently, the volume of voids is $V_v = 1000 - 552.4 = 447.6\ \text{cm}^3$. The maximum void ratio is then

$$e_{\max} = \frac{447.6}{552.4} = 0.810$$

Now, when the soil is at its densest state, the weight is 17.70 N and the volume of solids, in turn, is

$$V_s = \frac{17.70}{0.0267} = 662.9\ \text{cm}^3$$

The corresponding volume of voids is $V_v = 1000 - 662.9 = 337.1\ \text{cm}^3$. The minimum void ratio follows as

$$e_{\min} = \frac{337.1}{662.9} = 0.509$$

Finally, noting that the void ratio in the natural state is e = 0.63, the relative density is calculated as

$$D_r = \frac{e_{\max} - e}{e_{\max} - e_{\min}} = \frac{0.810-0.630}{0.810-0.509} = 0.598 = 59.8\% \approx \boxed{60\%}$$

► The correct answer is **A**.

P.12 ■ Solution

The dry unit weight can be calculated from the expression for relative density,

$$D_r = \frac{\gamma_d - \gamma_{d,\min}}{\gamma_{d,\max} - \gamma_{d,\min}}\left(\frac{\gamma_{d,\max}}{\gamma_d}\right) \quad \text{(I)}$$

where $\gamma_{d,\min}$ is the dry unit weight in the loosest condition, γ_d is the in situ dry unit weight, and $\gamma_{d,\max}$ is the dry unit weight in the densest condition. To calculate the dry unit weight in the densest condition, we write

$$\gamma_{d,\max} = \frac{G_s\gamma_w}{1+e_{\min}}$$

where e_{min} is the minimum void ratio of the soil. Substituting the data we were given, it follows that

$$\gamma_{d,\max} = \frac{(2.68\times 9.81)}{1+0.46} = 18.00\ \text{kN/m}^3$$

Next, the dry unit weight in the loosest condition is

$$\gamma_{d,\min} = \frac{G_s\gamma_w}{1+e_{\max}}$$

where e_{max} is the maximum void ratio of the soil. Substituting the data we were given, we have

$$\gamma_{d,\min} = \frac{(2.68\times 9.81)}{1+0.72} = 15.29\text{ kN/m}^3$$

Thence, we can return to Equation (I) and solve for γ_d,

$$0.82 = \left(\frac{\gamma_d - 15.29}{18.00-15.29}\right)\times\left(\frac{18.00}{\gamma_d}\right) \rightarrow \gamma_d = 17.45\text{ kN/m}^3$$

In order to compute the moist unit weight, we make use of the formula

$$\gamma_d = \frac{\gamma}{(1+w)}$$

Substituting γ_d = 17.45 kN/m³ and w = 0.11, we find that

$$\gamma = \gamma_d(1+w) = 17.45(1+0.11) = 19.37 \approx \boxed{19.4\text{ kN/m}^3}$$

► The correct answer is **B**.

► ANSWER SUMMARY

Problem 1	D
Problem 2	B
Problem 3	C
Problem 4	C
Problem 5	D
Problem 6	A
Problem 7	A
Problem 8	D
Problem 9	C
Problem 10	C
Problem 11	A
Problem 12	B

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